1/25/24 Will Legg

4 Week 2. Review

2) $u = \langle 1, 1, 1 \rangle$ & $v = \langle 2, 1, 5 \rangle$

a) find $w = u \times v$, write eq for plane containing pt $(1, 2, 3)$ and whose normal vec is w, (a, b, c) = normal

a) $\vec{u} \times \vec{v}$ $\begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 2 & 1 & 5 \end{vmatrix} = \langle 4, -3, -1 \rangle = \vec{w}$

$$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$$

$$\boxed{\text{Plane: } 4(x-1) - 3(y-2) - 1(z-3) = 0}$$

$$\hat{w} = \frac{w}{|w|} = \frac{(4,-3,-1)}{\sqrt{4^2+(-3)^2+(-1)^2}} = \quad a = \frac{4}{\sqrt{26}} \quad b = \frac{-3}{\sqrt{26}}$$

$$c = \frac{-1}{\sqrt{26}}$$

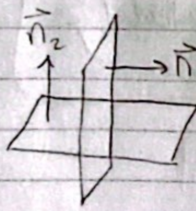
$$a+b+c = \boxed{\pm\left(\frac{4-3-1}{\sqrt{26}}\right)}$$

b) find eq of plane through line of the intersection of the planes $x - z = 1$ and $y + 2z = 3$ and perpend. to plane $x + y - 2z = 1$

$\vec{n} \perp \langle 1, 1, -2 \rangle$

$\vec{n} = \boxed{\langle \text{intersect} \rangle} \times \langle 1, 1, -2 \rangle$

$= \langle 1, 0, -1 \rangle \times \langle 0, 1, 2 \rangle$



normal vec of $x + y - 2z = -1$

$$\boxed{\vec{n} = \langle 3, 3, 3 \rangle}$$

$$\hat{n} = \pm\frac{\vec{n}}{|\vec{n}|} = \frac{\langle 3,3,3 \rangle}{\sqrt{3^2+3^2+3^2}} = \frac{3}{\sqrt{27}}$$

$$= 3 \cdot \frac{3}{\sqrt{27}} = \frac{9}{\sqrt{27}} \boxed{= 1.732}$$

Week 2 Review

Ex4) find eq of plane P_1 that passes through $A(2,1,1)$, $B(-1,-1,0)$, $C(1,3,4)$

a) $\vec{n} = \overline{AB} \times \overline{AC}$ $AB = \langle -3, -2, -1 \rangle$

$AC = \langle -1, 2, -5 \rangle$

$\vec{n} = \langle 12, -14, -8 \rangle$

eq of plane $= 12(x-2) - 14(y-1) - 8(z-1) = 0$

b) equation of plane through pt $D(2,0,4)$ and normal $v = (2,-4,-3)$

b) $2(x-2) - 4(y) - 3(z-4) = 0$

c) angle between two vectors

$$\theta = \cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}\right) = \cos^{-1}\left(\frac{24 + (4 \times 14) + 24}{\sqrt{12^2+14^2+8^2} \cdot \sqrt{2^2+4^2+3^2}}\right)$$

d) find parametric eq of the line of intersect of two planes

$\vec{r}(t) = \langle at + x_0, bt + y_0, ct + z_0 \rangle$

① Intersection $\vec{v} = \vec{n}_1 \times \vec{n}_2$

$= \langle 12, -14, -8 \rangle \times \langle 2, -4, -3 \rangle$

② find one pt on plane

$$\begin{cases} 12(x-2) - 14(y-1) - 8(z-1) = 0 \\ 2(x-2) - 4y - 3(z-4) = 0 \end{cases} = \begin{cases} 12x - 14y - 8z = 2 \\ \cdot(-6)(2x - 4y - 3z = -8) \end{cases}$$

$= -12x + 24y + 18z = 48$

$10y + 10z = 50$

$y + z = 5$

$@\, y = 0 \rightarrow \boxed{z = 5}$

plug into eq $\rightarrow x = \frac{42}{12}$

$$\boxed{\langle -5t + \tfrac{7}{2}, -10t, 10t + 5 \rangle}$$

Week 2 Review

5| find dist d_1 from origin to line $r(t) = (1+t, 2-t, -1+2t)$.

a| dist from $(1, 2, -1)$ to $(0,0,0)$

$$d_1 = \frac{|\vec{OP} \times v|}{|v|} = \frac{|\sqrt{6} \cdot \langle 1, 2, -1 \rangle|}{\| \langle 1, -1, 2 \rangle \|}$$

$$\vec{OP} = \sqrt{(1-0)^2 + (2-0)^2 + (-1-0)^2} = \sqrt{6}$$

$$d_1 = \frac{|1(\sqrt{6}) + 2(\sqrt{6}) + (-1)(\sqrt{6})|}{\sqrt{6}} = \frac{\sqrt{6} + 2\sqrt{6} - \sqrt{6}}{\sqrt{6}} =$$

$$= \boxed{d_1 = 2}$$

b| find dist d_2 between planes $3x + y - 4z = 2$ and $3x + y - 4z = 24$

$x = y = 0 \rightarrow -4z = 24 \rightarrow z = -6$ @ $(0, 0, -6)$

use $\frac{|ax + by + cz + d|}{\sqrt{ax^2 + b^2 + c^2}}$

$$d_2 = \frac{|3(0) + 1(0) - 4(-6) - 2|}{\sqrt{3^2 + 1^2 + (-4)^2}} \boxed{= \frac{22}{\sqrt{26}}}$$

c| whats $d_1 + d_2$

$$2 + \frac{22}{\sqrt{26}} = \boxed{\frac{26 + 11\sqrt{26}}{13}}$$