Group Project 1 - Will, Evan, Ben, Nicholas & Mikel

Facts:

- The velocity of the worm: 30 m/s
- The angle of the ravine: 50 degrees north of east
- Paul's distance to the ravine: 80 meters to north (+y)
- 7 meters to the top of the worm front the edge of the ravine (Δy)
- The worm center is 7 meters from the edge (Δx)
- The worm starts 500 meters from Paul directly to the west (-x)
- $x_{final} = x_{initial} + v_0 t + \frac{1}{2} a t$
- $a^2 + b^2 = c^2$
- $V^2 = V_0^2 + 2a\Delta x$
- $V = V_0 = at$

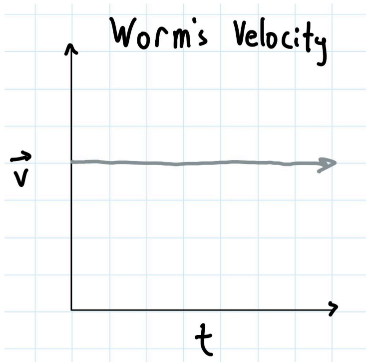
Lacking:

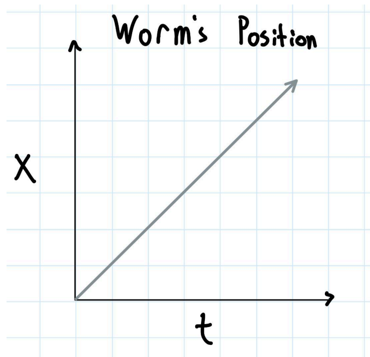
- The distance the worm travels to the jump point
- The time it takes the worm to travel to the jump point (aka, Paul's total time in freefall and running on the ground)
- Time Paul should start running to land on worm
- Velocity Paul needs when he leaves the cliff to land directly on the worm face
- Any acceleration Paul needs to get up to his launching velocity
- The time it takes Paul to fall on to the worm
- Amount of time Paul has until worm is at the point 80 meters directly North of Paul

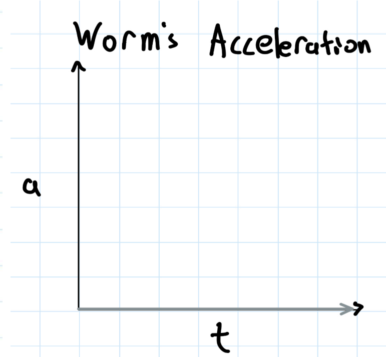
Assumptions:

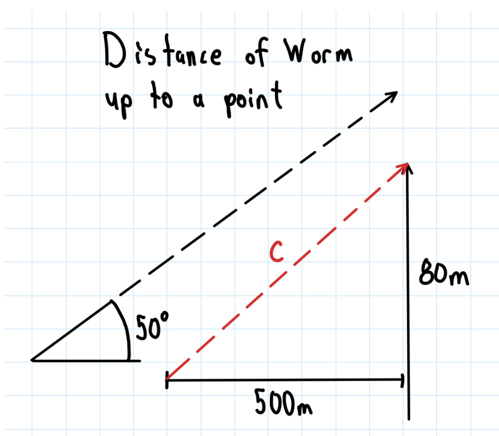
- We are assuming that the planet Arrakis has the same conditions as Earth and therefore has the same gravity and atmospheric makeup as Earth
- We are assuming that there is no air resistance, friction, or non-conservative forces
- We assume that the worm has a constant velocity of 30m/s, following a constant direction of 50 degrees north of east (angle of the ravine)
- Acceleration of worm is 0m/s^2
- Paul doesn't jump off cliff at an angle, he instead runs straight off
- Paul maintains a constant velocity in the x direction when he is falling

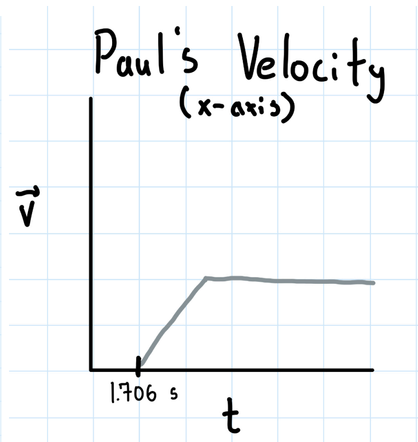
Representation:

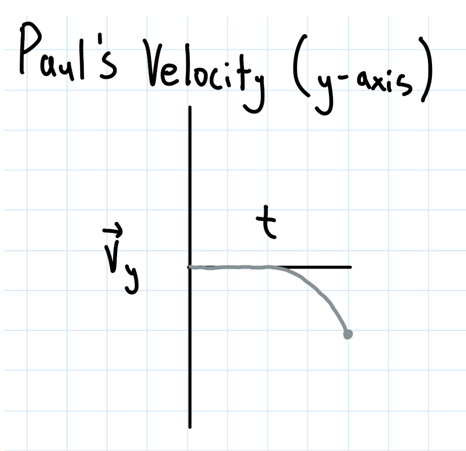


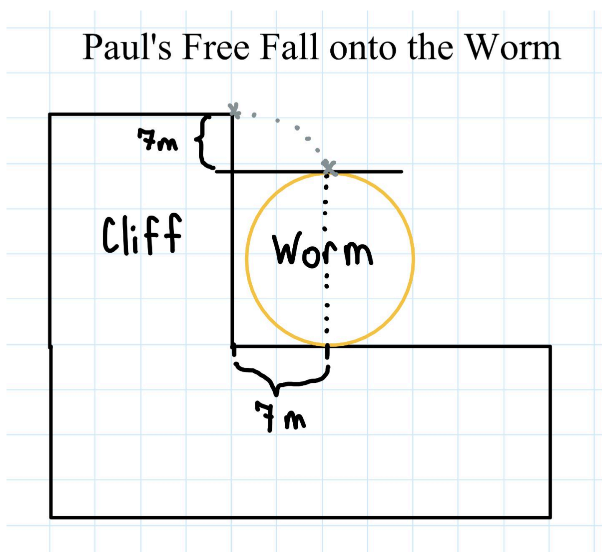












Goal:

For this problem, we first need to determine the distance that the worm is traveling to the defined jump point. We are given that the worm is 500 meters west of Paul and 80 meters south of him. From here, we need to calculate the time it takes the worm to reach this point, given that he is traveling at a constant velocity of 30 m/s and maintaining a constant angle of 50 degrees north of east. After this, we must determine the velocity at which Paul jumps, the time that it takes him to fall from the edge onto the worm, the time that he needs to start running, and the velocity (and acceleration if required) he needs to run at to make it onto the worm successfully.

Justification of Assumptions:

- The planet that we are on must have the same properties as Earth (gravity) so that the free-fall motion is the same. If this is not assumed, we have no basis on which to calculate Paul's free-fall time, therefore providing no solution to the problem. We are also assuming this because if the acceleration due to gravity was different, we were not given any devices in the problem to calculate these differences.
- Air resistance can be neglected, because the velocities we are using are at very small speeds, meaning that it will not have much of an impact on the calculations. Also, air resistance is a more complex idea that would be over-complicating the realm of the problem that we are looking at.
- The worm must have a constant velocity (and therefore an acceleration of zero). If this were not the case, he would be covering certain distances in different time intervals, on which basis we would have no way to calculate the time it takes it to reach the cliff.
- If Paul jumps off the cliff at an angle, then the time that it takes him to fall onto the worm changes. We must assume this so that we can properly calculate the time.
- Once Paul jumps off the cliff, we assume that he has no means of changing his velocity in the air. This means the only thing that is impacting his trajectory once he is in the air is gravity.

