Problem Set One, Suggested Solutions
ARE 264: Empirical Energy and Environmental Economics: Spring 2021

Problem 1: Clean subsidies versus dirty taxes

It is not possible to get the socially optimal allocation due to the fact that there are two margins of adjustment and the policy instrument targets only one of them. Specifically, firm j's problem (assuming competitive pricing) is:

$$\max_{x_j, a_j} \pi_j = px_j + sa_j - g_j(a_j) - c(x_j)$$

FOCs yield:

$$c'_j = p \qquad g'_j = s.$$

The planner's problem is:

$$\max_{\mathbf{x},\mathbf{a}} W = U\left(\sum_j x_j\right) - \phi\sigma_j\left(e_j(x_j) - a_j\right) - \sum_j c_j(x_j) - \sum_j g_j(a_j).$$

This has FOCs:

$$u' - c'_j = \phi e'_j \ \forall j$$
$$g'_j(a_j) = \phi \ \forall j.$$

In competitive markets, $p = u'$. So, the firm will set $c' = p$, and the abatement subsidy can do nothing to affect this margin. (It can do nothing "directly", and, in this model, the costs of abatement and production are completely separable; if the cost of abatement were related to quantities, then there would be some way to influence q via the incentive of a.)

A direct tax on the externality would generate efficient behavior on both margins, but the subsidy to abatement activities cannot decentralize the optimum because it can affect only the abatement decision margin, leaving the quantity choices suboptimal.

A subsidy to abatement can achieve the socially optimal levels of abatement activity at each firm. As shown above, the planner wishes every firm to abate until marginal cost (g'_j) equals the externality. This is achieved with a subsidy of $\phi = s$, which creates the right decision-rule for each firm in their FOC. Again, this result is "clean" because a and x are separable in the cost function for the firm, so not only are the marginal incentives aligned, by the first-best levels of a_j are achieved (despite the x_j being "wrong").

Problem 2: Variance and value

In past years I assigned my own version of this problem. That prompt and the questions I led students through is pasted here. The answers are described after that. This is certainly not the only (and surely not the very best) way to illustrate the concepts.

Old version of the question

The first-best Pigouvian tax level for a good that causes an externality does not depend on the elasticity of supply and demand. But, the welfare gain achieved by the implementation of the tax does.

Similarly, the first-best Pigouvian tax does not depend on the heterogeneity in the cost of abatement opportunities. But, the relative efficiency of a tax versus a command-and-control alternative does depend on the degree of heterogeneity.

This exercise asks you to illustrate these two results in a specific example.

The model is an abatement cost model, with no quantity margin. A continuum of firms with mass one have a constant marginal cost of abatement parameter c^i distributed uniformly on $[l, h]$. Each firm emits one unit of pollution, and it can abate at marginal cost $c^i \times a^i$), where $a^i \in [0, 1]$. That is, firms have constant marginal cost of abatement equal to their index number, and they can abate only up to 1 unit each (because that is their baseline emission). So the lowest cost firm can abate up to 1 unit; it would cost l to do so. If they abated 0.5 units, the cost would be $l/2$. Suppose that the social damage per unit of emission is $(l + h)/2$ throughout the problem.

To illustrate how the welfare gain depends on price sensitivities:

1. Derive the abatement cost curve (i.e., the marginal cost curve of abatement) of this model. What is the slope of this curve?
2. Calculate the welfare gain of the optimal Pigouvian subsidy.
3. Relate this gain to the Harberger triangle; in particular explain how the gain depends on the price derivative of abatement.
4. Draw a graph that illustrates all of the points of this exercise. (Note: drawing a graph may help you solve the prior steps.) The intended audience of this graph is not me; it is an economist unfamiliar with this point. Annotate the graph as necessary to make it clear as a stand alone diagram.

To illustrate how efficiency gain from a tax versus alternatives depends upon heterogeneity:

5. Calculate the cost of a command and control policy that requires all firms to abate 30% of their emissions ($a^i = 0.3\ \forall i$).
6. Find the subsidy level that leads to the same emissions reductions as the command and control policy. Calculate its cost.
7. Show how the difference in cost between command and control and the subsidy depends upon the variance of the cost distribution.

8. Draw a graph that illustrates this point. The intended audience of this graph is not me; it is an economist unfamiliar with this point. Annotate the graph as necessary to make it clear as a stand alone diagram.

