1 The Collision Probability of Y Set

Glossary The symbol, abbreviation and marks needed in expressing the shift stage and probability computation.

1.1 Preliminaries

The Equality of Two Sets Assume there are two sets, marked as X_A and X_B, having same NO. of elements and the number is M. Each element is given an index from 1 to M. The definition of Identical Index Equality is expressed here:

Definition 1.1. *Identical Index Equality(IIE): For any i∈[1,M], X_A[i]=X_B[i].*

The definition of Cross Index Equality is expressed here:

Definition 1.2. *Cross Index Equality(CIE): For any i,j∈[1,M], X_A[i] = X_B[j], where i≠j*

Any two sets that have same number of elements can be splited into several sub-sets and each sub-sets contains several (X_A[i],X_B[i]) pairs. The number of sub-sets various from 1 to M. We give the definition of set level equality of two sets with same number of elements:

Definition 1.3. *Set Level Equality(SLE): Each sub-set have one of the two kinds of Equality.*

For two sets X_A and X_B, if there is no way of split that at least one sub-set is of CIE and there are at least one way of split that all sub-sets are of IIE, then such X_A and X_B are IIE only Sets. The definition of CIE only Sets is given in same way.

1.1.1 Replay Attack and Y Set Collision

The Message Sets in CETD Under Replay Attack

- D_A and D_B are definitely IIE.
- X_A=D_A, X_B=D_B
- Y_A and Y_B are just SLE
- when Y_A and Y_B are SLE, tag collision

In replay attack, the adversary replace a data-tag pair on the memory with a pair copied from the same address at an old time point. That means for the two pairs at different time point, the two message block sets and related tags are identical respectively, while the nonce N_A and N_B are randomly generated and the equality is unpredictable if their generator is of high quality. That means the shifting bits parameter segment on the nonce R_A and R_B, are randomly generated.

In this scenario, the probability of a successful attack can be expressed as the equation 1.1:

Definition 1.4. *Pr[Successful Replay Attack] = Pr[Tag_a = Tag_b | (D_A = D_B) & (R_A and R_B are random)] = Pr[Y_A = Y_B | (D_A = D_B) & (R_A and R_B are random)]*

D_A and D_B sets are of Identical Index Equality. Y_A and Y_B is of Set Level Equality.

We can see that the set level equality of Y_A and Y_B will directly leads to the collision of tag and cause the succeed of replay attack. For easy understanding, we assume the shuffle stage does not work at first, which means D_A = X_A = D_B = X_B(IIE). Hence the Y set is the output of rotate shifting stage in CETD, we will analyze the properties of block rotate shifting and the cases of input sets that can result Y set collision(SLE).

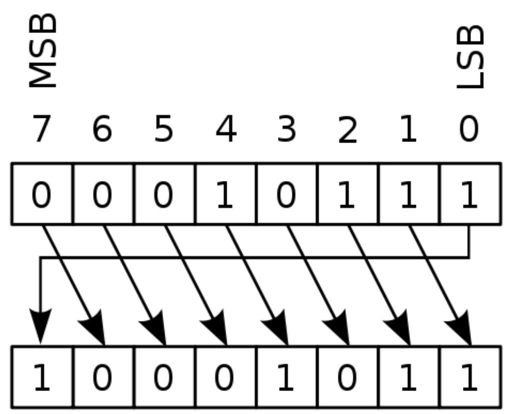


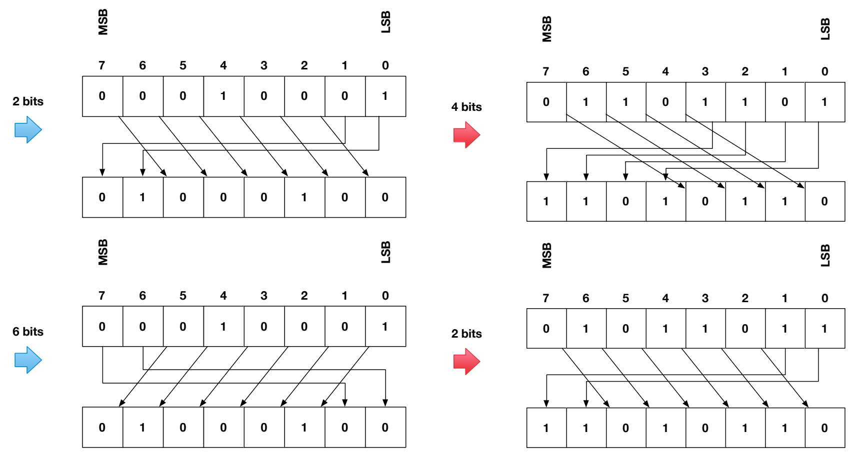
Figure 1: The Concept of Rotate Shifting(Right)

