<u>Group Project 3</u>

Goal: How much energy must batman put into heating the glue and how much energy into launching it to stop the disk. Find height batman should be at when launching the ball.

Facts:
- <u>Wax Ball</u>: mass: 20kg, must be 330K
- <u>Disk</u>: mass: 1000kg, radius: 2m, rotating counter-clockwise
- Motor pulls rope attached to disk at 500 N for 10 seconds
- The ladder is 15m from the center of the disk
- <u>System</u>: disk, car, wax ball

Lacking:
- How much energy is needed to heat up the glue
- How energy is needed to shoot the glue enough to stop the disk
- How much energy does the spinning disk have after 10 seconds
- Height that batman needs to climb on the latter

Assumptions:
- We are not taking air resistance or friction into account
- Net external force acting on the system is zero
- Assume heat capacity of paraffin wax is 2.20 J/K
- Assume the initial temperature of the wax is room temperature, 293 K
- Momentum is conserved, collision between wax and disk is maximally inelastic
- Kinetic energy of the system is not conserved if angular momentum is conserved
- Robins weight will not shift the center of mass considerably
- Assuming that the disk is not slowed down due to friction with some kind of axle
- The disk will not slow down due to air resistance
- Wax ball hits the disk perfectly at 90 degrees
- The wax sticks to the disk without rebounding
- Batman and the ladder will not have any force exerted on him from the wax canon

Equations:

$$\Delta E_{sys} = W_{surr} + Q$$

$$Q = MC\Delta T$$

$$\tau = F\,x\,r$$

$$I = \frac{MR^2}{2}$$

$$\alpha = \frac{\tau}{I}$$

$$\omega = \alpha * t$$

$$K_{rot} = \frac{1}{2} I\omega^2$$

$$I = \frac{L}{\omega}$$

$$L = mvr$$

$$E_K = \frac{1}{2}m\vec{v}^2$$

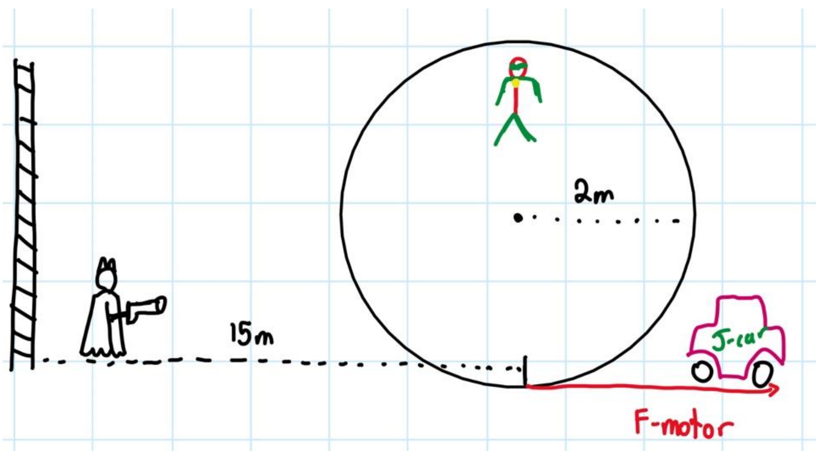
$$\Delta E_K = \vec{F}d$$

$$F_{\text{net},\perp} = \frac{m\vec{v}^2}{R}$$

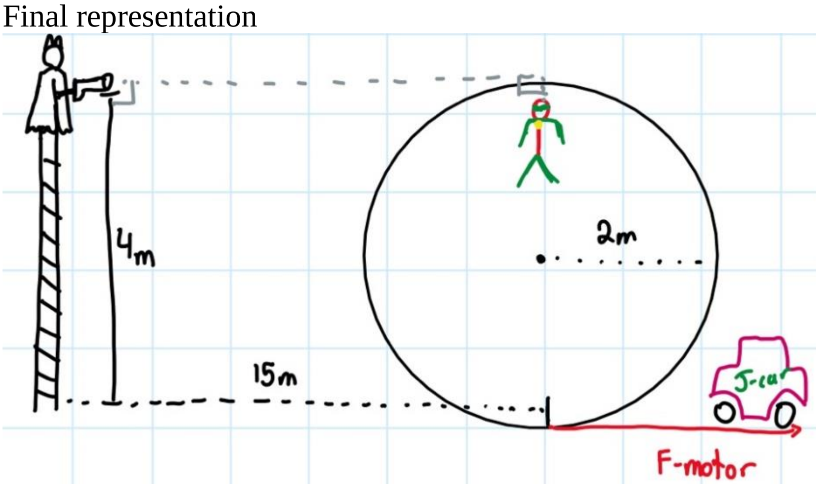
$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{v}$$

