# 1 Approach

In this article, we name the MAC scheme adopted in Cost-Effective Tag Design[] CETD-MAC.

## 1.1 Background of CETD-MAC and Security Definitions

**Notations of Symbols** Let $\{0,1\}^n$ be the set of all n-bit binary strings. The set of all binary string is expressed as $\{0,1\}^*$. For a string $X \in \{0,1\}^n$, —X— is its length in bits, and $\mid X \mid_l = \lceil \mid X \mid / l \rceil$ is the length of X in l-bit blocks. Let $0^l$ and $1^l$ denote bit strings of all zeros and all ones. For a bit string X and an integer l that $\mid X \mid \geq l$, $\text{msb}_l(X)$ denotes the most significant l bits(left most l bits) of X and $\text{lsb}_l(X)$ for least significant l bits(right most l bits) of X. For two bit string X and Y, we denote X‖Y or XY as the their concatenation. For bit string X whose length in bits is multiple of integer l, we denote X parted into l-bit sub-strings as $X = (X[1]X[2]...X[n])_l$, where X[1], X[2], ..., $X[n] \in \{0,1\}^l$. The number of bits in a string of X is denoted as len(X).

The block cipher encryption of a string X with a secret key K is denoted as $E_K(X)$. $E_K(X)$ expresses the String mapping of $\{0,1\}^n \rightarrow \{0,1\}^n$ where n is the len(X) and len(output).

**Specification of CETD-MAC** In this section, we depict the design details of CETD-MAC scheme. At the beginning we express notations required to understand CETD-MAC scheme.

**Definition of CETD-MAC Scheme** CETD-MAC scheme can be expressed as tag = CETD-MAC(M, nonce-input). The input arguments of CETD-MAC are message M and a tuple named nonce-input. The tuple nonce-input is the concatenation of the memory address of M, a counter and a random number, denoted as nonce-input=(address‖counter‖random). The length of nonce-input, len(nonce-input), is identical to the length of input to block cipher $E_K(X)$. The output of CETD-MAC is named tag, whose length is optional. We use Sub-BLK-No to express the value of len(M)/len(tag). One preliminary of CETD-MAC is that Sub-BLK-No should be no less than 2 to assure that swapping stage can work and 0s will be concatenated to the leftmost of input message when Sub-BLK-No is not integer. The concept of CETD-MAC is expressed in Figure **??**.

We follow the definition of CETD-MAC in [**?**].

- Message is splitted to sub-blocks each of whose length is tag length
- The first stage is several rounds of bit-segment swapping. For each round , two sub-blocks are randomly chosen. Introduce shuffle parameter V[i]
- The output sub-blocks from swapping stage, named X blocks, are sent to block-level rotate shifting stage. Each X block is rotate shifted for several

