ARE 213 PS 2a

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```

Problem 1

Question 10.3 from Wooldridge: For $T = 2$ consider the standard unobserved effects model:

$$y_{it} = \alpha + x_{it}\beta + c_i + u_{it} \tag{1}$$

Let $\hat{\beta}_{FE}$ and $\hat{\beta}_{FD}$ represent the fixed effects and first differences estimators respectively.

Part (a)

Show that $\hat{\beta}_{FE}$ and $\hat{\beta}_{FD}$ are numerically identical. Hint: it may be easier to write $\hat{\beta}_{FE}$ as the "within estimator" rather than the fixed effects estimator.

Writing $\hat{\beta}_{FE}$ as the within estimator, $\hat{\beta}_{FE}$ and $\hat{\beta}_{FD}$ are given by

$$\hat{\beta}_{FD} = \left(\Delta X' \Delta X\right)^{-1} \left(\Delta X' \Delta y\right) \qquad \text{and} \qquad \hat{\beta}_{FE} = \left(\ddot{X}' \ddot{X}\right)^{-1} \left(\ddot{X}' \ddot{y}\right)$$

Expanding the inner products, we have

$$\hat{\beta}_{FD} = \left(\sum_i \sum_t \Delta X_{it}' \Delta X_{it}\right)^{-1} \left(\sum_i \sum_t \Delta X_{it}' \Delta y_{it}\right)$$

and

$$\hat{\beta}_{FE} = \left(\sum_i \sum_t \ddot{X}_{it}' \ddot{X}_{it}\right)^{-1} \left(\sum_i \sum_t \ddot{X}_{it}' \ddot{y}_{it}\right)$$

Since there are only two periods, $\hat{\beta}_{FD}$ simplifies to

$$\hat{\beta}_{FD} = \left(\sum_i \Delta X_i' \Delta X_i\right)^{-1} \left(\sum_i \Delta X_i' \Delta y_i\right)$$

where

$$\Delta X_i \equiv X_{i2} - X_{i1} \qquad \text{and} \qquad \Delta y_i \equiv y_{i2} - y_{i1}$$

Now we note that

$$\ddot{X}_{i1} = X_{i1} - \frac{1}{2}(X_{i1} + X_{i2}) = \frac{1}{2}(X_{i1} - X_{i2}) = -\frac{1}{2}\Delta X_i$$

and similiarly

$$\ddot{X}_{i2} = \frac{1}{2}\Delta X_i, \quad \ddot{y}_{i1} = -\frac{1}{2}\Delta y_i \quad \ddot{y}_{i2} = \frac{1}{2}\Delta y_i$$

Then, $\hat{\beta}_{FE}$ becomes

$$\begin{aligned}
\hat{\beta}_{FE} &= \left(\sum_i \sum_t \ddot{X}'_{it} \ddot{X}_{it} \right)^{-1} \left(\sum_i \sum_t \ddot{X}'_{it} \ddot{y}_{it} \right) \\
&= \left(\sum_i \frac{1}{4} \Delta X'_i \Delta X_i + \frac{1}{4} \Delta X'_i \Delta X_i \right)^{-1} \left(\sum_i \frac{1}{4} \Delta X'_i \Delta y_i + \frac{1}{4} \Delta X'_i \Delta y_i \right) \\
&= \left(\frac{1}{2} \sum_i \Delta X'_i \Delta X_i \right)^{-1} \left(\frac{1}{2} \sum_i \Delta X'_i \Delta y_i \right) \\
&= \left(\sum_i \Delta X'_i \Delta X_i \right)^{-1} \left(\sum_i \Delta X'_i \Delta y_i \right) \\
&= \hat{\beta}_{FD}
\end{aligned}$$

So $\hat{\beta}_{FE}$ and $\hat{\beta}_{FD}$ are numerically identical.

Part (b)

Show that the standard errors of $\hat{\beta}_{FE}$ and $\hat{\beta}_{FD}$ are numerically identical. If you wish, you may assume that x_{it} is a scalar (i.e. there is only one regressor) and ignore any degree of freedom corrections. You are not clustering the standard errors in this problem.

The standard errors are estimates of the square root of the asymptotic variances of our estimators, so WLOG, we can compare the asymptotic variances. The asymptotic variances of our estimators are

$$\widehat{Avar(\hat{\beta}_{FE})} = \hat{\sigma}^2_{u,FE}\left(\ddot{X}'\ddot{X}\right)^{-1} \qquad \text{and} \qquad \widehat{Avar(\hat{\beta}_{FE})} = \hat{\sigma}^2_{u,FD}\left(\Delta X'\Delta X\right)^{-1}$$

where $\hat{\sigma}^2_{u,FE}$ and $\hat{\sigma}^2_{u,FD}$ are estimated from the residuals of the corresponding regressions and using the correct degrees of freedom:

$$\hat{\sigma}^2_{u,FE} = \frac{\sum_i \sum_t \hat{\ddot{u}}^2_{it}}{N(T-1)-K} = \frac{\sum_i \sum_t \hat{\ddot{u}}^2_{it}}{N-K} \qquad \text{and} \qquad \hat{\sigma}^2_{u,FD} = \frac{\sum_i \widehat{\Delta u_i}^2}{N(T-1)-K} = \frac{\sum_i \widehat{\Delta u_i}^2}{N-K}$$

Let $\hat{\beta} := \hat{\beta}_{FD} = \hat{\beta}_{FE}$. Then, from part (a), we can find the relationship between $\widehat{\Delta u_i}$ and $\hat{\ddot{u}}_{it}$:

$$\begin{aligned}
\hat{\ddot{u}}^2_{it} &= (\ddot{y}_{it} - \ddot{X}_{it}\hat{\beta})^2 \\
&= \left((-1)^t\left(\frac{1}{2}\Delta y_i - \frac{1}{2}\Delta X_i\hat{\beta}\right)\right)^2 \\
&= \frac{1}{4}\left(\Delta y_i - \Delta X_i\hat{\beta}\right)^2 \\
&= \frac{1}{4}\widehat{\Delta u_i}^2
\end{aligned}$$

