Tolerance in weighted coevolving network dynamics

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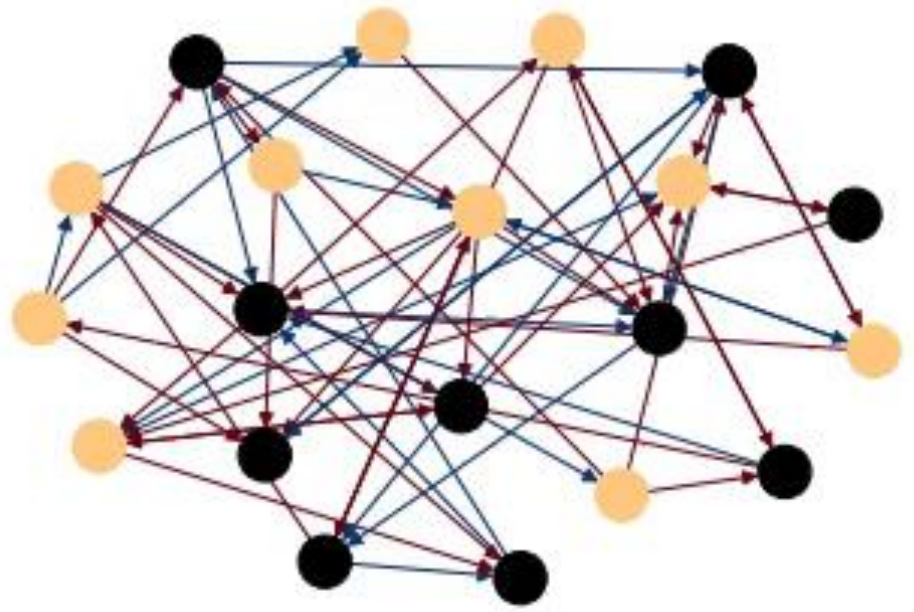
Alice Schwarze, Peter Mucha
Department of Mathematics, Dartmouth College

Co-evolving network dynamics



x_i

A_{ij}

Why weighted coevolving dynamics?

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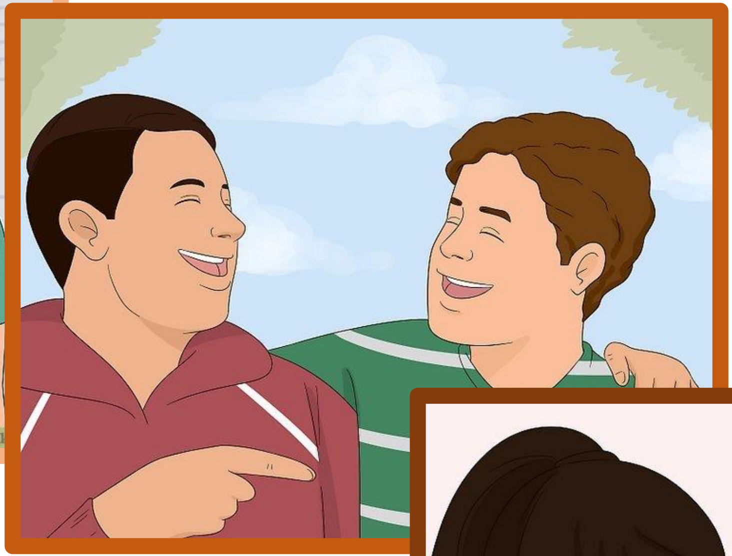
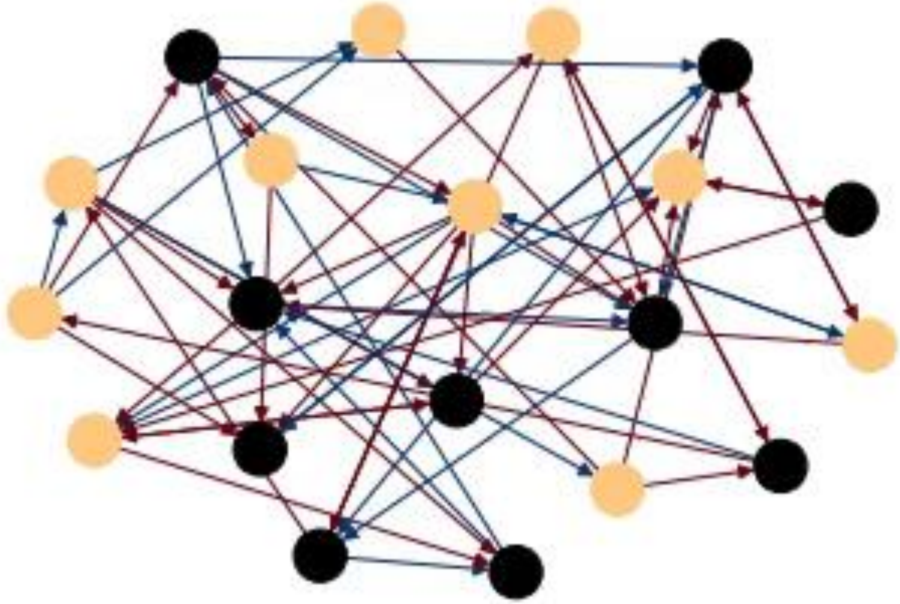


Why weighted coevolving dynamics?

Co-evolving network dynamics



$$x_i \in \mathbb{R}$$

$$A_{ij} \in \mathbb{R}$$

Applications of weighted coevolving dynamics

Opinion & norm formation

- Diffusion-type models
- Consensus vs. fragmentation

Neural plasticity

- Oscillators
- Global, clustered, or no synchronization

Example: Homophily & attention to novelty

Social Fragmentation Transitions in Large-Scale Parameter Sweep Simulations of Adaptive Social Networks*

Hiroki Sayama[1,2][0000−0002−2670−5864]

[1] Center for Collective Dynamics of Complex Systems, Binghamton University, State University of New York, Binghamton, NY 13902-6000, USA
[2] Waseda Innovation Lab, Waseda University, Shinjuku, Tokyo 169-8050, Japan
sayama@binghamton.edu