Answers to the old version

1. The marginal cost of abatement curve is a straight line. Moving from a marginal cost of l to a marginal cost of h spans 1 unit of aggregate abatement, so the slope of the line is $h-l$; it's y-intercept is l. Thus, the marginal abatement cost curve denoted, c' is: $c'(Q) = l + (h-l)Q$.

2. The optimal Pigouvian tax is equal to marginal damages, assumed to be $(h-l)/2$. This will generate abatement of $1/2$. The total cost of abating $1/2$ is the integral of the cost function from 0 to 1/2: $\int_0^{1/2} l + (h-l)Qdq$. This evaluates to:

$$\left[\frac{h-l}{2}Q^2 + lQ\right]_0^{1/2} = \frac{h-l}{8} + \frac{l}{2}.$$

   The benefits are just the marginal benefit times quantity: $\frac{h+l}{2} \times 1/2$. Subtracting costs from benefits (with a little fractions work) yields: $(h-l)/8$.

3. This is a Harberger triangle. The height of the Harberger triangle is the gap between the minimum cost and the social benefit ($(h+l)/2 - l = (h-l)/2$); the width is the total abatement (1/2). So, $1/2\times$ width $\times$ height is $1/2 \times (h-l)/2 \times 1/2 = (h-l)/8$. Steeper slopes will imply larger welfare gains, as the triangle will be larger (holding constant the optimal abatement at 1/2; so the base/width of the triangle is constant but the height will rise with the steepness of the slope).

4. See graph:

5. If all firms abate 30%, then you just calculate the cost of all abatement and multiply by .3: $0.3 \times \int_0^1 (h-l)q + ldq = 0.3\frac{h+l}{2} = .15(h+l)$.

6. The subsidy level will be the marginal cost of abatement at 0.3 under efficiency; or, t^* solves $t^* = (h-l)0.3 + l$. Or, $t^* = 0.3h + 0.7l$. The cost of abating this amount is the same integral evaluated at the new quantity 0.3:

$$\left[\frac{h-l}{2}Q^2 + lQ\right]_0^{0.3} = 0.045(h-l) + 0.3l.$$

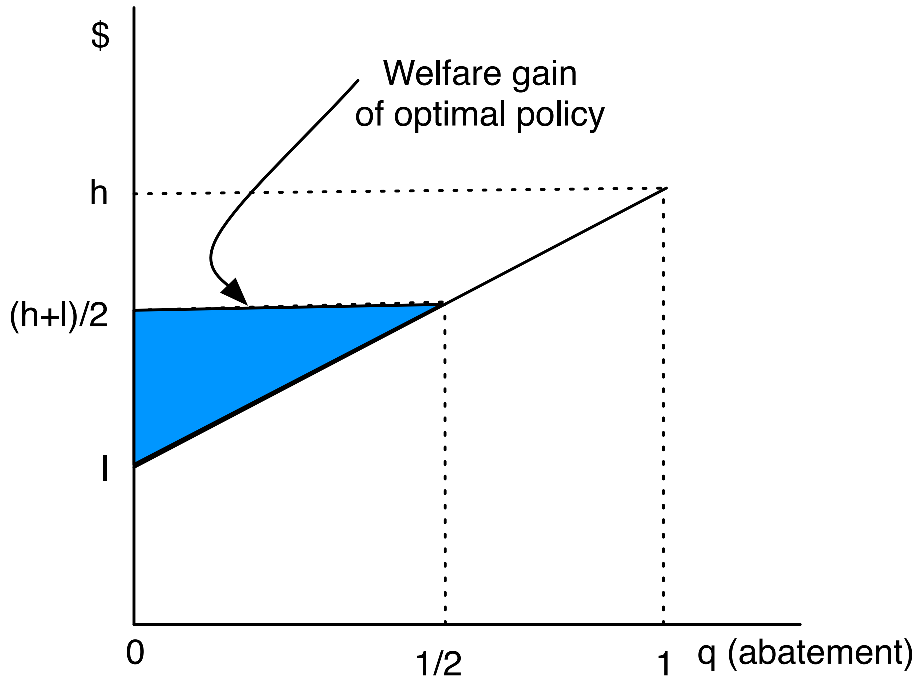
7. The difference in costs is found by subtracting the two prior results:

$$D = .15(h+l) - 0.045(h-l) + 0.3l = .105(h-l).$$

   The variance of the cost distribution is isomorphic to the slope of the marginal cost curve $(h-l)$. Specifically, the variance of the uniform distribution is $1/12 \times (h-l)^2$. So, the standard deviation is a scalar times $(h-l)$. Thus, we can illustrate the effect of a greater variance in the diagram via a steeper slope (which we already know from above implies a larger Harberger triangle...).

