1 The Collision Probability of Y Set

Glossary The symbol, abbreviation and marks needed in expressing the shift stage and probability computation.

Replay Attack and Y Set Collision In replay attack, the adversary replace a data-tag pair on the memory with a pair copied from the same address at an old time point. That means for the two pairs at different time point, the two message block sets and related tags are identical respectively, while the nonce N_A and N_B are randomly generated and the equality is unpredictable if their generator is of high quality. That means the shifting bits parameter segment on the nonce R_A and R_B, are randomly generated.

In this scenario, the probability of a successful attack can be expressed as the equation 1.1:

Definition 1.1. *Pr[Successful Replay Attack] = Pr[Tag_a = Tag_b | (D_A = D_B) & (R_A and R_B are random)] = Pr[Y_A = Y_B | (D_A = D_B) & (R_A and R_B are random)]*

We can see that the collision of Y set will directly leads to the collision of tag and cause the succeed of replay attack. For easy understanding, we assume the shuffle stage does not work at first, which means D_A = X_A = D_B = X_B. Hence the Y set is the output of rotate shifting stage in CETD, we will analyze the properties of block rotate shifting and the cases of input sets that can result Y set collision.

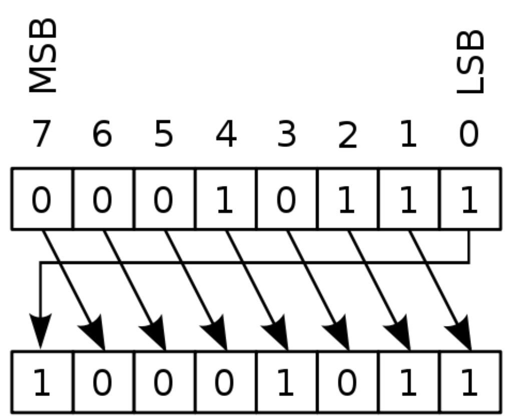
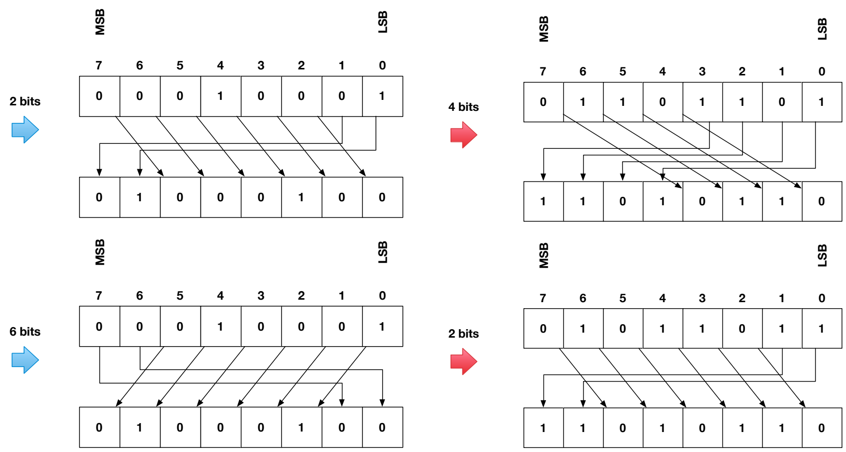


Figure 1: The Concept of Rotate Shifting(Right)

Rotate Shifting Introduction Unlike logical shifting and arithmetic shifting, rotate shifting behaves like whirling a wheel. The empty position in a



(a) Same Block Shifted Different Bits (b) Different Blocks Shifted Different Bits

Figure 2: The Examples of Y Block Collision

message blk shifted is filled by the bits shifted out. Figure 1 express the concept of rotate shift.

We can refer that the result of rotate shifting a message block depends on the value of blk and the bits shifted. If two message blocks whose value are identical are shifted with distinct R_is, the result blocks may be identical.That means when R_i fixed,the mapping from message blocks to the shifted result blocks is not injection. This case is expressed in Figure 2(a).

For two distinct message blocks, however, their result blocks may be identical when their R_is are distinct. This case is expressed in Figure 2(b). When the R_is of two message blocks are identical, the equality of result blocks is same as the equality of their input blocks.

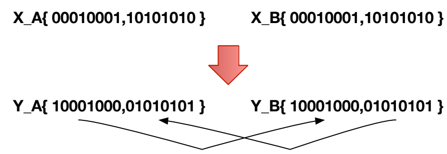
1.1 Rotate Shifting and Y Set Collision

Assume the shuffle stage does not work, then D_A= X_A=D_B=X_B,which means the following properties exist in X_A and X_B:

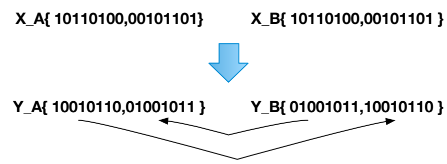
- X_A[i] = X_B[i] for all i∈[1,M], M is the number of elements
- The equality of elements in a X set is uncertain.