So the estimated error variances are related by

$$\begin{aligned}
\hat{\sigma}^2_{u,FE} &= \frac{\sum_i \sum_t \hat{\ddot{u}}^2_{it}}{N-K} \\
&= \frac{\sum_i \sum_t \frac{1}{4}\widehat{\Delta u_i}^2}{N-K} \\
&= \frac{\sum_i \frac{1}{2}\widehat{\Delta u_i}^2}{N-K} \\
&= \frac{1}{2}\frac{\sum_i \widehat{\Delta u_i}^2}{N-K} \\
&= \frac{1}{2}\hat{\sigma}^2_{u,FD}
\end{aligned}$$

We know from part (a) that

$$\begin{aligned}
\left(\sum_i \sum_t \ddot{X}'_{it}\ddot{X}_{it}\right)^{-1} &= \left(\frac{1}{2}\sum_i \Delta X'_i \Delta X_i\right)^{-1} \\
&= 2\left(\sum_i \Delta X'_i \Delta X_i\right)^{-1}
\end{aligned}$$

And putting all these together, we have

$$\begin{aligned}
\widehat{Avar(\hat{\beta}_{FE})} &= \hat{\sigma}^2_{u,FE} \left(\ddot{X}' \ddot{X} \right)^{-1} \\
&= \frac{1}{2} \hat{\sigma}^2_{u,FD} 2 \left(\sum_i \Delta X_i' \Delta X_i \right)^{-1} \\
&= \hat{\sigma}^2_{u,FD} \left(\sum_i \Delta X_i' \Delta X_i \right)^{-1} \\
&= \widehat{Avar(\hat{\beta}_{FE})}
\end{aligned}$$

Because the estimates of the asymptotic variances are equal, the standard errors (the square roots) will be equal.

Problem 2

Question 21-3 from Cameron-Trivedi (enhanced): Consider the fixed effects, two-way error component panel data model:

$$y_{it} = \alpha + x_{it}\beta + \mu_i + \lambda_t + \epsilon_{it} \tag{2}$$

Part (a)

Show that the fixed effects estimator of β can be obtained by applying two within (one-way) transformations on this model. The first is the within transformation ignoring the time effects followed by the within transformation ignoring the individual effects. Assume the panel is balanced. (Hint: it may be easier to analyze the fixed effects regression using partitioned regression.)

We want to show that

$$\ddot{\ddot{y}}_{it} = \ddot{\ddot{x}}_{it}\beta + \ddot{\ddot{e}}_{it}$$

Ignoring time effects, we get

$$\ddot{y}_{it} = y_{it} - \overline{y_i} = y_{it} - \frac{1}{T}\sum_t y_{it}$$

$$\implies \ddot{y}_{it} = y_{it} - \frac{1}{T}\sum_t (\alpha + x_{it}\beta + \mu_i + \lambda_t + \epsilon_{it})$$

Applying the second within transformation, we get that

$$\begin{aligned}
\ddot{\ddot{y}}_{it} &= \ddot{y}_{it} - \overline{\ddot{y}_t} \\
&= \ddot{y}_{it} - \frac{1}{N}\sum_i \ddot{y}_{it} \\
&= y_{it} - \overline{y_i} - \overline{y_t} + \overline{y} \\
&= \alpha + X_{it}\beta + \mu_i + \lambda_t + \epsilon_{it} \\
&\quad - \left(\frac{1}{T}\sum_t (\alpha + X_{it}\beta + \mu_i + \lambda_t + \epsilon_{it})\right) \\
&\quad - \left(\frac{1}{N}\sum_i (\alpha + X_{it}\beta + \mu_i + \lambda_t + \epsilon_{it})\right) \\
&\quad + \left(\frac{1}{NT}\sum_i\sum_t (\alpha + X_{it}\beta + \mu_i + \lambda_t + \epsilon_{it})\right) \\
&= \beta\left(x_{it} - \overline{x_i} - \overline{x_t} + \overline{x}\right) + \epsilon_{it} - \overline{\epsilon_i} - \overline{\epsilon_t} + \overline{\epsilon}
\end{aligned}$$

Since

$$\ddot{\ddot{x}}_{it} = x_{it} - \overline{x_i} - \overline{x_t} + \overline{x}$$

and

$$\ddot{\ddot{e}}_{it} = e_{it} - \overline{e_i} - \overline{e_t} + \overline{e}$$

We get that

$$\ddot{\ddot{y}}_{it} = \ddot{\ddot{x}}_{it}\beta + \ddot{\ddot{e}}_{it}$$

Part (b)

Show that the order of the transformations is unimportant. Give an intuitive explanation for why.