Example: Homophily & attention to novelty

$$\frac{dx_i}{dt} = c\left(\langle x \rangle_i - x_i\right) + \epsilon$$

$$\frac{dw_{ij}}{dt} = hF_h(x_i, x_j) + aF_a(\langle x \rangle_i, x_j)$$

$$\langle x \rangle_i = \frac{\sum_j w_{ij} x_j}{\sum_j w_{ij}}$$

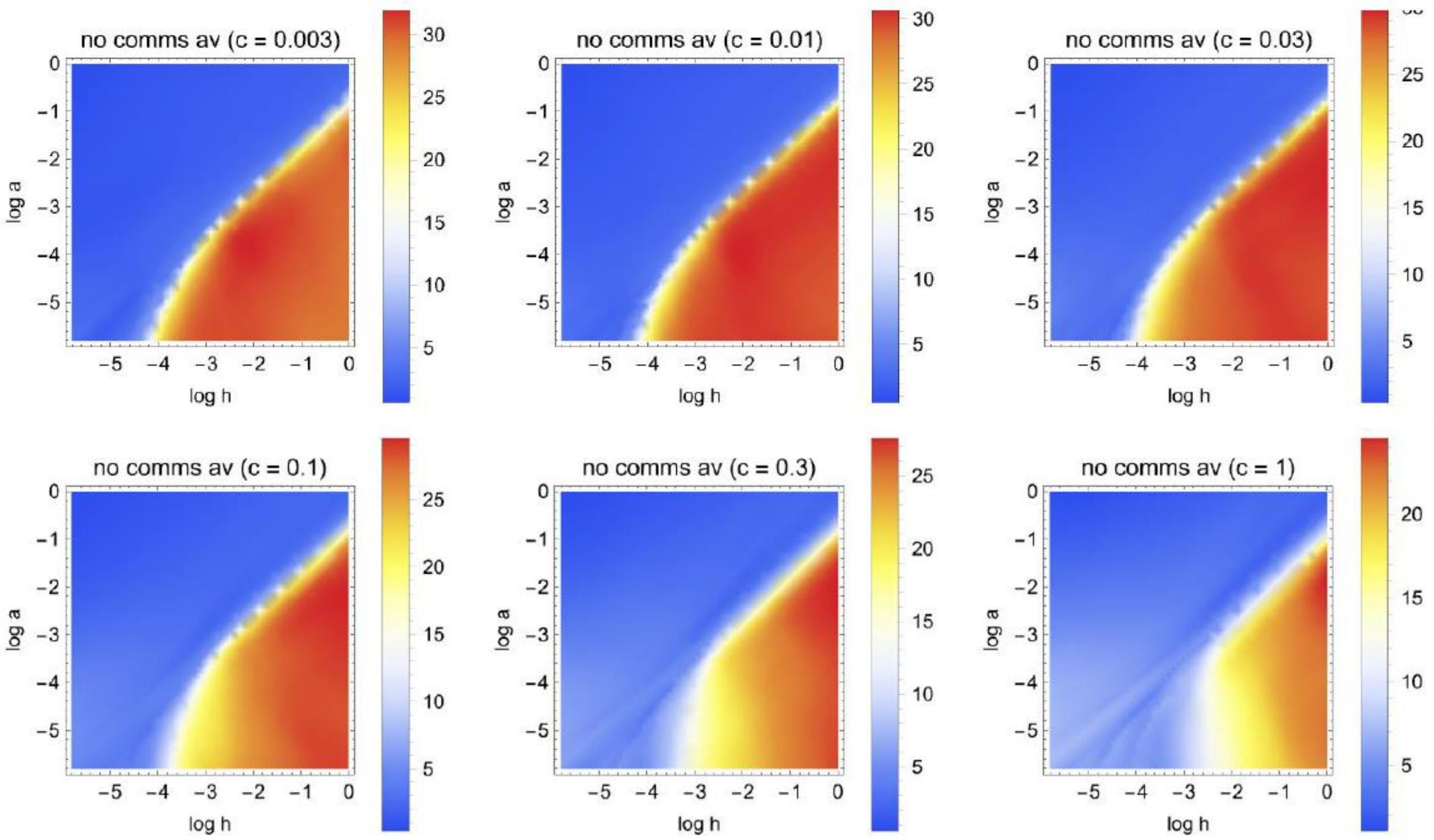
$$F_h(x_i, x_j) = \theta_h - |x_i - x_j|$$

$$F_a(\langle x \rangle_i, x_j) = |\langle x \rangle_i - x_j| - \theta_a$$

H. Sayama (2022)

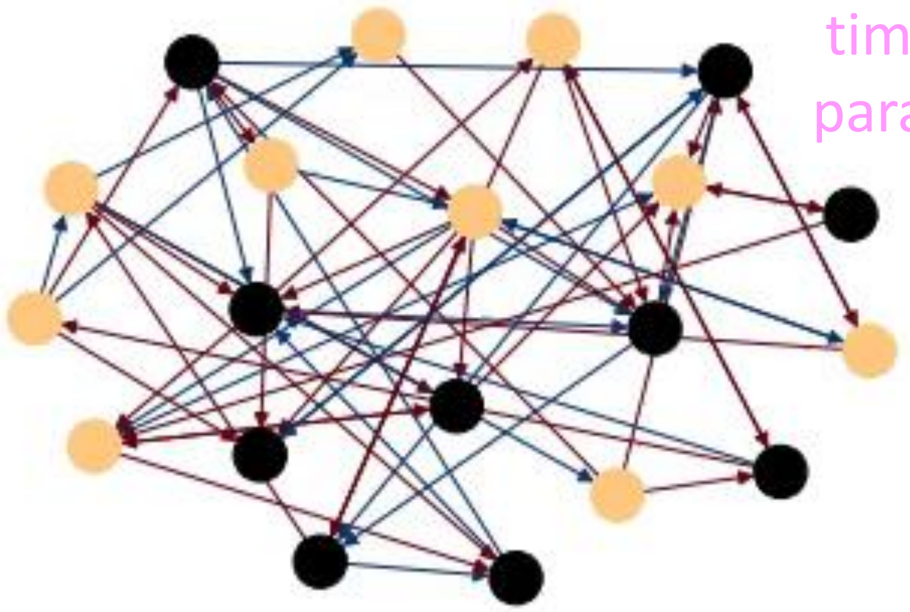
Example: Homophily & attention to novelty

