Economics 202A
Macroeconomics
Fall 2021

Problem Set 9
Due by 11:59pm on November 22, 2022
Please upload solutions to Gradescope

You are allowed to work in groups. But please write up your own solutions. Please note that copying from old solutions constitutes plagiarism. Please write up your solutions as clearly as possible. Your grade will be reduced if your solution is unreasonably difficult to follow.

In problem set 8, you solved a (modified) consumption-savings problem using the methods exposited in Alisdair McKay's numerical analysis notes. It turned out that these methods did not work well for the consumption-savings problem we are studying.

In this problem set, you will solve a variant of this same problem with three differences. First, the problem in this problem set will include a much tighter borrowing constraint: the household is not allowed to borrow at all. Second, we incorporate unemployment into the problem. Third, I ask you to use a somewhat different numerical method. In certain ways, the numerical method you will use in this problem set is simpler than the one we used in problem set 8. It is also more accurate. You may ask: Why didn't we use this method from the get go? There are two reasons. First, it is useful for you to know about polynomial approximations for other numerical applications you may encounter. Second, pedagogically, it was useful to follow McKay's notes as closely as possible the first time around.

We start with a description of the problem. Consider the following household consumption-savings problem with uninsurable idiosyncratic income risk. The household's utility function is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma},$$

where C_t denotes consumption, β is the household's subjective discount factor, and γ is the coefficient of relative risk aversion and also the reciprocal of the elasticity of intertemporal substitution. The household starts off with assets A_0 and each period receives labor income Y_t. Labor income is affected by two types of risks. First, the household may become unemployed and receive only unemployment benefits b. When the household is employed, its labor income follows an AR(1) process. These assumptions can be represented mathematically as

$$Y_t = \begin{cases} \tilde{Y}_t, \text{if employed} \\ b, \text{if unemployed} \end{cases}$$

where

$$\log \tilde{Y}_t = (1-\rho)\log\mu + \rho\log\tilde{Y}_{t-1} + \epsilon_t$$

and $\epsilon_t \sim \mathrm{N}(0, \sigma^2)$. (Here $\log$ denotes the natural logarithm.) The parameter μ denotes the unconditional mean of labor income when employed, ρ is the persistence of fluctuations in labor income when employed, and σ^2 is the variance of innovations to labor income when employed. When a household is employed, its probability of becoming unemployed next period is p. When a household is unemployed, its probability of becoming employed again next period is q.

The household has access to a risk-free technology for borrowing and lending. We assume for simplicity that the interest rate is the same for borrowing and landing and use r denote that interest rate. Each period, the household faces a choice of how much to consume and how much to borrow or save. Its budget constraint is therefore

$$C_t + \frac{A_{t+1}}{1+r} = Y_t + A_t.$$

and the borrowing constraint is

$$A_t \geq 0.$$

The rest of the problem set asks you to solve this problem using value function iteration. The solution will be quite similar to the solution to problem set 8, but there are a few differences, which I will describe.

When solving the problem, please consider the following set of parameter values to be the baseline parameter values and use these unless otherwise instructed: $\gamma = 2$, $\beta = 0.98$, $\mu = 1$, $b = 0.4$, $\rho = 0.9$, $\sigma^2 = 0.05$, $p = 0.05$, $q = 0.25$, and $r = 0.01$.

A. Write the household's problem recursively. Be sure to state what variables are chosen and all the constraints. Hints: This problem has three state variables: assets brought into the period A, income if employed $\tilde{Y}$, and whether the household is employed. But one of these state variables only takes two values (whether the household is employed). The easiest way to solve the problem is therefore to think of the household has having two value functions V_e and V_u representing the value if employed and the value if unemployed. Each of these functions is then a function of two variables (A and $\tilde{Y}$).

B. The most important difference between the solution method we use in this problem set and in problem set 8 is that we will not use the polynomial functional approximation we used in problem set 8. We will therefore not create a PolyBasis function this time around.

C. We will also not create a PolyGetCoeff function this time around.

D. Start your main Matlab program by reading in the parameter values into a structure.

E. Create a grid on A and $\tilde{Y}$. Feel free to use McKay's tauchen function as needed. Use 7 grid points for $\tilde{Y}$ and 1,000 grid points for A. (Please create the grid for $\tilde{Y}$, not $\log \tilde{Y}$. Uniformity will make grading easier.) Choose reasonable values for the size of the grid (i.e., min and max points for each dimension). **Note that the lower bound of the A grid in this case should be zero, since the borrowing constraint prevents A from being negative.** As in problem set 8, it is important to choose a large enough value for the maximum of the A grid to get an accurate solution. But at the same time, choosing something excessively large is inefficient. Some trial and error is probably necessary here.

F. Write a Bellman matlab function for this problem. Since we have two value functions (V_e and V_u), the best way to go here is to write two functions BellmanE and BellmanU, one for each value function.

These Bellman functions should be quite similar to the Bellman functions you used in problem set 8. The main difference is that you will not be passing the polynomial coefficients (which McKay denotes by `b`) to the function. Instead, you will have arguments that you may want to call `EVe` and `EVu` (which will replace the `be` and `bu` you had in problem set 8).

