The importance of measurement uncertainty in ecological management

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Abstract

Ecological management and decision-making typically focus on uncertainty about the future, but surprisingly little is known about how to account for uncertainty of the present: that is, the realities of having only partial or imperfect measurements. Our two primary paradigms for handling decisions under uncertainty – the precautionary principle and optimal control – have so far given entirely controdictory results. This paradox is best illustrated in the example of fisheries management, where many ideas that guide our thinking about ecological decision making were first developed. There, we find that simplistic optimal control approaches have repeatedly concluded that a manager should increase catch quotas when faced with greater uncertainty about the standing stock biomass. Meanwhile, most current best practices take a more precautionary approach, decreasing catch quotas by a fixed amount to account for uncertainty. Using comparisons to both simulated and historical catch data, we find that neither approach is sufficient to avoid stock collapses under moderate observational uncertainty. Using cutting-edge point-based partially observed Markov decision process (POMDP) methods, we demonstrate how this paradox arises from flaws in the standard management theory, which contributes to over-exploitation of fisheries and increased probability of economic and ecological collapse. In contrast, we find POMDP-based management avoids such over-exploitation while also generating higher economic value through adaptive learning. These results have significant implications for how we handle uncertainty in both fisheries and ecological management more generally. Dealing with uncertainty in decisions requires approaches which are both adaptive and explicit about uncertainty, such as POMDPs.
Key words: Measurement uncertainty, POMDP, Decision theory, Fisheries, Conservation,

Imperfect information is ubiquitous in ecological management and conservation decision making. While the pressing concerns of global change have put the spotlight on *forecasting*, i.e. the uncertainty of the future (e.g. Petchey et al. 2015), management decisions must also contend with uncertainty of the *present*: How many fish are in the sea today? What regions harbor the most fragile biodiversity? How have invasive species transformed native communities? A recognition of the importance of uncertainty as well as sophisticated methods and tools to do so has become increasingly widespread in the context of estimating ecological models from available data, thanks in large part to the rise of approaches such as Hierarchical Bayesian

Modeling, along with convenient software tools for implementing these methods. In sharp contrast to that shift, the treatment of uncertainty in decisions – how the uncertainty in models should be reflected in scientific recommendations, remains far less developed, more qualitative, and under-served by available tools. In this paper we illustrate how uncertainty in measurement has a dramatic negative impact on outcomes under a variety of these existing approaches, which can be avoided through the use of more formal approaches that have since been developed in other fields that can likewise be implemented in software tools. This insight both resolves a long-standing paradox in fisheries management and has important implications for how ecosystem management deals with uncertainty more generally.

Approaches for decision making under uncertainty in ecological systems can be divided into two camps: approaches based in "optimal control" and usually favored by natural resource economists, and approaches based in heuristic methods such as scenario planning, resilience thinking, and precautionary rules of thumb, more commonly found in both ecological literature and actual practice (Fischer et al. 2009; Polasky et al. 2011). While researchers have for some time recognized the need to unify the transparent and quantitative algorithmic approach of optimal control with the greater complexity and uncertainty of real ecosystems that is acknowledged by heuristic methods, computational barriers to doing so have hither-to stymied this progress. Here, we illustrate how the limitations of these approaches has been manifested in the example of fisheries management, and present a new approach that can combine the reality of measurement uncertainty and the rigor of optimization to resolve a long-standing paradox and suggest a more robust approach to management.

Fisheries conservation and management has long been both a crucible and proving ground for the theory of ecological management more generally, including topics such as adaptive management (Walters and Hilborn 1978), ecosystem-based management (Levin and Lubchenco 2008) and resilience thinking (Holling 1973; May 1977), while also giving rise to the sub-discipline of resource economics (Gordon and Press 1954; Schaefer 1954; Beverton and Holt 1957), thanks to its global relevance, long history, and readily available data. Consequently, understanding the impact of measurement uncertainty in this rich and well-studied context will have implications for both resource management and conservation efforts in other domains, many of which draw on concepts established or tested in fisheries ecosystems. In particular, the approach here illustrates the limitations of both over-simple optimal control approaches and rule-of-thumb precautionary approaches, and offers instead a more general strategy for dealing with measurement uncertainty adaptively.

We compare the results of managing simulated fish stocks under the existing optimal control approach, a heuristic approach that more closely resembles today's management practices, and a newly proposed approach that provides a more rigorous treatment of uncertainty.

The Decision Problem

We consider the management problem of setting catch quotas for a marine fishery in the face of imperfect information about the current stock size and uncertainty about future recruitment.

We seek to determine the sequence of actions h_t for $t \in [1, ...\infty]$ that maximize the net present value (discounted sum of all future profits) of the fishery. We will denote the discount rate γ. For simplicity we will once again follow classic theory and assume a fixed price for fish, (equivalently, measuring our value in units of discounted fish rather than discounted dollars), though again this assumption is easy to relax. Each year, the manager also obtains an estimate y_t of the true x_t stock size subject to some measurement uncertainty ξ_t

$$y_t = \xi_t x_t$$

For simplicity we will assume measurement uncertainty is normally distributed with standard deviation σ_m, $\xi \sim \mathcal{N}(1, \sigma_m)$. Given an estimated recruitment model along with this measurement of the stock size, the manager must choose the set of a harvest quotas $\{h_t\}$ for $t \in [0, 1, \infty]$ that maximizes expected long term (net) present value:

$$\max_{\{h_t\}} \mathbb{E} \left\{ \sum_{t=0}^{\infty} \gamma^t \cdot h_t \right\}$$

A few subtleties arise in this seemingly simple problem statement that we must address. First, given our discrete-time formulation of the recruitment process, it is necessary to decide if the measurement y_t happens before or after harvest h_t: that is do we: measure, recruit, harvest, or measure, harvest, recruit? Following convention in the optimal control literature (e.g. (Reed 1979; Clark 1990; Sethi et al. 2005)), we will assume the latter; measurement occurs immediately before harvest, followed by recruitment.

A second subtly is in the nature of the decision problem itself: note that the manager in not myopic, thinking only about the next year, but rather considers the sequence of all future actions h_t. While the manager will revisit this sequence each year in light of new observations, it is this ability of the decision problem to think ahead that allows it to tolerate short-term costs (e.g. reduced harvests in the current season) for larger future payoffs. In this, ecological management is a game of chess, always thinking several moves ahead. This differs from the problem solved in theory as well as the practice of maximum sustainable yield, which seeks to identify only a fixed mortality F_{MSY} applied for all time, rather than a *policy* of varying quotas h_t depending on the stock assessment y_t. Understanding this distinction is key both to understanding the differences between current theory and current best practice in fisheries management.

