State transition matrix is about graph traversal. Consider this graph

$$
\begin{tikzcd}
& A \arrow[dl] & \\
B \arrow[rr] & & C \arrow[ul]
\end{tikzcd}
$$

We can represent this w/ matrix

$$
\begin{array}{c}
\\ A \\ B \\ C
\end{array}
\begin{array}{c}
\begin{array}{ccc} A & B & C \end{array} \\
\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}
\end{array}
$$

so if we start at B, we can encode our current position as $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

Apply the matrix

$$
\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}
$$

now @ node C

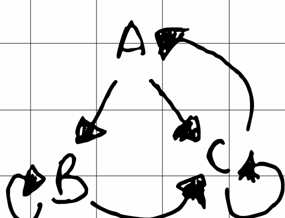
note we can keep track of multiple positions simultaneously.

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

@ nodes B and C

@ nodes A and C

Now consider the more complicated graph where every node has 2 children




$$\begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$\begin{array}{c|ccc} & A & B & C \\ \hline A & 0 & 0 & 1 \\ B & 1 & 1 & 0 \\ C & 1 & 1 & 1 \end{array} \qquad \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

start @ A

end @ B & C

We can apply the transformation again

$$\begin{array}{c|ccc} & A & B & C \\ \hline A & 0 & 0 & 1 \\ B & 1 & 1 & 0 \\ C & 1 & 1 & 1 \end{array} \qquad \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

after each transforms we double the number of nodes.

We could also create a matrix for the number of nodes occupied after 2 moves directly

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

In general, if $A$ is our inital state transition matrix, the $A^k$ will be a transition matrix that shows the number of each node occupied after $k$ moves.