1.1.2 Rotate Shifting Introduction

- the concept of rotate shift
- The examples of X,R,Y

Unlike logical shifting and arithmetic shifting, rotate shifting behaves like whirling a wheel. The empty position in a message block shifted is filled by the bits shifted out. Figure 1 express the concept of rotate shift.

We can refer that the result of rotate shifting a message block depends on the value of block and the bits shifted. If two message blocks whose value are identical are shifted with distinct R_is, the result blocks may be identical.That means when R_i fixed,the mapping from message blocks to the shifted result blocks is not injection. This case is expressed in Figure 2(a).



(a) Same Block Shifted Different Bits (b) Different Blocks Shifted Different Bits

Figure 2: The Examples of Y Block Collision

For two distinct message blocks, however, their result blocks may be identical when their R_is are distinct. This case is expressed in Figure 2(b). When the R_is of two message blocks are identical, the equality of result blocks is same as the equality of their input blocks.

1.2 Rotate Shifting and Y Set Collision

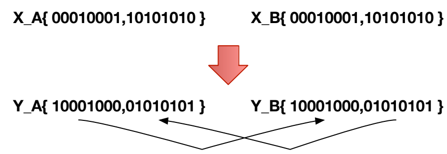
- distinct pairs in R sets: IIE pairs and CIE pairs
- X block pairs causing IIE Y block pairs
- X block pairs causing CIE Y block pairs

Assume the shuffle stage does not work, then D_A= X_A=D_B=X_B,which means the following properties exist in X_A and X_B:

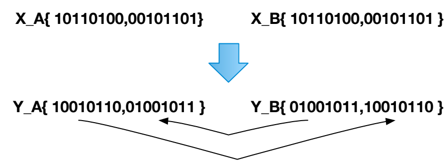
- X_A[i] = X_B[i] for all i∈[1,M], M is the number of elements
- The equality of elements in a X set is uncertain.

That means X_A and X_B are of Identical Index Equality. All the analysis in this section is based on the assumption that shuffle stage does not work. If such X_A and X_B result two Y sets of Set Level Equality using two identical shifting bits parameter sets R_A and R_B, the the related tag $T_a = T_b$.

Figure 3 and Figure 4 express the examples of X sets that lead to SLE Y sets with distinct R sets. In this paper, we analyze the cases of the input set pair (X_A and X_B) of shifting stage in CETD under replay attack that can lead to Y set pair collision(SLE).

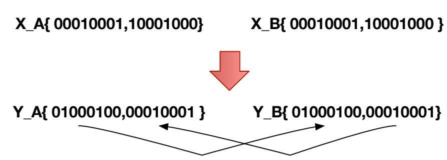


(a) Only Single Block Equality

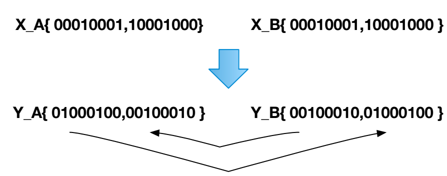


(b) Only Set Level Equality

Figure 3: X Set Pairs with Only One Type of Y Equality



(a) Single Block Equality Case



(b) Set Level Equality Case

Figure 4: A X Set Pair with Two Types of Y Equality

1.2.1 Case of X Sets Resulting Y Set Collision

- If R_A $\neq$ R_B, there is at least one pair distinct
- the related x blocks pairs should generate a SLE y pairs.
- if the Y pairs are iie, then each X block pair has pattern
- if the Y pairs are cie, then each X block in a sub-set is from a same base

Assume the number of distinct block pairs in R_A and R_B is R_d. For these R block pairs, their related X_A and X_B blocks formed a sub-group, marked as X_A_s and X_B_s, and this sub-group is IIE. The related sub-group in Y_A is marked as Y_A_s and the one in Y_B for Y_B_s.