Population model

To facilitate tractability and interpretation, we will focus on the well-studied Gordon-Schaefer model (Gordon and Press 1954; Schaefer 1954) of logistic population growth (stock recruitment),

$$x_{t+1} = \varepsilon_t (x_t - h_t) r \left(1 - \frac{(x_t - h_t)}{K}\right)$$

where x_t is the current stock size, h_t the harvest chosen that year, with parameters r giving the individual growth rate, and K the carrying capacity and ε representing stochastic recruitment in a variable environment. We have written this in terms of $x_t - h_t$ to underscore that we assume the convention of observe, harvest, recruit. We will assume for simplicity $\varepsilon \sim \mathcal{N}(1, \sigma_g)$. This and similar models are widely used in large scale analyses across diverse stocks (e.g. (Costello et al. 2016; Britten et al. 2017)), and form the basis for much of bioeconomic theory (Clark 1990). As such, it will be easier to compare our results against intuition and classic theory, and avoid the possibility of differences arsing only because of some particular subtle assumption hidden in a more complex model.

It is important to remember that both existing methods we discuss and the POMDP approach proposed here can or are already applied to more complex models as well, including those with age or stage structure. Likewise, while we consider the model and it's parameters as "given," it should be understood to mean that this in practice comes as the result of some model choice and statistical estimation from historical data, frequently accounting for uncertainty in both the process and the measurements (e.g. Dichmont et al. 2016). As we shall see here, having included uncertainty in the model *estimation* in no way excuses us from having to also deal once again with that uncertainty in the decision process.

Current Theory and Practice

Gordon and Press (1954) and Schaefer (1954) independently showed in the same year that the maximum sustainable yield for the model bearing their names is achieved by reducing the stock to the population to the size at which it obtains the maximum growth rate which is known as Biomass at Maximum Sustainable Yield, B_{MSY}. For the Gordon-Schaefer model (and many others), this is achieved at $B_{MSY} = K/2$. They observed that fishing at a *constant yield* (that is, an individual fish mortality, F, such that harvest $H = F \cdot B$) of

$F_{MSY} := \frac{H_{MSY}}{B_{MSY}}$) will eventually lead to a population that converges to the biomass B_{MSY} and produces the maximum sustainable harvest, $H_{MSY} = rK/4$ in this model. From here, theory and practice diverge.

This *constant yield* (constant mortality) solution thus corresponds to an equilibrium analysis which does not solve the time-dependent optimization problem above. In particular, this maximum sustainable yield (MSY) strategy will not be optimal whenever the stock is away from B_{MSY}. Clark (1973) demonstrated (assuming no uncertainty in measurement or stochasticity in population dynamics) that the optimal time-dependent strategy is not one of constant yield, but rather of *constant escapement*. We will return to this approach in a moment.

Uncertainty in Practice: MSY & TAC

Maximum Sustainable Yield (MSY) remains the basis of international law (including the UN, IWC, IATTC, ICCAT, ICNAF; Mace (2001)) and a familiar standard of management in terrestrial ecosystems as well as aquatic (Clark 1990). Critics have for some time observed the limitations of harvesting at MSY in face of uncertainty in stock sizes and population dynamics (Larkin 1977; Botsford et al. 1997). Many US fisheries reflect this uncertainty through a series of adjustments that effectively reduce the target fishing mortality level to reflect this uncertainty. Typically, a stock assessment model provides a best estimate of B_{MSY} and corresponding mortality F_{MSY} is used to define the stock Over-fishing Limit, (OFL). Based on this, a somewhat lower level is set as the Allowable Biological Catch (ABC), reflecting uncertainty in the stock assessment. To reflect possible uncertainty between reported and actual catch, the ABC is reduced somewhat further to define the Total Allowable Catch, (TAC), which forms the basic unit of management for many such fisheries. To reflect this process, our analysis will also consider policies in which the harvest quota is set at 80% of the level expected under the MSY policy: $H_{TAC}(t) = 0.8 \cdot F_{MSY} \cdot B_t$, for a biomass estimated at $B(t)$ in year t. To distinguish this approach from MSY, we will refer to this approach as a TAC policy. This more closely represents a heuristic (e.g. Hilborn 2010) or resilience-based approach (sensu Fischer et al. 2009) than an optimization-based policy. Importantly, this approach shares the fundamentally stationary assumptions of an MSY policy by defining a constant mortality rather than a dynamic policy. Thus, despite being more cautious overall and generating lower economic yield than a constant escapement policy, TAC policies continue to harvest at non-zero rates even if a stock falls below B_{MSY}, while the constant escapement policy does not.

Optimal management

So far, attempts to provide a more formal basis for managing uncertainty than the heuristic adjustment of catch limits described above have largely foundered in a series of paradoxes. Without uncertainty, the theoretical policies are quite intuitive: The result derived by Clark (1973) for an optimal dynamic strategy to replace the equilibrium solution of MSY can be summarized as: If the stock is most productive at B_{MSY}, then obtain B_{MSY} as quickly as possible. Thus, at any level below B_{MSY}, the economically optimal thing to do is to completely shut down the fishery, while above this stock, harvests greater than H_{MSY} will be needed to bring the stock back to B_{MSY}. (This intuition must be adjusted slightly in the case of economic discounting of future profits, but is otherwise quite general, see Clark (1990)). This strategy is known as constant escapement, since a constant sock size B_{MSY} escapes harvest each year. While both converge to the same long-term biomass and long-term yield, these differences away from equilibrium between the mortality-based approach that dominates fisheries management practice and the escapement-based approach that became the focus of theory will result in different behaviors under uncertainty. Intuitively, any uncertainty, either in the population growth model (stochasticity), or measurement error, should seem to justify harvesting fewer fish, as is built into the TAC limits discussed above. Yet formal treatments of these issues have so far required approximations, and these approximations have driven paradoxical conclusions which we briefly summarize here.

Reed's Paradox: $S = D$

The first of these we shall refer to as Reed's Paradox for stochastic growth, owing to the mathematical proof provided in Reed (1979) which demonstrates that, under sufficiently general assumptions, the optimal escapement S for a population under stochastic growth, is identical to the optimal escapement D of a deterministic population, $S = D = B_{MSY}$. This surprising result suggests that in going from a world where a manager has perfect knowledge of absolutely everything into a scenario where the manager faces considerable uncertainty about the future state of the world, no additional precaution is needed. This also provides the manager with a remarkably convenient mechanism for determining the optimal harvest policy: rather than rely on computationally intensive Stochastic Dynamic Programming to determine the optimal policy, a simple derivative will suffice, which is all that is needed to solve the maximum of the deterministic problem.

