Economics 202A
Macroeconomics
Fall 2022

Problem Set 1

Due by 9:00 on September 6, 2022
Please upload solutions to Gradescope

You are allowed to work in groups. But please write up your own solutions. Please note that copying from old solutions constitutes plagiarism. Please write up your solutions as clearly as possible. Your grade will be reduced if your solution is unreasonably difficult to follow.

1. **Basic Properties of Growth Rates**: Do problem 1.1 in Romer (2019).

2. **Constant-Elasticity-of-Substitution Production Functions**: Consider the production function

$$Y = F(K, L) = A\left[aK^{\phi} + (1-a)L^{\phi}\right]^{1/\phi} \tag{1}$$

where $0 < a < 1$ and $\phi < 1$.

Recall that the elasticity of substitution between capital and labor is the curvature of the isoquants of the production function. The slope of an isoquant is

$$\text{Slope} = \frac{\partial F(\cdot)/\partial K}{\partial F(\cdot)/\partial L}$$

The elasticity is then

$$\left[\frac{\partial(\text{Slope})}{\partial(L/K)}\frac{L/K}{\text{Slope}}\right]^{-1}$$

a. Show that the elasticity of subtitution of the production function (1) is constant and equal to $1/(1-\phi)$.

Notice that when the elasticity of substitution approaches zero ($\phi \to -\infty$) this production function approaches the "Leontief" production function $Y = A\min[K, L]$ and when the elasticity of substitution becomes infinite ($\phi = 1$) this production function becomes linear $Y = A[aK + (1-a)L]$ so that K and L are perfect substitutes.

b. Show that when the elasticity of substitution approaches 1 ($\phi \to 0$) this production function approaches the Cobb-Douglas form $Y = (\text{constant})K^{a}L^{1-a}$. (Hint: You will need to use l'Hopital's rule.)

It is common to refer to the case where the elasticity of substitution is larger than one as the case where labor and capital are "gross substitutes" (or just "substitutes" for short) and the case where the elasticity of substitution is smaller than one as the case where labor and capital are "gross complements" (or just "complements" for short).

c. Divide through equation (1) by L and rewrite the production function in terms of output per person y and capital per person k. Let's refer to this production function as $y = f(K)$.

d. Derive expressions for $f'(k)$ and $f(k)/k$.

e. Consider the case where capital and labor are gross substitutes ($0 < \phi < 1$). Does the production function satisfy the Inada conditions?

Recall that

$$\frac{\dot{k}}{k} = \frac{sf(k)}{k} - (n + \delta)$$

where s is the savings rate, n is the population growth rate, and δ is the depreciation rate of capital.

f. Can capital accumulation result in sustained growth in this economy given a constant savings rate? (Hint: Plot $sf(k)/k$ and $(n + \delta)$ as a function of k on the same figure.)

3. **The Harrod-Domar Model**: Prior to the development of the Solow model, work on development and growth often used a Leontief production function

$$Y = F(L, K) = \min(AK, BL), \tag{2}$$

where $A = 0$ and $B = 0$. As we discussed above, this corresponds to a CES production function with $\phi \to -\infty$. Well known papers to use this type of production function are Harrod (1939) and Domar (1946). Growth models that use this production function have since usually been referred to as Harrod-Domar models. Harrod and Domar predicted that capitalist economies would suffer from persistent problems of unemployment of either labor or machines. We now explore how this conclusion follow from the production function (2).

a. Rewrite the production function (2) in terms of per capita output y and capital per capita k. Plot the resulting production function as a function of k. Also plot the marginal product of capital as a function of k.

b. Recall that with a constant savings rate s we have

$$\frac{\dot{k}}{k} = \frac{sf(k)}{k} - (n + \delta)$$

Consider first the case where $sA < (n + \delta)$. Plot $sf(k)/k$ and $n + \delta$ as a function of k. Discuss the dynamics of this economy starting from some initial capital stock k_0. Comment in particular on the degree of unemployment of labor in the long run in such an economy.

c. Now consider the case were $sA > (n + \delta)$. Again, plot $sf(k)/k$ and $n + \delta$ as a function of k. Discuss the dynamics of this economy starting from some initial capital stock k_0. Comment on the degree of unemployment of labor in the long run as well as the possible idleness of capital in the long run.

d. Is it possible for both capital and labor to be fully employed in a Harrod-Domar economy in the long run?

References

DOMAR, E. D. (1946): "Capital Expansion, Rate of Growth, and Employment," *Econometrica*, 14, 137–147.

HARROD, R. F. (1939): "An Essay in Dynamic Theory," *Economic Journal*, 49, 14–33.