CSSS 2016 Pollinator Network Project

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Abstract

1 Generalized Model, applied to plant-pollinator network

Investors in a market aim to maximize their profits and to minimize losses. This optimization problem can be naturally recast using a Lagrangian. Terms in the Lagrangian represent possible interactions between dynamical variables, and we include terms based on the behavior we expect to see in the derived equations of motion.

Let's see if there could be an analogy with a plant-pollinator network. We now consider a number of bee species each with an available number of visits E_i(t), to distribute among a set of flowers μ. Each flower has a finite resource, nectar, which determines the total number of bees that populate the flower at a given time, $n_\mu(t)$. We assume that a high n_μ implies a large number of pollinators. Note that in the case of a market, a low stock price does not imply a large amount of investment–prices are set by supply and demand, and price formation is dual to an integer programming problem. In this ecological context, it seems reasonable to assume that pollinators (demand curve) respond linearly to the supply curve (nectar availability)–that is, they populate a flower with available nectar, and don't exhibit behavior analogous to a trader rejecting low price assets due to concerns about exposure to unsafe liabilities. Basically, I'm saying that at this level of approximation, all nectar is equivalently delicious, and the bee's aren't tiny little divas that are choosy about where they eat.

$A_{i\mu}(t)$ is the adjacency matrix of this bipartite network, with rows indexing species and columns indexing plants. Each entry in this matrix is a weight that counts the number of vists from species i to plant μ. The Lagrangian that we wrote down will be used to derive equations that describe the time evolution of our dynamical variables, E_i(t), $A_{i\mu}(t)$, and $n_\mu(t) = 1/p_\mu(t)$. In order for this to work, the Lagrangian must include terms that couple these variables and their derivatives.

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Table 1: Notation

symbol	denotes
$n_\mu(t)$	Number of bee's at flower μ at time $t \approx$ Nectar available in flower μ at time t
$1/n_\mu(t) = p_\mu(t)$	Analogous to inverse price; *assumption: low price attracts investors*
$E_i(t)$	Number of visits available to species i at t
$A_{i\mu}(t)$	Visit's made to flower μ by species i at t

(Include details of Lagrangian formulation, and equations of motion)

Ingredients:

The matrices and vectors we need are

- The column vector $E_i(t)$
- The column vector $\delta E_i = E_i(t) - E_i(t-1)$
- The column vector $p_\mu(t)$
- The column vector $\delta p_\mu(t) = p_\mu(t) - p_\mu(t-1)$
- The matrix $A_{i\mu}(t)$
- The matrix $\delta A_{i\mu}(t) = A_{i\mu}(t) - A_{i\mu}(t-1)$

From our dynamical variables we can define,

$$B_{i\mu} \equiv \delta E_i A_{i\mu} p_\mu, \quad C_{i\mu} \equiv E_i \delta A_{i\mu} p_\mu, \quad D_{i\mu} \equiv E_i A_{i\mu} \delta p_\mu, \tag{1}$$

and via simple substitutions derive,

$$\begin{aligned} \sum_i D_{i\mu} &\approx \alpha \sum_i C_{i\mu}, & \sum_i C_{i\mu} &\approx \beta \sum_i B_{i\mu}, \\ \sum_\mu D_{i\mu} &\approx \alpha \sum_\mu C_{i\mu} & \sum_\mu C_{i\mu} &\approx \beta \sum_\mu B_{i\mu} \end{aligned} \tag{2}$$

Equal row and column sums are not enough to uniquely define matrix equivalence, but if the equations are to hold for random systems configurations, then one simple way to satisfy them is for

$$D_{i\mu} = \alpha C_{i\mu}, \qquad C_{i\mu} = \beta B_{i\mu}$$

. From these, we can simplify:

$$\frac{\delta p_\mu}{p_\mu} \approx \alpha \frac{\delta A_{i\mu}}{A_{i\mu}} \qquad \frac{\delta A_{i\mu}}{A_{i\mu}} \approx \beta \frac{\delta E_i}{E_{i\mu}} \tag{3}$$

The above two equations, along with $E = Ap$ are the simplest thing we can check to see if my model may apply (in its current form) to this data. Just using "populations" (of visits on the species side and bees on the flower side) seems not quite right, and mostly a test of self consistency and closedness; need some parametrization of the amount of nectar a bee can consume/amount of nectar in a flower.