(a) Number of communities



H. Sayama (2022)

Co-evolving network dynamics



timescale parameter

$$\frac{d}{dt}x_i = -\sum_j L_{ij}x_j$$

growth function

$$\boxed{\alpha\frac{d}{dt}}A_{ij} = \boxed{(\tau} - |x_i - x_j|^p)\boxed{f(A_{ij})} + \boxed{g(A,x)}$$

conservation function

$$L(t) = D_{in}(t) - A(t)$$

tolerance parameter

Examples of edge dynamics

$$\alpha \frac{d}{dt} A_{ij} = (\tau - |x_i - x_j|^p) \boxed{f(A_{ij})} + \boxed{g(A, x)}$$

growth function

conservation function

Examples of growth functions:

$$f(A_{ij}) = 1$$

$$f(A_{ij}) = A_{ij}$$

$$f(A_{ij}) = A_{ij}(1 - A_{ij})$$

$$f(A_{ij}) = (1 - A_{ij})(1 + A_{ij})$$

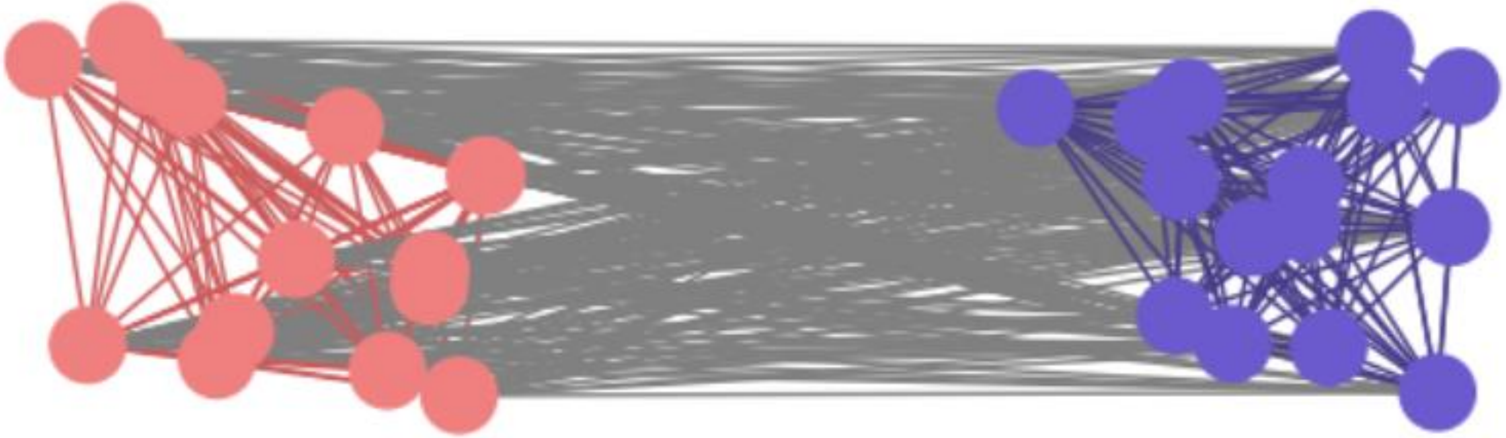
Examples of conservation functions:

$$g(A, x) = 0$$

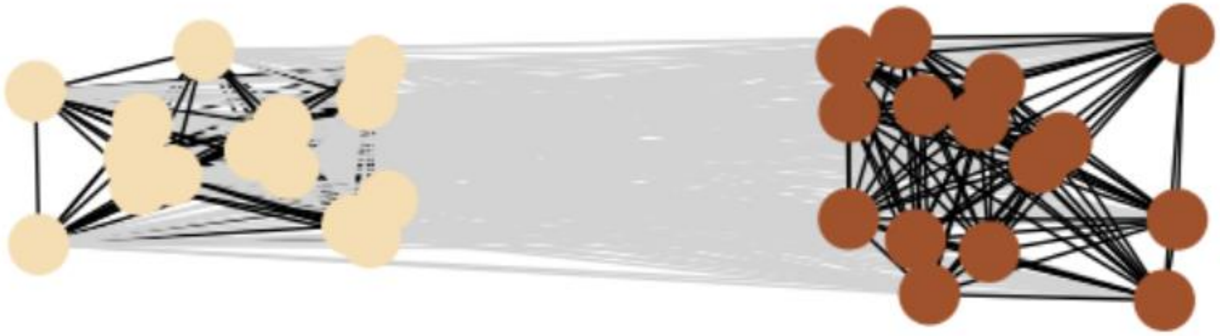
$$g(A, x) = \sum_{ij} A_{ij}(\tau - |x_i - x_j|^p)$$

$$g(A, x) = -\sum_{j} A_{ij}(\tau - |x_i - x_j|^p)$$

2-block-model case

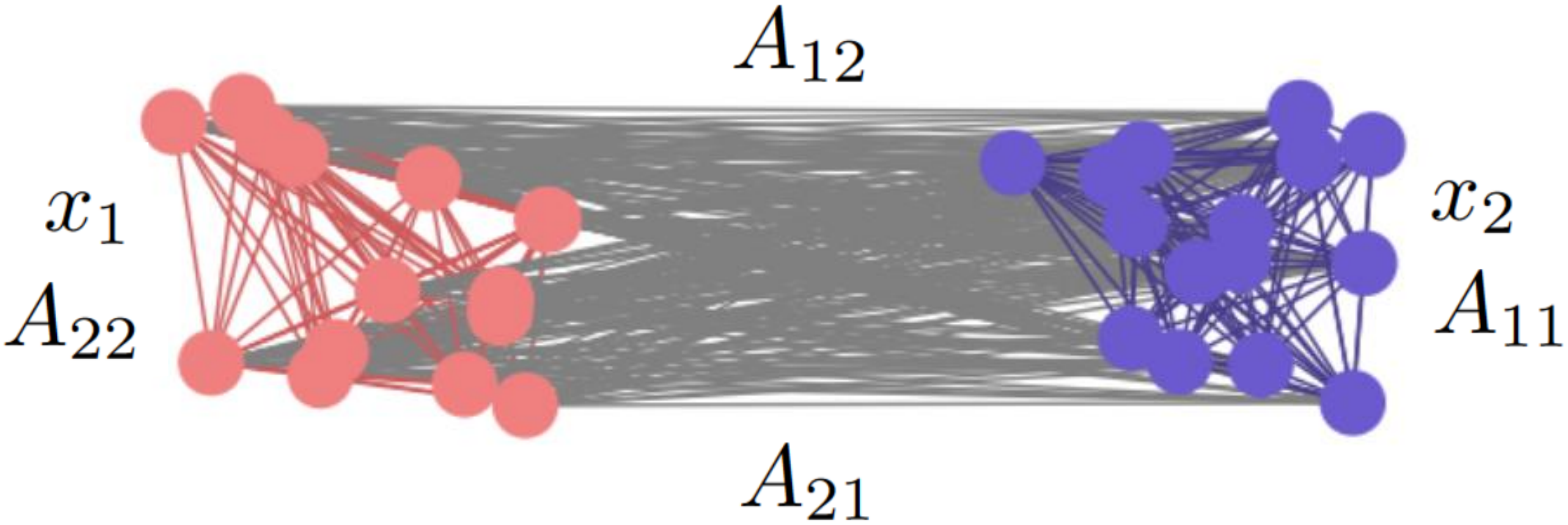


start close to fragmentated state



start close to consensus state

2-block-model case



Linear or exponential growing tie strengths:

$$\chi = x_1 - x_2$$
$$s_1 := A_{12} + A_{21}$$
$$d_1 := A_{12} - A_{21}$$

Logistically growing tie strengths:

$$\chi = x_1 - x_2$$
$$\mu := \frac{1}{2}(\mathrm{logit}(A_{12}) + \mathrm{logit}(A_{21}))$$
$$\delta := \frac{1}{2}(\mathrm{logit}(A_{12}) - \mathrm{logit}(A_{21}))$$

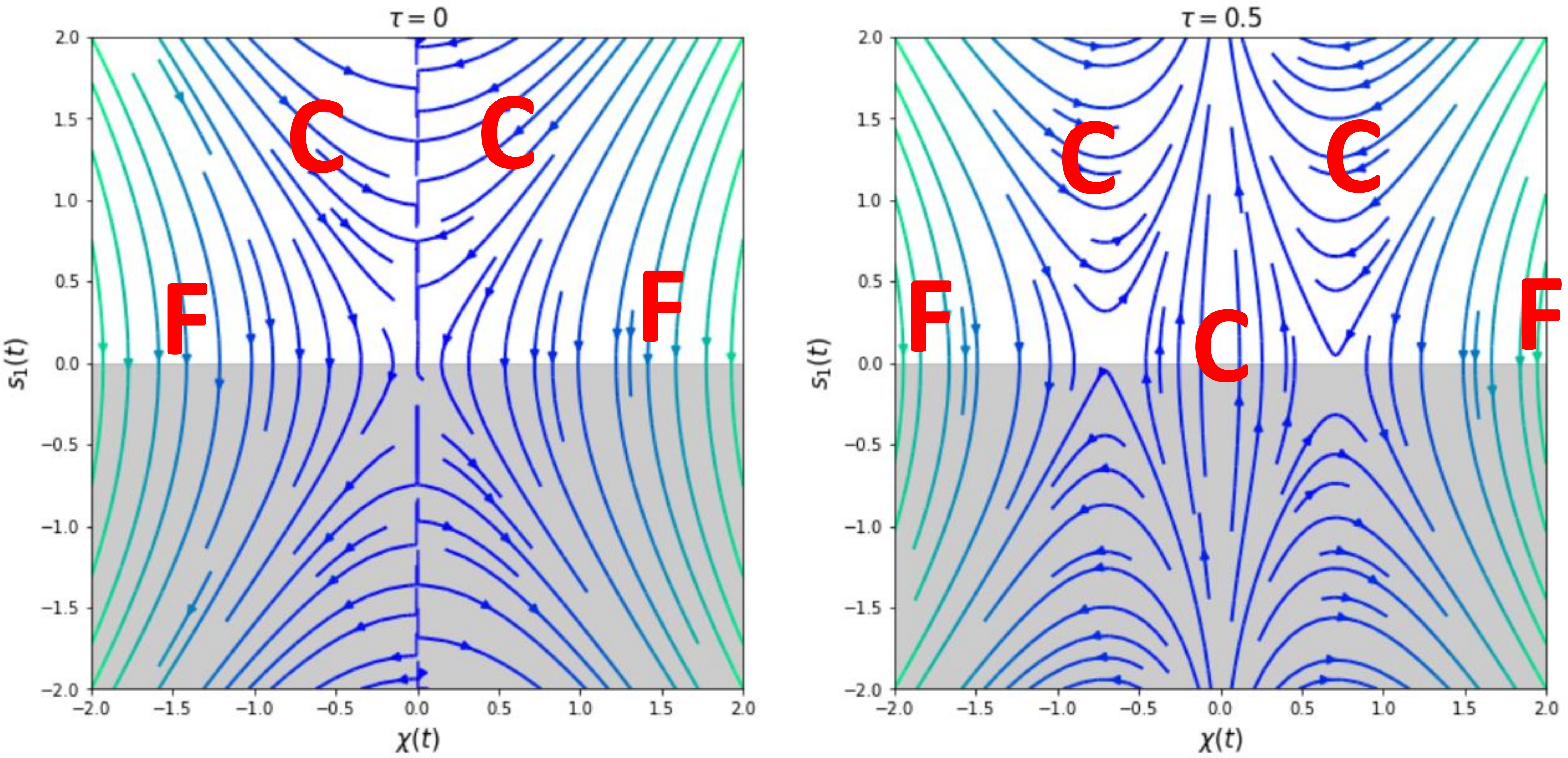
Results: Linear growth, no conservation

