

# STRUCTURAL ROBUSTNESS IN NETWORKS

CSSS Tutorial  
June 18, 2018  
Alice C. U. Schwarze

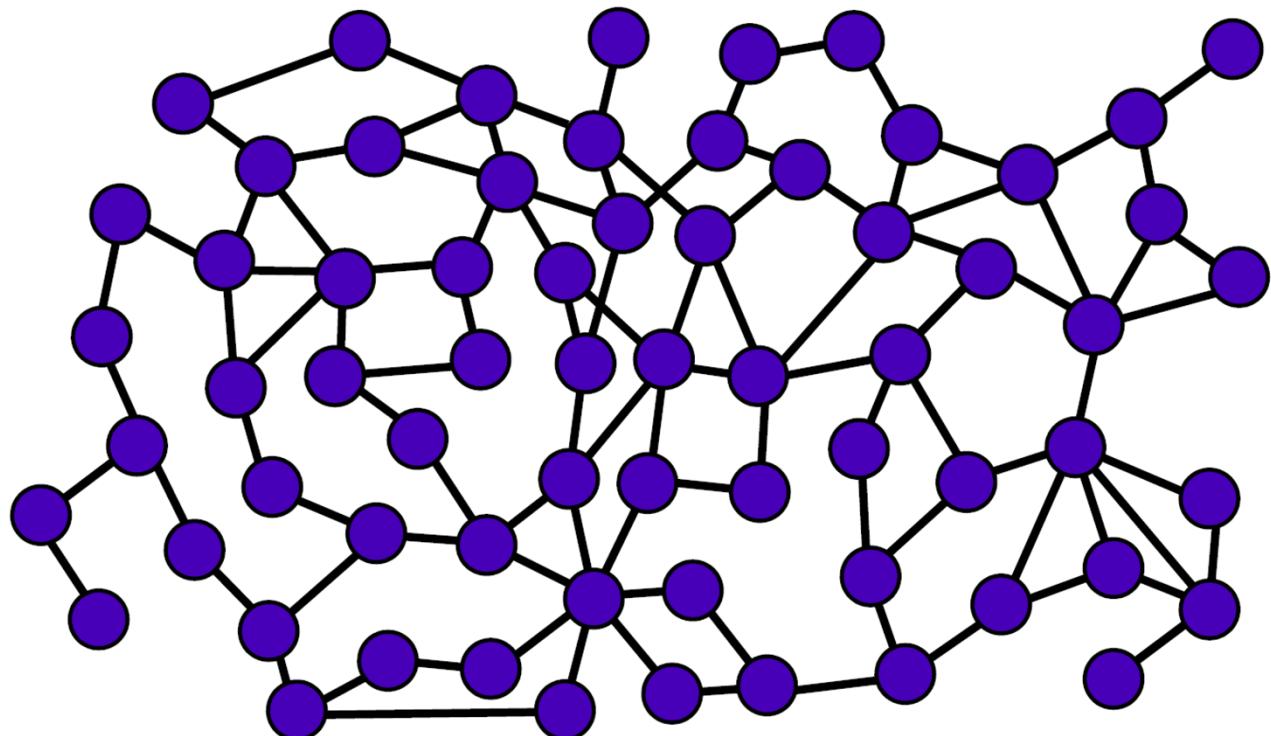
# WHY DO WE CARE ABOUT NETWORK STRUCTURE?

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Network = Reduction of complex system to a structure of pairwise interactions



# WHY DO WE CARE ABOUT NETWORK STRUCTURE?

- ❖ Limited ability to model robustness mathematically, computationally, experimentally
- ❖ Availability of structural data
- ❖ Expectation that network structure should affect robustness



# WHY DO WE CARE ABOUT ROBUSTNESS?

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ROBUSTNESS ... HOW THINGS REACT TO THINGS



# ROBUSTNESS, RESILIENCE, STABILITY, ...?

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Robustness = Insensitivity/Stability of a **system** **PROPERTY** under **perturbation**

**Example:** Biological networks

**system** = organism/cell

**PROPERTY** = viability,

**perturbation** = environmental changes or mutations

# ROBUSTNESS, RESILIENCE, STABILITY, . . . ?

We construct a frequency distribution,  $f(D)$ , of the values of  $D_k$  obtained from the whole map. The entropy is then defined as

$$S(D) = - \int f(D) \log[f(D)] dD \quad (5)$$

where the integral is taken over all values of  $D$ , that is, from 0 to  $2\pi$ . The use of  $D$ , rather than  $\phi$  itself, to define entropy is one way of accounting for the lack of translation invariance of  $\phi$ , a problem that was missed in previous attempts to quantify phase entropy<sup>16</sup>. A uniform distribution of  $D$  is a state of maximum entropy (minimum information), corresponding to gaussian initial conditions (random phases). This maximal value of  $S_{\max} = \log(2\pi)$  is a characteristic of gaussian fields. As the system evolves, it moves into states of greater information content (that is, lower entropy). The scaling of  $S$  with clustering growth displays interesting properties<sup>5</sup>, establishing an important link between the spatial pattern and the physical processes driving clustering growth. This phase information is a unique 'fingerprint' of gravitational instability, and it therefore also furnishes statistical tests of the presence of any initial non-gaussianity<sup>17–19</sup>. □

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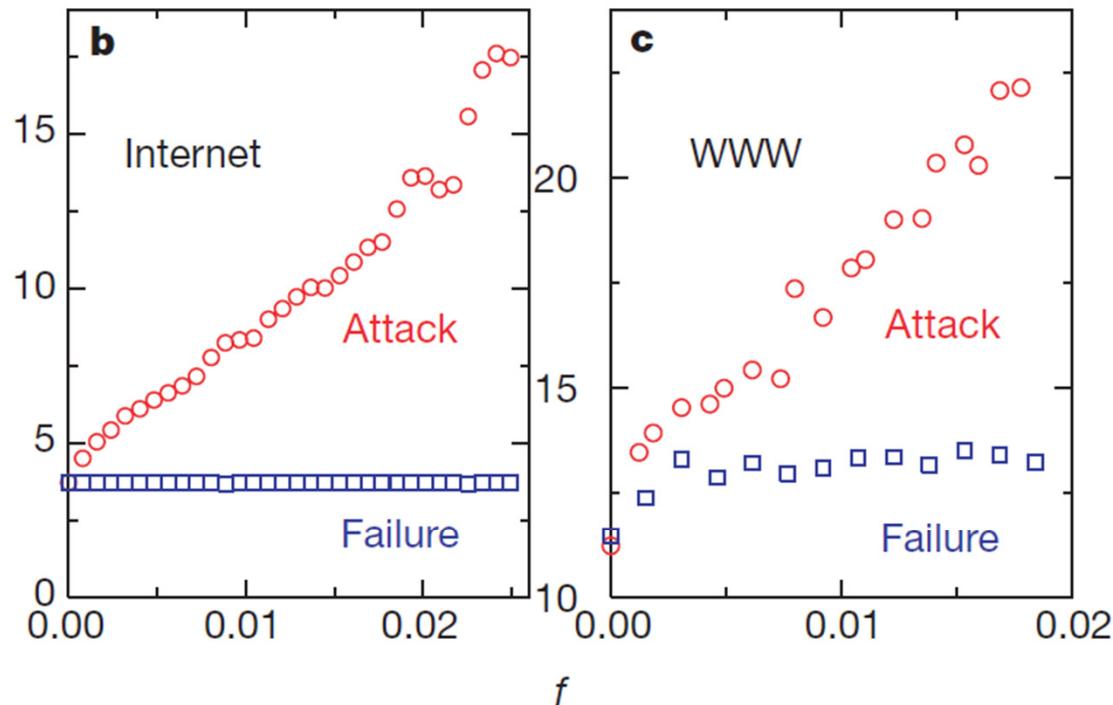
## Error and attack tolerance of complex networks