Clark's Paradox

Clark and Kirkwood (1986) was among the first attempts to resolve Reed's Paradox. Clark and Kirkwood (1986) (quite correctly, as we will see), identified the crux of Reed's Paradox as the absence of measurement uncertainty:

> An important tacit assumption in Reed's analysis, as in the other works referred to above, is that the recruitment level X is known accurately prior to the harvest decision, [...] In the case of fishery resources, the stock level X is almost never known very accurately, owing to the difficulty of observing fish in their natural environment.

Unfortunately, Clark and Kirkwood (1986) was unable to solve the resulting problem exactly, but had to adopt an almost equally troublesome assumption:

> For reasons of tractability, we shall adopt the simplifying assumption that the escapement level S, is known exactly at the end of that period. (The mathematical difficulty of the problem increases markedly if this assumption is relaxed.)

Unfortunately for Clark and Kirkwood (1986), this "simplifying assumption" serves to squash most of the measurement error, and their results instead only deepened the paradox, finding policies that become even *less* cautious as uncertainty increases:

> [Our] results appear to contradict the conventional wisdom of renewable resource management, under which high uncertainty would call for increased caution in the setting of quotas.

Relying on a quite different but still flawed assumption nearly two decades later, Sethi et al. (2005) largely confirm Clark's Paradox, which they likewise observed with some concern:

> It may seem counter-intuitive that a measurement error causes lower expected escapements below the deterministic fishery closure threshold.

Despite these notes of caution, both Clark and Kirkwood (1986) and Sethi et al. (2005) ultimately attempt to rationalize this counter-intuitive conclusion rather than reject it. Others have also attempted to introduce measurement error, but always relying on various approximations that similarly alter the problem. The most common such assumption is to assume from the outset that the solution must be of 'constant-escapement' type (Ludwig and Walters 1981; Roughgarden and Smith 1996; Engen et al. 1997; Moxnes 2003). Here we will illustrate the use of powerful modern algorithms that will permit a more direct solution to the case of measurement uncertainty. In addition to resolving this paradox, we will demonstrate this direct solution also out-performs current more heuristic rules of thumb such as those currently used in fisheries management to account for uncertainty.

POMDPs: An optimal treatment of measurement uncertainty

As discussed above, absent the short-cut provided by Reed's theorems – solving for the optimal management policy in a stochastically fluctuating population – involves a Markov Decision Process (MDP), commonly referred to in ecological literature by its solution method, Stochastic Dynamic Programming (SDP). The constant escapement (CE) strategy considered here is the result of such an MDP or SDP analysis. Once measurement uncertainty is introduced, the problem becomes a Partially Observed Markov Decision Process (POMDP, not be confused with a partially observed or hidden Markov model, HMM, in which no decision process is involved). The POMDP problem is posed identically to that of the MDP problem, with the addition of an observation process, but cannot be solved using SDP directly. The POMDP problem for fisheries question considered here can be summarized as follows:

- Transition function (state equation): $T(x_t, x_{t+1}, a_t)$: the probability that a system is in state x_{t+1} at time $t+1$ given that it began in state x_t at time t and the manager took action a_t. In our context, this relationship is given by the Gordon-Schaefer stock recruitment function f with normally distributed growth uncertainty $x_{t+1} \sim \mathcal{N}(f(x_t, a_t), \sigma_g)$, truncated at zero to exclude negative population sizes.

- Observation function: $O(x_t, y_t, a_{t-1})$ the probability of observing state y_t given a system in state x_t. In principle, the action chosen can influence the precision of the observation. In our case, we simply assume normally distributed errors around the true state, $y_t \sim \mathcal{N}(x_t, \sigma_m)$, truncated at zero to exclude negative population sizes.

- Utility function: $U(x_t, a_t)$, the utility received at time t for taking action a_t, given that the system is in state x_t. For simplicity of analysis, we will simply set the utility to be equal to the harvested stock: $U(x_t, a_t) = \min(x_t, a_t)$, indicating that realized harvest cannot be negative. This choice ensures that in the case of no uncertainty ($\sigma_g = \sigma_m = 0$), the optimal solution matches that expected under a simple MSY calculation. More realistic utility functions may include diminishing returns with increasing harvest (supply and demand effects), and the cost of fishing, both of which act to suppress large harvests. By focusing on a simple utility we can be sure that our comparison to MSY is driven by the treatment of uncertainty rather than merely differing economic assumptions.

The optimization problem is to select the action a_t that will maximize the net present utility over all time. Future utility may be discounted by a factor γ, so that a value V in t year is valued at $\gamma^t V$ today. Numerically, each of these functions are defined over a discrete set of possible states, observations, and actions, and can thus be represented as a collection of matrices or tensors. While the solution method for

MDP problems, stochastic dynamic programming (SDP) is well known and readily implemented in ecological problems (e.g Mangel and Clark 1988; Marescot et al. 2013), the POMDP problem cannot be efficiently solved using SDP. While not unknown to the conservation literature (Williams 2011), algorithms for POMDP have historically scaled quite poorly, and their application has been restricted to contexts with only a handful of possible states and actions (e.g. Chadès et al. 2008, 2011; Fackler and Haight 2014; Fackler and Pacifici 2014). POMDP algorithms have a substantial literature starting almost contemporaneously to the work of Reed (1979) (see foundational papers by Smallwood and Sondik (1973) and Sondik (1978)) and remain an active area in artificial intelligence (e.g. Kaelbling et al. 1998; Pineau et al. 2003). By adapting recent algorithmic developments (Kurniawati et al. 2008) from that field we are able to find solutions for the considerably more complex problems such as the case of fisheries management, which requires on the order of 100 states and actions to provide sufficient numerical resolution. As the details for solving such problems numerically are already well documented we will not rehash them here, but instead provide a preliminary R package (Boettiger et al. 2018) which provides an implementation of the SARSOP algorithm and the fisheries POMDP problem, which can be used to replicate any of the results presented here and explore other variations. Details of the analysis presented here including annotated code to reproduce and further explore all of the following results is provided in the supplementary material.

