Exam I

1] length of curve $r(t) = \langle 2t+5, \frac{1}{2}t^2 - 2, \frac{4}{3}\sqrt{t^3} \rangle$

$0 \le t \le 3$ $\quad r'(t) = \langle 2, t, \frac{12}{6}\sqrt{t} \rangle$

$$L = \int_a^b |r'(t)| = \int_0^3 \sqrt{2^2 + (t)^2 + (2\sqrt{t})^2} = \int_0^3 \sqrt{t^2+4t+4}$$

$$\int_0^3 \sqrt{(t+2)^2} = \int_0^3 t+2 = \frac{1}{2}t^2 + 2t \Big|_0^3$$

$$= \frac{1}{2}(3)^2 + 2(3) - \left(\frac{1}{2}(0)^2 + 2(0)\right) \boxed{= \frac{21}{2} \approx 10.5} = L$$

2] parametrics of paraboloid $z = 3x^2 + y^2$ and cylinder $y = x^2$

$x = t$

$y = t^2$

$z = 3t^2 + (t^2)^2 \rightarrow 3t^2 + t^4$

$$\boxed{r(t) = \langle t, t^2, 3t^2 + t^4 \rangle}$$

3] find dist from pt $(0,7,1)$ and plane $x + y + z = 1$

$$D = \frac{|Ax + By + Cz + D|}{\sqrt{a^2+b^2+c^2}} = \frac{|1(0) + 1(7) + 1(1) - 1|}{\sqrt{1^2+1^2+1^2}} =$$

$$= \frac{8}{\sqrt{3}} \approx \boxed{4.619}$$

4) find area $\triangle ABC$ $A(1,0,-2)$, $B(3,1,0)$, $C(0,5,1)$

$A = \frac{1}{2}|u \times v|$ $\quad u = \overrightarrow{AB} = \langle 3-1, 1-0, 0-(-2)\rangle$

$v = \overrightarrow{AC} = \langle 0-1, 5-0, 1-(-2)\rangle$

$A = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$ $\quad \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & k \\ 2 & 1 & 2 \\ -1 & 5 & 3 \end{vmatrix}$

$= (3(1)-2(5))i - (2(3)-(2(-1)))j + (2(5)-(1)(-1))k$

$= -7i - 8j + 11k$

$A = \frac{1}{2}|\langle -7, -8, 11\rangle| = \frac{1}{2}\sqrt{(-7)^2+(-8)^2+(11)^2} = \frac{3\sqrt{26}}{2}$

$= 7.649$