Réka Albert, Hawoong Jeong & Albert-László Barabási

Department of Physics, 225 Nieuwland Science Hall, University of Notre Dame, Notre Dame, Indiana 46556, USA

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Journal of Operations Management

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## Supply network disruption and resilience: A network structural perspective

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### ABSTRACT

Increasingly, scholars recognize the importance of understanding *supply network disruptions*. However, the literature still lacks a clear conceptualization of a network-level understanding of supply disruptions. Not having a network level understanding of supply disruptions prevents firms from fully mitigating the negative effects of a supply disruption. Graph theory helps to conceptualize a supply network and dif-

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Y. Kim et al. / Journal of Operations Management 33–34 (2015) 43–59

**Table 3**  
Network metrics and resilience for four basic supply network structures.

Network metrics	Block-diagonal	Scale-free	Centralized	Diagonal
Node/arcs	12/18	12/25	12/29	12/25
Network density	.14	.19	.22	.19
Average degree	1.50	2.08	2.42	2.08
Resilience	.11	.30	.16	.13
Walks	8	19	27	21
Average walk length	6.5	6.89	5.44	8.62
Max. walk length	9	11	7	13
Min. walk length	5	3	3	5
Connectivity	3	4	2	3
Betweenness centrality	1.33	1.67	.75	2.58
Centralization (%)	10.91	52.75	67.27	30.91

## Supply network resilience

$$\text{total number of node/arc disruptions, which does not result in a supply network disruption} = \frac{\text{total number of node/arc disruptions}}{\text{total number of node/arc disruptions}}$$

We exclude the source and sink node in this calculation because this would automatically lead to a disruption, and because our focus

The *diagonal* structure has the second most walks (21), but with an even lower resilience score (.13). The average, maximum, and minimum walk lengths of each structure also did not predict network resilience. On these metrics, the *diagonal* structure has the highest scores.

The network-level metrics of *betweenness centrality* and *centralization* also did not correlate with resilience. One might assume that the more often the nodes bridge other nodes in a network or the more concentrated the network connections around a few nodes in a network, the more resilient the supply network will be to random failures. However, *diagonal* structure scores highest on the betweenness centrality metric but second lowest on resilience. The *centralized* structure scores highest on centralization, but again is not the most resilient. The *connectivity* metric appears to be more predictive of network resilience when compared with the other metrics. The *scale-free* structure has the highest connectivity (4). However, the *centralized* structure has the lowest connectivity (2), albeit the second most resilient. Further, the *diagonal* and *block-diagonal* structures tie for the connectivity at 3, while the former is relatively more resilient. This metric cannot distinguish among the three less resilient network structures. We also examined other network archetypes (after structural adaptations to the supply net-

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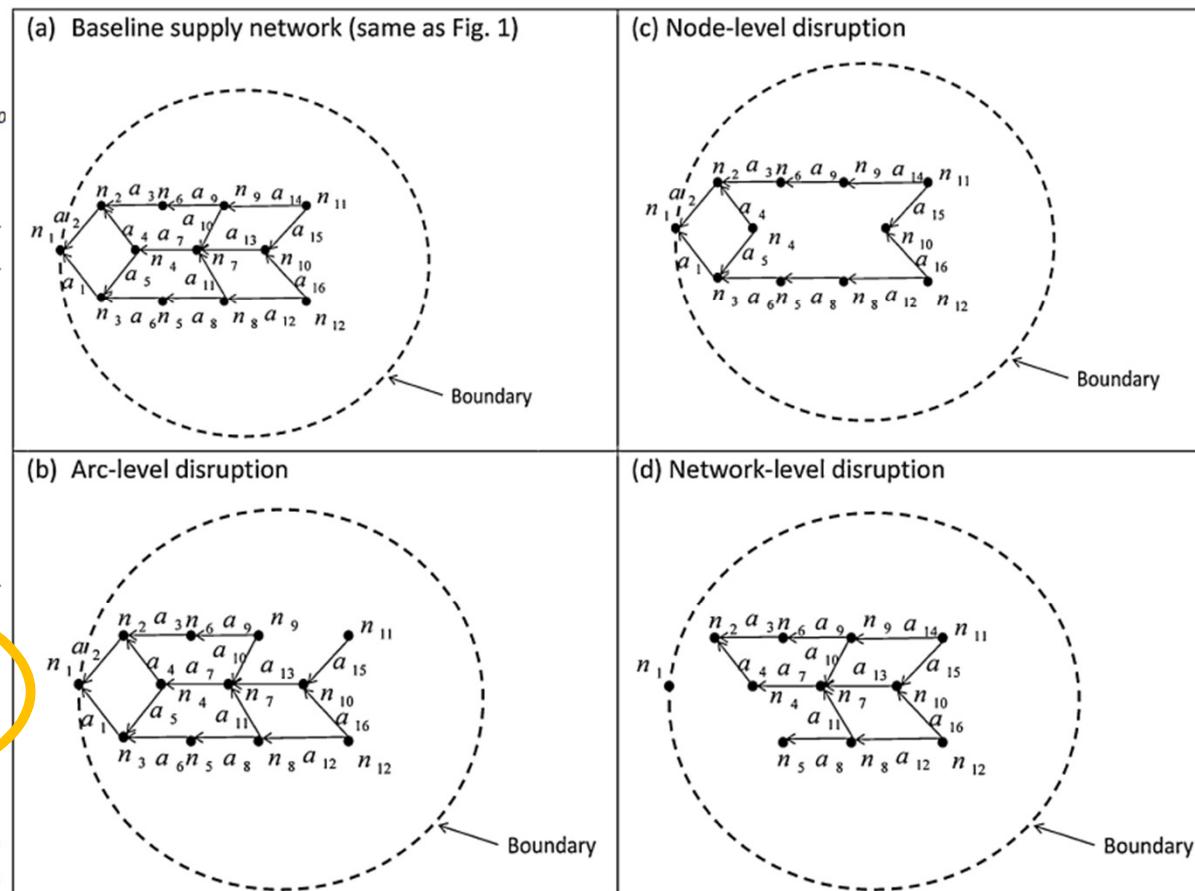
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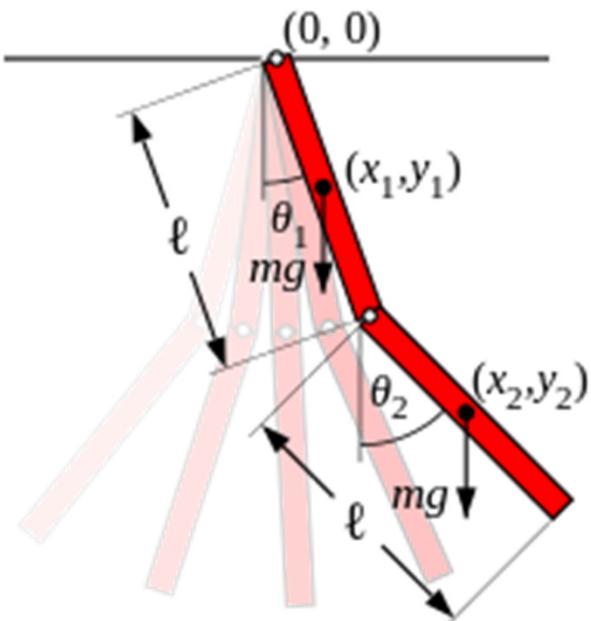
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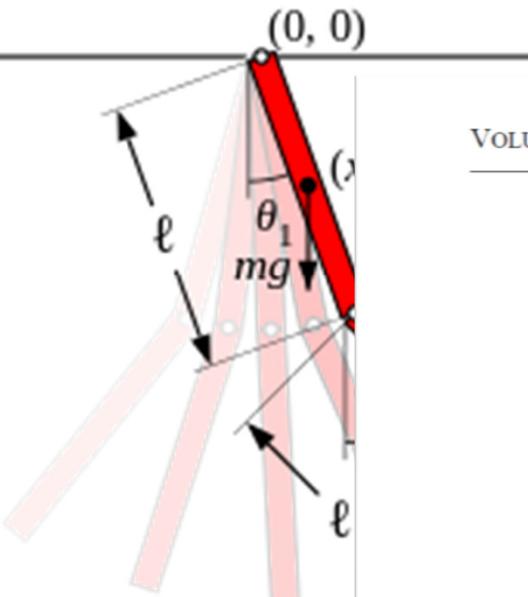
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VOLUME 80, NUMBER 10