$$\frac{d}{dt}x_i = -\sum_j L_{ij}x_j$$

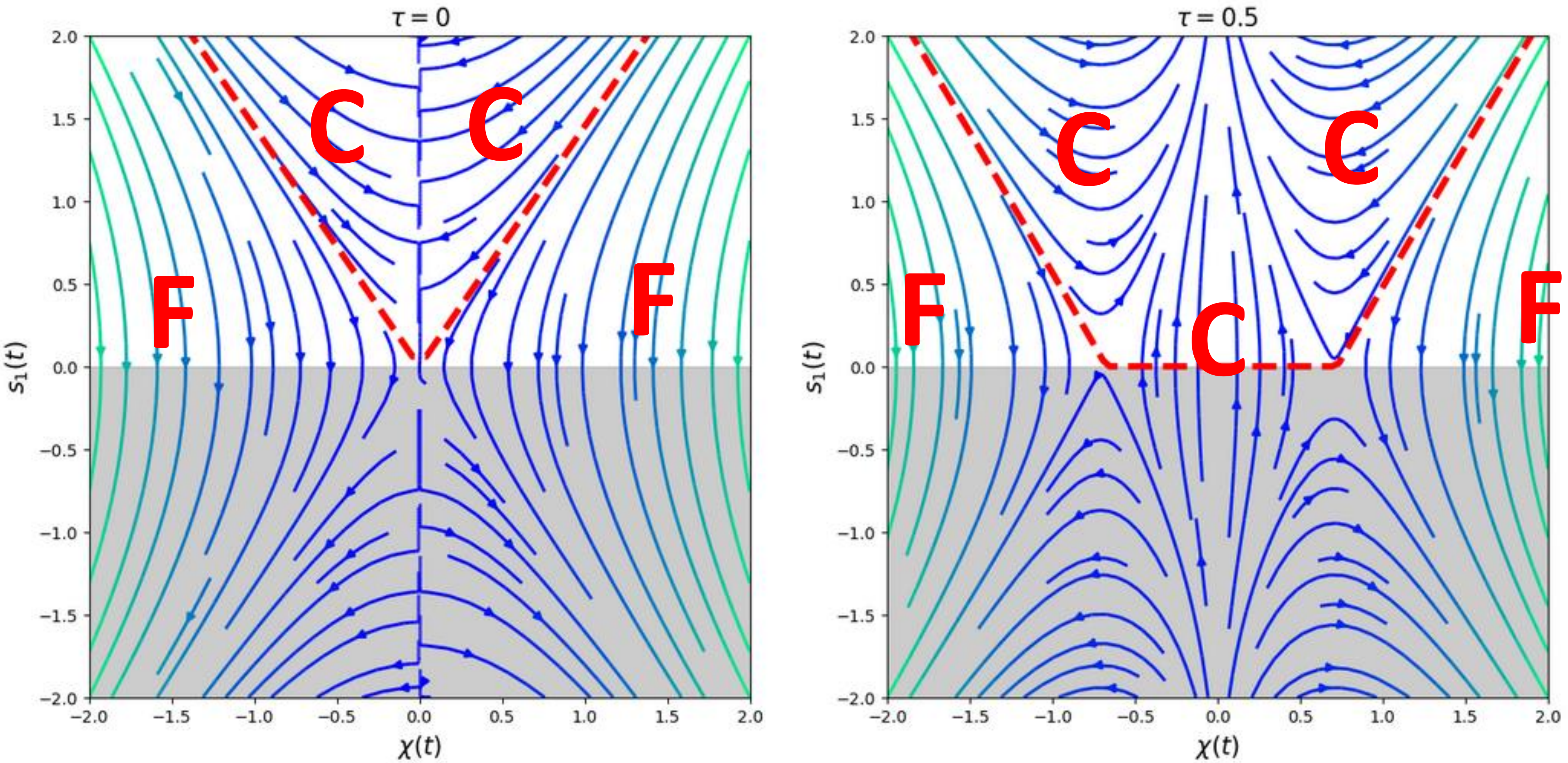
$$\alpha\frac{d}{dt}A_{ij} = (\tau - |x_i - x_j|^p)$$

$$L = D_{in} - A$$

Results: Linear growth, no conservation



Results: Linear growth, no conservation



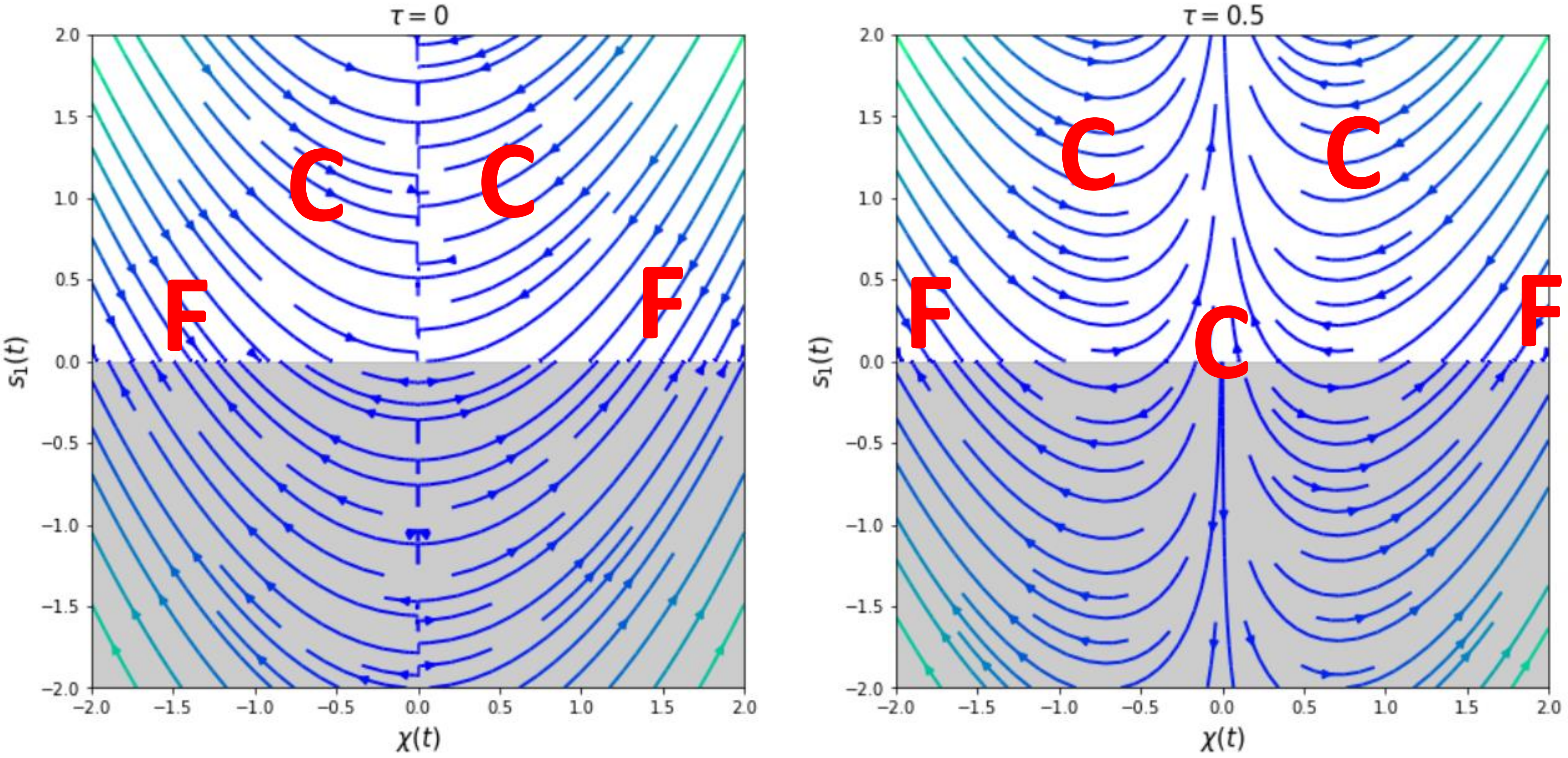
Results: Exponential growth, no conservation