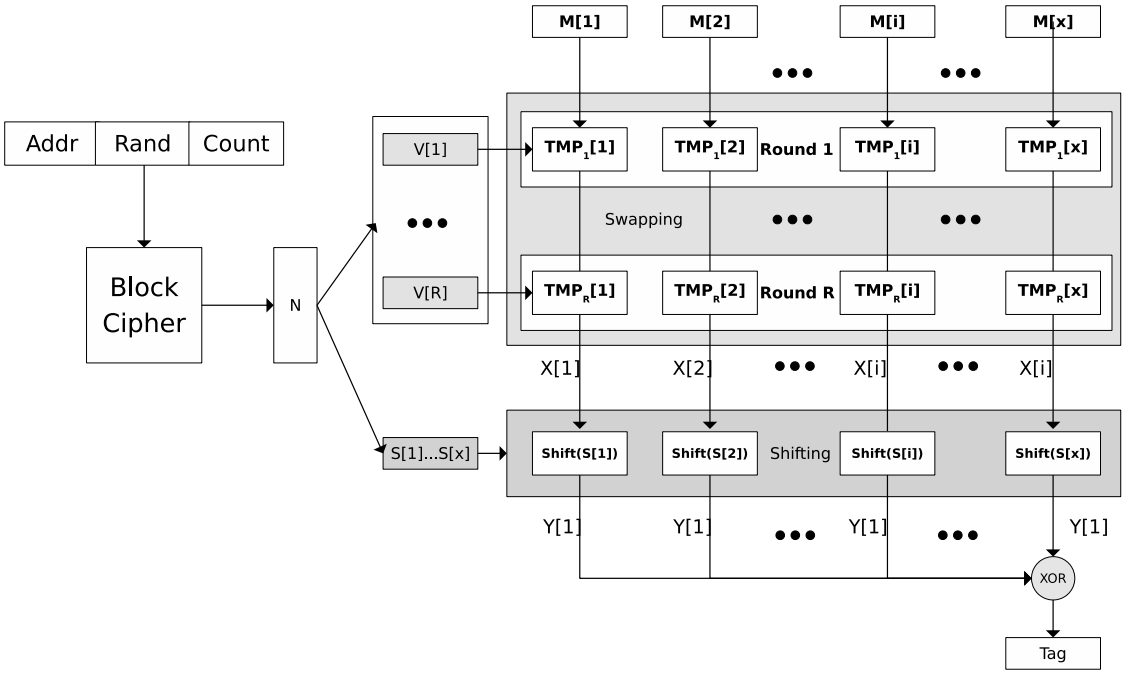



Figure 1: The CETD-MAC Scheme

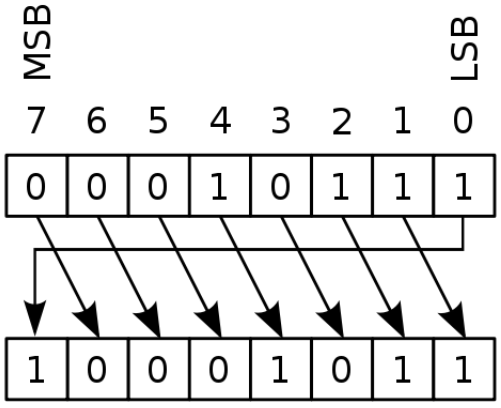



Figure 2: The Concept of Rotate Shifting(Right)

bits. The concept of rotate shifting is expressed in Fig 2 Introduce shift parameter S[i]

- The output sub-blocks from shifting stage, named Y blocks, are XORed to form the final tag

As the nonce is generated by block cipher encryption $E_K$. Assume $E_K$ performance like a ideal random value generator, for two distinct input nonce-input1 and nonce-input2, the corresponding nonce values N1 and N2 should be random and the probability that N1 is identical to N2 should be $1/2^n$, where n is len(N1).

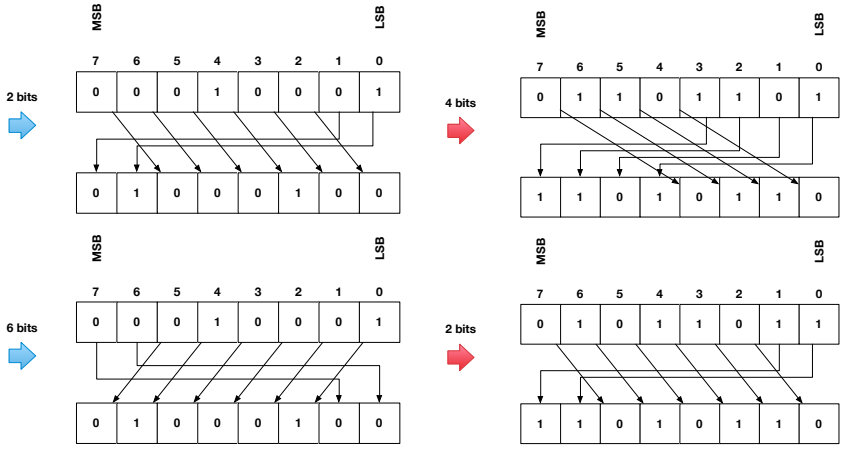
**Security Definitions of MAC Schemes under Replay Attacks** Our security definition of MAC schemes under replay attack follows the definition chosen-message attack. An adversary is given access to a tag generation black-box(named oracle) and a message-tag pair verification oracle. The encryption key is maintained unchanged while the nonce value is updated for each calling of tag generation oracle no matter whether the value of input message collide with a value in old timestamp. Secret key and nonces are kept in trusted area and cannot be acquired by the adversary. The adversary can copy the message-tag pairs in old time stamp. When conducting the replay attack, the adversary replaces a memory frame containing a data-tag pair in new timestamp with a copy from his storage. The forgery advantage $F_{CETD-MAC}$ under replay attack is the probability that the adversary can get the verification oracle to accept the replace data-tag pair.