Reversing the order, we can show that we get the same result

Again, we want to show that

$$\ddot{\ddot{y}}_{it} = \ddot{\ddot{x}}_{it}\beta + \ddot{\ddot{e}}_{it}$$

Ignoring individual effects, we get

$$\ddot{y}_{it} = y_{it} - \overline{y_t} = y_{it} - \frac{1}{N}\sum_i y_{it}$$

$$\implies \ddot{y}_{it} = y_{it} - \frac{1}{N}\sum_i (\alpha + x_{it}\beta + \mu_i + \lambda_t + \epsilon_{it})$$

Applying the second within transformation, we get that

$$\begin{aligned}
\ddot{\ddot{y}}_{it} &= \ddot{y}_{it} - \overline{\ddot{y}_{it}} \\
&= \ddot{y}_{it} - \frac{1}{T}\sum_i \ddot{y}_{it} \\
&= y_{it} - \overline{y_t} - \overline{y_i} + \overline{y} \\
&= \alpha + X_{it}\beta + \mu_i + \lambda_t + \epsilon_{it} \\
&\quad - \left(\frac{1}{N}\sum_i (\alpha + X_{it}\beta + \mu_i + \lambda_t + \epsilon_{it})\right) \\
&\quad - \left(\frac{1}{T}\sum_t (\alpha + X_{it}\beta + \mu_i + \lambda_t + \epsilon_{it})\right) \\
&\quad + \left(\frac{1}{NT}\sum_i\sum_t (\alpha + X_{it}\beta + \mu_i + \lambda_t + \epsilon_{it})\right) \\
&= \beta\left(x_{it} - \overline{x_t} - \overline{x_i} + \overline{x}\right) + \epsilon_{it} - \overline{\epsilon_t} - \overline{\epsilon_i} + \overline{\epsilon}
\end{aligned}$$

Since

$$\ddot{\ddot{x}}_{it} = x_{it} - \overline{x_t} - \overline{x_i} + \overline{x}$$

and

$$\ddot{\ddot{e}}_{it} = e_{it} - \overline{e_t} - \overline{e_i} + \overline{e}$$

We get that

$$\ddot{\ddot{y}}_{it} = \ddot{\ddot{x}}_{it}\beta + \ddot{\ddot{e}}_{it}$$

Intuitively, the order of the transformations is unimportant because in the end, were still manage to difference out the individual and time effects. There is nothing particular to individual or time effects that would warrant removal in any particular order.

Part (c)

Does your answer to part (a) change if the panel becomes unbalanced (i.e., contains different numbers of observations for each individual i). Why or why not?

Yes – attrition from the panel may be related to the dependent variable and we might get selection bias. To address selection bias caused by non-ignorable attrition, we would need to weight by the inverse propensity score.

Problem 3

We now begin with an actual analysis of the data. The goal here is to determine what effect, if any, primary belt laws have on the log of traffic fatalities per capita (we log the LHS variable because we believe the effect of safety belt laws should be proportional to the overall level of fatalities per capita).

```
data = read_dta('traffic_safety2.dta') %>%
    filter(state != 99) %>%
    mutate(fatal_per_cap = fatalities / population,
           vmt_per_cap = totalvmt/population)
```

Part (a)

Run pooled bivariate OLS. Interpret. Add year fixed effects. Interpret. Add all covariates that you believe are appropriate. Think carefully about which covariates should be log transformed and which should enter in levels. What happens when you add these covariates? Why?

```
# Bivariate with lm
reg_a_bivariate_lm = lm(log(fatal_per_cap) ~ primary , data = data)
# Bivariate with plm
reg_a_bivariate_plm = plm(log(fatal_per_cap) ~ primary,
                          data = data,
                          model = "pooling")
# FE with lm
reg_a_yfe_lm = lm(log(fatal_per_cap) ~ primary + factor(year), data = data)
# FE with plm
reg_a_yfe_plm = plm(log(fatal_per_cap) ~ primary,
                    data = data,
                    index = c("state", "year"),  # order matters: group-var, time-var
                    model = "within",
                    effect = "time")  # only do time

# FE + covars with lm
reg_a_full_lm = lm(log(fatal_per_cap) ~ primary + factor(year) + college + beer
                   + secondary + unemploy + log(vmt_per_cap) + log(precip) + snow32
                   + log(rural_speed) + log(urban_speed), data = data)
# FE + covars with plm
reg_a_full_plm = plm(log(fatal_per_cap) ~ primary + college + beer
                     + secondary + unemploy + log(vmt_per_cap) + log(precip) + snow32
                     + log(rural_speed) + log(urban_speed),
                     data = data,
                     index = c("state", "year"),  # order matters: unit-var, time-var
                     model = "within",
                     effect = "time")  # only do time
```

```
stargazer(reg_a_bivariate_plm, reg_a_yfe_plm, reg_a_full_plm, type=table_type,
          header=FALSE, dep.var.labels.include = FALSE, model.names = FALSE,
          column.labels = c("Bivariate", "Year FE", "Year FE + Controls"))
```

We included covariates that we believe to be correlated (both statistically and theoretically) with both the outcome (log fatalities per capita) and the treatment variable (primary). This includes weather variables, education, beer, unemployment, speed limits, and tvmt. Since tvmt is a level variable, we thought it best to divide it by population and take its log. We also logged total vehicle miles traveled per capita because we think the increase in fatalities per capita would be proportional to a percentage point increase in vehicle miles traveled per capita; not absolute levels, since the percentage point increase is a better measure of deviation

Table 1:

	Dependent variable:		
	Bivariate	Year FE	Year FE + Controls
	(1)	(2)	(3)
primary	-0.155^{***}	-0.091^{***}	0.031
	(0.027)	(0.028)	(0.020)
college			-2.387^{***}
			(0.146)
beer			0.176^{***}
			(0.025)
secondary			0.038^{**}
			(0.017)
unemploy			0.017^{***}
			(0.004)
log(vmt_per_cap)			1.154^{***}
			(0.041)
log(precip)			-0.051^{***}
			(0.012)
snow32			-0.171^{***}
			(0.014)
log(rural_speed)			0.554^{***}
			(0.127)
log(urban_speed)			0.182^{**}
			(0.091)
Constant	-1.705^{***}		
	(0.011)		
Observations	1,104	1,104	1,104
R^2	0.029	0.010	0.744
Adjusted R^2	0.028	−0.011	0.736
F Statistic	32.799^{***} (df = 1; 1102)	10.848^{***} (df = 1; 1080)	311.366^{***} (df = 10; 1071)
Note:			*p<0.1; **p<0.05; ***p<0.01