Results

Figure 1 shows average fish biomass across 100 replicate simulations under three different management strategies: constant escapement (CE), total allowable catch (TAC; equal to 80% MSY), and partially observed Markov decision process (POMDP) management. Each successive panel shows a subsequently higher level of measurement error, from $\sigma_m = 0$, $\sigma_m = 0.1$, and $\sigma_m = 0.15$, as indicated. Standard deviation from the mean across replicate simulations is shown as faint colored bands, indicating significant variation due to stochasticity between individual replicates. In each panel shown in Figure 1 stochastic recruitment (environmental noise) is set to a moderate $\sigma_g = 0.15$. In the absence of either environmental noise or measurement error (Supplemental material, Figure S1) POMDP, CE, and MSY would converge to stock at the B_{MSY}, while TAC would maintain the stock at a slightly higher level. The first panel of Figure 1 shows that the the introduction of stochastic growth has a significant negative impact on the TAC strategy (This impact is even more severe for MSY, which always harvests strictly more than TAC by definition, as, shown in Supplemental material Figure S2), which results in an average biomass significantly lower than B_{MSY}. Without measurement error, the CE and POMDP strategies are nearly identical with both approximately

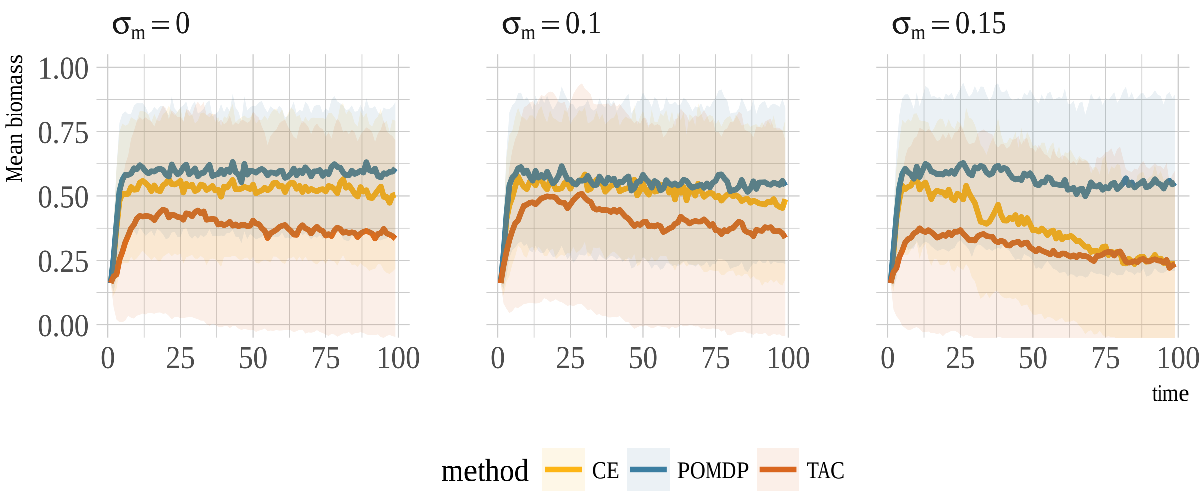

Figure 1: Average fish biomass under different management strategies under increasing levels of measurement uncertainty. Each plot the mean stock size over time across 100 replicate simulations under each policy: constant escapement (CE), Total allowable catch (TAC = 80% MSY), and the proposed partially observed Markov decision process (POMDP) method. Measurement error increases as a normal distribution with standard deviation 0, 0.1, or 0.15, as indicated at the top of the panel. Environmental stochasticity is fixed a standard deviation of 0.15 in each panel. Carrying capacity K normalized to 1, r = 0.75. Additional environmental noise levels and comparison to MSY rather than TAC can be found in the supplementary material.

maintaining the stock at B_{MSY} despite the significant environmental stochasticity. As measurement error increases in the subsequent panels, CE and TAC strategies perform increasingly poorly, while the POMDP continues to maintain the average stock close to B_{MSY}. Notably, CE is significantly more impacted by measurement uncertainty than TAC, with CE averaging even lower biomass than TAC under moderate measurement uncertainty.

Figure 1 confirms that the precautionary approach represented by the TAC does indeed prove more robust to the problem of measurement error than the optimization solution represented by CE. In general, we expect that the rigid assumptions required of optimization will lead to worse outcomes when those assumptions are not met than we see under the corresponding heuristic approach. Yet it is important to bear in mind that the opposite pattern was observed in the case of environmental stochasticity, in which it was the heuristic TAC strategy rather than the CE strategy that lead to the highest over-fishing rate. In contrast, the POMDP approach successfully handles both stochasticity and observation error, maintaining the stock near B_{MSY} levels that produce the highest yield.

This pattern in management success in ecological terms (the relatively recovery and maintenance of fish biomass) is also borne out in terms of economic performance. Figure 2 shows the mean net present value (averaging across replicates and discounting future profits by the discount factor γ) for these same simulations at increasing levels of measurement uncertainty. As before, environmental stochasticity is set at $\sigma_g = 0.15$. In the absence of measurement uncertainty, constant escapement (CE) is optimal (as per Reed

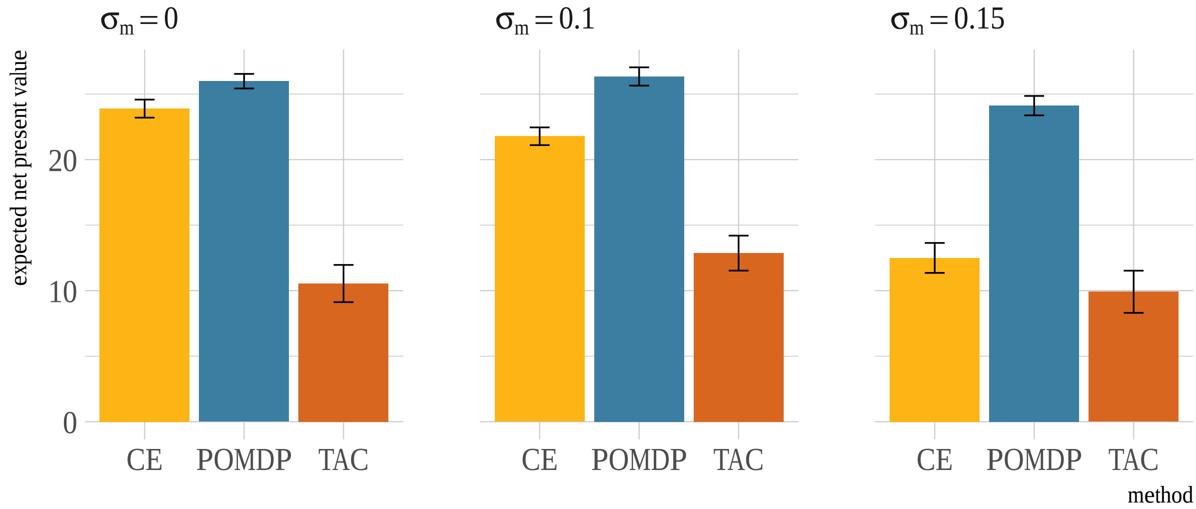

Figure 2: Expected net present value of the fishery across strategies under increasing levels of measurement error σ_m. Net present value is th sum of all future harvests discounted over time and averaged across 100 replicates simulations under each strategy, as shown in Figure 1. The value under constant escapement (CE) is optimal when measurements are perfect but decreases rapidly with increasing measurement error. The more cautious TAC is not economically optimal but largely unimpacted by increasing error, while POMDP attains consistently high economic yield despite the increasing uncertainty.

