$S_n = \sup_{x \neq 0} \frac{\|B^n x\|}{\|x\|} = \|B\|^n < \varepsilon$

$\Downarrow$

$(\|B\|^{-1})^n > 1/\varepsilon$

$n \geq n_0(\varepsilon) = \left[\ln(\varepsilon^{-1}) / \ln(\|B\|^{-1})\right] + 1$

$\|x^n - x^{n-1}\| < \varepsilon$

МПЙ $\frac{q^n}{1-q} < \varepsilon \Rightarrow n_0(\varepsilon)$

$q|a_{ii}| > \sum |a_{ij}|$, $i = \overline{1, n}$

$x = Bx + c \qquad (1)$

$\|B\| \leq q < 1$

4. Методом прогонки розв'язати СЛАР $A\vec{y} = \vec{b}$, де

$$A = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{pmatrix}, \ \vec{y} = \begin{pmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix}, \ \vec{b} = \begin{pmatrix} 1 \\ -2 \\ -3 \\ -4 \\ 3 \end{pmatrix}.$$

$y_i = \alpha_{i+1} y_{i+1} + \beta_{i+1}, \quad i = \overline{n-1, 0}$

$\alpha_{i+1} = \frac{b_i}{z_i} = \frac{b_i}{c_i - a_i \alpha_i}$

$\beta_{i+1} = \frac{f_i + a_i \beta_i}{z_i}$

$\alpha_1 = \frac{b_0}{c_0} \qquad \beta_1 = \frac{f_0}{c_0}$

$|a_{ii}| \geq \sum |a_{ij}|, \ \forall > (V)$

$\alpha_1 = \frac{-1}{-1} = 1 \qquad \beta_1 = \frac{-1}{-1} = 1$

$\alpha_2 = \frac{b_1}{c_1 - a_1 \alpha_1} = \frac{-1}{\underbrace{-2-(-1)\cdot 1}} = 1, \ z_1 = -1$

$\beta_2 = \frac{2 + (-1)1}{-1} = -1$

$z_2 = c_2 - a_2 \alpha_2 = -2 - (-1) \cdot 1 = -1$

$\alpha_3 = \frac{-1}{-1} = 1, \quad \beta_3 = \frac{3 + (-1)^2}{-1} = -4$

$z_3 = c_3 - a_3 \alpha_3 = -2 - (-1)1 = -1$

$\alpha_4 = \frac{-1}{-1} = 1, \quad \beta_4 = \frac{4 + (-1)(-4)}{-1} = -8$

$z_4 = c_4 - a_4 \alpha_4 = -2 - (-1)1 = -1$

$y_n = \frac{f_n + \alpha_n \beta_n}{z_n}$

$y_4 = \frac{-3 - 8}{-1} = 11$

$y_3 = 1 \cdot 11 + (-8) = 3$

$y_2 = 3 + (-4) = -1$

$y_1 = -1 \cdot 1 + (-1) = -2$

$y_0 = -2 \cdot 1 + 1 = -1$

$\det(A) = -c_0(-z_1) \ldots (-z_4) =$

$= -1 \cdot (+1)^4 = -1$

$A x_j = e_j,\ j=\overline{1,n}$

$A^{-1} = \{x_j\}_{j=\overline{1,n}} \qquad E = \{e_j\}$

$x_1 \qquad A x_1 = e_1 \qquad AA^{-1} = E$

$x_2 \qquad A x_2 = e_2$

$\vdots$ 8h - 2

$\nu(A) = \|A\| \cdot \|A^{-1}\|$

748

$\begin{pmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 1/3 & 1/4 \\ 1/3 & 1/4 & 1/5 \end{pmatrix}$ $\quad 1+\frac{1}{3}+\frac{1}{2} = \frac{11}{6}$

$\frac{1}{2}+\frac{1}{3}+\frac{1}{5} = \frac{13}{12}$

$\frac{1}{3}+\frac{1}{4}+\frac{1}{5} = \frac{47}{60}$

$\mathrm{cond}(A) = \|A\| \cdot \|A^{-1}\| = \frac{11}{6} \cdot 408 = \underline{748}$

$\|A\|_\infty = \max_{j=\overline{1,n}} \sum_{k=1}^{n} |a_{jk}|$

$\|A\|_\infty = \frac{11}{6}$

$$\left(\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{3} & 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & 0 & 1 & 0 \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{3} & 1 & 0 & 0 \\ 0 & \frac{1}{12} & \frac{1}{12} & -\frac{1}{2} & 1 & 0 \\ 0 & \frac{1}{12} & \frac{4}{45} & -\frac{1}{3} & 0 & 1 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{3} & 1 & 0 & 0 \\ 0 & \frac{1}{12} & \frac{1}{12} & -\frac{1}{2} & 1 & 0 \\ 0 & 0 & \frac{1}{180} & \frac{1}{6} & -1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{3} & 1 & 0 & 0 \\ 0 & 1 & 1 & -6 & 12 & 0 \\ 0 & 0 & 1 & 30 & -180 & 180 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & \frac{1}{2} & 0 & -9 & 60 & -60 \\ 0 & 1 & 0 & -36 & 192 & -180 \\ 0 & 0 & 1 & 30 & -180 & 180 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 9 & -36 & 30 \\ 0 & 1 & 0 & -36 & 192 & -180 \\ 0 & 0 & 1 & 30 & -180 & 180 \end{array}\right)$$

75

408

390

$\|A^{-1}\|_\infty = 408$

$\frac{5}{108} \qquad -\frac{9}{80}$

$\frac{1}{15} + \frac{2}{9 \cdot 3} - \frac{2}{27} - \frac{1}{16} - \frac{1}{20}$

$-\frac{1}{4}\left(\frac{1}{4}+\frac{1}{5}\right) = -\frac{9}{80}$

$\frac{1}{3}\left(\frac{1}{4}-\frac{1}{9}\right) = \frac{5}{36} \cdot \frac{1}{3} = \frac{5}{108}$

$\frac{1}{5}\left(\frac{1}{3}-\frac{9}{16}\right) = \frac{-11}{5 \cdot 48}$

$\boxed{\text{Дз: ст. 40: № 8, 9}}$

$3 \cdot q > 2 \qquad q > \frac{2}{3}$

$2 \cdot q > \frac{3}{2} \qquad q > \frac{3}{4}$

$3 \cdot q > \frac{3}{2} \qquad q > \frac{1}{2}$