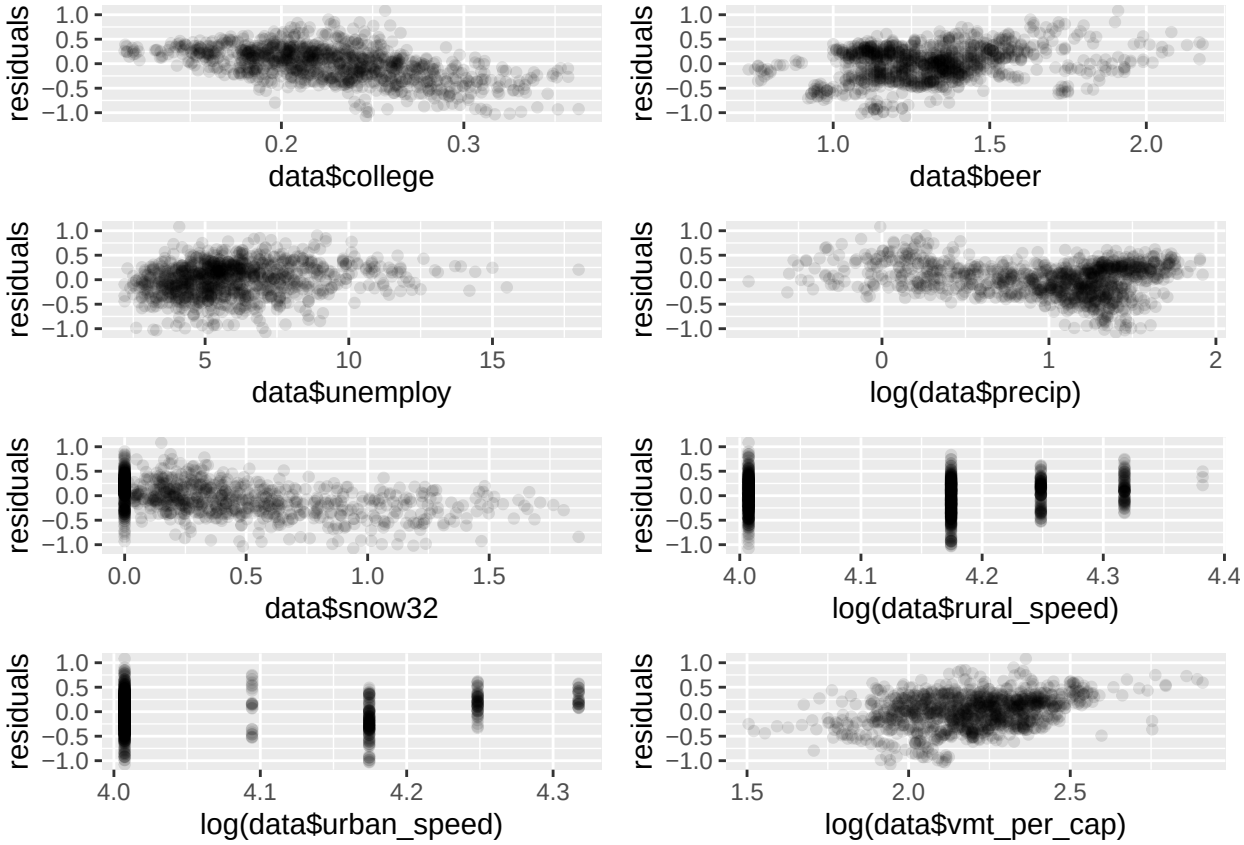
from the norm, and drivers are more likely to adjust poorly to a deviation from the norm than to an absolute increase in vehicle miles traveled. In addition, we logged precip because we care about percentage-point deviation from the norm. We did not log snow because it has zeros. Since we used per-capita measures, we did not include population. We included "secondary" in one of the regressions to identify the marginal effect of having a primary seatbelt law in addition to the secondary one where that is already in place.

Including these covariates drastically changes our point estimate. When the covariates other than secondary are include, the effect essentially disappears though remains negative. This is due to the fact that variation in fatalities per capita that we previously attributed to "primary" in the OLS regression without covariates is actually attributable to these other characteristics. Furthermore, once secondary is included, the effect remains insignificant but turns positive. If this is actually a true zero, then this could be because the secondary policy already prevents fatalities. If this is a true positive (we are less convinced), then it could be that the primary policy causes people to take less responsibility of one's own actions and depend on law enforcement. Laws can cause rebellion from teens and also unintentionally shift responsibility from the citizen to the government.

In addition, it is worth noting that when adding state fixed effects, the coefficient on primary becomes negative and significant, even with and without controlling for "secondary." In fact, it is even more negative when controlling for "secondary." This, to us, seems to identify the treatment effect since it controls for any intrinsic state characteristics that were not included and could have cause omitted variable bias. However, we are not convinced this is the correct treatment effect since the treatment may be time-varying and these policies were enacted at different times (see section notes 11/1).

