Will Legg 2-22-24

Week 6 Review

31 find three pos numbers whose sum is 12 and sum of squares is as small as possible

$f(x,y,z) = x^2+y^2+z^2$ $g(x,y,z) = x+y+z=12$

$\nabla f = \lambda \nabla g$

$f_x = \lambda g_x$
$f_y = \lambda g_y$
$f_z = \lambda g_z$
$x+y+z=12$

$2x = \lambda$
$2y = \lambda$
$2z = \lambda$
$x+y+z=12$

$z=x$, $y=x$
$x+x+x=12$
$3x = 12$
$\boxed{x=4 \rightarrow y=4,\ z=4}$

$f(4,4,4)=4^2+4^2+4^2=48$ $2x=\lambda \rightarrow \boxed{\lambda = 8}$

$x=y=z=4$ $\boxed{f=48}$ check 48 is smallest

for $f(1,1,10) = 1^2+1^2+10^2 = 102$
$\downarrow$ $\rightarrow 102 > 48$
$x+y+z=12$ so 48 is smallest
$\boxed{x=y=z=4}$

Week 6 Review

Ex 4) find volume of largest rectangular box in Q1 w/ 3 faces in coord plane and whose final vertex is contained on surface
$x+2y+3z=6$

$(x, y, z > 0)$

$f = V$ of box
$= xyz$

$g(x,y,z) = x+2y+3z-6$

set $\nabla f = \lambda \nabla g$

$f_x = yz = \lambda g_x = \lambda$
$f_y = xz = \lambda g_y = 2\lambda$
$f_z = xy = 3\lambda$
$x+2y+3z-6=0$

$yz = \lambda = \frac{xz}{2}$ ①
$yz = \lambda = \frac{xy}{3}$ ②
$x+2y+3z=6$ ③

① $yz = \frac{xz}{2} \rightarrow y = \frac{x}{2}$

② $z = \frac{x}{3} \longrightarrow x + 2\left(\frac{x}{2}\right) + 3\left(\frac{x}{3}\right) = 6$

$x = 2$

$y = 1 \quad z = 2/3$

$f(2, 1, 2/3) = 4/3$

Week 6 Review

1) Let $f(x,y) = e^{2x}\cos(y)$ & let $Q_0 = Q_0(0, \frac{\pi}{6})$

a) $\nabla f = \langle f_x, f_y \rangle = \langle 2e^{2x}\cos(y), e^{2x}(-\sin y) \rangle$

$\nabla f(0, \frac{\pi}{6}) = \langle 2e^0 \cos(\frac{\pi}{6}), e^0(-\sin\frac{\pi}{6}) \rangle$

b) $D_{\vec{u}} f = \nabla f \cdot \frac{\vec{u}}{|u|}(0, \frac{\pi}{6})$

$= \nabla f(0, \frac{\pi}{6}) \cdot \frac{(1,2)}{\sqrt{1^2+2^2}} =$

2) find pts on surface $y^2 = 9 + xz$ that are closest to origin

$f(x,y,z) = \sqrt{(x-0)^2 + (y-0)^2 + (z-0)^2}$
$=$

$f(x,y,z) = x^2 + y^2 + z^2$ set $\nabla f = 0$

$\nabla f = 0$ $\quad y^2 = 9 + xz$

$f_x = 2x = 0$

$f_y = 2y = 0$

$g(x,z) = x^2 + (9+xz)^2 + z^2$

$\nabla g = 0$

$g_x = 2x + z = 0$

$g_z = x + 2z = 0$

solve sys

$x=0 \quad z=0$

check if min (Hassain Matrix)

$g_{xx} g_{zz} - (g_{xz})^2$

$g_{xx} = 2$

$g_{xz} = 1 \quad = 2 \cdot 2 - 1 \boxed{= 3 > 0}$

$g_{zz} = 2$

so $x=0$ & $z=0$ is a min