In this part, we discuss the equality case of Y_A_s and Y_B_s

Identical Index Equality Case If Y_A_s and Y_B_s are IIE, then for all i$\in$[1, R_d] the following two properties are met:

- Y_A_s[i] = Y_B_s[i]
- R_A_s[i] $\neq$ Y_B_s[i]
- X_A_s[i] = X_B_s[i]

Figure 2(a) provides a instance of IIE Y sets. The identical block pairs (X_A_s[i],X_B_s[i]) that lead to IIE Y_A_s and Y_B_s have the property in Theorem 1.1:

Theorem 1.1. *Assume X_A[i] and X_B[i] are two identical block from two sub-group X_A_s and X_B_s with same index i. X_A[i] and X_B[i] have same number of bits $N=2^n$. The result of rotate shifting X_A[i] and X_B[i] with distinct shifting bit parameter R_A[i] and R_B[i] are marked as Y_A[i] and Y_B[i].*

Then Y_A[i] and Y_B[i] can be identical only when X_A[i] and X_B[i] are formed by repeating a binary pattern P, which is a binary segment. The length of this pattern P can be expressed as:

P_L = 2^p, p$\in$[0,n-1]

Cross Index Equality Case If Y_A_s and Y_B_s are CIE, then for all distinct (i,j) that i,j$\in$[1, R_d] the following two properties are met:

- Y_A_s[i] = Y_B_s[j]
- R_A_s[i] $\neq$ Y_B_s[i]
- X_A_s[i] = X_B_s[i]

Figure 2(b) provides an instance of CIE Y sets. The blocks in X_A_s and X_B_s that lead to CIE Y_A_s and Y_B_s have the property in Theorem 1.2:

Theorem 1.2. *If Y_A_s and Y_B_s are CIE, each element in X_A_s can be formed by rotate shifting another element in X_A_s.*

If each element in X_A_s is formed by a pattern can be formed by rotate shifting another element in X_B_s then the related Y_A_s and Y_B_s can be either IIE or CIE or SLE. The proof of Theorem 1.1 and Theorem 1.2 can be refered in appendix.

1.3 The Probability of Y Set Collision

- Assume the sub-set contaions M blocks, the probability that they are both pattern
- if they are both pattern, the probability that Y set are iie
- the probability that they are from same base
- if they are same base, the pro that Y set is cie
- the probability that bothe pattern and same base
- if both, the pro of Y SLE

Hence the shift bit parameter set R_A and R_B are randomly generated, the probability of Y set collision for each input case is determined by the number of specific combinations of R_A and R_B. The computation of Pr[Y_A = Y_B] is based on this idea.

1.3.1 Identical Index Equality Case

For ease analysis, we assume R_A[i] $\neq$ R_B[i] for all i$\in$[1, M], where M is the No. of elements in R_A. From Theorem 1.1 we know that Y_A and Y_B are IIE if each element in X_A is formed by a pattern P whose length P_L is 2^p. Assume two identical sets X_A and X_B satisfy the following properties:

- Each element is formed by a pattern with same length P_L = 2^p
- Each element cannot be formed by rotate shifting any other element.

We call this kind of X_A and X_B Identical Pattern Length(IPL) X Sets. IPL X sets can form IIE Y sets. Assume D_A set is randomly generated, then the probability of a IPL D_A is expressed as:

Definition 1.5. *Pr[IPL D(X)] =*

The probability of IIE of Y_A and Y_B is marked as Pr[IIE Y sets] and expressed as the following way:

Definition 1.6. *Pr[IIE Y sets] =*

$$\prod_{i=1}^{M} Pr[Y\_A[i] = Y\_B[i]]$$

M is the No. of elements in a set

As D_A[i]=X_A[i]=X_B[i]=D_B[i] for all i∈[1,M], Pr[Y_A[i] = Y_B[i]] can be expressed as Pr[Y_A[i] = Y_B[i] — X_A[i] = X_B[i] & R_A and R_B are randomly generated]. We use Pr[Y block collision] to express this probability.