Assume the copy with old timestamp from the adversary is expressed as (M,T1) where M is the message and T1 is the corresponding tag, and the nonce of (M,T1) is denoted as N1. When applying M to the tag generation at a new time point, we mark the corresponding tag T2 and the nonce N2. If the adversary can succeed a replay attack, then T1 is identical to T2 regardless of the equality of N1 and N2. Then $F_{CETD-MAC}$ under replay attack can be denoted as the probability Pr[T1 = T2 | Message value is M & N1,N2 are randomly generated].

According to the design of CETD-MAC, the tag can be expressed as $Y[1]\oplus Y[2]\oplus\ldots\oplus Y[x]$, x is the number of input sub-blocks to CETD-MAC. If the following equation is met: $Y_1[1]\oplus Y_1[2]\ldots\oplus Y_1[x]\oplus Y_2[1]\oplus Y_2[2]\ldots\oplus Y_2[x] = 0$, where $Y_1$ blocks produce T1 and $Y_2$ blocks generate T2, then T1 is identical to T2. This equation indicates that the behaviour of block-level rotate shifting stage effect the probability of tag collision directly. For easy understanding, we firstly analyze the case that shuffle stage does not work under replay attack, which means $X_1[i]$ is identical to $X_2[i]$ for any $i\in[1,x]$. The analysis of effect from shuffle stage on X blocks follows then.

## 1.2 Cases of Input Causing High Tag Collision Probability

In replay attack, the adversary replace the memory frame written in new time point $Time_2$ with a copy from old time stamp $Time_1$. The content in memory




(a) Same Block Shifted Different Bits (b) Different Blocks Shifted Different Bits

Figure 3: The Examples of Y Block Collision

frame is a data-tag pair. We denote the data M and the tag in the forgery frame T1. The output blocks from shuffle stage in generating T1 is denoted as X1 blocks and the output from shifting stage are Y1 blocks. When the forgery frame is read to be verified, the output or shuffle stage is X2 blocks and Y2 blocks for the output from shifting stage. The tag generated by verification is T2.

In this section, we depict the weaknesses we found in CETD-MAC under replay attack. These weaknesses in shifting stage let the CETD-MAC to produce tags with high collision probability with some chosen inputs. The tags with highly collision probability ease the adversary‘s attack. Figure **??** gives the examples of the outputs from shifting stage that lead tag collision.

### 1.2.1 The Sequence-level Pattern Collision

Firstly, we treat blocks as a block sequences, which means in a block sequence, each block has a unique index. The definition of block sequence equality is expressed below:

**Definition 1.1.** *Sequence Equality: Two block sequences X1 and X2 are equal in sequence-level if $X_1[i]$ is identical to $X_2[i]$ for any i∈[1,x].*

Assume the shuffle stage does not work, then the following fact exist:

- X1 and X2 are two identical block sequences in sequence-level

In block-level rotate shifting stage, a segment from nonce, marked as S, is adopted as the parameter of shifting bits. S is a concatenation of sub-blocks and is denoted as S=(S[1]||S[2]...S[x]), where x is the number of blocks in X

sequence. The value of S[i] the bits shifted for X[i] block. For two identical blocks $X_1$[i] and $X_2$[i], the output blocks $Y_1$[i] and $Y_2$[i] are identical if $S_1$[i] is identical to $S_2$[i]. We found that $Y_1$[i] and $Y_2$[i] still have probability to be identical if $X_1$[i] is formed by a repeated short bit segment named pattern. This property is depicted in Proposition 1.1:

**Proposition 1.1.** *Let $X_1$[i] and $X_2$[i] be two identical block and len($X_1$[i])=N, $N=2^n$. Let $S_1$[i] and $S_2$[i] be two distinct shifting bit parameters for $X_1$[i] and $X_2$[i]. Then $Y_1$[i] and $Y_2$[i] can be identical only when $X_1$[i] is formed by repeating a binary bit segment named pattern and marked as P. The length of P len(P) has the following format: $P\_L = 2^p$, $p \in [0,n-1]$*

Base on Proposition 1.1, we got the following corollaries:

**Corollary 1.1.** *If a pattern P is not formed by a sub-pattern with shorter length, we call P a distinct pattern with length P_L. The No. of distinct patterns with length $P\_L=2^p$ is $2^{2^p}$-$2^{2^{p-1}}$*

**Corollary 1.2.** *Assume the length of a pattern-forming X block is $N=2^n$, then the No. of all possible distinct patterns is $2^{N/2}$*

**Corollary 1.3.** *For two pattern-formed identical X blocks $X_1$[i] and $X_2$[i], the probability that $Y_1$[i] is identical to $Y_2$[i] when $S_1$[i] and $S_2$[i] are randomly generated is expressed as Pr[$Y_1$[i]=$Y_2$[i]]=*

From Corollary 1.3 we can see that when the adversary conducts the replay attack on the data concatenated by pattern-formed blocks, the probability that Y block collision is high when shuffle stage does not work. The proof of Proposition 1.1 and Corollary 1.1 to 1.3 is expressed in the appendix.

**The Probability of Tag Collision Under Sequence Pattern Attack** For two identical X blocks X_A[i] and X_B[i], the probability that Y_A[i]=Y_B[i] when R_A[i] and R_B[i] are randomly generated is marked as Pr[Y block collision]. Pr[Y block collision] is expressed in Theorem 1.3:

**Theorem 1.4.** *Assume for two identical blocks $X_1$[i] and $X_2$[i] the length is $N=2^n$, and the pattern length $P_l=2^p$ where $p \in [0,n-1]$. If the pattern contains no internal sub-pattern, then :*

$$Pr[Y_1[i]=Y_2[i]] = 1/2^p \tag{1}$$

*If the pattern length in X[i] is identical for any $i \in [1,x]$ where x is the number of blocks in X sequence, then:*

$$Pr[Y_1[i]=Y_2[i]] = (1/2^p)^x \tag{2}$$

### 1.2.2 The Set-level Pattern Collision

Secondly we treat blocks as a set allowing the existence of identical elements. We give the definition of block set equality below:

**Definition 1.2.** *Set Equality: Two block sets X1 and X2 are identical in set-level if the following properties are met:*

- *X1 and X2 have same number of elements*
- *The number of elements for each distinct value in X1 is identical to the number in X2*

When none of block in a X block sequence is formed by repeated pattern, it is impossible to make two Y sequences with block sequence equality by using two distinct s sequences. This assertion can be proved based on the proof of Proposition 1.1. However, we found that it is possible to form two Y sets with block set equality by using two distinct s sequences. The set-level identical Y sets can lead to tag collision directly. This attack is expressed in Proposition 1.2:

**Proposition 1.2.** *Let $X_1[i]$ and $X_2[j]$ be two distinct block and len($X_1[i]$)=len($X_2[j]$)=N, $N=2^n$. Let $S_1[i]$ and $S_2[j]$ be two distinct blocks of shifting bit parameters for $X_1[i]$ and $X_2[j]$. Then $Y_1[i]$ and $Y_2[j]$ can be identical if and only if $X_1[i]$ can be formed by rotate shifting $X_2[j]$.*

Based on Proposition 1.2, we got the following corollary:

**Corollary 1.5.** *For the rotate shifting stage, when the distinct input blocks $X_1[i]$ and $X_2[j]$ have the property that $X_1[i]$ can be formed by rotate shifting $X_2[j]$,and the shifting bits parameter $S_1[i]$ and $S_2[j]$ are distinct, the probability that the output blocks $Y_1[i]$ and $Y_2[j]$ are identical is $Pr[Y_1[i]=Y_2[j]]=$*

Assume the shuffle stage does not work. When the adversary conducts replay attack on the memory frame whose data part is concatenated by blocks that are formated by shifting a common base block, then each element in X1 sequence can be formed by rotate shifting another element and the probability of forming identical Y sets is high, which will lead to tag collision.

**The Probability of Tag Collision Under Set Pattern Attack** Assume two identical sets X_A and X_B satisfy the following properties:

- Each element can be formed by rotate shifting any other element in the set
- None of the elements is formed by pattern.

As each element in the X set can be formed by rotate shifting a base block, we call such X set Same Base Block(SBB) X sets, short for SBB X sets. If none of the element is formed by a pattern ,we call such X set SBB only sets.

From Corollary 1.3 we know that for a block that is N bits long, the No. of possible values that is formed by a pattern is $2^{N/2}$, which also means the No. of distinct values that is not formed by a pattern is $2^{N/2}$. Assume D_A and D_B are randomly generated, the probability of generating SBB only D set is expressed:

**Definition 1.3.** *Pr[ SBB only D(X)] =*

$$2^{n/2} * N^{M-1}/(2^N)^M \tag{3}$$

Using SBB only X_A and X_B, the probability of CIE of Y_A and Y_B is marked as Pr[CIE Y Sets] and expressed as the following way:

**Definition 1.4.** *Pr[CIE Y Sets] =*

$$\prod_{i=1,j=1}^{M} Pr[Y\_A[i] = Y\_B[j]]$$

*where M is the No. of elements in a set, i,j∈[1,M] and i≠j*

For two randomly generated R_A and R_B, the probability that Y_A and Y_B are CIE when X_A and X_B are SBB only is expressed in the following theorem:

**Theorem 1.6.** *Assume X_A and X_B are SBB only, R_A and R_B are randomly generated,then:*
*Pr[CIE Y Sets]:*

$$(\sum_{K=1}^{min(N,M)} \binom{N}{K} * (M!/v1!v2!v3!\cdots vk!)^2)/(N^M)^2$$

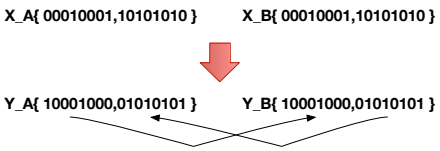
**The Case of Input Trigging Two Pattern Attacks** If the blocks in X set is formed by rotate shifting a base block and this base block is formed by a pattern, then such X set pair X_A and X_B can result either IIE or CIE Y set pairs.

Assume the R_A and R_B are randomly generated, the probability that Y_A and Y_B are SLE is expressed in the following theorem:

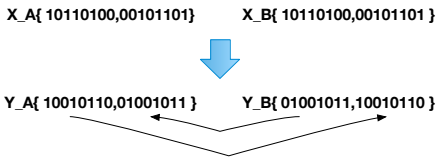
**Theorem 1.7.** *Assume X_A and X_B are both SBB and IPL, R_A and R_B are randomly generated,then:*
*Pr[SLE Y Sets]=*

$$(\sum_{K=1}^{min(N,M)} \binom{N}{K} * (M!/v1!v2!v3!\cdots vk!)^2)/(N^M)^2$$

The proof of theorems is expressed in the appendix.