PHYSICAL REVIEW LETTERS

9 MARCH 1998

## Master Stability Functions for Synchronized Coupled Systems

Louis M. Pecora and Thomas L. Carroll

Code 6343, Naval Research Laboratory, Washington, D.C. 20375

(Received 7 July 1997)

We show that many coupled oscillator array configurations considered in the literature can be put into a simple form so that determining the stability of the synchronous state can be done by a master stability function, which can be tailored to one's choice of stability requirement. This solves, once and for all, the problem of synchronous stability for any linear coupling of that oscillator. [S0031-9007(98)05387-3]

PACS numbers: 05.45.+b, 84.30.Ng

A particularly interesting form of dynamical behavior occurs in networks of coupled systems or oscillators when all of the subsystems behave in the same fashion; that is, they all do the same thing at the same time. Such behavior of a network simulates a continuous system that has a uniform movement, models neurons that synchronize and coupled synchronized lasers and

hyperplane in the phase space and number (2) makes the stability diagram specific to our choice of oscillators and the components. Number (4) is the choice of many studies of coupled systems since it is often a good approximation and can be considered prototypical.

In determining the stability of the synchronous state, various criteria are possible. The weakest is that the

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*Ecology Letters*, (2002) 5: 558–567

REPORT

## Network structure and biodiversity loss in food webs: robustness increases with connectance

### Abstract

Jennifer A. Dunne,<sup>1,2\*</sup> Richard J. Williams<sup>1</sup> and Neo D. Martinez<sup>1</sup>

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Santa Fe,  
NM 87501, USA

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Food-web structure mediates dramatic effects of biodiversity loss including secondary and ‘cascading’ extinctions. We studied these effects by simulating primary species loss in 16 food webs from terrestrial and aquatic ecosystems and measuring robustness in terms of the secondary extinctions that followed. As observed in other networks, food webs are more robust to random removal of species than to selective removal of species with the most trophic links to other species. More surprisingly, robustness increases with food-web connectance but appears independent of species richness and omnivory. In particular, food webs experience ‘rivet-like’ thresholds past which they display extreme sensitivity to removal of highly connected species. Higher connectance delays the onset of this threshold. Removing species with few trophic connections generally has little effect though there are several striking exceptions. These findings emphasize how the *number* of species removed affects ecosystems differently depending on the trophic functions of species removed.

# WHICH NETWORKS ARE ROBUST?

- ❖ Many edges

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## ECOLOGY LETTERS

*Ecology Letters*, (2015) 18: 144–152

doi: 10.1111/ele.12394

LETTER

### A simple stochastic model for complex coextinctions in mutualistic networks: robustness decreases with connectance

#### Abstract

Marcos Costa Vieira<sup>1\*</sup> and  
Mário Almeida-Neto<sup>2</sup>

Understanding and predicting species extinctions and coextinctions is a major goal of ecological research in the face of a biodiversity crisis. Typically, models based on network topology are used to simulate coextinctions in mutualistic networks. However, such topological models neglect two key biological features of species interactions: variation in the intrinsic dependence of species on the mutualism, and variation in the relative importance of each interacting partner. By incorporating both types of variation, we developed a stochastic coextinction model capable of simulating extinction cascades far more complex than those observed in previous topological models. Using a set of empirical mutualistic networks, we show that the traditional topological model may either underestimate or overestimate the number and likelihood of coextinctions, depending on the intrinsic dependence of species on the mutualism. More importantly, contrary to topological models, our stochastic model predicts extinction cascades to be more likely in highly connected mutualistic communities.