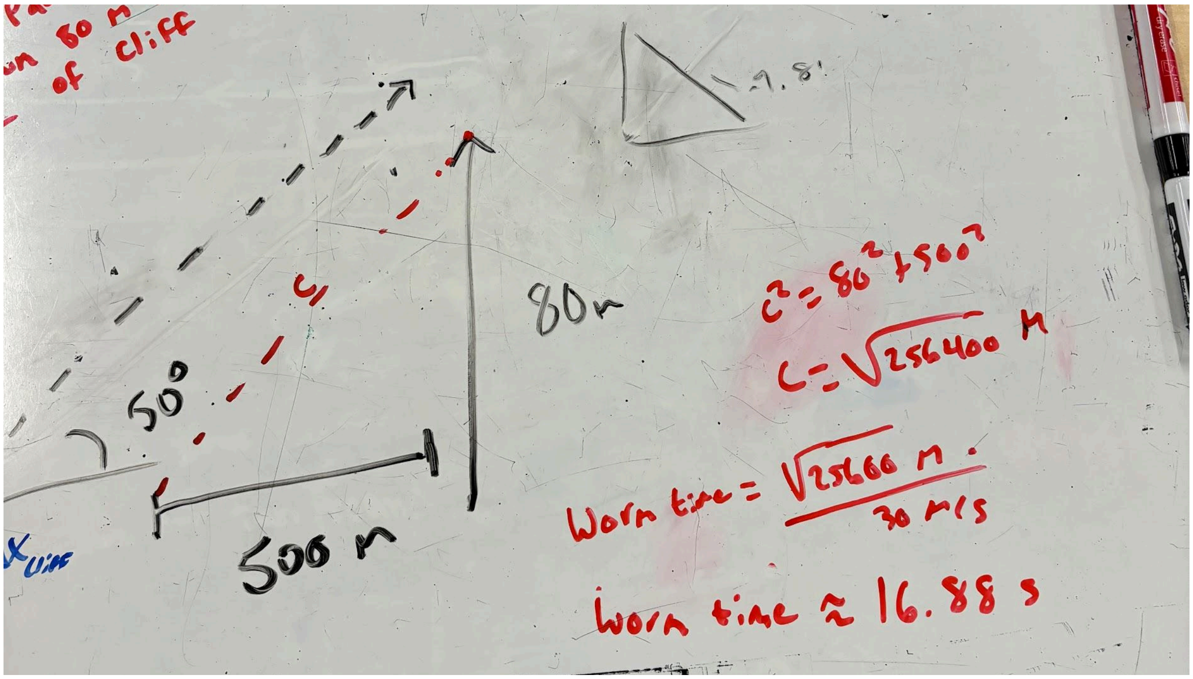
Plan of Approach:

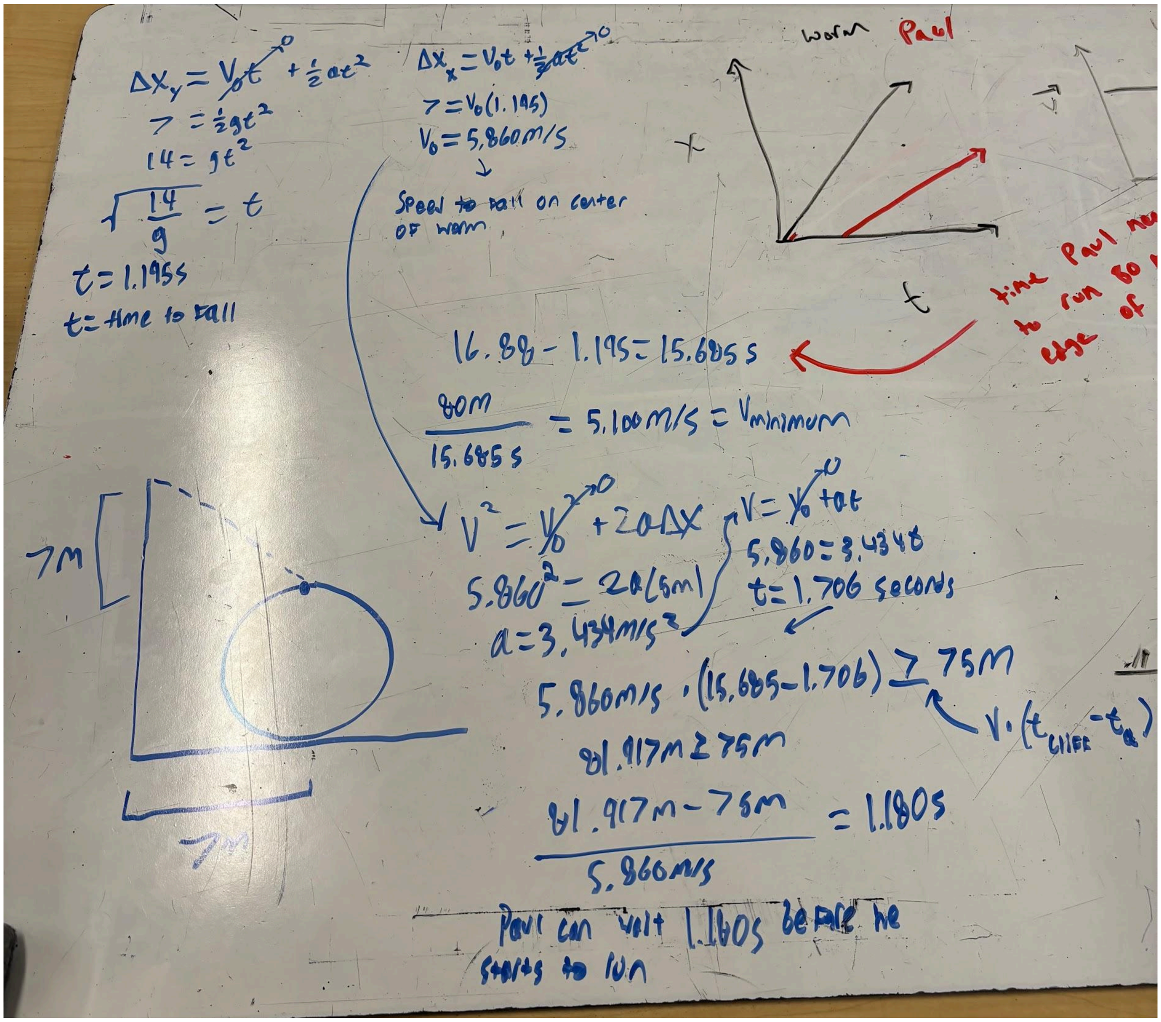
1. Determine the distance that the worm travels
2. Determine the time it would take the worm to cover this distance (the point 80 meters directly north of Paul)
3. Determine the time Paul would be falling 7 meters to land on the worm face
4. Find the velocity Paul needs to be traveling as he leaves the ravine edge to hit the center of the worm face
5. Find the time Paul needs to travel 80 meters to the edge of the cliff
6. Find Paul's required velocity (and if needed acceleration) to make this time interval up to the edge update: Paul's velocity at the jump point was greater than a constant velocity scenario, so Paul needed to have an acceleration.
7. If necessary: determine the time Paul would wait to start his run after determining the time it would take for Paul to get to the worm after accelerating and traveling at the needed velocity to land on the worm update: Paul needed to wait 1.180 seconds before he started running.

Calculation w/ Explanation:

Find the time it takes for the worm to travel to the position directly North of Paul:

1. Utilize Pythagorean theorem ($a^2+b^2 = c^2$) to determine the distance the worm would travel to be at the position directly north of Paul (at the jump point, 80 meters north of Paul's current location) starting from its current position of 500 meters west
2. Using the worm's velocity, determine the time it'd take for the worm to travel this distance





- The time it takes Paul to go from the edge of the cliff to the center of the worm is the time it takes Paul to fall 7m. Using kinematic equations the time was determined to be 1.195 seconds. With that time the same kinematic equation but in the x-direction can be used to determine the velocity Paul will need to leave the cliff to land 7m away from the cliff on top of the worm.The velocity Paul needs when leaving the cliff is 5.860

- Once we had our time to fall we had to subtract that from the time it takes the worm to reach Paul in order to determine the time he has to reach the end of the cliff. With that time we found the minimum average velocity Paul must have in order to reach the end of the cliff within the time frame.

- Since we knew our final velocity(5.860m/s) and our initial velocity(0m/s) we could determine Paul's acceleration by also including the distance it takes to accelerate. 5m was a seemingly reasonable distance to accelerate so it was used to calculate acceleration. The acceleration was 3.434m/s^2. Paul only accelerates until he reaches 5.680m/s. Using kinematics it was determined Paul will accelerate for 1.706 seconds before keeping a constant velocity.

- After determining Paul's time to accelerate and distance traveled during acceleration we had to make sure the velocity he held afterward was enough to reach the cliff in time. It was determined he would reach the cliff with time to spare.