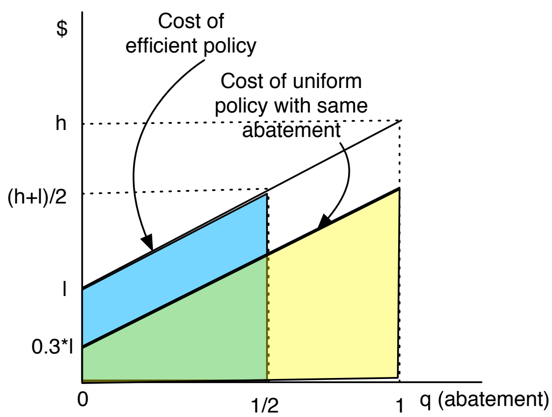
   In the limit, if there is no heterogeneity (there is a mass of identical firms at some cost, so $h = l$), the two policies would have the same welfare costs.

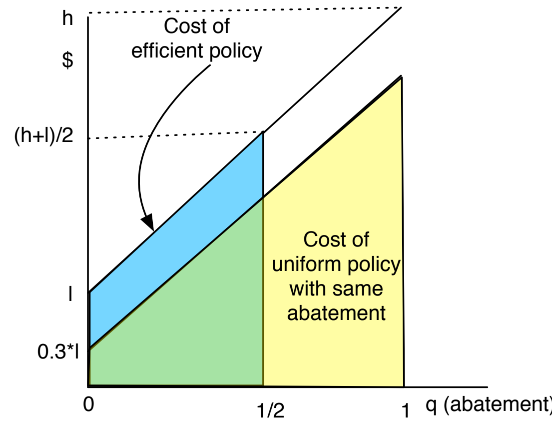
Figure 1: Abatement curve and efficiency gains



8. See graph. (Green areas are costs included in both policies.)

Figure 2: Implications of variance for relative efficiency of policies:
left graph has less heterogeneity than right





Problem 3: Two related dirty goods

As prompted, I think there is one way to make this problem very straightforward. Just assume there are two goods and utility depends on both $U(X,Y)$. This setup is generic in terms of the substitution pattern between the goods. Let those goods be supplied by a competitive market at fixed prices P_X and P_Y and let each have an externality ϕ_X and ϕ_Y. There is a fixed tax on Y that is below marginal damages and is not a choice variable for the planner t_Y. Then, the question is how to find the second-best tax on X.

To keep things simple, just assume there is some other numeraire z and fixed income I for a representative consumer. Then, the consumer will choose so that marginal utility of each good is equal to the tax inclusive price, which means $t_X = U_X - P_X$.

The planner's problem is then:

$$\max_{t_X} U(X,Y) - \phi_X X - \phi_Y Y + (I - P_X X - P_Y Y)$$

You can take the FOC wrt t_X, just writing the change in behavior as the reduced form demand curve:

$$(U_X - \phi_X - P_X)\frac{\partial X}{\partial t_X} + (U_Y - \phi_Y - P_Y)\frac{\partial Y}{\partial t_X} = 0.$$

Substituting for the consumer's FOC yields wedges between tax rates and externalities:

$$(t_X - \phi_X)\frac{\partial X}{\partial t_X} + (t_Y - \phi_Y)\frac{\partial Y}{\partial t_X} = 0.$$

Rearrange to solve for t_X:

$$t_X^{SB} = \phi_X + (t_Y - \phi_Y)\frac{-\frac{\partial Y}{\partial t_X}}{\frac{\partial X}{\partial t_X}}.$$

This says that the optimal tax on X is marginal damages, plus a term that deals with the other good. That second term disappears if either (a) the tax on the other good is set to marginal damages, or (b) there is no substitution, so that $\partial Y/\partial t_X = 0$. In those cases, the model collapses back to the Pigouvian benchmark, which makes sense because in (a) there is no distortion in the other market, so substitution does not affect the optimal policy or (b) there is no substitution so the other market is unaffected.

Otherwise, with substitute goods, the ratio of derivatives should be positive (with the negative sign is included; an increase in the tax on X raises Y but lowers X). That means that when Y is "undertaxed", then the additional term will be negative, implying that the optimal tax is less than the Pigouvian benchmark of marginal damages. If the substitution is very strong and the externality in Y is sufficiently large and undertaxed, it is possible that the optimal tax on X is actually a subsidy.