$$\frac{d}{dt}x_i = -\sum_j L_{ij}x_j$$

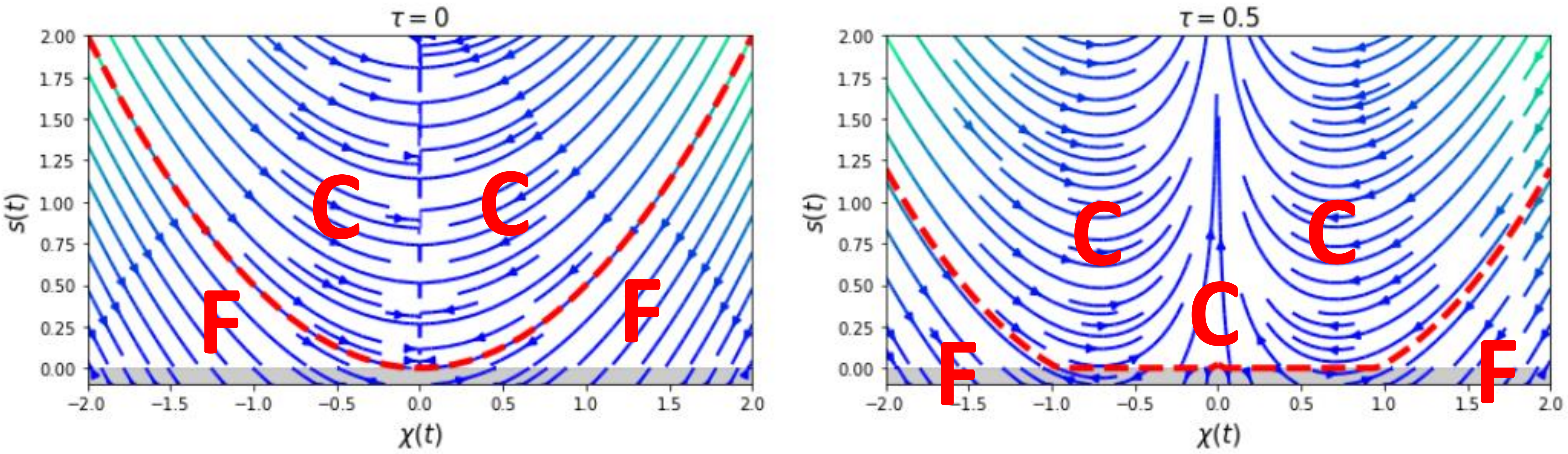
$$\alpha\frac{d}{dt}A_{ij} = (\tau - |x_i - x_j|^p)A_{ij}$$

$$L = D_{in} - A$$

Results: Exponential growth, no conservation



Results: Exponential growth, no conservation



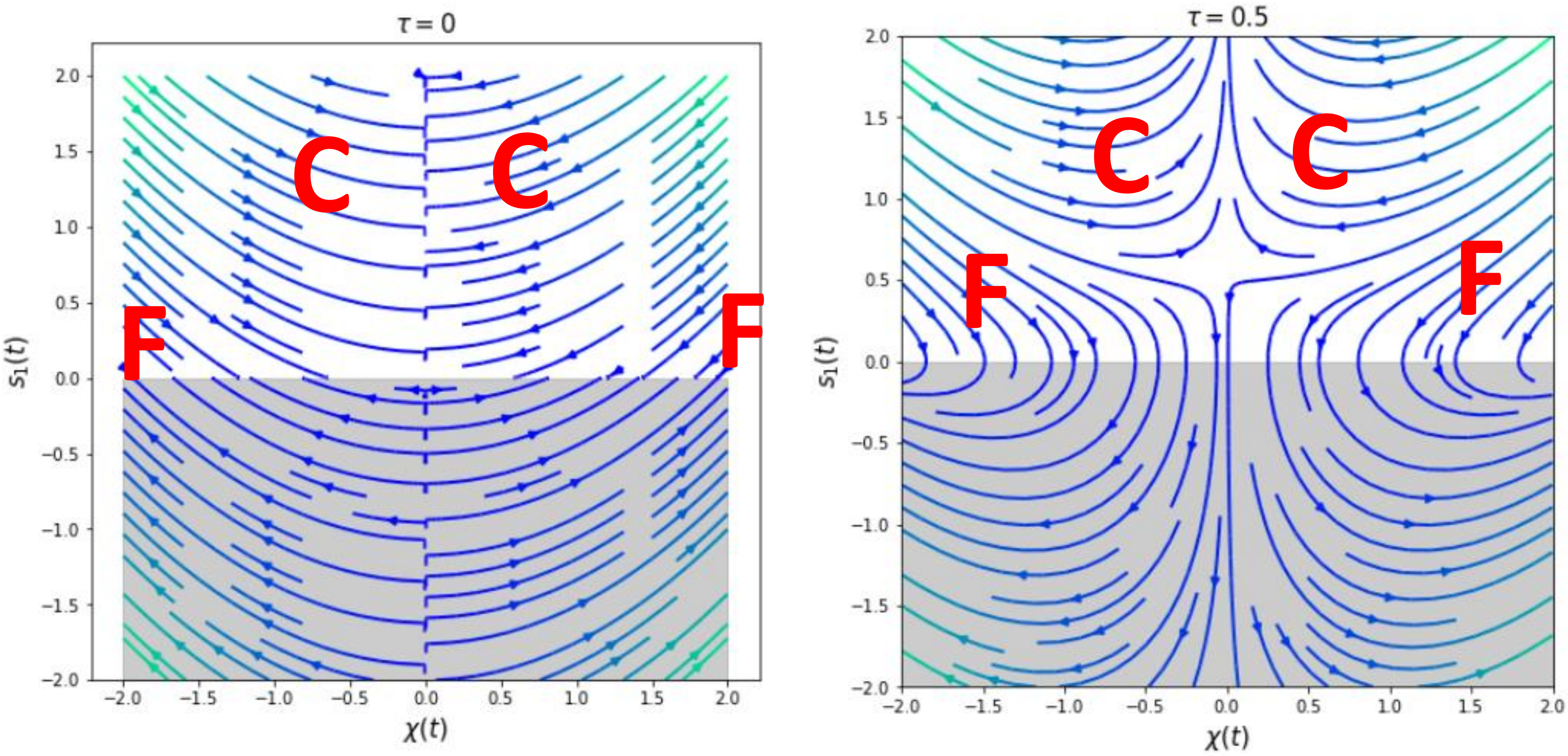
Exp. growth, total edge weight conserved