(1979)), while the heuristic caution built into the total allowable catch (TAC) strategy (at 80% MSY) results in a sub-optimal economic yield. The presence of stochasticity in recruitment ($\sigma_g = 0.15$), also contributes to the reduction in yield under TAC. Though CE is quite robust to the environmental stochasticity level, this strategy proves very sensitive to increasing measurement error, showing sharp declines in economic value, falling below the TAC economic value at $\sigma_m = 0.15$. Though the risk of stock collapse does increase with increasing measurement error under TAC (as seen by the mean declines in Figure 1), these have little impact on the economic value due to the discount rate. A smaller discount rate would penalize unlikely but not improbable collapses more, since a significant amount of time is required to realize those rare events. Supplemental Figure S3 summarizes these economic trends across different noise values σ_g and includes comparison to a simple MSY policy. In contrast to the declining economic performance of CE and the consistently sub-optimal economic yield for TAC, the POMDP strategy continues to generate economic yields at approximately the optimal level (as attained by CE in the absence of measurement error) despite the increasingly uncertain measurements. This demonstrates that the reduced risk of stock collapse and higher average stock biomass attained by the POMDP strategy in the presence of high measurement error, Figure 1) is not the result of a trivial reduction in harvesting under all circumstances, but rather, evidence of a more nuanced strategy that manages to account for the uncertainty in measurement while maintaining a reasonable harvest.

The POMDP strategy is also robust to overestimation of the level of measurement uncertainty. In the

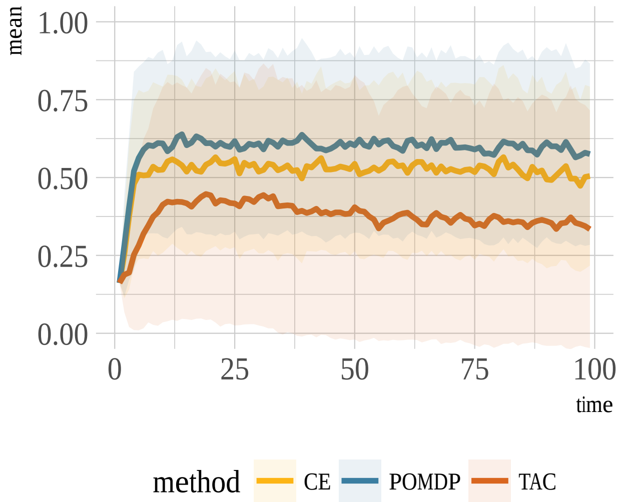

Figure 3: Average fish biomass when the POMDP strategy assumes a high level of measurement uncertainty, while simulations reflect perfect measurements. Despite this overestimation, biomass under POMDP strategy closely tracks the optimal biomass under CE in the absence of measurement error.

simulation results shown in Figures 1 and 2, we have assumed that the level of measurement uncertainty σ_m was known, and saw that ignoring this uncertainty (as the CE policy does) has significant negative impact on ecological and economic outcomes. If the level of measurement uncertainty is not known precisely, *overestimating* the measurement error while using the POMDP strategy provides a precautionary approach that can nevertheless achieve nearly optimal ecological and economic outcomes. Figure 3 summarizes the results of the same simulations as before, but under the scenario in which the POMDP approach assumes a measurement error of $\sigma_m = 0.15$, when in fact all simulated measurements are made without error ($\sigma_m = 0$). This represents an extreme case of overestimating the measurement error. Stochastic growth remains the same as before, $\sigma_g = 0.15$, and simulations under TAC and CE policies are shown for comparison. When measurement error is absent, Reed's proofs hold and the CE strategy is optimal. Figure 3 shows that the POMDP outcomes track almost exactly the CE outcomes despite the misplaced assumption that measurements are quite poor. (As we have already seen, the TAC strategy is insufficiently cautious for this level of stochasticity in recruitment, resulting in over-exploitation and long-term decline). The ability of the POMDP solution to perform nearly optimally even when significantly overestimating the level of measurement uncertainty contrasts sharply to the significant declines from ignoring measurement uncertainty seen in the CE solutions in Figure 1. This demonstrates that while we may not know precisely the level of measurement error, we achieve far better outcomes overestimating measurement uncertainty than underestimating it. This also underscores the observation that POMDP policy is quite robust to the details of the uncertainty. We can get a better understanding for the performance of POMDP in these simulations by looking more closely

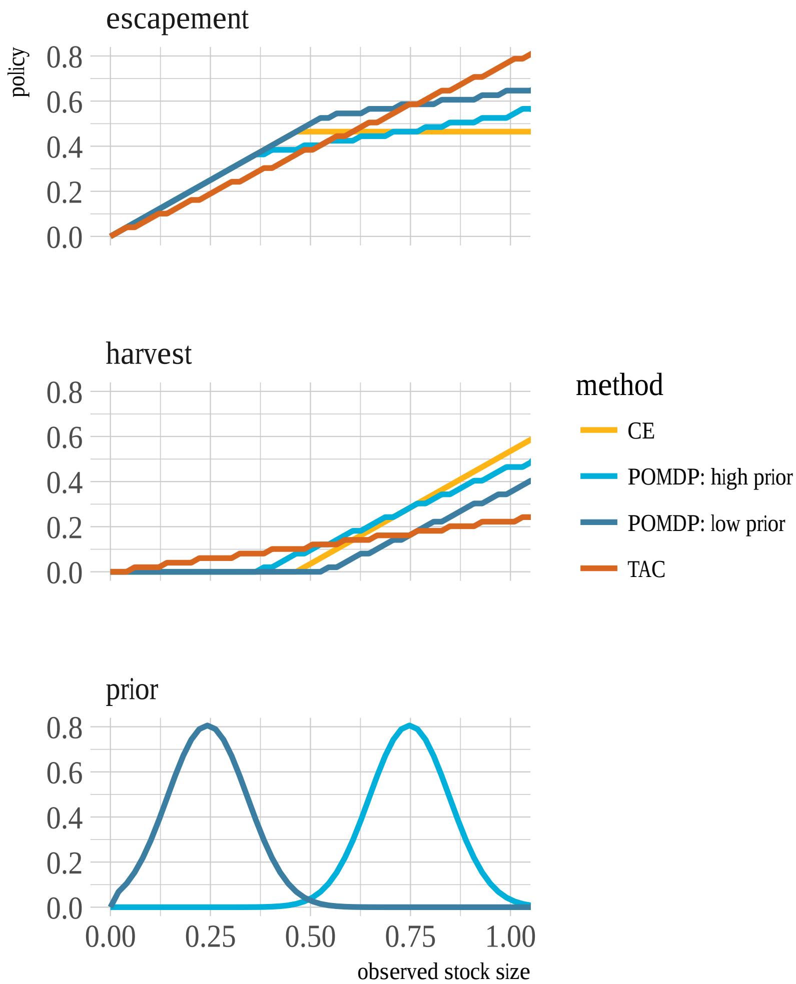

Figure 4: Comparison of the harvest and escapement policies under each strategy as a function of the observed stock size. Escapement refers to the expected fraction of fish left in the sea, $x - h$, while harvest refers to the target catch; two different conventions for plotting the same action. Uniquely, the POMDP policy plot will depend not only on the observed stock size, but also depends on prior information (bottom panel) as determined from any prior observations and actions. Depending on this prior it may harvest less or more than the other policies given an identical observation.

at how any individual decision under a POMDP strategy compares to the action chosen by the current alternatives.