Part (b)

Ignore omitted variables bias issues for the moment. Do you think the standard errors from above are right? Compute the Huber-White heteroskedasticity robust standard errors (e.g., ", robust"). Do they change much? Compute the clustered standard errors that are robust to within-state correlation (e.g., ", cluster(state)"). Do this using both the "canned" command and manually using the formulas we learned in class. Do the standard errors change much? Are you surprised? Interpret.




```
# Bivariate, Robust SEs (manual)
Sigma = diag(resid(reg_a_bivariate_plm)^2)
X = model.matrix(~ primary, data = data)
XpX = t(X)%*%X
XpXinv = solve(XpX)
XpSigmaX = t(X)%*%Sigma%*%X
var_b = XpXinv %*% XpSigmaX %*% XpXinv
SE_3b_1 = sqrt(diag(var_b))
SE_3b_1
```

```
## (Intercept)     primary
##  0.01084099  0.02829574
```

```
# Bivariate, Robust SEs (canned)
coeftest(reg_a_bivariate_lm, vcov = vcovHC(reg_a_bivariate_lm, method="white1", type="HC0"))
```

```
##
## t test of coefficients:
##
```

```
##              Estimate Std. Error   t value  Pr(>|t|)
## (Intercept) -1.704923   0.010841 -157.2664 < 2.2e-16 ***
## primary     -0.154902   0.028296   -5.4744 5.435e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Yay! We got the same as the canned standard errors.

```
# Year FE, Robust SEs (manual)
d_temp = data %>%
    mutate(resids = reg_a_yfe_lm$residuals) %>%
    dummy_cols(., select_columns = 'year') %>%
    select(primary, resids, state, contains('year_'))
k = ncol(d_temp) - 2
meat = matrix(0, nrow=k, ncol=k)
# sum over all states
# for each state, subset just that state and calc X'(ee')X
for (s in unique(data$state)) {
    X = as.matrix(d_temp %>% filter(state==s) %>% select(-state, -resids))
    e = as.matrix(d_temp %>% filter(state==s) %>% select(resids))
    meat = meat + t(X)%*% (e%*%t(e)) %*% X
}
# X'X full data
X = as.matrix(d_temp %>% select(-state, -resids))
XpX = t(X)%*%X
XpXinv = solve(t(X)%*%X)
N = length(unique(data$state))
T_ = length(unique(data$year))
var_b = XpXinv %*% meat %*% XpXinv * (N*T_ - 1) * T_ / ((N*T_ - k)*(T_ - 1))
SE_b3_2 = sqrt(diag(var_b))
rm(d_temp)
# Cluster robust standard errors
SE_b3_2[1]
```

```
##  primary
## 0.115595
```

```
# Year FE, Robust SEs (canned)
coeftest(reg_a_yfe_lm, vcov = vcovCL, cluster = ~state)[[2,2]]
```

```
## [1] 0.1142505
```

```
# Year FE + covariates, Robust SEs (manual)
d_temp = data %>%
    mutate(resids = reg_a_yfe_lm$residuals) %>%
    dummy_cols(., select_columns = 'year') %>%
    select(primary, resids, state, contains('year_'))
k = ncol(d_temp) - 2
meat = matrix(0, nrow=k, ncol=k)
# sum over all states
# for each state, subset just that state and calc X'(ee')X
for (s in unique(data$state)) {
    X = as.matrix(d_temp %>% filter(state==s) %>% select(-state, -resids))
    e = as.matrix(d_temp %>% filter(state==s) %>% select(resids))
    meat = meat + t(X)%*% (e%*%t(e)) %*% X
}
```

```
# X'X full data
X = as.matrix(d_temp %>% select(-state, -resids))
XpX = t(X)%*%X
XpXinv = solve(t(X)%*%X)
N = length(unique(data$state))
T_ = length(unique(data$year))
var_b = XpXinv %*% meat %*% XpXinv * (N*T_ - 1) * T_ / ((N*T_ - k)*(T_ - 1))
SE_3b_3 = sqrt(diag(var_b))
rm(d_temp)
# Cluster robust standard errors
SE_3b_3[1]
```

```
##  primary
## 0.115595
```

```
# Year FE + covariates, Robust SEs (canned)
coeftest(reg_a_full_lm, vcov = vcovCL, cluster = ~state)[[2,2]]
```

```
## [1] 0.04932284
```

Standard errors from specifications in part (a) is likely incorrect given we have potential correlation across observations from same state.

We observe that for simple bivariate regression and spepcification with only year fixed effects, robust standards are slightly larger. In model with controls and time fixed effects, robust standard errors are slightly smaller.

We observe that clustered SE are significantly higher than the original as well as robust SE. This suggest that correlation within state over time is seriously biasing our conventional standard errors.

From the below plots of the residuals, we can see that the variance of the residuals is not constant along many of our covariates – especially speed limits, unemployment rates, and snow levels. Heteroskedasticity is an issue we need to correct for.

Part (c)

Compute the between estimator, both with and without covariates. Under what conditions will this give an unbiased estimate of the effect of primary seat belt laws on fatalities per capita? Do you believe those conditions are met? Are you concerned about the standard errors in this case?

The between estimator is consistent in the random effects model (under the RE assumptions) but not under fixed effects. Thus, in order for this between estimator to be unbiased, individual state time-invariant effects, which now rests within the error term, cannot be correlated with the independent variables (including 'primary' and other time-varying covariates). We do not think this assumption holds here, given each states' time-invariant characteristics regarding average driving styles or fatality rate would likely impact state adoption of primary and secondary laws as well as driving conditions captured by time-varying covariates. With the between estimator, we am worried about SE, given we are down to 48 observations, one for each state, by averaging each state's observation over time.