Representation:

Initial representation



Final representation



Justification of Assumptions:

- Air resistance and friction are not accounted for in our system for us to be able to reasonably solve for the energy to stop the disk within class time
- We are assuming that forces from outside the system will not change the energy of the system a considerable amount
- In order to find the velocity, the ball would need to travel to stop the disk, we needed to assume that angular momentum was conserved to use our momentum conservation equation, in doing this we're also implying that the kinetic energy of the system is not conserved
- We assumed that the center of mass of the disk would not change with the additional mass of the wax balls as they stick to the disk and that the center would not change due to the mass of Robin
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Plan of Approach:

1. Solve for energy needed to heat the wax to 330 kelvin
2. Solve for torque generated by the disk
3. Find the moment of inertia of the disk
4. Solve for the angular acceleration of the disk
5. Find the angular velocity of the disk
6. Find angular momentum of the disk after the force from the motor is applied
7. Find velocity of the ball needed to stop the disk
8. Convert velocity of the ball into a translational energy that the ball will be launched at

Calculation w/ Explanation:

1. Find Energy required to melt the paraffin wax
   a. Used the $Q = MC\Delta T$ equation with the given and assumed variables to find the thermal energy needed to heat the wax to 330K

1. Solve for thermal energy needed to heat wax

$M_w = 20$

$Q = MC\Delta T$

$C_w = 2.20$

$Q_w = M_w C_w \Delta T_w$

$\Delta T_w = (330 - 293)$

$Q_w = 20 \times 2.20 \times (330 - 293)$

$Q_w = 1628\ J$

   b. Batman would need to input 1628 J into the cannon to get the wax ball to be melted and sticky

2. Find the torque of the spinning disk
   a. Used the equation $T = F_r \times r_d$ with the given variables to find the torque

2. Solving for torque of disk

$T = F_r \times r_d$

$F_r = 500N$

$T = 500 \times 2$

$r_d = 2m$

$T = 1000\ Nm$

3. Find the moment of Inertia of the disk
   a. Using the equation $I = \frac{m \times R^2}{2}$ and the given values to calculate the Moment of Inertia for the disk

3. Solve for the moment of Inertia of the disk

$m_d = 1000$

$I = \frac{m R^2}{2}$

$r_d = 2$

$I = \frac{1000 \times (2)^2}{2}$

$I = 2000\ kgm^2$

4. Find the angular acceleration of the disk
    a. Used the equation $\alpha = {}^{T}/_{I}$ and the calculated values to solve for angular acceleration

4. Solve for the angular acceleration of the disk

$T = 1000$

$\alpha = \frac{T}{I}$ $\quad I = 2000$

$\alpha = \frac{1000}{2000}$

$\alpha = \frac{1}{2}\ \text{rad}^2/s$

5. Find the Angular Velocity of the disk after the motor force is applied
    a. Used the equation $\omega = \alpha \times t$ with the calculated values to find the angular velocity

5. Find the Angular Velocity of the disk after the force is applied

$w = \alpha \times t$ $\quad t = 10$

$w = 0.5 \times 10$ $\quad \alpha = 0.5$

$w = 5\ \text{rad}/s$

6. Find the angular momentum of the disk after the force is applied
    a. Used the equation $L = I\omega$ with calculated vales to find the angular momentum of the disk after the force is applied

6. Find the angular momentum of the disk after the motor force is applied

$L = Iw$ $\quad I = 2000$

$L = 2000 \times 5$ $\quad w = 5$

$L = 10{,}000\ kg\ m^2/s$

7. Find the velocity of the ball needed to stop the disk.
    a. Used equation of momentum and conservation of momentum equations with calculated variables to solve for velocity of the ball.