Recall that `be` and `bu` were vectors of polynomial coefficients, which you used to describe your approximations of the expected value functions $E[V_e(A_{t+1}, \tilde{Y}_{t+1})|\tilde{Y}_t]$ and $E[V_u(A_{t+1}, \tilde{Y}_{t+1})|\tilde{Y}_t]$. In this problem set, you are not using the polynomial functional approximation you used in problem set 8. Instead, we will calculate the value of the expected value function for each point on the asset and income grid. This means that `EVe` and `EVu` will have the same dimension as `grid.A` and `grid.Y` (or what McKay's notes call `grid.K` and `grid.Z`)).

Something that may be a little bit confusing is that `EVe` and `EVu` (i.e., our discrete representations of $E[V_e(A_{t+1}, \tilde{Y}_{t+1})|\tilde{Y}_t]$ and $E[V_u(A_{t+1}, \tilde{Y}_{t+1})|\tilde{Y}_t]$) are functions of A_{t+1} and Y_t (i.e., `Ap` and `Y`, rather than `A` and `Y`. Just as in problem set 8, we will pass an argument to BellmanE and BellmanU with the values for A_{t+1} at which we want to evaluate `EVe` and `EVu`. Let's call this argument `Ap` (this is the equivalent to the argument `Kp` in McKay's Bellman function).

A complication that arises is that the values `Ap` that we want to evaluate `EVe` and `EVu` at may not be on the grid (i.e., they may be – typically will be – between gridpoints). We must thus interpolate to find reasonable appoximations for the value of `EVe` and `EVu` at these values for `Ap`. For this, we use the Matlab function `interp2`. In particular, we need the following code:

```
EVe = reshape(EVe,Grid.nA,Grid.nY)
```

And similar reshape statements for `A`, `Y`, and `Ap`.

```
EVe = interp2(Y,A,EVe,Y,Ap,'spline')
```

and similarly for `EVu`.

In the last line of the Bellman function where McKay's code has `PolyBasis(Kp,Z) * b`, you need to put instead a probability-weighted average of the `EVe` and `EVu` variables.

G. Write a version of McKay's MaxBellman function for this problem. Again, you will have two functions MaxBellmanE and MaxBellmanU. These should be quite similar to the analogous function in problem set 8. The main difference is that you will be passing the entire grids on assets and income to the Bellman function. (So, you will have something like `grid.AA` as an argument that you pass to Bellman as opposed to `grid.AA(I)` in problem set 8 (and the same for the grid on income).)

H. Write the value function iteration for-loop for this problem. This should be the same as in problem set 8, except that your version of McKay's

```
b = PolyGetCoef(Grid.KK,Grid.ZZ,EV(:));
```

can be deleted. The new `EVe` and `EVu` that you calculate in your version of McKay's

```
EV = reshape(V,Grid.nK,Grid.nZ) * Grid.PZ;
```

simply are your new guesses for the expected value functions.

I. (Optional) Implement the Howard acceleration for this problem. Report the speed improvement that you are able to achieve. (You will need to experiment with the number of iterations to figure out what works well in terms of giving a speed improvement.)

J. Adapt McKay's Simulate function for this problem. You will need to get rid of the

```
PolyBasis(Sim.K(t-1),Sim.Z(t-1)) * bKp;
```

in a similar way to the way you got rid of this part in the Bellman function. Note also that you need to simulate the evolution employment/unemployment in addition to A, $\tilde{Y}$, and C.

Now you can view some results:

K. Using your new Simulate function, produce a 10,000 period simulation of the evolution of A, $\tilde{Y}$, C, and employment status for a single household. Report a histogram of A, Y, and C. Report the mean and standard deviation of each variable. Plot the evolution of A, Y, and C over a 100 period stretch starting from period 1000. (Note that I am asking you to report Y, not $\tilde{Y}$.)

L. Plot consumption as a function of A for several values of $\tilde{Y}$. Do this for the a range of values for A that encompasses most of the mass in the histogram you report in part K. On the same figure, also plot consumption when unemployed as a function of A. Do this for the average value of $\tilde{Y}$.

M. Plot change in assets $Y - C$ when employed as a function A for several values of $\tilde{Y}$. Do this for the same range of values for A as part L. On the same figure, also plot change in assets when unemployed as a function of A. Do this for the average value of $\tilde{Y}$.

N. Plot the marginal propensity to consume when employed as a function of A for several values of $\tilde{Y}$. On the same figure, also plot the marginal propensity to consume when unemployed as a function of A (again for the average value of $\tilde{Y}$). Do this for the entire range of A on your grid. You can approximate the marginal propensity to consume as the extra consumption in the period that results from a windfall gain of 1 unit of A.

O. No need to explore how accuracy depends on polynomial basis.

P. (Optional) Explore how the accuracy of the solution depends on the number of grid points used. In particular, reproduce the figures above with only 10 grid points for A. Compare the results with those produced before and comment on any results that look "puzzling" in the less accurate case. You may also want to vary the bounds of the grid for A and see how this affects the accuracy of the solution.

Q. (Optional) Explore how changing the value of β changes the answer to parts K-N above. In particular, try a value of $\beta = 0.9$.