All the analysis in this section is based on the assumption that shuffle stage does not work. If these two X sets generate two identical Y sets with distinct R_A and R_B, then the two Y sets contains at least one of the following properties:

- Single Block Equality: Y_A[i] = Y_B[i] for all i∈[1,M] where M is the number of element

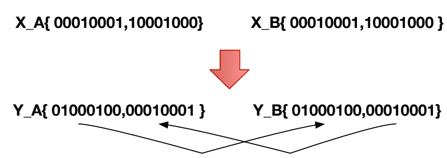


(a) Only Single Block Equality

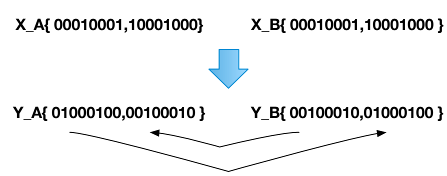


(b) Only Set Level Equality

Figure 3: X Set Pairs with Only One Type of Y Equality



(a) Single Block Equality Case



(b) Set Level Equality Case

Figure 4: A X Set Pair with Two Types of Y Equality

- Set Level Equality: Any element in Y_A, marked as Y_A[i], is identical to an element in Y_B, marked as Y_B[j] and i≠j.

Figure 3 and Figure 4 express the examples of X set pair that lead to idential Y set pair with distinct R set pair. In this paper, we analyze the cases of the input set pair (X_A and X_B) of shifting stage in CETD under replay attack that can lead to Y set pair collision.

1.1.1 Case of X Sets Resulting Y Set Collision

Single Block Equality Case As shown in Figure 2(a), two identical X blocks can result two identical Y blocks while their relative shifting bit parameter R[i] and R[j] are distinct. We found that the identical X block pair X_A[i]=X_B[i] resulting identical Y blocks with distinct R_A[i] and R_B[i] have a common property, this property is expressed in Theorem 1.

Theorem 1.1. *Assume X_A[i] and X_B[i] are two identical block from two set X_A and X_B with same index i. X_A[i] and X_B[i] have same number of bits N=2^n. The result of rotate shifting X_A[i] and X_B[i] with distinct shifting bit parameter R_A[i] and R_B[i] are marked as Y_A[i] and Y_B[i]. Then Y_A[i] and Y_B[i] can be identical only when X_A[i] and X_B[i] are formed by repeating a binary pattern P, which is a binary segment. The length of this pattern P can be expressed as:*

P_L = 2^p, p∈[0,n-1]

Assume the two X sets X_A and X_B are identical and can form single block equality. Base on Theorem 1.1, we can conclude the corollary of the relationship between X set pair and Y set pair for single block case.

Corollary 1.2. *Assume there is M elements in X_A and X_B. If there is at least one i∈[1,M], X_A[i] = X_B[i] is formed by repeating a pattern P_i whose length is P_L_i = $2^p i$, then Y_A and Y_B can be single block identical with some two distinct shifting bits parameter sets R_A and R_B.*

Set Level Equality Case From Figure 2(b) we can see that two distinct X block X_A[i] and X_B[j] can result two identical Y blocks Y_A[i]=Y_B[j] when the relative R_A[i] ≠ R_B[j].

Assume the element in a set with distinct indexes are distinct. We found that two distinct blocks X_A[i] and X_B[j] resulting identical Y blocks with distinct R_A[i] and R_B[i] have a common property. This property is expressed in Theorem 2.

Theorem 1.3. *Assume X_A[i] and X_B[j] are two distinct blocks from two set X_A and X_B and i≠j. X_A[i] and X_B[i] have same number of bits N=2^n. The result of rotate shifting X_A[i] and X_B[j] with distinct shifting bit parameter R_A[i] and R_B[i] are marked as Y_A[i] and Y_B[i].*

Then Y_A[i] and Y_B[i] can be identical only when X_A[i] can be rotate shifted to X_B[i]

Assume the two X sets X_A and X_B are identical and can form set level equality. Base on Theorem 1.2, we can conclude the corollary of the relationship between X set pair and Y set pair for set level equality.