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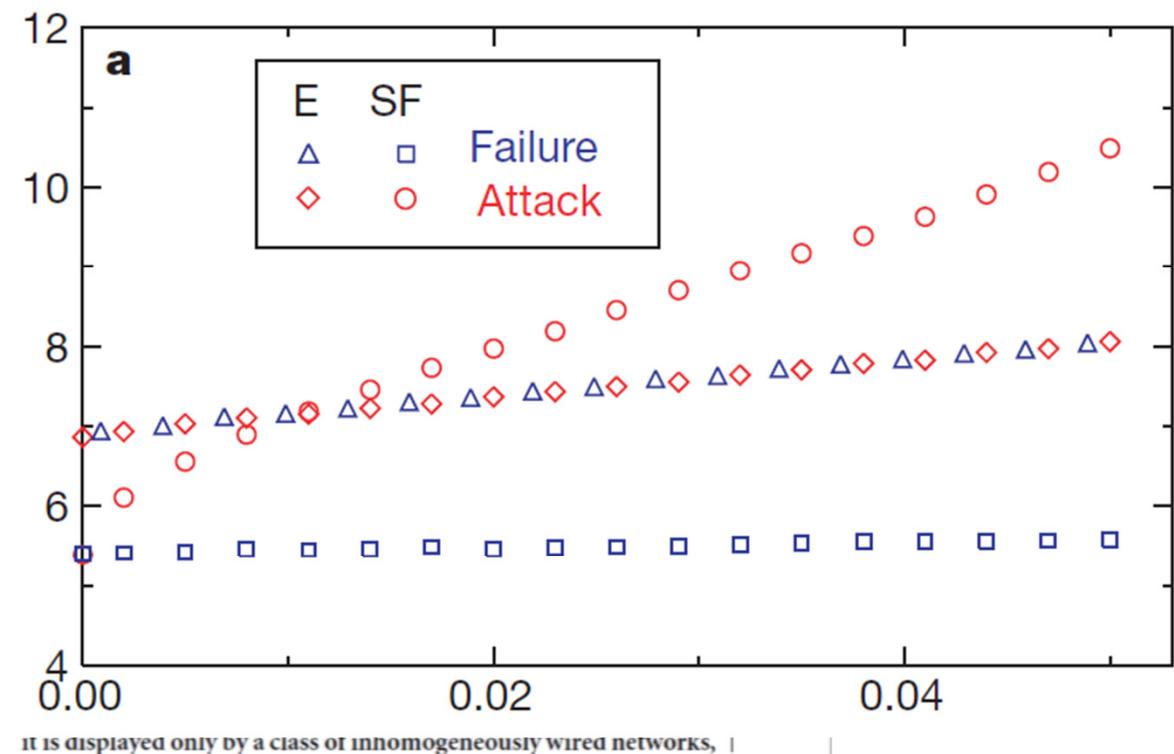
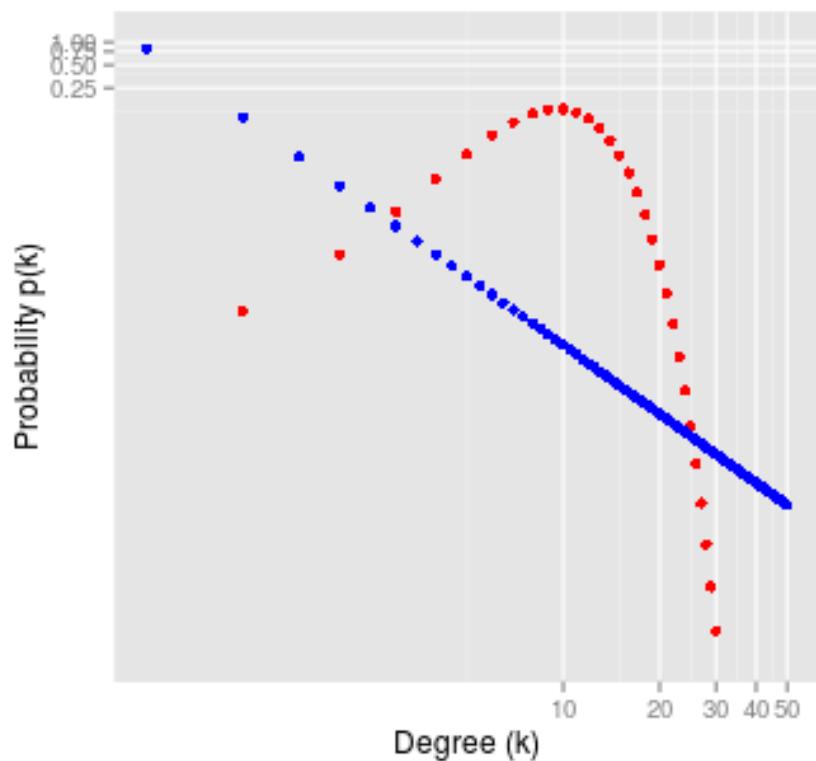
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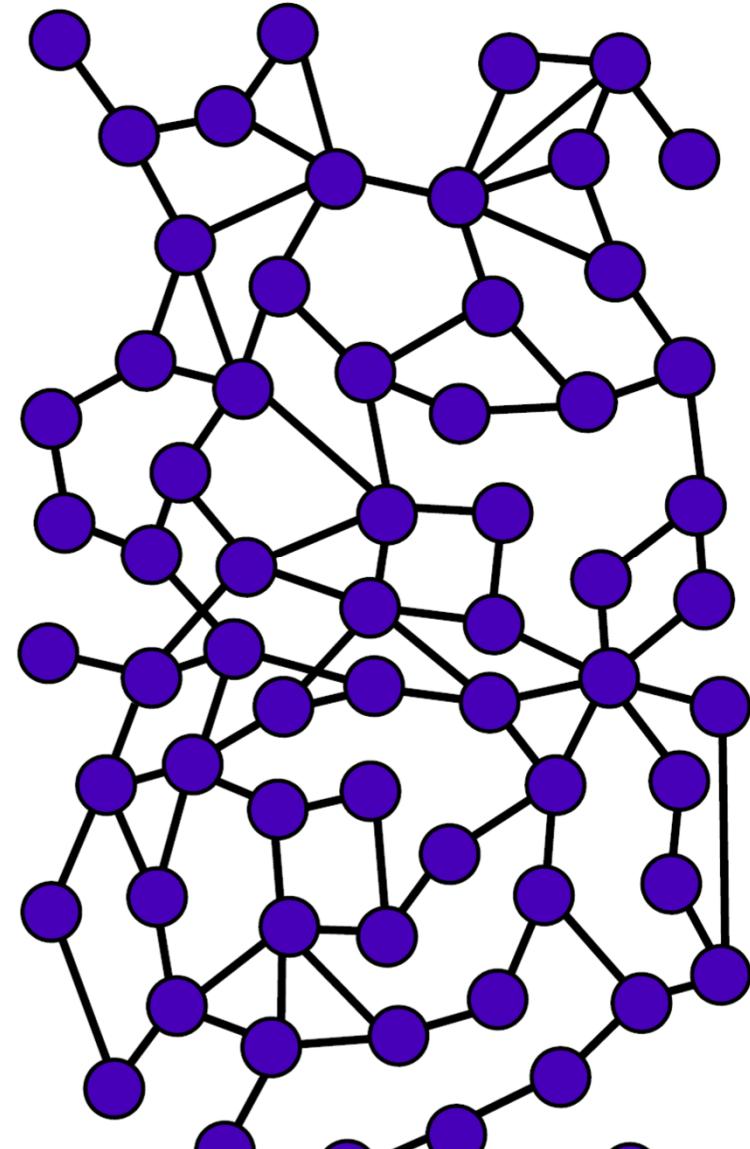


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- ❖ Heterogeneous degree distribution?