- In order to make sure Paul didn't jump too early we had to determine the amount of time he would have to wait in order to reach the cliff's edge at the right time. We took the spare distance Paul would have traveled in the time frame and divided it by his velocity to get the spare time. This spare time was 1.180s. This means Paul can wait 1.180s before he has to start running in order to reach the cliff edge on time.

Final Answer:

In order for Paul to land on the head of the worm **he must wait 1.18 seconds before running**. He will accelerate at 3.434 m/s^2 until he reaches a velocity of 5.68 m/s, he will be accelerating for 1.786 seconds. He will then jump off the cliff and land on the head of the worm fulfilling the prophecy.

Evaluation of Solution:

In order to solve this problem we first determined the time at which the worm would be at the position 80 meters directly north of Paul's position. Our results were consistent with our expectations as we had the vertical and horizontal distances of the worm as well as the worm's acceleration so we were able to calculate the time through the use of the Pythagorean theorem as well as the general equation for velocity. We were only considering the worm's face which we took to be a single point so the calculations were easy to determine.

Next, we would have to determine how fast Paul would have to be traveling to hit the face of the worm as it passes him at the position 80 meters directly north of his position. Our results were within our expectations as we knew the forces (gravity) which needed to be accounted for and were able to solve for the velocity and we only considered gravity to be the only force acting on Paul. After determining the time it would take for Paul to fall 7 meters and the velocity he would have to be traveling at to move 7 meters horizontally, we would need to determine the time it would take for Paul to travel the 80 meters to get to the position of the worm 80 meters north of him, considering the fact that he would have to accelerate to the velocity needed because he was starting from rest.

This would allow us to determine the time Paul would have to start his run by subtracting from the time it would take for the worm to get to that position 80 meters north. We assumed that it would reasonably take him 5 meters to accelerate to a velocity of 5.86 m/s. We thought that this would be the normal time it would take for a normal human to accelerate to this speed. After determining the time it'd take for Paul to travel to the position we could subtract that time from the time it'd take for the worm to reach that position and it gave us the time that Paul would have to wait to start his run to jump onto the worm. We're satisfied that our result is a reasonable answer to the problem with the assumptions that we made to do our calculations.

Evaluation of Model:

- Our model gave a rough approximation of this real-world scenario. We assumed constant velocity and direction for the worm and for Paul (which in the real world nothing can ever be exactly constant). Also, we assume there to be no friction in our calculations. Friction is a necessary force in the world for any motion to occur, so this is another hindering point in our model. However, our model did a good job of creating calculations for Paul's time, given his distance and time. The numbers that we got for his time (the total time to run 80m and fall is 15.7 s), acceleration (he accelerates at 3.434 m/s^2 for 1.706 seconds), and velocity (after he stops accelerating, he runs at a constant velocity of 5.86 m/s to reach worm) for the given distances make sense, and are reasonable for any human to achieve. Another limitation in our model was the assumption that this planet had the same fundamental properties as Earth (which for other planets in our solar system we know is not a good approximation). While this was a big assumption, it was one that was necessary to solve the problem. However, in a real application of this scenario, this would cause problems with different accelerative forces (due to gravity), which would impact Paul's time he falls and the speed at which he needs to run off the cliff to make it to the worm.

Improvements to Model:

- To improve our model and make it more accurate to the actual situation, we could do a couple of things. First and foremost, we could find the force of gravity on Arrakis instead of assuming that this was the same as on Earth. It is a very big stretch to assume two planets have the same gravitational acceleration. For example, Venus (the closest planet to Earth in our solar system) has a gravitational acceleration of 8.87 m/s^2 which is considerably different from that of Earth (9.81m/s^2). By getting the actual gravitational acceleration of this planet, we would be able to better model the free-fall motion of objects (and in this situation humans). Another improvement to this model would be to consider air resistance and any other friction forces that would affect motion. By doing this, we would give motion models that are accurate to this situation.
- Take a sample of the atmosphere on Arrakis to find out what the true air resistance is. This would change the time it took Paul to fall. This would also affect the velocity that Paul would have to travel at to get to the worm because if it took him more time to fall, he would have to get to the edge of the cliff at a faster rate.