```
data <- data %>%
    mutate(ln_fat_pc = log(fatal_per_cap), ln_vmt_pc = log(vmt_per_cap),
           ln_precip = log(precip), ln_rspeed = log(rural_speed),
           ln_uspeed = log(urban_speed))
df_within <- data %>%
    group_by(state) %>%
    summarise(across(c(college, beer, primary, secondary, population, unemploy,
                       fatalities, totalvmt, precip, snow32, rural_speed, urban_speed, fatal_per_cap,
                       ln_fat_pc, ln_vmt_pc, ln_precip, ln_rspeed, ln_uspeed), mean, na.rm = TRUE))

reg_3c_btw_nocov <- plm(ln_fat_pc ~ primary,
                        data = df_within,
                        model = "between")

# Is between estimator same as simply including time fixed effects
#reg_3c_time <- plm(ln_fat_pc ~ primary,
#                        data = df,
#                        index = c("state", "year"), model = "between", effects = "time")

reg_3c_btw_cov <- plm(ln_fat_pc ~ primary + secondary + beer + college + unemploy
                      + ln_vmt_pc + ln_precip + snow32 + ln_rspeed + ln_uspeed,
                      data = df_within,
                      index = c("state"), model = "between")

# Same Regression with lm() to check
#reg_3c_btw_cov_checkwithlm <- lm(ln_fat_pc ~ primary + secondary + beer + college + unemploy
#                                    + ln_tvmt_pc + precip + snow32 + rural_speed + urban_speed,
#                                    data = df_within)

stargazer(reg_3c_btw_nocov, reg_3c_btw_cov,
          title = "Between Estimator",
          dep.var.caption = "Log(Fatality per Population)",
          dep.var.labels.include = FALSE, model.names = FALSE,
          column.labels = c("Binary", "Covariates"),
          keep = c("primary"),
          add.lines=list(c('Covariates', 'No', 'Yes')),
          font.size = "footnotesize", column.sep.width = "1pt", no.space = TRUE, omit.stat=c("f", "ser"),
          digits = 4, type = table_type, header = FALSE)
```

Table 2: Between Estimator

	Log(Fatality per Population)	
	Binary	Covariates
	(1)	(2)
primary	−0.0936	0.0789
	(0.1641)	(0.1366)
Covariates	No	Yes
Observations	48	48
R^2	0.0070	0.8598
Adjusted R^2	−0.0146	0.8220
Note:	*p<0.1; **p<0.05; ***p<0.01	

Part (d)

Compute the Random Effects estimator (including covariates). Under what conditions will this give an unbiased estimate of the effect of primary seat belt laws on fatalities per capita? What are its advantages or disadvantages as compared to pooled OLS?

```
reg_3d_RE_bin <- plm(ln_fat_pc ~ primary,
                     data = data,
                     model = "random")

reg_3d_RE_cov <- plm(ln_fat_pc ~ primary + secondary + college + beer + unemploy
                     + ln_vmt_pc + ln_precip + snow32 + ln_rspeed + ln_uspeed,
                     data = data,
                     model = "random")

reg_3d_pooled_bin <- plm(ln_fat_pc ~ primary,
                         data = data,
                         model = "pooling")

reg_3d_pooled_cov <- plm(ln_fat_pc ~ primary + secondary + college + beer + unemploy
                         + ln_vmt_pc + ln_precip + snow32 + ln_rspeed + ln_uspeed,
                         data = data,
                         model = "pooling")

stargazer(reg_3d_RE_bin, reg_3d_RE_cov, reg_3d_pooled_bin, reg_3d_pooled_cov,
          title = "Random Effects vs Pooled",
          dep.var.caption = "Log(Fatality per Population)",
          dep.var.labels.include = FALSE, model.names = FALSE,
          column.labels = c("RE", "RE", "Pooled", "Pooled"),
          keep = c("primary"),
          add.lines=list(c('Covariates', 'No', 'Yes', 'No', 'Yes')),
          font.size = "footnotesize", column.sep.width = "1pt", no.space = TRUE,
          omit.stat=c("f", "ser"), type = table_type, header = FALSE, digits = 4)
```

Random Effects model will be unbiased when state-level unobserved heterogeneity is uncorrelated with the independent variables. If this were the case, then RE is more efficient than FE. However, as evidenced by 3a, this assumption does not not appear to hold.

Compared to pooled OLS, random effects assumes that within-state residuals are equally correlated with each other (as opposed to assuming no correlation between them in OLS). Random effects then used these residuals to compute weighted least squares, using the inverse of the variance-covariance matrices as weights. Similar to OLS, RE still assumed that across-state residuals are uncorrelated.

Table 3: Random Effects vs Pooled

	Log(Fatality per Population)			
	RE	RE	Pooled	Pooled
	(1)	(2)	(3)	(4)
primary	−0.2243***	−0.1493***	−0.1549***	−0.1407***
	(0.0167)	(0.0161)	(0.0270)	(0.0210)
Covariates	No	Yes	No	Yes
Observations	1,104	1,104	1,104	1,104
R^2	0.1411	0.5807	0.0289	0.6743
Adjusted R^2	0.1403	0.5768	0.0280	0.6713
Note:				*p<0.1; **p<0.05; ***p<0.01

The advantage of using RE is that it is more efficient than pooled OLS in the case when we have reason to think that states have exist time-invariant unobserved characteristics (which islikely the case here). However, when we think those unobservables are correlated with the independent variables, we would be better off using FE.

Part (e)

Do you think the standard errors from RE are right? Compute the clustered standard errors. Are they substantially different? If so, why? (i.e., what assumption(s) are being violated?)