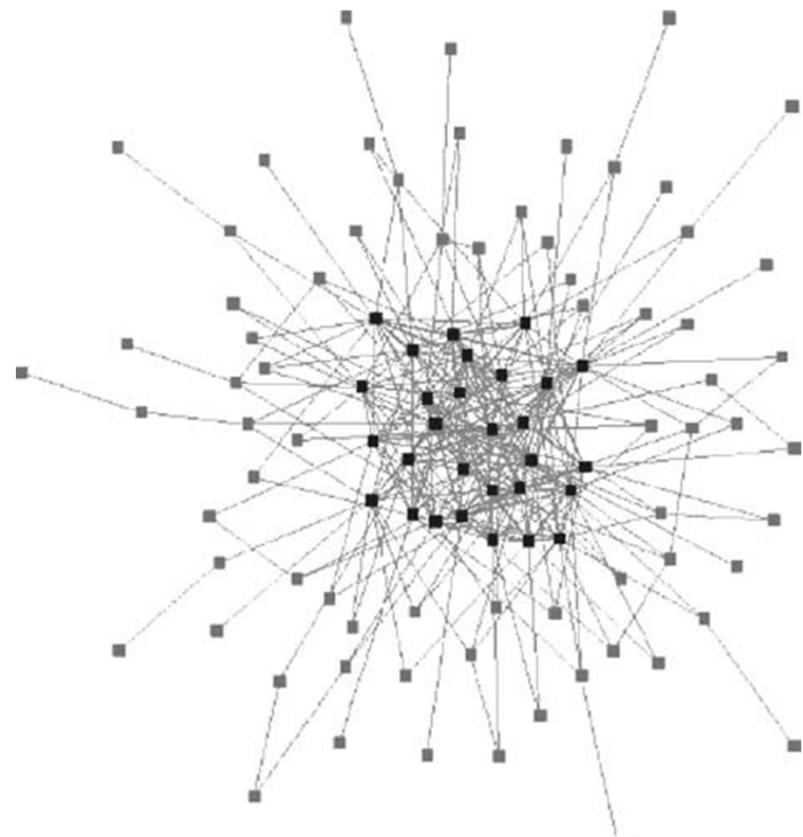
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- ❖ Core-periphery structure/ onion structure



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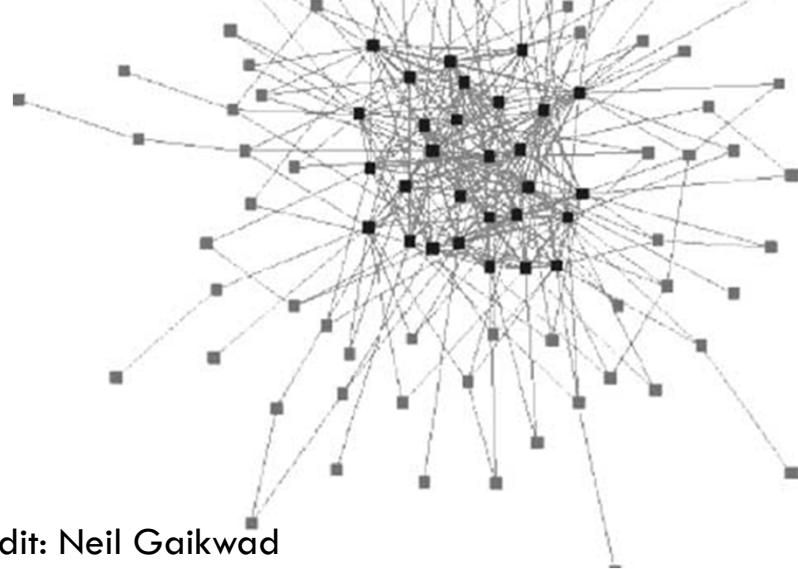


Photo Credit: Neil Gaikwad

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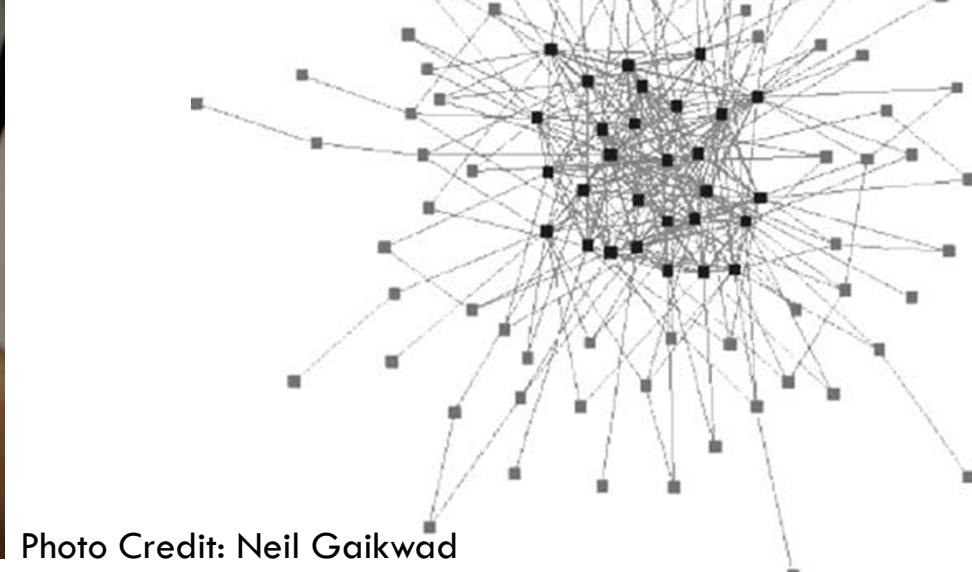


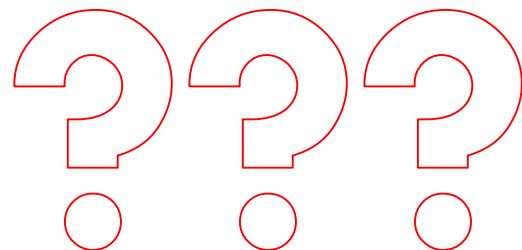
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- ❖ Redundancy
- ❖ ....

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**Example:** Biological networks

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# SPECIFYING THE NETWORK ROBUSTNESS PROBLEM

- ❖ What **network or data** do we have?
- ❖ What kind of **perturbation or abnormal event** do we have?
- ❖ How do we **measure network robustness**?
- ❖ (What is our **null model**?)

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DAMN!