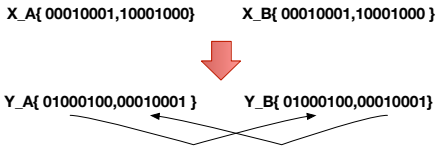



(a) Only Single Block Equality

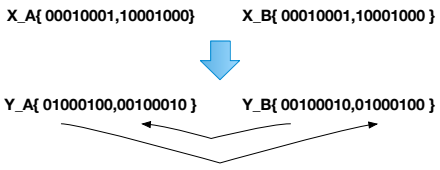



(b) Only Set Level Equality

Figure 4: X Set Pairs with Only One Type of Y Equality




(a) Single Block Equality Case




(b) Set Level Equality Case

Figure 5: A X Set Pair with Two Types of Y Equality

### 1.2.3 The Behaviour of X Blocks from Shuffling Stage

In each round of Shuffle stage, two blocks from the output of previous round of shuffle are randomly selected then operate bit segments swapping, which will effect the distribution of bits in the two blocks. After the shuffle stage, the output, X blocks, can be classified to four categories:

1. Formed by repeated patterns
2. Formed by rotate shifting another block in this category
3. Contains the properties from categories 1 and 2
4. No properties from categories 1 or 2

Assume the block cipher used in nonce generation performances like PRF, the number of blocks in each category is effected by the input message M and the rounds of shuffle stage.

## 1.3 A Solution to Eliminate the Collision

In this section we provide an modification of original CETD-MAC. Our approach does not require additional component or information to the original CETD-MAC. We then prove that the optimized CETD-MAC can fix the weakness and protect input message from replay attack.

**Question** Using only shuffle and shift with random nonce, is it possible to ensure the range of tag to $2^n$? If not, why?

# 2 Experiments and Results

# A Proof of Assertions

## A.1 Proof of Theorem 1.1

This part proves that if two identical block X_A[i] and X_B[i] are shifted different bits and the result blocks remain identical, X_A[i] is formed by pattern. The pattern length and $\delta$=|R_A[i]-R_B[i] | has such correlation:

- If $\Delta$ = P_L then Y_A[i] = Y_B[i] where P_L the length of pattern

Assume the length of X_A[i] is N bits, where N= $2^n$. When X_A[i] is formed by a pattern whose length is P_L = $2^p$ bits, then the domain of Y_A[i] has P_L distinct values. There are N/P_L distinct R_A[i] values that shift X_A[i] to a Y_A[i].

### A.2 Proof of Theorem 1.2

If two distinct X blocks result two identical Y block. Assume the shift bits parameter blocks are R_A[i] and R_B[i]. Then X_A[i] can be formed by rotate shifting Y_A[i] for (N-R_A[i]) mod N bits. X_B[i] cna be formed for (N-R_B[i]) mod N bits. X_B[i] can be formed by rotate shifting X_A[i] for (R_A[i] + N-R_B[i]) mod N bits. Theorem 1.2 proved.

## B Computation Procedure of Probabilities

### B.1 Proof of Theorem 1.3

For identical each block pair X_A[i] and X_B[i], the No. of combination of R_A[i] and R_B[i] = N*N, where N is the length a X block. If X_A[i] is formed by pattern, then Y_A[i]=Y_B[i] if |R_A[i]-R_B[i]| = P_L * K, where P_L is the pattern length and K is a positive integer. Then for two random R_A[i] and R_B[i], Pr[Y_A[i]=Y_B[i] | X_A[i] = X_B[i] & pattern = P_L](shoft for Pr[Y_A[i] = Y_B[i]]) can be expressed in the following way:

- Pr[Y_A[i]=Y_B[i]] = N * (N/P_L)/(N*N) = 1/P_L

### B.2 Proof of Theorem 1.4

From Theorem 1.1 we can see that if a X block is formed by a pattern, then the No. of distinct values in the range of Y block is P_L. While the domain of a R block has N distinct values, then for a block X, there are N/P_L distinct R values that lead to a Y value.