```
reg_3d_RE_cov_cluster <- coeftest(reg_3d_RE_cov, vcov = vcovHC(reg_3d_RE_cov, type = "sss", cluster = "
reg_3d_pooled_cov_cluster <- coeftest(reg_3d_pooled_cov, vcov = vcovHC(reg_3d_pooled_cov, type = "sss",

stargazer(reg_3d_RE_cov, reg_3d_RE_cov_cluster, reg_3d_pooled_cov, reg_3d_pooled_cov_cluster,
          title = "Regression with Clustered SE",
          dep.var.caption = "Log(Fatality per Population)",
          dep.var.labels.include = FALSE, model.names = FALSE,
          column.labels = c("RE", "RE(Cluster)", "Pooled", "Pool(Cluster)"),
          keep = "primary",
          add.lines=list(c('Covariates', 'Yes', 'Yes', 'Yes', 'Yes')),
          font.size = "footnotesize", column.sep.width = "1pt", no.space = TRUE, omit.stat=c("f", "ser"
          type = table_type, header = FALSE, digits = 4)
```

Table 4: Regression with Clustered SE

	Log(Fatality per Population)			
	RE	RE(Cluster)	Pooled	Pool(Cluster)
	(1)	(2)	(3)	(4)
primary	−0.1493***	−0.1493***	−0.1407***	−0.1407***
	(0.0161)	(0.0324)	(0.0210)	(0.0490)
Covariates	Yes	Yes	Yes	Yes
Observations	1,104		1,104	
R^2	0.5807		0.6743	
Adjusted R^2	0.5768		0.6713	
Note:				*p<0.1; **p<0.05; ***p<0.01

```
# Test for serial correlation
# serialCorrelationTest(reg_3d_RE_cov, test = "rank.von.Neumann",
#     alternative = "two.sided", conf.level = 0.95)

pdwtest(reg_3d_RE_cov, alternative="two.sided")
```

```
Durbin-Watson test for serial correlation in panel models
```

data: ln_fat_pc ~ primary + secondary + college + beer + unemploy + ln_vmt_pc + ln_precip + snow32 + ln_rspeed + ln_uspeed DW = 0.90152, p-value < 2.2e-16 alternative hypothesis: serial correlation in idiosyncratic errors

The standard errors are different with and without clustering because there exists serial correlation in the error term (as evidenced by the Durbin-Watson test above). Clustered standard errors correct for these estimates.

Part (f)

Compute the FE estimator using only primary and year fixed effects as the covariates. Compute the normal standard errors and the clustered standard errors. If they are different, why?

```
reg_a_yfe_plm_cluster <- coeftest(reg_a_yfe_plm, vcov = vcovHC(reg_a_yfe_plm, method = "white1", type =

stargazer(reg_a_yfe_plm, reg_a_yfe_plm_cluster,
          title = "FE with Clustered SE",
          dep.var.caption = "Log(Fatality per Population)",
          dep.var.labels.include = FALSE, model.names = FALSE,
          column.labels = c("FE", "FE(Cluster)"),
          keep = "primary",
          add.lines=list(c('Covariates', 'No', 'No')),
          font.size = "footnotesize", column.sep.width = "1pt", no.space = TRUE, omit.stat=c("f", "ser"
          type = table_type, header = FALSE, digits = 4)
```

Table 5: FE with Clustered SE

	Log(Fatality per Population)	
	FE	FE(Cluster)
	(1)	(2)
primary	−0.0906***	−0.0906***
	(0.0275)	(0.0296)
Covariates	No	No
Observations	1,104	
R^2	0.0099	
Adjusted R^2	−0.0111	
Note:	*p<0.1; **p<0.05; ***p<0.01	

```
# Test for serial correlation
# serialCorrelationTest(reg_3d_RE_cov, test = "rank.von.Neumann",
#     alternative = "two.sided", conf.level = 0.95)

pdwtest(reg_a_yfe_plm, alternative="two.sided")
```

```
Durbin-Watson test for serial correlation in panel models
```

data: log(fatal_per_cap) ~ primary DW = 0.12953, p-value < 2.2e-16 alternative hypothesis: serial correlation in idiosyncratic errors

The estimate of FE with only primary and year FE is computed in 3a. The standard errors are surprisingly similar with and without clustering, though still the SE is larger in the clustered case. The Durbin-Watson test indicates that there is indeed positive autocorrelation and that it is stronger here than under the previous RE model. (Also note that we interpreted the question to be that we were asked to only estimate year FE; not state FE with year FE covariates.)

Part (g)

Add the same range of covariates to the FE estimator that you did to the OLS estimator. Are the FE estimates more or less stable than the OLS estimates? Why?

```
# OLS + covars with lm
reg_g_ols_lm = lm(log(fatal_per_cap) ~ primary + college + beer
                  + secondary + unemploy + log(vmt_per_cap) + log(precip) + snow32
                  + log(rural_speed) + log(urban_speed), data = data)

stargazer(reg_a_bivariate_lm, reg_g_ols_lm,reg_a_yfe_lm, reg_a_full_lm,
          title = "OLS and FE with covariates",
          dep.var.caption = "Log(Fatality per Population)",
          dep.var.labels.include = FALSE, model.names = FALSE,
          column.labels = c("OLS", "OLS", "FE", "FE"),
          keep = "primary",
          add.lines=list(c('Covariates', 'No', 'Yes', 'No', 'Yes')),
          font.size = "footnotesize", column.sep.width = "1pt", no.space = TRUE,
          omit.stat=c("f", "ser"), type = table_type, header = FALSE, digits = 4)
```

Table 6: OLS and FE with covariates

	Log(Fatality per Population)			
	OLS	OLS	FE	FE
	(1)	(2)	(3)	(4)
primary	−0.1549***	−0.1407***	−0.0906***	0.0309
	(0.0270)	(0.0210)	(0.0275)	(0.0204)
Covariates	No	Yes	No	Yes
Observations	1,104	1,104	1,104	1,104
R^2	0.0289	0.6743	0.1072	0.7692
Adjusted R^2	0.0280	0.6713	0.0881	0.7623
Note:				*p<0.1; **p<0.05; ***p<0.01

The FE estimates are less stable than the OLS estimates when adding covariates. This is likely due to the fact that the FE estimate without covariates is exploiting variation between states with very different characteristics. Once those characteristics are controlled for with covariates, the effect disappears. On the other hand, OLS is exploiting variation across both time and state. When the covariates are added, it takes some variation away from the state dimension, but is still (perhaps unjustifiably) exploiting the time dimension.

Part (h)

Estimate a first-differences estimator, a 5-year differences estimator, and a long differences estimator, including year fixed effects (when feasible) and the appropriate covariates in each case. Briefly describe the pattern that emerges from the three differencing estimates. Where does the FE estimate fall in this pattern? Are you surprised?

