Will Legg

Week 5 review

$\boxed{Ex\ 4}$ $\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial s}$

$\frac{\partial z}{\partial x}$ $z = x^2y^3 \rightarrow 2xy^3$ $\frac{\partial x}{\partial s}$ $x = s\sin(5t+3)$ $\sin(5t)$

$\frac{\partial z}{\partial y} = 3x^2y^2$ $\frac{\partial y}{\partial s}$ $y = 2t^2 + 4s^2 = 8s$

$\frac{\partial z}{\partial s} = (2xy^3)(\sin(5t)) + (3x^2y^2)(8s)$

$@(s,t) = (1,0)$

$x(1,0) = 1 \cdot \sin(0) + 3 = 3$

$y(1,0) = 2 \cdot 0 + 4 \cdot 1 = 4$

$(x,y) = (3,4)$

$\frac{\partial z}{\partial s} = 2xy^3(\sin(0)) + 3 \cdot (3)^2 \cdot (4)^3 \cdot 8(1) \boxed{= 3456}$

Week 5 Review

Ex 1 $A = \frac{1}{2}ab\sin(C)$

$$dA = \frac{\partial A}{\partial a}da + \frac{\partial A}{\partial b}db + \frac{\partial A}{\partial C}dC$$

$$= \left(\frac{1}{2}b\sin C\right)da + \left(\frac{1}{2}a\sin C\right) + \left(\frac{1}{2}ab\cos(C)\right)dC$$

$$\Delta A \approx \left(\frac{1}{2}(100)\sin\left(\frac{\pi}{3}\right)\right)(0.5) + \left(\frac{1}{2}\left(150\sin\frac{\pi}{3}\right)\right)0.5 + \left(\frac{1}{2}(150)(100)\cos\frac{\pi}{3}\right)\left(\frac{\pi}{90}\right)$$

$$\frac{\Delta A}{\frac{1}{2}(150)(100)\sin\left(\frac{\pi}{3}\right)} \approx \boxed{2.849\%}$$

Ex 3 find $\frac{\partial z}{\partial x}$ if $e^z = xyz$

$$F(x,y,z) = e^z - xyz = 0 \rightarrow \boxed{\frac{\partial z}{\partial x} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}}}$$

$$dF = \frac{\partial F}{\partial z}\cdot\frac{\partial z}{\partial x} + \frac{\partial F}{\partial x}\frac{\partial x}{\partial x}$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$$

$F_x = -yz$

$F_z = e^z - xy$

$$= \boxed{\frac{yz}{e^z - xy}}$$

Week 5 Review

$\boxed{Ex\,2}$ ellipsoid : $x^2+2y^2+3z^2=1$

$x^2+(\sqrt{2}\,y)^2+(\sqrt{3}\,z)^2=1$ standard 4 $\boxed{x^2+y^2+z^2=1}$

$$\begin{cases} x=\sin\phi\cos\theta \\ y=\frac{1}{\sqrt{2}}\sin\theta\sin\phi \\ z=\frac{1}{3}\cos\phi \end{cases}$$

while $x \le 0$
$\theta \in (\frac{\pi}{2}, \frac{3\pi}{2})$
$\cos\theta \le 0$
$\cos\theta$ when $\theta \in [\frac{\pi}{2}, \frac{3\pi}{2}]$

$x = \sin\phi\cos\theta$
$\sqrt{2}y = \sin\phi\sin\theta$
$\sqrt{3}z = \cos\phi$

$\vec{r}(\phi,\theta) =$

① $\vec{r}_\phi$

② $\vec{r}_\phi \times \vec{r}_\theta \; (\frac{1}{\sqrt{6}}, \frac{1}{2}, \frac{1}{3}) =$ normal vector

$=(\frac{1}{3\sqrt{6}}, \frac{1}{3}, \frac{1}{3})$

tan plane $= \boxed{\frac{1}{3\sqrt{6}}(x-\frac{1}{\sqrt{6}})+\frac{1}{3}(y-\frac{1}{2})+\frac{1}{3}(z-\frac{1}{3})}$