To better understand the differences in performance of these strategies and resolve the paradox of uncertainty, we must take a closer look at how the specific action recommended by each policy compares given the same observation. Figure 4 shows the action taken by each strategy in response to a measurement of the stock size (biomass estimate). These plots show, for any possible observations, which strategy will attempt to harvest most and which will attempt to harvest least. For comparison purposes, we plot policies both in terms of expected escapement, $S = Y - H$ as is typical in the optimal control literature, (e.g. Reed 1979; Clark and Kirkwood 1986; Sethi et al. 2005), and also directly in terms of the harvest quota H. These plots clearly illustrate the contrast between the constant escapement (CE) strategy and the precautionary Total Allowable Catch (TAC) strategy: CE sets harvest strictly to zero for stocks estimated at biomass below B_{MSY}, while TAC permits a modest harvest of even very small stocks. (Comparable plots with MSY can be found in the Supplemental Material). In principle, the CE policy could depend on σ_g, but as Reed (1979) proved, the constant escapement level with stochasticity is the same as in the deterministic case, S = D, unless the noise level is quite high (comparison plots in Supplemental material.) In contrast to this, our POMDP plots depend very much on both the choice of σ_m and σ_g. If $\sigma_m = 0$, they reduce exactly to the CE solution. Here we show the corresponding policies for POMDP solutions focusing on $\sigma_g = 0.15$ as above, with a modest measurement uncertainty $\sigma_m = 0.1$. Alternate combinations of measurement uncertainty can be found in the appendix but do not change the general pattern. In addition to an explicit dependence on the measurement error, our POMDP solutions depend on another piece of information: any prior observations of the stock size.

Differences in attention to prior beliefs,determined by prior observations, drive the differences between the POMDP strategy and the other strategies and can resolve the uncertainty paradox. While the catch quota under both TAC and CE strategies can be completely determined given the most recent observation of the stock size by using the policy curves shown in Figure 4, this is not the case for the POMDP approach. This fundamental difference is key to understanding the difference in performance and resolving the paradox of uncertainty. The POMDP policy cannot be specified by the most recent observation alone. Instead, the POMDP policy depends on all prior observations, not just the most recent. The reason for this complexity comes from the Markov property. Observations of the state in the perfectly observed system satisfy the Markov property: once we have measured the current biomass exactly, we cannot get any better estimate of the current stock size by studying older measurements. When measurements are uncertain this is no longer the case: intuitively, by comparing the most recent measurement to previous observations we may be able to infer when any given measurement is unusually high or unusually low. POMDP formalizes this intuition by capturing the information from all previous observations into a *prior belief*. This prior belief is updated after every subsequent action and observation in accordance with Bayes Law. The mechanics of this process are well documented in the extensive literature on POMDPs (e.g. Smallwood and Sondik 1973; Sondik 1978; Kurniawati et al. 2008; Williams 2011), but for our purposes it is sufficient to observe how this evolving prior belief serves to continually adjust the POMDP policy. Figure S8 illustrates how the prior belief evolves in response to subsequent observations and actions over the course of an individual simulation.

Figure 4 shows two separate policy curves (in terms of harvest and escapement) for the same POMDP solution given two different prior belief distributions (panel 3, priors). While both priors shown express considerable uncertainty about the precise stock size prior to the most recent observation, the lower prior is centered at a value $\frac{1}{4}K$ stock size, while the high prior is centered at a value of $\frac{3}{4}K$, relative to a (post-harvest, before observation) target size of $B_{MSY} = K/2$. The POMDP policy is determined by the combination of this prior information and the most recent observation, as indicated by the two different POMDP curves for harvest/escapement shown corresponding to the different priors. As with the other policies, the higher the most recent observation (x axis) the higher the POMDP recommended harvest. Yet unlike the alternative strategies, the POMDP solution always reflects the prior information. Consequently, relative to constant escapement (that is, no measurement error), the POMDP with low prior starts harvesting only at higher stock sizes and always harvests less. In contrast under the high prior, the POMDP always harvests at the same or higher level than the constant escapement solution.

A resolution to the paradox.

Herein lies our resolution to the paradox of uncertainty. Previous work created this paradox by suggesting that increased harvest rates (decreased target escapement) would often be the rational response to increased uncertainty. The exact solutions from the POMDP reveal that this is only an accident of the assumptions: it is indeed true that under certain circumstances, harvest levels should increase relative to the case of no uncertainty, but *only when prior knowledge suggests the stock size should be much higher than the most recent estimate would suggest*. In the POMDP solution, all information must be put into its historical (and constantly updated) context. When a measurement roughly matches the expectation of this prior context, Figure 4 shows that the optimal response from POMDP is roughly comparable or slightly more cautious to the harvest under no uncertainty, and not more aggressive as the paradox would suggest. Measurements that exceed expectations are tempered with some skepticism: while the CE solution is willing to meet a high stock measurement with a large harvest, the POMDP solutions increase harvest more cautiously.

This difference between underestimating and overestimating accounts for the poor performance of the CE solution under large measurement uncertainty, where it over-harvests whenever measurements are too large. Even though underestimating is equally likely under the measurement uncertainty model, sooner or later a run of “heads”, a sequence of overestimations relative to the true stock, can drive stocks to very low levels where the chance of local extinction becomes possible. It is precisely this asymmetry: that too much over-harvesting leads to an irreversible state of extinction, while too much under-harvesting is always reversible (modulo some lost revenue) that lay behind Clark’s original intuition that there was something fishy about Reed’s result that $S = D$: that uncertainty required no extra caution. Constant escapement