$$\frac{d}{dt}x_i = -\sum_j L_{ij}x_j$$

$$\alpha\frac{d}{dt}A_{ij} = (\tau - |x_i - x_j|^p)A_{ij} - \binom{N}{2}^{-1} \sum_{\substack{1\leq i,j\leq N \\ j\neq i}} A_{ij}(\tau - (|x_i - x_j|)^p)$$

$$L = D_{in} - A$$

Exp. growth, total edge weight conserved



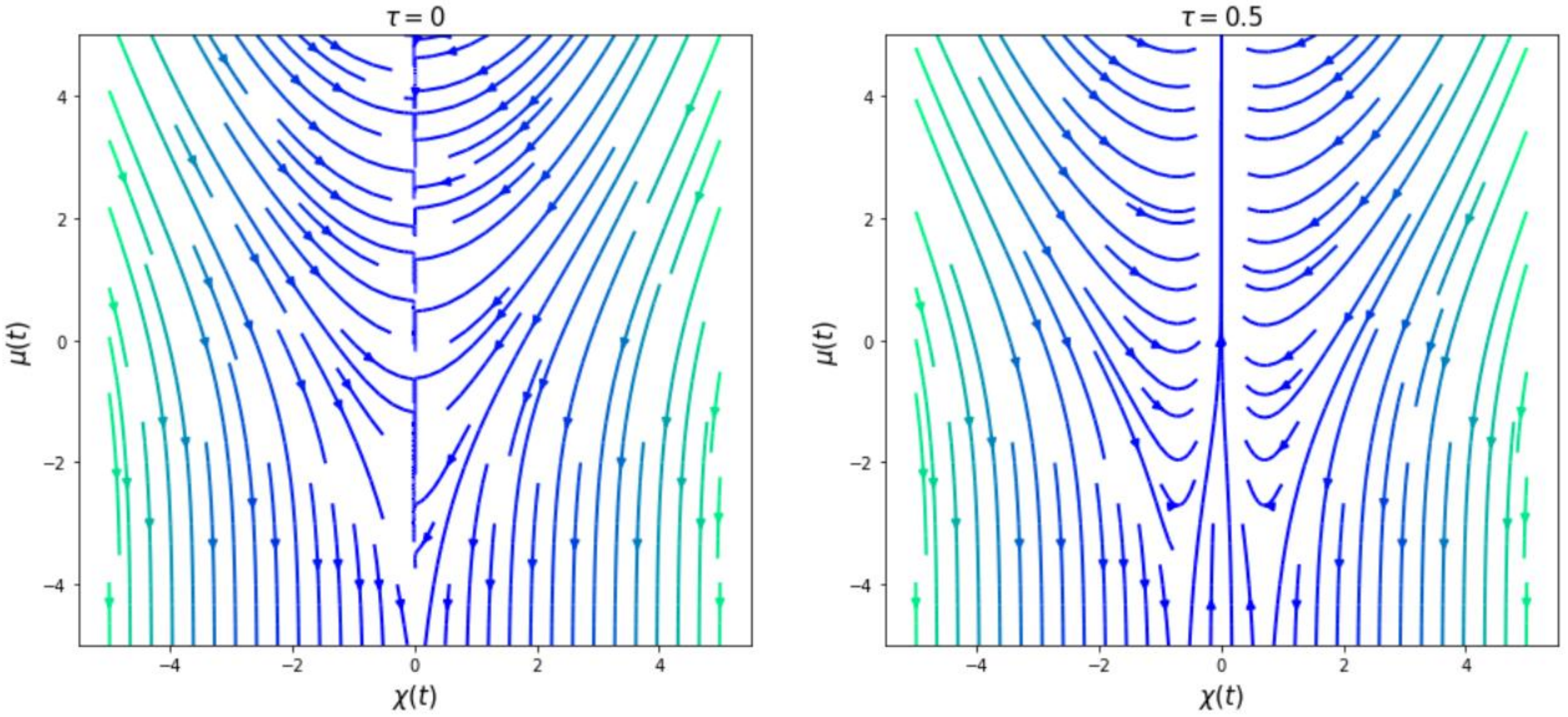
Results: Logistic growth, no conservation

$$\frac{d}{dt}x_i = -\sum_j L_{ij}x_j$$

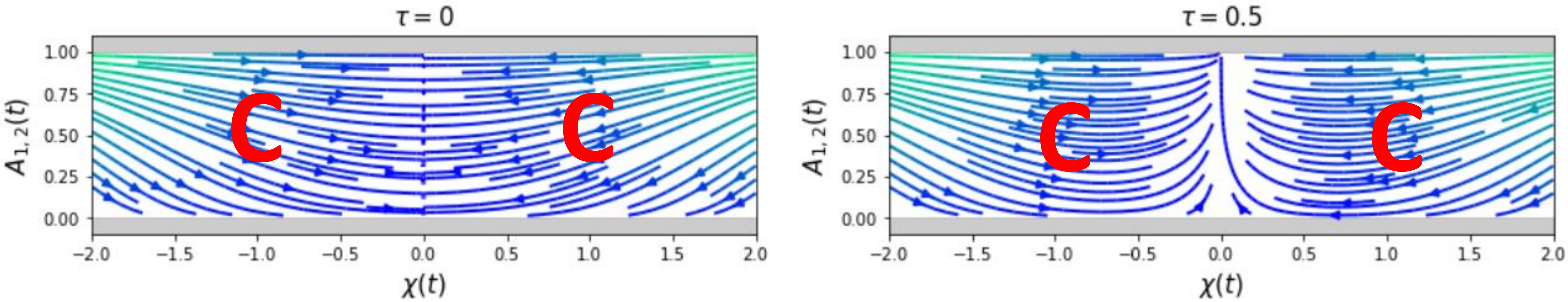
$$\alpha\frac{d}{dt}A_{ij} = (\tau - |x_i - x_j|^p)A_{ij}(1 - A_{ij})$$