If a X block is not formed by a pattern, then for each distinct R value, there is a distinct Y value. That means for a given X set that none of the blocks is formed by a pattern, each R set will lead to a distinct Y set. The map between R set and Y set is bijection.

When each R block is randomly generated, the possible combination of R sets is $N^M$. That means for a given X set, the No. of possible Y set is $N^M$.

Assume X_A and X_B are IIE and each block can be formed by rotate shifting another block in the set. On the other hand, none of blocks is formed by a pattern. Then the sub-group of blocks in X_A and X_B can form CIE with specific R_A and R_B set.

Assume set Y_A and Y_B are SLE, then Y_B is a permutation of Y_A. This concept can be modeled in the following way:

- Assume the M elements in Y_A contain K distinct values. The No. of the elements that have each value are marked as v1,v2...vk.
- IF the elements in Y_B are a permutation of the elements in Y_A, then Y_A and Y_B are SLE.

Based on the basic concept in Combinatorics, the No. of a permutation of Y_A that contains K distinct value can be expressed in the following equation:

**Definition B.1.** *No. of Permutation with K distinct Values:*

$$\binom{M}{v1} * \binom{M-v1}{v2} * \binom{M-v1-v2}{v3} \cdots * \binom{vk}{vk} = M!/v1!v2!v3! \cdots vk!$$

As any one of the elements in X set can be formed by rotate shifting another element, then the No. of distinct values of M elements, K, various from 1 to M. If Y_A has K distinct values, the No. of possible combination of these K elements in Y_A can be expressed as Com_Y_A:

**Definition B.2.** *Com_Y_A:*

$$M!/v1!v2!v3! \cdots vk!$$

For each case of Y_A in Com_Y_A, the No. of Y_B to form SLE is also Com_Y_A. Then if Y_A has K distinct values, Pr[CIE Y sets] can be expressed as:

**Definition B.3.** *Pr[CIE Y Sets]:*

$$\binom{N}{K} * (M!/v1!v2!v3! \cdots vk!)^2/(N^M)^2$$

If R_A and R_B are randomly generated, then the value of K various from 1 to M. We do th sum to get the expression in Theorem 1.4

**Definition B.4.** *Pr[CIE Y Sets]:*

$$(\sum_{K=1}^{min(N,M)} \binom{N}{K} * (M!/v1!v2!v3! \cdots vk!)^2)/(N^M)^2$$

## B.3 Proof of Theorem 1.5

If the element contains both the properties of CIE and IIE, then the following properties:

- For two distinct sets R_A and R_B, Y_A and Y_B can be IIE
- For each value of a Y block, there are N/P_L distinct R block that can shift X to this Y. That means for a distinct Y set, the No. of R sets is not one.
- For two distinct sets R_A and R_B, Y_A and Y_B can be CIE

Base on the Theorem 1.1, if the X block is formed by pattern, then the No. of values in the range of Y is P_L, base on theorem 1.4, the Pr[SLE Y sets] can be expressed as:

**Definition B.5.** *Pr[SLE Y Sets:]:*

$$\binom{P\_L}{K} * (M!/v1!v2!v3! \cdots vk!)^2)/(P\_L^M)^2$$

If R_A and R_B are randomly generated, then the value of K various from 1 to M. We do th sum to get the expression in Theorem 1.5

**Definition B.6.** *Pr[SLE Y Sets]:*

$$(\sum_{K=1}^{min(P\_L,M)} \binom{P\_L}{K} * (M!/v1!v2!v3! \cdots vk!)^2)/(P\_L^M)^2$$