Pr[Y block collision] is expressed in Theorem 1.3:

Theorem 1.3. *If the No. of bits of each block in X_A[i]-X_B[i] pair is $N=2^n$, the pattern length $P_l=2^p$ where $p\in[0,n-1]$. The pattern contains no internal sub-pattern, then Pr[Y block collision] = $1/2^p$*

If each X set contains M elements, Pr[SBE] = $(1/2^p)^M$. The proof of theorem can be referred in appendix.

1.3.2 Cross Index Equality Case

What to say in this section

- In general condition, Y_A and Y_B are multisets. If Y_A and Y_B are identical in set level, then Y_B is a permutation of Y_A.
- how to make two identical Y block with two distinct X block and R block: if there is no pattern, for each value of block, the responding Y block is definite. 1 r map to 1 Y value
- How to represent the expression of Pr[set level collision | X collision & R distinct]
- If Y_A is identical to Y_B in set level, element in Y_B can be regarded as a multiset permutation of set Y_A.

Assume two identical sets X_A and X_B satisfy the following properties:

- Each element can be formed by rotate shifting any other element in the set
- None of the elements is formed by pattern.

Such two sets X_A and X_B can only result set level identical Y_A and Y_B. The probability of set level equality of Y_A and Y_B is marked as Pr[SLE] and expressed as the following way:

Definition 1.7. *Pr[SLE] =*

$$\prod_{i=1,j=1}^{M} Pr[Y\_A[i] = Y\_B[j]]$$

where M is the No. of elements in a set, i,j∈[1,M] and i≠j

what to say next

- for any Y_A, the set level identical Y_B is a permutation of Y_A
- The No. of Y_B is effected by the value distribution of elements in Y_A

1.3.3 The Intersection Case

What to say in this section

- If there is pattern in the block, for each element, the mapping between r and Y is not 1-to-1.
- Compute set level equality first, then solve the severl r to 1 Y problem
- express the Probability with combinatorics

1.4 A General Condition of X Set and Related Y Set Collision

what to say in this section

- If shuffle stage works, the properties in some block pairs in D_A and D_B can be eliminated, while the remaining blocks maintain the pattern
- the no-pattern block pairs follow the analysis in Main Case 3.
- these patterned block pairs can be splited into several, each group is a kind of pattern.
- the pattern in any two group is distinct
- assume the No. of element in each group is expressed as M_i, then the theorem of each case of X sets can be applied in each group

A Proof of Pattern Introduction

A.1 Proof of Theorem 1

This part proves that if two identical block X_A[i] and X_B[i] are shifted different bits and the result blocks remain identical, X_A[i] is formed by pattern. The pattern length and δ=|R_A[i]-R_B[i] | has such correlation:

- If δ = P_L then Y_A[i] = Y_B[i] where P_L the length of pattern

Proof of Corollary 1.2 According to the definition of set block equality, Y_A[i]=Y_B[i] for all i∈[1,M]. As R_A $\neq$ R_B, there is at least one pair (R_A[i],R_B[i]) is distinct. As the related X_A[i] is identical to X_B[i], X_A[i] is formed by pattern based on Theorem 1.1.

A.2 Proof of Theorem 2

Assume Y_A[i] and Y_B[j] are identical. That means the following equations are met:

- Y_A[i][b] = X_A[i][(b+R_A[i]) mod N]
- Y_B[j][b] = X_B[j][(b+R_A[j]) mod N]
- Y_A[i]$_b$ = Y_B[j]$_b$
- X_A[i][(b+R_A[i]) mod N] = X_B[j][(b+R_A[j]) mod N]

Proof of Corollary 1.3 According to the definition of set level equality, Y_A[i]=Y_B[j] for i,j∈[1,M], i≠j and X_A[i] ≠ . As R_A ≠ R_B, there is at least one pair (R_A[i],R_B[i]) is distinct. As the related X_A[i] is identical to X_B[i], X_A[i] is formed by pattern based on Theorem 1.1.

B Proof of Probability Computation

B.1 Proof of Theorem 1.4

B.2 Proof of Theorem 1.5