Problem Set Two — Solution Comments
ARE 264: Environmental and Resource Economics: Spring 2022

Problem 1: Co-benefits and pre-existing policies

I don't have a single setup in mind for this case, so rather than push a single model think it is worth commenting a bit on what people did.

One version of this problem involves thinking about two externalities, that are both tied directly and in the same way to a given outcome. In its simplest form, some good x could have (for example) a climate externality equal to ϕ and a local air pollution externality equal to γ. One could interpret the prompt as saying that there is some policy that targets local air pollution but set a tax different than γ, then ask what is the optimal tax when you are thinking about carbon.

This is not a very "rich" problem, in that the solution is very simple. As described here, that problem is the same as a generic externality problem with marginal damages equal to $\phi + \gamma$ with a preexisting tax equal to τ. In this case, you can get the first best by setting a tax equal to $t^{FB} = \phi + \gamma - \tau$. This just says to put a tax on whatever portion of the externality isn't already taxed. This is not what I had in mind, but it is a reasonable interpretation of the prompt! And, this in fact yields some of the key insight, which is that there if there are co-benefits *that are not taxed already* then they should be incorporated into the tax on the primary pollutant of interest. In contrast, if there are co-benefits, but they have already been "priced" by some policy, then this does not affect the optimal tax (back to the Pigouvian benchmark).

What makes the problem richer? Lots of ways to make the problem more interesting. One is to allow multiple margins of behavior/abatement. If there is an activity that abates one of the pollutants and not the other, then the problem gets a bit more interesting. One person had a Diamond model where there were two different goods that each generated heterogenous externalities; with two separate derivatives, this yields an extended and intuitive version of Diamond's result.

Some students are explicit in modeling a cap on one of the externalities, i.e., there is a pre-exiting cap on an externality and then there is a related externality that we might be taxing. This is an interesting situation. Note critically here that the shadow price on the cap for pollutant y would be a function of the tax on pollutant z, because a tax that changes z will change the private choice of y, which affects the shadow price.

Problem 2: Externalities with monopoly

Given the equations, you can first describe the monopolist's optimality condition by differentiating profits and arranging the FOC:

$$\frac{\partial \pi}{\partial q} = p'q + p - c' - t = 0$$

$$\Rightarrow p - c' = t - p'q.$$

Then, the planner's first-order comes from differentiation:

$$\frac{\partial W}{\partial t} = \frac{\partial q}{\partial t}\left[p(q) - c'(q) - \phi\right] = 0.$$

$$\Rightarrow p - c' = \phi$$

Substituting in the monopolist's efficiency condition eliminates $p - c'$:

$$t - p'q = \phi$$

$$\Rightarrow t^{SB} = \phi + p'q,$$

where q and $p'(q)$ are evaluated at the second-best value of q.

This means that the optimal tax is the sum of marginal damages and a second term. $p'q < 0$, assuming downward sloping demand. So, the optimal tax will be below the Pigouvian benchmark. That is, the existence of market power attenuates the Pigouvian tax. (It would accentuate the subsidy if q yielded a positive externality.)

It is possible to overturn the tax, so that the optimal policy is a subsidy, even when the good generates a negative externality. This occurs when $|p'q| > \phi$; recall that $p'q = p - c'$, that is, this wedge represents the gap between price and marginal cost. So, a subsidy is optimal when the harm from the dirty good is smaller than the (marginal) welfare loss from restricting quantities. If these happen to offset exactly, then it could be that the monopolist is restricting supply by the exact amount desired by the planner, in which case no policy is optimal.

Problem 3: Optimal subsidy for two goods

1. You can setup and solve a model here if you like, but the answer is immediately obvious and it is okay to jump to it. The Pigouvian tax is optimal: $s_1^* = s_2^* = \phi$.

2. No, this would not change my answer to part (a) because this is still a first-best setting. In the first-best, if both externalities are corrected, then the Pigouvian tax is still optimal, regardless of cross-price derivatives. (The answer would differ if one or the other product were untaxed.)