$$L = D_{in} - A$$

Logistically growing tie strengths



Logistically growing tie strengths



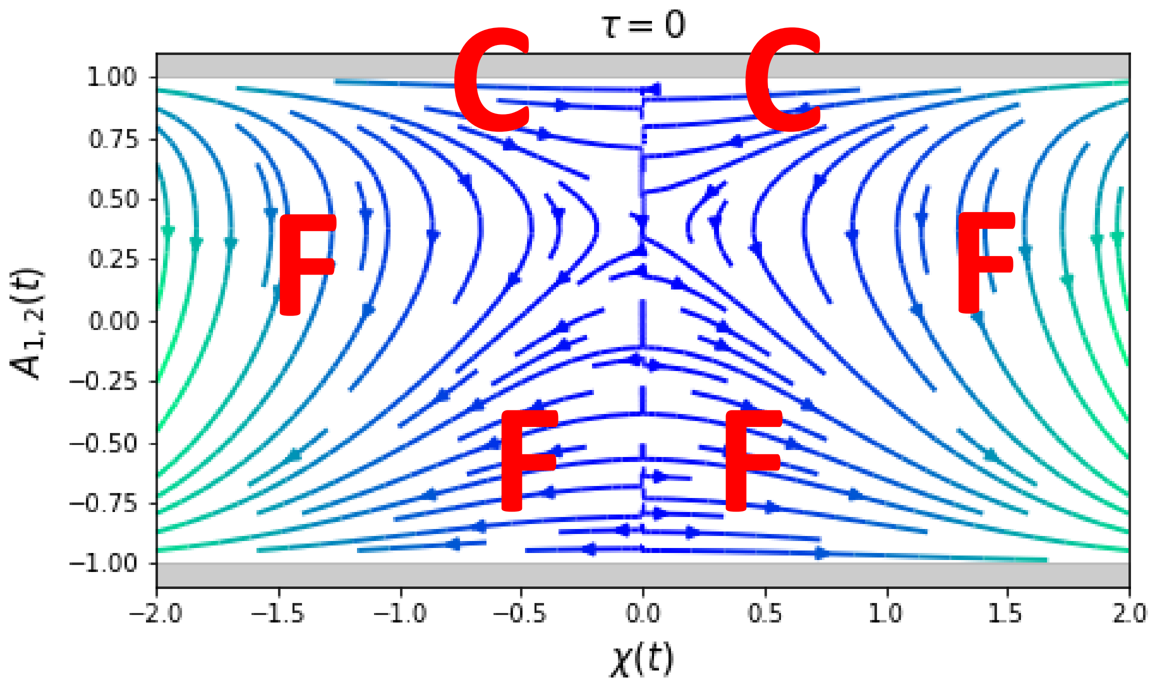
Results: Gen. logistic growth, no conservation

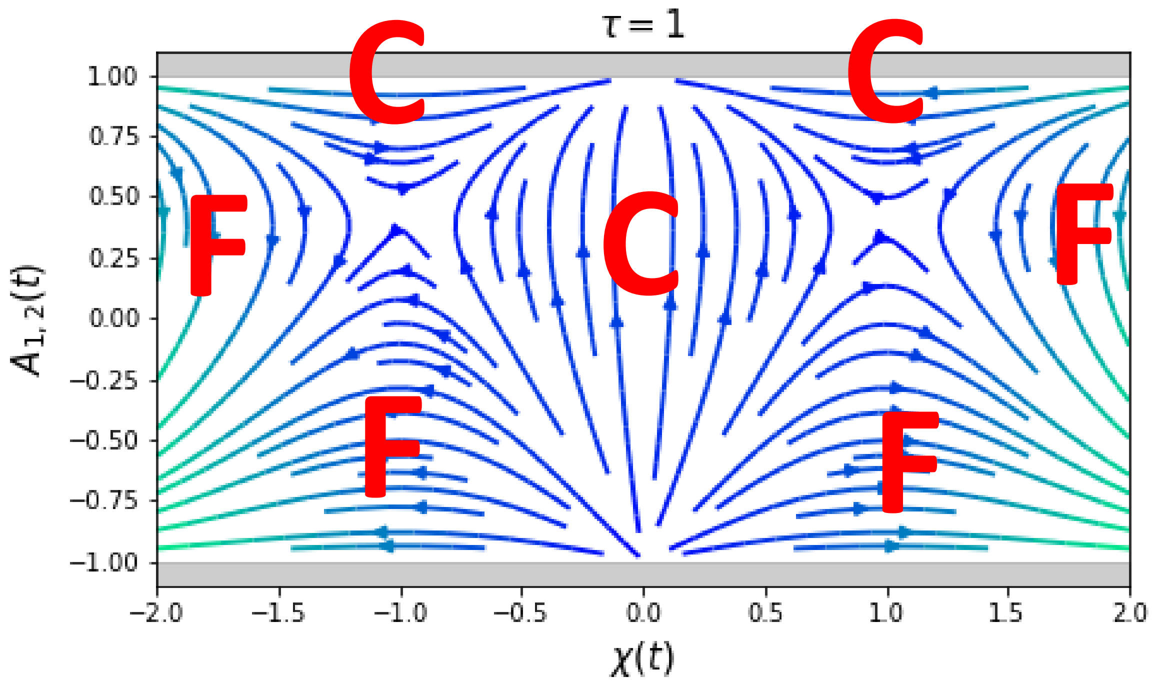
$$\frac{d}{dt}x_i = -\sum_j L_{ij}x_j$$

$$\alpha\frac{d}{dt}A_{ij} = (\tau - |x_i - x_j|^p)(1 - A_{ij})(1 + A_{ij})$$

$$L = D_{in} - A$$

Results: Gen. logistic growth, no conservation





Conclusions and Outlook

- **Tolerance** can lead to **rich dynamics**
- Even a small tolerance value can **qualitatively change the set of absorbing states**
- Tractability of weighted coevolving network models
- Comparison to dynamics of edge probabilities in discrete models of coevolution
- Model validation/selection

Thank you!