```
df_diff <- data %>% arrange(state, year)
varlist <- names(df_diff)[3:21]

# First Differences
for (i in varlist) {
    df_diff <- df_diff %>%
        group_by(state) %>%
        mutate("{i}_diff" := eval(as.symbol(i)) - dplyr::lag(eval(as.symbol(i))))
}

reg3h_1 = plm(ln_fat_pc_diff ~ primary_diff + college_diff + beer_diff
              + secondary_diff + unemploy_diff + ln_vmt_pc_diff + ln_precip_diff + snow32_diff
              + ln_rspeed_diff + ln_uspeed_diff,
              data = df_diff,
              index = c("state", "year"),
              model = "within",
              effect = "time")

# Five Year Difference Estimator
df_diff <- data %>% arrange(state, year)

for (i in varlist) {
    df_diff <- df_diff %>%
        group_by(state) %>%
        mutate("{i}_diff" := eval(as.symbol(i)) - dplyr::lag(eval(as.symbol(i)), n = 5))
}

reg3h_2 = plm(ln_fat_pc_diff ~ primary_diff + college_diff + beer_diff
              + secondary_diff + unemploy_diff + ln_vmt_pc_diff + ln_precip_diff + snow32_diff
              + ln_rspeed_diff + ln_uspeed_diff,
              data = df_diff,
              index = c("state", "year"),
              model = "within",
              effect = "time")


# Long Differences Estimator
df_diff <- data %>% arrange(state, year)

for (i in varlist) {
    df_diff <- df_diff %>%
        group_by(state) %>%
        mutate("{i}_diff" := eval(as.symbol(i)) - dplyr::lag(eval(as.symbol(i)), n= 22))
}

reg3h_3 = plm(ln_fat_pc_diff ~ primary_diff + college_diff + beer_diff
              + secondary_diff + unemploy_diff + ln_vmt_pc_diff
              + ln_precip_diff + snow32_diff
```

```
              + ln_rspeed_diff + ln_uspeed_diff,
              data = df_diff,
              index = c("state", "year"),
              model = "pooling")

# Table
stargazer(reg3h_1, reg3h_2, reg3h_3,
          title = "First, Second, and Long Difference Estimator\\label{tab:differences}",
          dep.var.caption = "Differences in Log(Fatality per Population)",
          dep.var.labels.include = FALSE, model.names = FALSE,
          column.labels = c("1st", "5th", "Long"),
          add.lines=list(c('Time FE', 'Yes', 'Yes', 'No')),
          font.size = "footnotesize", column.sep.width = "1pt", no.space = TRUE,
          omit.stat=c("f", "ser"), type = table_type, header = FALSE, digits = 4)
```

Table 7: First, Second, and Long Difference Estimator

	Differences in Log(Fatality per Population)		
	1st	5th	Long
	(1)	(2)	(3)
primary_diff	−0.0456**	−0.0764***	−0.0910
	(0.0225)	(0.0146)	(0.1622)
college_diff	−0.6400	−0.4247	−0.2613
	(0.5299)	(0.2647)	(0.9933)
beer_diff	0.2648***	0.5018***	0.6559***
	(0.0697)	(0.0528)	(0.1725)
secondary_diff	−0.0377***	−0.0405***	0.0335
	(0.0136)	(0.0094)	(0.1624)
unemploy_diff	−0.0140***	−0.0278***	−0.0450**
	(0.0039)	(0.0027)	(0.0168)
ln_vmt_pc_diff	0.2896***	0.2930***	0.3315
	(0.0908)	(0.0670)	(0.2394)
ln_precip_diff	−0.0815***	−0.0465**	−0.1113
	(0.0132)	(0.0182)	(0.1405)
snow32_diff	0.0127	−0.0008	0.1690**
	(0.0100)	(0.0132)	(0.0773)
ln_rspeed_diff	−0.2438**	−0.0923	−0.0345
	(0.1158)	(0.0749)	(0.6765)
ln_uspeed_diff	0.1176	0.1706***	0.1655
	(0.0888)	(0.0604)	(0.2812)
Constant			−0.4790*
			(0.2693)
Time FE	Yes	Yes	No
Observations	1,056	864	48
R^2	0.0855	0.3409	0.6480
Adjusted R^2	0.0578	0.3196	0.5528
Note:			*p<0.1; **p<0.05; ***p<0.01

From Table 7, we observe that 1st and 5th difference estimators give very similar or slightly more negative coefficients estimates, while standard errors go down significantly. Perhaps, this may mean that effect of implementing primary law is more volatile but stabilizes over longer term period. Long term difference estimator has the most negative coefficient. Another key feature to notice is that for the long difference estimator, we are taking the difference between the first and last year for each state, and we are left with one observation per state. Hence, all of the standard errors increase, and some, quite significantly.

The fixed effects estimator from earlier parts seems to fall between the first and long differences estimator; however, it is very close to the long difference estimator.

Part (i)

Make the case that the first-differences estimate is superior to the 5-year or long differences estimates.

First differences provides the researcher with more observations than the 5-year long differences and therefore potentially more power.

Part (j)

Make the case that the 5-year or long differences estimates are superior to the first-differences estimate.

Five-year differences could potentially identify a larger effect than first-differences, though with less power (fewer observations). This may also be the strategy a researcher would want to take if they suspect that the treatment to take a while to fully take effect.