3. Planner's Lagrangean, and FOC are below. Planner chooses s_1 and s_2. **Note that s_1 has no effect on x_2 (and vice-versa) because of separability and quasi-linearity** (assuming interior solutions; you can see this by looking at the consumer's FOCs):

$$\mathcal{L} = u(x_1) + v(x_2) + \phi(x_1 + x_2) + M - c_1 x_1 - c_2 x_2 + \lambda(R - s_1 x_1 - s_2 x_2)$$

$$\begin{aligned}
&(u' + \phi - c_1 - \lambda s_1)\frac{\partial x_1}{\partial s_1} - x_1 \lambda = 0 \\
\Rightarrow &\tilde{s}_1 = \phi \frac{1}{1+\lambda} - \frac{\lambda}{1+\lambda}\left(1 + \frac{x_1}{\partial x_1 / \partial s_1}\right)
\end{aligned}$$

The transformation uses the consumer's FOC: $u' - c_1 = -s_1$. Then, rearranging yields the result. This formulation is not properly closed form, but it shows additivity; the optimal subsidy is marginal benefits plus some other term. The other term has to do with the revenue constraint. In this form, it is a little tricky, but starting to see that where the revenue constraint does not bind, $\lambda = 0$ and the result collapses to Pigou, but you can see that immediately in the FOC (it is harder later because you have divided by zero...). Obviously, the formula for $\tilde{s}_2$ is parallel.

4. Without algebra: the ideal answer here invokes the lessons we've learned about inframarginal subsidy takers (free riders), in the spirit of the Boomhower and Davis paper from class. We want to maximize the marginal increases, and raising the subsidy requires us to give an increase subsidy to all the inframarginals. So, we would put more subsidy towards the product with lower market share, all else equal.

   Algebraically, we can write $\tilde{s}_1 - \tilde{s}_2$:

$$\begin{aligned}
\tilde{s}_1 - \tilde{s}_2 &= \left[\phi \frac{1}{1+\lambda} - \frac{\lambda}{1+\lambda}\left(1 + \frac{x_1}{\partial x_1/\partial s_1}\right)\right] - \left[\phi \frac{1}{1+\lambda} - \frac{\lambda}{1+\lambda}\left(1 + \frac{x_2}{\partial x_2/\partial s_2}\right)\right] \\
&= \frac{\lambda}{1+\lambda}\left(\frac{x_2}{\partial x_2/\partial s_2} - \frac{x_1}{\partial x_1/\partial s_1}\right)
\end{aligned}$$

   If the denominators are equal (same own-price derivative), then:

$$\tilde{s}_1 - \tilde{s}_2 = \frac{\lambda}{1+\lambda}\frac{1}{\partial x_1/\partial s_1}(x_2 - x_1).$$

   λ is positive. The own-price derivative is negative (must be true, as there is no income effect). So, the multiplier is negative. Thus, as x_1 rises, $\tilde{s}_1$ falls (relative to $\tilde{s}_2$). This is the same result as the rough intuition above: because of the revenue constraint, you want to offer a smaller subsidy to the more popular product (all else equal).

5. Without algebra: the intuition here is that you want to subsidize the more responsive product. So, this will cause a relatively larger subsidy to product one. The reasoning is the same, in a second-best setting with a revenue constraint, you want to subsidize the product that generates a larger response.

   Algebraically, this version is:

$$\tilde{s}_1 - \tilde{s}_2 = x_1 \frac{\lambda}{1+\lambda} \left(\frac{1}{\partial x_2 / \partial s_2} - \frac{1}{\partial x_1 / \partial s_1} \right).$$

   The term out front is positive. As the inside denominator on the left (good 1) gets larger, that term shrinks, which raises the total inside the parentheses (the larger negative term is on the right, which is subtracted, so the whole thing becomes larger and positive as good one becomes relatively more price sensitive).