7. Find Angular momentum of the ball that is needed to stop the disk

$P_b = m \times v$ $\quad$ $r_b = 2$

$L_{id} - L_{ib} = L_{fd} - L_{fb}$ $\quad$ $m_b = 20$

$L_{ib} = r_b \times P_b$ $\quad$ $L_{id} = 10000$

$$L_{id} - L_{ib} = 0$$

$$L_{id} - (r_b \times m_b \times v_b) = 0$$

$$v_b = \frac{-L_{id}}{r_b \times m_b}$$

$$v_b = \frac{10000}{2 \times 20}$$

$$v_b = 250 \, m/s$$

8. Solve for energy needed to shoot the wax ball
    a. Used the equation $KE = \frac{1}{2} \times m \times v^2$ with calculated values to solve for KE

8. Solve for Energy needed for gun

$$KE_g = \frac{mv^2}{2}$$

$v = 250$

$m = 20$

$$KE_g = \frac{20 \times (250)^2}{2}$$

$$KE_g = 625{,}000 \, J$$

Evaluation of Solution:

- Overall given our assumptions the solution our team came to is extremely accurate.
- We began by ensuring strict adherence to our established assumptions. The torque generated by the motor was calculated using the force exerted and the radius of the disk, leading to an angular acceleration consistent with the physical properties of the disk. Momentum conservation principles were applied correctly, assuming a closed system with no net external torque after the motor ceased. The omission of air resistance and

friction was justified by their negligible effect on the system's energy given its short time span and the scale of forces involved.

- However, the assumption of uniform heating of the wax ball warrants further examination, as real-world conditions may cause a temperature gradient, affecting the wax's adhesive properties upon contact.
- Initially, when solving the thermal energy required to heat up the wax, our team had to look up room temperature and its heat capacity. However, with this information our solution for the heat capacity is very accurate due to the equation Q = m*c*T
- Given the fact that we assumed Robin would not affect the mass or center mass of the disk our evaluation of the moment of inertia of the disk is accurate because of the equation I = (mr^2)/2. In addition to this all our calculations directly used the moment of inertia or used a number that was calculated from the moment of inertia.
- Also assuming no air resistance, the velocity of the ball needed to stop the disk from spinning would be the same as the initial velocity of the ball once it is launched out of the gadget.
- Assuming no friction or air resistance allows us to use the gadget at any point and have the same angular momentum of the disk.

Evaluation of Model:

- Our model is not realistic as it makes many assumptions that do not work in the real world.
- We assumed that robin did not affect the center of mass of the disk however in reality his mass would shift the center of mass of the disk and change the moment of inertia which would change all calculations after step 3
- We assumed that the wax was going to hit the disk perfectly perpendicular, but that is unrealistic as it would be very hard to hit it at the correct angle
- We assumed that the distance the wax would fall after being shot was negligible, however in reality it would fall a certain distance as it travels the 15m to hit the disk

Improvements to Model:

- The wax ball in our model does not lose height after being shot from the canon. To account for this, we could determine distance that the ball would fall vertically due to gravity
- Mass of Robin and mass added by the wax balls colliding and sticking to the disk were not accounted for. The added masses would change the total mass of the disk as well as the center of mass of the disk, this would change the radius that we would need to use for our equations used to solve for the moment of inertia. Accounting for the added masses would make the velocity slower and probably more realistic
- We're not accounting for any kind of force that would be applied to Batman as he fired the canon, in order to make our model more accurate we could find this force and see if it would affect his ability to balance on the later while firing the canon.
- Our model doesn't account for the force that Robin would experience from being spun and then stopped instantly, and whether or not this force is survivable for him.

- The model doesn't account for energy that would be lost due to friction or air resistance. Our model would be more realistic if we found a way to account for the lowered speed of the disk due to friction against the axle that it is spinning on, the disk also slow down in a non-linear fashion over time if friction and air resistance were accounted for