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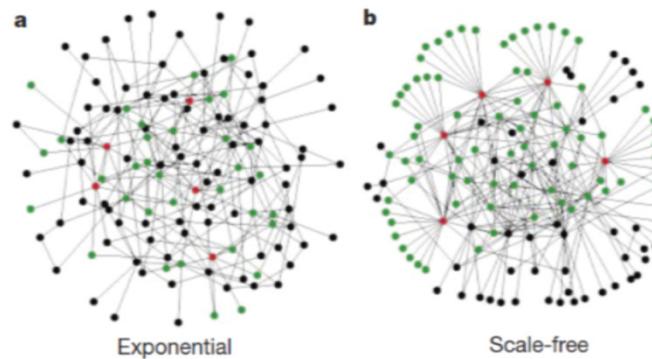
where the integral is taken over  $D$ . The use of  $D$ , rather than  $\phi$  is accounting for the lack of translation invariance that was missed in previous attempts. The uniform distribution of  $D$  is a measure of information (mutual information), corresponding to random phases. This may be characteristic of gaussian fields in states of greater informa- tion. The scaling of  $S$  with cluster properties<sup>5</sup>, establishing an universal pattern and the physical process phase information is a unique property, and it therefore also furnishes any initial non-gaussianity<sup>17–19</sup>.

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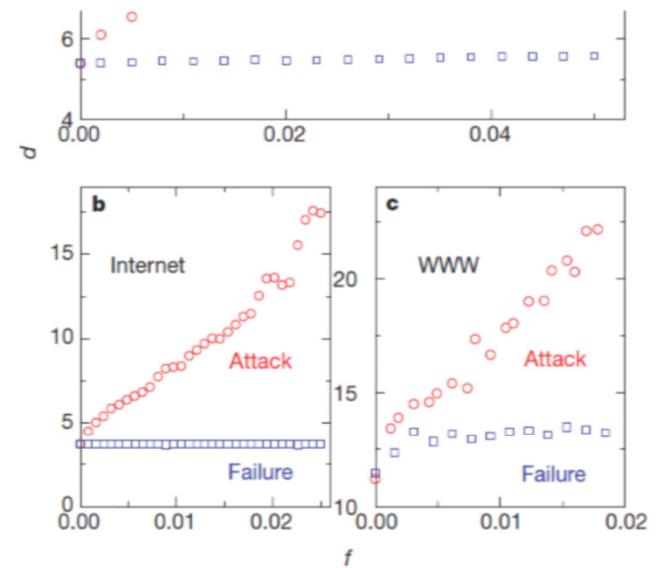
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ability that a node has a very large number of connections ( $k \gg \langle k \rangle$ ) is practically prohibited in exponential networks, highly connected nodes are statistically significant in scale-free networks (Fig. 1).

We start by investigating the robustness of the two basic connectivity distribution models, the Erdős–Rényi (ER) model<sup>9,10</sup> that produces a network with an exponential tail, and the scale-free model<sup>17</sup> with a power-law tail. In the ER model we first define the  $N$  nodes, and then connect each pair of nodes with probability  $p$ . This algorithm generates a homogeneous network (Fig. 1), whose connectivity follows a Poisson distribution peaked at  $\langle k \rangle$  and decaying exponentially for  $k \gg \langle k \rangle$ .



**Figure 1** Visual illustration of the difference between an exponential and a scale-free network. **a**, The exponential network is homogeneous: most nodes have approximately



**Figure 2** Changes in the diameter  $d$  of the network as a function of the fraction  $f$  of the removed nodes. **a**, Comparison between the exponential (E) and scale-free (SF) network models, each containing  $N = 10,000$  nodes and 20,000 links (that is,  $\langle k \rangle = 4$ ). The blue symbols correspond to the diameter of the exponential (triangles) and the scale-free (squares) networks when a fraction  $f$  of the nodes are removed randomly (error tolerance). Red symbols show the response of the exponential (diamonds) and the scale-free (circles) networks to attacks, when the most connected nodes are removed. We determined the  $f$  dependence of the diameter for different system sizes ( $N = 1,000; 5,000; 20,000$ ) and found that the obtained curves, apart from a logarithmic size correction, overlap with those shown in **a**, indicating that the results are independent of the size of the system. We

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## Supply network disruption and resilience: A network structural perspective

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*Y. Kim et al. / Journal of Operatio*

**Table 3**

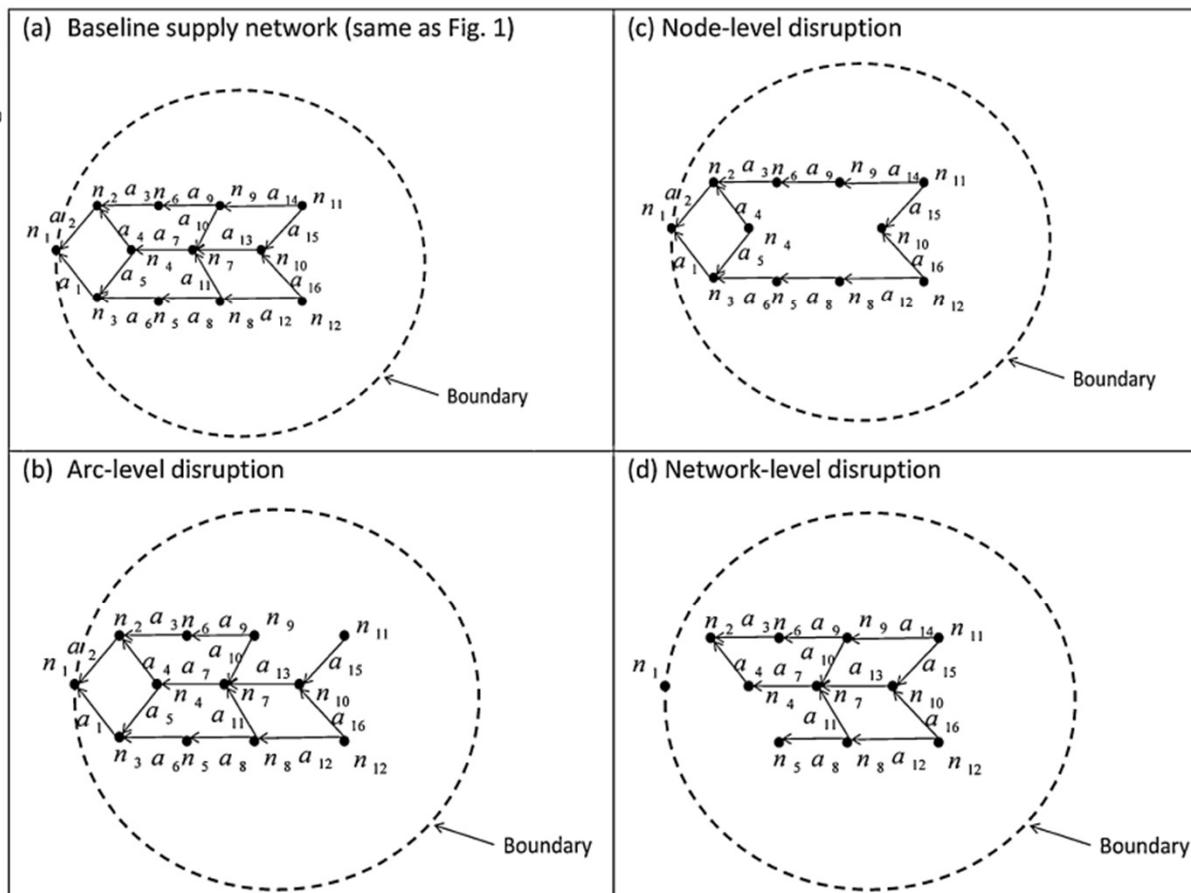
Network metrics and resilience for four basic supply network structures.

