Week 8 Review

Ex2 find crit pts of $f(x,y) = 4 + x^3 + y^3 - 3xy$ and classify

$\nabla f = 0$

$f_x = 3x^2 - 3y = 0 \rightarrow y = x^2$

$f_y = 3y^2 - 3x = 0$

$x^4 - x = 0 \rightarrow x(x^3 - 1) = 0 \nearrow x = 0$ or $x = 1$

| $x = 0$ | $x = 1$ |
|---|---|
| $y = (0)^2$ | $y = 1^2$ |
| $y = 0$ | $y = 1$ |
| $(0,0)$ | $(1,1)$ |

$f(0,0) = 4$

$f(1,1) = 3$

$D = f_{xx} f_{yy} - f_{xy}^2$

$f_{xx} = 6x$

$f_{yy} = 6y$

$f_{xy} = -3$

$D(0,0) = 6(0) \cdot 6(0) - (-3)^2 = -9 < 0 \rightarrow \boxed{(0,0) = \text{saddle pt}}$

$D(1,1) = 25 > 0 \rightarrow \boxed{(1,1) \text{ is local min}}$

Ex 1 closed : boundary included
($a \le x \le b$)

| $x, y$ : | closed | bounded |
|---|---|---|
| $0 < x$ | F | F |
| $0 \le x$ | T | |
| $0 \le x \le 1$ | T | F (no y) - y unbound |
| $0 < x < 1$ | F | F |
| $0 < x < 1,\ 0 < y < 1$ | F | T |
| $0 \le x \le 1,\ y \le 1$ | T | T |

bounded

$-N \le R \le N$

$0 < x \le 1$
↳ not bound

bound

$0 \le x \le 1$

$2 \le x \le 3$

↳ can find sufficiently large # (real #) to bound region
must have lower bound

Ex 3 Eval $\int_C x\,ds$ where C is part of curve $y = x^2 + 1$ from (0,1) to (3,10)

$$\int_0^3 x\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_0^3 x\sqrt{1+(2x)^2}\,dx$$

$ds = \|r'(x)\| = \sqrt{1+(2x)^2}$

↳ $\frac{1}{8}\cdot\int \sqrt{u}\,du = \frac{1}{8}\cdot\frac{2}{3}u^{3/2}\Big|_0^3$

$$\boxed{= \frac{1}{12}(37)^{3/2} - \frac{1}{12}}$$

Scanned with CamScanner

Ex 4 Eval $\int_C \langle x, 0\rangle$ where c is part of $x^2+1=y$ from $(0,1)$ & $(3,10)$

$\vec{r}(x) = \langle x, x^2+1\rangle \quad 0 \le x \le 3$

$\int_C \langle x, 0\rangle \cdot \vec{r}' \, dx$

$\vec{r}'(x) = \langle 1, 2x\rangle$

$\langle x, 0\rangle \cdot \vec{r}' = x$

$$= \int_0^3 x \, dx = \frac{1}{2}x^2 \Big|_0^3 \boxed{= \frac{9}{2}}$$

Ex 5 find work done by vec field $F(x,y) = (2x+y^2, 2xy)$ around circle $x^2+y^2=4$ in counter clockwise

Work $= \int_C \vec{F} \cdot d\vec{r}$

$\vec{F} = \langle P, Q\rangle$

if $P_y = Q_x$ then $\vec{F}$ is conservative

Property $\boxed{\int_C \vec{F} \cdot d\vec{r} = 0 \text{ if } \vec{F} \text{ is conservative}}$

a loop if conservative

$\boxed{P_y = Q_x = 2y}$

alternate

$C: \vec{r}(t) = \langle 2\cos t, 2\sin t\rangle$

$\vec{r}'(t) = \langle -2\sin t, 2\cos t\rangle$

$0 \le t \le 2\pi$, counterclockwise

$$\int_0^{2\pi} \langle 4\cos t + (4\sin t)^2, 2(2\cos t)(2\sin t)\rangle \cdot \langle -2\sin t, 2\cos t\rangle$$

$= 0$