Corollary 1.4. *Assume there are M elements in X_A and X_B. If there is at least two pairs of data, marked as (X_A[i],X_B[j]) and (X_A[j]),X_B[i]) where i,j∈[1,M] and i≠j, that X_A[i] can be formed by shifting X_A[j], then Y_A and Y_B can be set level identical with some two distinct shifting bits parameter sets R_A and R_B.*

An Intersection Case Base on Corollary 1.3 and Corollary 1.4, we can draw the following corollary for the X set pairs that can form both set level and single block equality:

Corollary 1.5. *Assume there is M elements in X_A and X_B. If there are at least two pairs of data, marked as (X_A[i],X_B[j]) and (X_A[j]),X_B[i]) where i,j∈[1,M] and i≠j, that X_A[i] can be formed by shifting X_A[j] and X_A[i] = X_B[i] or X_A[j]=X_B[j] is formed by repeating a pattern P_i whose length is P_L_i = $2^p i$, then Y_A and Y_B can be either set level or single block identical with some two distinct shifting bits parameter sets R_A and R_B.*

The proof of Theorem 1.1 and Theorem 1.2 and Corollary 1.2 to 1.5 can be refered in appendix.

1.2 The Probability of Y Set Collision

Hence the shift bit parameter set R_A and R_B are randomly generated, the probability of Y set collision for each input case is determined by the number of specific combinations of R_A and R_B. The computation of Pr[Y_A = Y_B] is based on this idea.

1.2.1 Single Block Equality Case

Assume two identical sets X_A and X_B satisfy the following properties:

- Each element is formed by a pattern with same length P_L = 2^p
- Each element cannot be formed by rotate shifting any other element.

Such two sets X_A and X_B can only result single block identical Y_A and Y_B. The probability of single block equality of Y_A and Y_B is marked as Pr[SBE] and expressed as the following way:

Definition 1.2. *Pr[SBE] =*

$$\prod_{i=1}^{M} Pr[Y\_A[i] = Y\_B[i]]$$

M is the No. of elements in a set

As D_A[i]=X_A[i]=X_B[i]=D_B[i] for all i∈[1,M], Pr[Y_A[i] = Y_B[i]] can be expressed as Pr[Y_A[i] = Y_B[i] — X_A[i] = X_B[i] & R_A and R_B are randomly generated]. We use Pr[Y block collision] to express this probability.

If R_A[i] = R_B[i] for all i∈[1,M], then Pr[Y_A[i] = Y_B[i]] = 1. When R_A and R_B is distinct, then at least one (R_A[i],R_A[i]) pair int two R sets is distinct. Pr[Y block collision] is expressed in Theorem 1.3:

Theorem 1.6. *If the No. of bits of each block in X_A[i]-X_B[i] pair is N=2^n, the pattern length P_l=2^p where p∈[0,n-1]. The pattern contains no internal sub-pattern, then Pr[Y block collision] = 1/2^p*

If each X set contains M elements, Pr[SBE] = $(1/2^p)^M$. The proof of theorem can be referred in appendix.

1.2.2 Set Level Equality Case

What to say in this section

- In general condition, Y_A and Y_B are multisets. If Y_A and Y_B are identical in set level, then Y_B is a permutation of Y_A.
- how to make two identical Y block with two distinct X block and R block: if there is no pattern, for each value of block, the responding Y block is definite. 1 r map to 1 Y value
- How to represent the expression of Pr[set level collision | X collision & R distinct]
- If Y_A is identical to Y_B in set level, element in Y_B can be regarded as a multiset permutation of set Y_A.

Assume two identical sets X_A and X_B satisfy the following properties:

- Each element can be formed by rotate shifting any other element in the set
- None of the elements is formed by pattern.

Such two sets X_A and X_B can only result set level identical Y_A and Y_B. The probability of set level equality of Y_A and Y_B is marked as Pr[SLE] and expressed as the following way:

Definition 1.3. *Pr[SLE] =*

$$\prod_{i=1,j=1}^{M} Pr[Y\_A[i] = Y\_B[j]]$$

where M is the No. of elements in a set, i,j∈[1,M] and i≠j

what to say next

- for any Y_A, the set level identical Y_B is a permutation of Y_A
- The No. of Y_B is effected by the value distribution of elements in Y_A

1.2.3 The Intersection Case

What to say in this section

- If there is pattern in the block, for each element, the mapping between r and Y is not 1-to-1.
- Compute set level equality first, then solve the severl r to 1 Y problem
- express the Probability with combinatorics

1.3 A General Condition of X Set and Related Y Set Collision

what to say in this section

- If shuffle stage works, the properties in some block pairs in D_A and D_B can be eliminated, while the remaining blocks maintain the pattern
- the no-pattern block pairs follow the analysis in Main Case 3.
- these patterned block pairs can be splited into several, each group is a kind of pattern.
- the pattern in any two group is distinct
- assume the No. of element in each group is expressed as M_i, then the theorem of each case of X sets can be applied in each group

A Proof of Pattern Introduction

A.1 Proof of Theorem 1

A.2 Proof of Theorem 2

B Proof of Probability Computation

B.1 Proof of Theorem 1.4

B.2 Proof of Theorem 1.5