Network metrics	Block-diagonal	Scale-free	Centralized	Diagonal
Node/arcs	12/18	12/25	12/29	12/25
Network density	.14	.19	.22	.19
Average degree	1.50	2.08	2.42	2.08
Resilience	.11	.30	.16	.13
Walks	8	19	27	21
Average walk length	6.5	6.89	5.44	8.62
Max. walk length	9	11	7	13
Min. walk length	5	3	3	5
Connectivity	3	4	2	3
Betweenness centrality	1.33	1.67	.75	2.58
Centralization (%)	10.91	52.73	67.27	30.91

## Supply network resilience

$$= \frac{\text{total number of node/arc disruptions, which does not result in a supply network disruption}}{\text{total number of node/arc disruptions}}$$

We exclude the source and sink node in this calculation because this would automatically lead to a disruption, and because our focus



# WHICH NETWORKS ARE ROBUST?

# WHICH NETWORKS ARE ROBUST?

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PHYSICAL REVIEW LETTERS

9 MARCH 1998

## Master Stability Functions for Synchronized Coupled Systems

Louis M. Pecora and Thomas L. Carroll

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(Received 7 July 1997)

We show that many coupled oscillator array configurations considered in the literature can be put into a simple form so that determining the stability of the synchronous state can be done by a master stability function, which can be tailored to one's choice of stability requirement. This solves, once and for all, the problem of synchronous stability for any linear coupling of that oscillator.  
[S0031-9007(98)05387-3]

PACS numbers: 05.45.+b, 84.30.Ng

A particularly interesting form of dynamical behavior occurs in networks of coupled systems or oscillators when all of the subsystems behave in the same fashion; that is, they all do the same thing at the same time. Such behavior of a network simulates a continuous system that has a uniform movement, models neurons that synchronize and coupled synchronized lasers and

hyperplane in the phase space and number (2) makes the stability diagram specific to our choice of oscillators and the components. Number (4) is the choice of many studies of coupled systems since it is often a good approximation and can be considered prototypical.

In determining the stability of the synchronous state, various criteria are possible. The weakest is that the

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*Ecology Letters*, (2002) 5: 558–567

REPORT

## Network structure and biodiversity loss in food webs: robustness increases with connectance

### Abstract

Jennifer A. Dunne,<sup>1,2\*</sup> Richard J. Williams<sup>1</sup> and Neo D. Martinez<sup>1</sup>

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Food-web structure mediates dramatic effects of biodiversity loss including secondary and ‘cascading’ extinctions. We studied these effects by simulating primary species loss in 16 food webs from terrestrial and aquatic ecosystems and measuring robustness in terms of the secondary extinctions that followed. As observed in other networks, food webs are more robust to random removal of species than to selective removal of species with the most trophic links to other species. More surprisingly, robustness increases with food-web connectance but appears independent of species richness and omnivory. In particular, food webs experience ‘rivet-like’ thresholds past which they display extreme sensitivity to removal of highly connected species. Higher connectance delays the onset of this threshold. Removing species with few trophic connections generally has little effect though there are several striking exceptions. These findings emphasize how the number of species removed affects ecosystems differently depending on the trophic functions of species removed.

# WHICH NETWORKS ARE ROBUST?

## REPORT

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St. Martin (Goldwasser & Roughgarden 1993); (11) Little Rock Lake: pelagic and benthic species, particularly fishes, zooplankton, macroinvertebrates, and algae of a small Wisconsin lake (Martinez 1991); (12) Lake Tahoe: detailed benthic and pelagic species in a large California lake, including hundreds of planktonic autotrophs (Martinez unpublished data); (13) Mirror Lake: detailed benthic and pelagic species in a small New Hampshire lake (Martinez unpublished data); (14) Bridge Brook Lake: pelagic species from the largest of a set of 50 New York Adirondack lake food webs (Havens 1992); (15) Coachella Valley: a wide range of highly aggregated taxa in a southern California desert (Polis 1991); and (16) Skipwith Pond: invertebrates in an English pond (Warren 1989). In the case of the New Zealand stream food webs, we report results for 2 of 10 published webs, selecting the most speciose webs from each of two land-use types sampled.

All food-web data are limited, since some species, links, functional groups, and even taxonomic kingdoms are usually left out, and the included taxa are often unevenly resolved. However, studies of resolution, aggregation, and sampling effort of taxa and trophic links in food webs have provided detailed understanding of many effects of these methodological aspects of food-web structure (e.g. Martinez 1991, 1993, 1994; Martinez *et al.* 1999), and we feel that such empirical limitations are unlikely to change our general

1992; Warren 1994).

For each food web we simulated species loss by sequentially removing species using one of four criteria: removal of (1) the most connected species; (2) randomly chosen species (1000 random deletion sequences initiated for each web); (3) the most connected species excluding basal species (species with predators but no prey); and (4) the least connected species. Rather than determining the most or least connected species at each removal step based on the original web, it was determined based on the web remaining after all previous primary removals and secondary extinctions. Both predator and prey links were counted to determine total trophic connections ('degree') for each species. A previous study of species removals that looked at both total degree and prey-directed degree found little difference between the two (Solé & Montoya 2001). Removal of the most connected (1) and random (2) species follows research on network tolerance of 'attacks' and 'errors' (Albert *et al.* 2000; Jeong *et al.* 2000, 2001; Solé & Montoya 2001). To our knowledge, criteria (3) and (4) have not been assessed for networks before.

We examined the impact of species loss on one aspect of food web stability: the number of potential secondary extinctions. A secondary extinction occurs when a non-basal species loses all of its prey items, and also when a cannibalistic species loses all of its prey items except itself.

# WHICH NETWORKS ARE ROBUST?

Paper	Network or data	Perturbation/abn. event	Meas. of robustness	Null model
Albert et al.	Preferential Attachment Model	Remove nodes randomly or by degree	Change in diameter	Exponential random graph model
Kim et al.	Various models	All node or edge removal	Fraction of viable perturbations	Comparative study
Pecora et al.	Coupled oscillator equations	Small shifts in phase space	Synchrony of oscillators	n/a
Dunne et al.	Food web	Various	Extinction cascade size	Comparative study

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## ECOLOGY LETTERS

*Ecology Letters*, (2015) 18: 144–152

doi: 10.1111/ele.12394

LETTER

### A simple stochastic model for complex coextinctions in mutualistic networks: robustness decreases with connectance

#### Abstract

Marcos Costa Vieira<sup>1\*</sup> and  
Mário Almeida-Neto<sup>2</sup>

Understanding and predicting species extinctions and coextinctions is a major goal of ecological research in the face of a biodiversity crisis. Typically, models based on network topology are used to simulate coextinctions in mutualistic networks. However, such topological models neglect two key biological features of species interactions: variation in the intrinsic dependence of species on the mutualism, and variation in the relative importance of each interacting partner. By incorporating both types of variation, we developed a stochastic coextinction model capable of simulating extinction cascades far more complex than those observed in previous topological models. Using a set of empirical mutualistic networks, we show that the traditional topological model may either underestimate or overestimate the number and likelihood of coextinctions, depending on the intrinsic dependence of species on the mutualism. More importantly, contrary to topological models, our stochastic model predicts extinction cascades to be more likely in highly connected mutualistic communities.