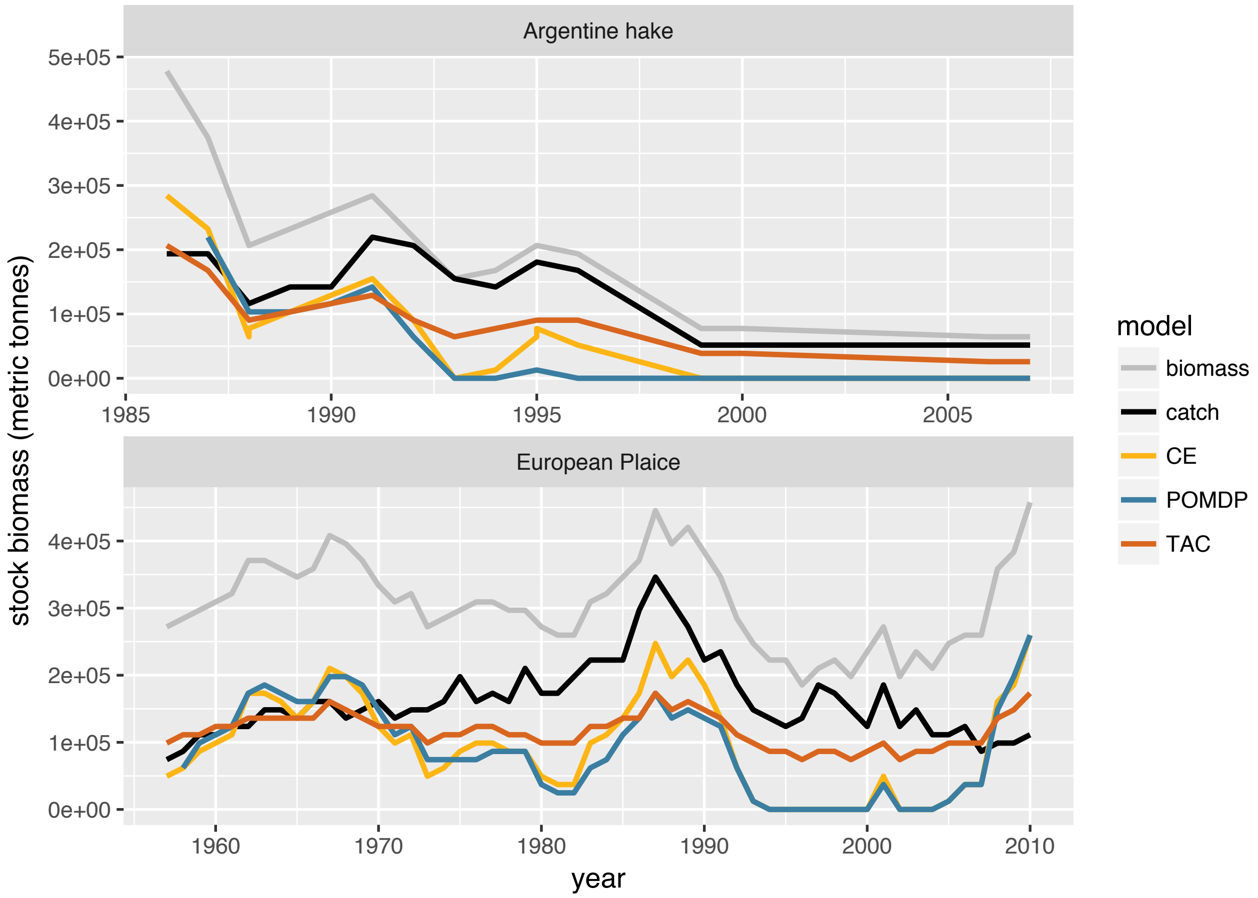

Figure 5: Comparisons of harvest level that would be recommended by policies considered here relative to historical harvest levels in two commercial fish stocks. POMDP solution assumes a measurement error of 10%.

is particularly susceptible to over-harvesting starting from stock sizes much higher than B_{MSY} since its bang-bang optimal solution attempts to bring stocks back to target level as fast as possible. CE is not so vulnerable to collapse from small stock sizes, since it shuts down all harvests once estimates fall below B_{MSY}. Stock collapse under TAC can also be driven by such a string of heads, but is unlikely at high stock sizes since harvest never exceeds H_{MSY}. In both cases, some measurement uncertainty interacts with inherent stochasticity, which provides a continued source of variation to sizes above B_{MSY}, where CE strategy is most vulnerable, and below B_{MSY}, where TAC is most vulnerable.

Historical examples and implementation.

So far we have focused on simulation and an examination of the relative policies under different prior information. Figure 5 compares the harvest level that would have been recommended by each of the strategies we have considered here against the historically observed catch recorded for two commercially fished stocks: Argentine Hake and European Plaice. Historical estimates of biomass and catch are taken from the R.A.

Myers Legacy Stock Assessment database (Ricard et al. 2011). Posterior distributions for parameters for the Gordon-Schaefer model are estimated with uncertainty through Markov Chain Monte Carlo using `nimble` (de Valpine et al. 2017) on the historical data, illustrating how this process might be done in more complex models as well (details and code in Supplement, Appendix B). B_{MSY} and corresponding TAC and CE policies are calculated based on posterior mean estimates of the model parameters, along with POMDP solution assuming a 10% measurement error rate. Though measurement error could be estimated directly from the raw data, this would not reflect the true measurement uncertainty arising from the stock assessment process. Each strategy is then compared to historical observations to determine a recommended harvest. As we have seen, the TAC and CE harvest policies are uniquely determined by the observed stock size relative to B_{MSY}, but the POMDP policy must be re-calculated each time step to reflect both the prior observation and prior action.

These historical examples provide another useful lens to compare how the different strategies respond in the face of fluctuations in real data, rather than model simulations. In both stocks, historical catch almost always exceeds that recommended by any of our strategies, falling closer to the MSY value estimated (see Appendix B). In Argentine Hake, the persistent declines result in both POMDP and CE strategies quickly closing down the fishery in an attempt to let the stock recover to a more productive target biomass, while TAC persists with merely a reduced harvest. A small recovery of biomass in 1995 is met by an immediate uptick in harvest under CE, while the POMDP response is more conservative. The example of European Plaice illustrates more volatility in the stock size, revealing further differences in the strategies. Despite this volatility, the TAC level remains relatively level across all five decades, rising or dipping only slightly with changes in stock size. In contrast, the CE harvest tracks this volatility almost exactly. Here, the POMDP solution falls in between these extremes, almost mirroring which-ever of the two policies is more conservative: the POMDP solution matches the dips in harvest taken by CE to allow the stock to recover most quickly, but does not track the doubling of harvests recommended by CE in the mid 1980, increasingly only to the more modest harvest recommended by the TAC policy. Once again, we see that POMDP solution provides a more consistently precautionary policy than either the over-simplified optimal control solution of CE or the more rule-of-thumb approach represented by TAC. Though we have seen that the POMDP solution need not always be more conservative (lower harvest) than these strategies, it makes use of the all prior observations to tune the level of caution appropriately.

