

### Note on Roback's definition of productive amenities

In the N=2, homogeneous firm, homogeneous consumer model, Roback (1982) defines an "unproductive amenity" such that **the partial derivative of the firm cost function is negative**. I believe that is a typo and the partial derivative **should be positive**. The main reason I bring this up is for internal consistency – Roback describes several important results in terms of the productive or unproductive amenity, and the definition of a productive amenity in the original paper leads to inconsistencies later in the paper when describing the unambiguous and ambiguous signs of the effects of changes in the amenity. Below I document these inconsistencies to see if you agree that the definition is incorrect.

#### Inconsistency 1: disagrees with original 1980 dissertation

Roback (1982) says on page 1260 that **If the amenity is unproductive, then  $C_s < 0$ .**

In Roback's 1980 dissertation, Chapter II (The Simple Model) is basically the same as the 1982 paper. The same paragraph from the dissertation says

If the amenity is productive, then  $C_s$  is negative.

Figure 1 also changed from showing an outward shift in the cost function to showing an inward shift in the cost function. The discrepancy with  $C_s < 0$  vs  $C_s > 0$  just appears to be a typo, where Roback was changing everything from productive to unproductive and forgot to switch the inequality. This makes sense when combined with the reasons below.

#### Inconsistency 2: intuitive effects on the cost function

Roback (1982) says on page 1260 that **If the amenity is unproductive, then  $C_s < 0$ .**

Then goes on to say

An example of an "unproductive amenity" is clean air, because firms must spend resources to use a nonpolluting technology. An example of a "productive amenity" might be "lack of severe snow storms" because blizzards may be as costly to the firm in inconvenience and lost production as they are unpleasant to consumers.

By the description above, an unproductive amenity is one that increases costs (spending resources to be at a location with higher levels of the unproductive amenity). A productive amenity is one that decreases costs – an increase in the "lack of severe snow storms", or a decrease in severe snow storms, decreases costs. Thus, holding wages and rental costs fixed, increasing an unproductive amenity must increase costs, implying that  $C_s > 0$  (not  $< 0$ , as stated in the paper).

### Inconsistency 3: disagreement between the textual description and the mathematical note

On page 1261, figure 1 displays cost and utility curves. Fig 1 is explained in the text to be displaying changes in the curves for an **unproductive** amenity. It is also clearly explained in the text on pg 1262 that

The figure clearly shows that in more amenable places, the wages should be lower while the change in rents is uncertain. The intuitive reason for this is that with  $s$  unproductive, firms prefer low  $s$  locations while workers prefer high  $s$  locations.

This description implies that  $dw/ds < 0$  and  $dr/ds$  is ambiguous, for an unproductive amenity.

However, below equation 4, the inequalities are explained with this description:

... with  $C_s > 0$ ,  $dw/ds < 0$ , while  $dr/ds$  depends on the relative strengths of the productivity and amenity effects.

By taking total derivatives of the utility and cost indifference conditions, I have verified the mathematical description above for the case of  $C_s > 0$ . So, **the case of  $C_s > 0$  matches the textual and figure descriptions of an “unproductive” amenity.** Thus  $C_s > 0$  should be the unproductive definition, and  $C_s < 0$  should be the productive definition.

I have also verified the inequalities in the case that  $C_s < 0$ : here  $dr/ds > 0$  and  $dw/ds$  is ambiguous. This matches the following textual description of the productive amenity:

... if  $s$  were productive, the rents would rise while the change in wages would be ambiguous.

### Conclusion

I suggest that this was just a small typo in the last paragraph of page 1260. If the sentence was changed to say “If the amenity is unproductive, then  $C_s > 0$ .”, it would solve all the small inconsistencies.