# WHICH NETWORKS ARE ROBUST?

four additional pollinator species, we define this event as a *third-degree extinction cascade*. Note that the degree does not necessarily correspond to the *total number of species lost* in the extinction cascade (which is seven in the example above).

## Compensation-mediated persistence under the TCM and the SCM

The SCM and the TCM postulate different rules for determining the outcome of primary extinctions (i.e. whether or not coextinctions occur). Those rules may be interpreted in terms of assumptions regarding the ability of species to compensate the loss of a mutualistic partner and thus persist. The TCM assumes, often implicitly, that a species is able to compensate the loss of a partner by interacting more frequently with its remaining partners. For example, if a plant species loses one of its seed dispersers, other species of seed dispersers should interact more frequently with the plant due to reduced interspecific competition in the absence of the extinct seed disperser. If a pollinator loses one of its partner plant species, it might compensate the loss by visiting its remaining plant species more frequently. Such compensatory mechanisms, which also include evolutionary responses, might explain part of the difference between the large numbers of coextinctions predicted by models and the small number of coextinction events observed empirically (Dunn *et al.* 2009). The TCM assumes that compensation-mediated persistence occurs independently

*et al.* (2007) and by Vieira *et al.* (2013) (see Table S1 for details and sources).

We simulated extinction cascades in the empirical mutualistic networks according to the SCM. In each simulation, the original network was subjected to a single extinction cascade in which the initial, primarily extinct species was chosen randomly from either trophic level and coextinctions occurred according to the equation  $P_{ij} = R_i d_{ij}$ . Starting  $d_{ij}$  values were calculated from the original interaction matrices.  $R_i$  was assumed to be equal for all species and was uniformly sampled in each simulation from three intervals representing low ( $0 < R_i \leq 0.3$ ), intermediate ( $0.3 < R_i \leq 0.6$ ) and high ( $0.6 < R_i \leq 1$ ) intrinsic demographic dependence on the mutualistic interaction for persistence. For each network, we performed  $10^4$  simulations for each interval of  $R_i$  and constructed empirical frequency distributions for the total number of extinctions in an extinction cascade. We also quantified the degree of each extinction cascade and constructed its corresponding frequency distribution. From this frequency distribution, we calculated, for each network and  $R_i$  level, the probability that a primary extinction would result in second-, third- and fourth-degree-or-higher extinction cascades.

In addition to performing simulations under the SCM, we used simulations to obtain the frequency distribution for the total number of extinctions under the TCM, which constrains the coextinction of a species to the loss of all of its mutualistic

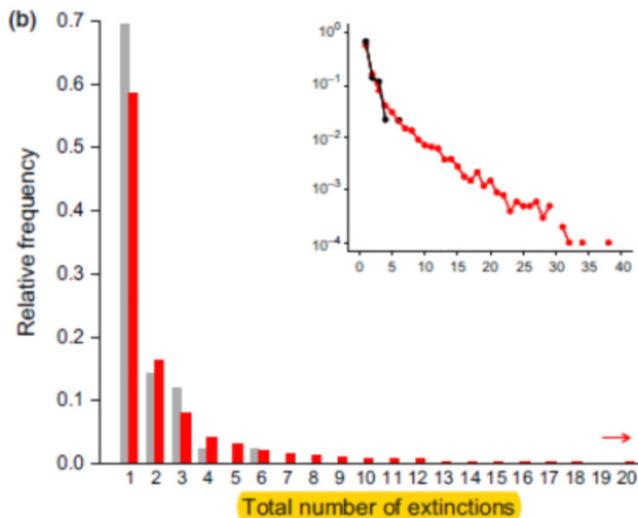
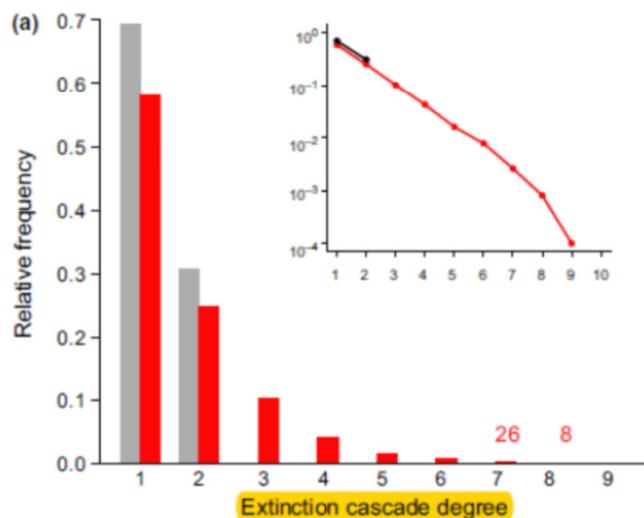
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# WHICH NETWORKS ARE ROBUST?

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*et al.* (2007) and by Vieira *et al.* (2013) (see Table S1 for details and sources).

Letter



**Figure 1** Typical frequency distributions of the degree (a) and total number of extinctions (b) of extinction cascades simulated in an empirical mutualistic network ('Albrecht' network, 49 species, see Table S1) using the topological coextinction model (grey bars and black points) and the stochastic coextinction model (SCM; red bars and points) under high values of  $R_i$ . Red numbers indicate the number of observations for rare, extremely large degree values obtained under the SCM. The red arrow indicates large observed values of total number of extinctions that were omitted to improve visualisation. Inset plots show the same data with a logarithmic y-axis.

empirical mutualistic simulation, the extinction cascade was chosen randomly. Extinctions occurred using  $d_{ij}$  values were matrices.  $R_i$  was uniformly sampled representing low (0.6) and high (1.0) ice on the mutualistic network, we performed of  $R_i$  and is for the total  $R$ . We also quantified constructed its this frequency and  $R_i$  level, the result in second-, cascades.

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Dunne et al.	Food web	Various	Extinction cascade size	Comparative study
Vieira et al.	Mutualistic species network	Random node removal	Size & degree of extinction cascades	Comparative study
...				

# CONCLUSIONS

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Don't despair!

# CONCLUSIONS

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DAMN all robustness papers!

# CONCLUSIONS

Don't despair!

DAMN all robustness papers!

Your system has its own robustness!

# WHAT IS ROBUST IN *YOUR* NETWORK?

Paper	Network or data	Perturbation/abn. event	Meas. of robustness	Null model
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