Discussion

Using modern algorithms we have been able to crack the nut of measurement uncertainty in the optimal management of marine fisheries and debunk long-standing paradoxes that increased measurement uncertainty should be met with larger harvests. We have demonstrated that both existing optimal control approaches and existing precautionary approaches result in over-exploitation and long term declines of fish stocks, as well as reduced economic value, while in contrast, the POMDP approach is able to deliver both ecological and economic recovery despite uncertainty in measurements. These results have important implications for both global fisheries and ecological management more generally. In marine fisheries, evidence of such over-exploitation has already been well documented (Worm et al. 2006, 2009; Costello et al. 2016). Our results suggest that measurement uncertainty may contribute to this pattern, as such errors lead to over-exploitation under current best practices such as MSY, TAC or CE, which have previously been thought to be sufficient to ensure the recovery of stocks world-wide (Costello et al. 2016).

Our results also have important consequences for ecological management more generally. The study and application of management strategies such as MSY and CE underpin much of conservation and natural resource management in forest management and many other terrestrial ecosystems (e.g. Clark and Mangel 2000). Descision making under uncertainty is a central challenge for ecology and conservation biology (Polasky et al. 2011), which we have long attempted to address through either simplified optimal control models or more heuristic approaches such as the precautionary principle and resilience thinking (Fischer et al. 2009). We have illustrated how both of these approaches can fail to provide an adequate strategy when faced with measurement uncertainty. To avoid this pitfall, a strategy must be dynamic to make best use of the available information without assuming that information is perfect. The simplisitic optimal control approach of CE fails by placing too much confidence in measurements even when they are significantly at odds with prior expectations. Similarly, the TAC approach also fails due to it's reliance on a simple one-size-fits-all strategy, not reducing catch quotas sufficiently when stocks are too low but also failing to increase them when stocks are higher. It is precisely this ability to adapt the policy in response to available information, as illustrated in Figure 4, that makes the POMDP approach successful. Nevertheless, it may be tempting to see these results as either trivial or impractical, but a closer inspection reveals they are neither.

First, it may seem obvious that POMDP would out-preform more primitive optimal control solutions such as CE, but this was hardly a forgone conclusion. Indeed, the previous example of Reed (1979) had proven that the complex stochastic dynamic programming solutions required to accommodate stochastic dynamics would perform no better than the simple maximization based purely on the deterministic model. If stochasticity doesn't matter, it is easy to believe that measurement error would be minor at best. In fact, we find that once we account for measurement error, the effects of both stochasticity and measurement have a significant influence over the optimal policy. It is similarly tempting to dismiss the better performance of POMDP against a more rule-of-thumb[^1] based strategy like the TAC scenario considered here (fishing at 80% MSY), but once again this was not a forgone conclusion. While a better economic return might reasonably be expected of the optimization approach, it was not obvious that POMDP would also be at times more cautious than the rule of thumb, able to deliver both higher economic returns by harvesting more when it was safe to do so, and higher ecological recovery by harvesting even less than TAC when necessary. These results highlight the difficulty of finding effective heuristic rules and underscore the power and utility of more sophisticated strategies such as POMDP which can make the best use of all available information.

^1: Given the sophisticated policies often in place to determine actual TAC levels, one might object to our characterization of this as merely a rule-of-thumb. We use this term merely to distinguish it from the optimal control approach – that despite technical precision, TAC adjustments are set by a heuristic instead of the result of some dynamic optimization algorithm as in CE and POMDP.

Second, we must ask if the policies derived from POMDP are practical. As with constant escapement, the POMDP solutions we considered here result in more year-to-year variation in harvest, and include the possibility of fishing moratorium to allow stocks to recover – decisions that many view as economically or socially unacceptable. While resolving this debate is beyond the scope of this paper, a few observations are relevant: (A) Stocks already managed under constant escapement, such as US salmon fisheries, are already subject to the possibility of these moratoriums. (B) It is straight-forward to introduce constraints on the minimal acceptable harvest level and/or include the additional costs of closing a fishery in the utility function optimized by POMDP. (C) Fisheries today are not managed at either their maximum ecological or economic potential. Taken as a whole, most ocean fisheries yield a net negative economic return (World Bank and FAO 2009), with fishing persisting only with the support of government subsidies (Arnason 2012), meanwhile some 68% of global fisheries are in poor biological condition (Costello et al. 2016). While the causes for this are both complex and numerous, we have seen that both the economic inefficiency and ecological health suffer under TAC/MSY and CE based management. Arnason (2012) and Costello et al. (2016) have both argued that a rights-based approach to fisheries management (RBFM) would lead to self adoption of precisely such optimal policies.

Third: are the examples considered here too simple? The strategies we have compared against underpin both current practice and management aspirations. Together, MSY and TAC-based management reflects much of modern best practice as it is understood in fisheries management and elsewhere. While real-world application of these approaches frequently involves more complex models, the models we have considered here are the very ones under which these strategies were designed and tested, and there is little reason to believe they would perform better under more complex cases such as age-structured models that do not always meet those assumptions (e.g. Holden and Conrad 2015). The relatively poor ecological and economic performance of these common best-practices relative to the alternative approach of POMDP is thus reason enough for concern. This issue is further underscored by the reality of both long-running ecological deterioration (Worm et al. 2006, 2009) and economic costs of global fisheries.

Fourth: is the approach taken here too complex to be feasible? The importance of simple and effective rules of thumb in conservation management has been well documented (e.g. Chadès et al. 2011). Sophisticated and computationally intensive approaches such as POMDP may seem improbable at a time when many areas of natural resource management, even simple, rule-of-thumb based methods struggle to take root. And yet, in this same age we rely on far more complex and computationally intensive algorithms to perform far more trivial tasks such as determining what advertisements we see. Surely a planet that can afford such complex optimization for how advertising space is allocated on a small screen can one day manage it's natural resources and conservation efforts with such tools?

Conclusions and Future directions

Several limitations that have been studied in fully observed (MDP) optimal decision problems, such as parameter uncertainty (e.g. Ludwig and Walters 1982), model uncertainty (Williams 2001; Boettiger et al. 2015) and adaptive management (e.g. Walters and Hilborn 1976) remain largely open challenges for partially observed systems. Future work could extend this analysis to more complex models, such as those with age structure, as (Holden and Conrad 2015) does for the fully observed case. Another limiting assumption common to MDPs and POMDPs is that of stationary dynamics: that the population dynamics equation itself is not changing over time. In reality, forces such as climate change and other forms of environmental variations violate this assumption (Britten et al. 2017). Direct approaches such as Fackler and Pacifici (2014)'s adaptation of Mixed Observability MDP (Ong et al. 2010) do not scale to the number of states and actions considered here. A value of information (VOI) analysis for POMDP (e.g. Johnson and Williams 2015; Memarzadeh and Pozzi 2016), could identify when it is worthwhile to actively reduce measurement error.

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