ECON 202A (Second Half) 2021 PS 1 Solution

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Question 1

a

```
function B  = PolyBasis(X,Y)
% B  = PolyBasis(X,Y)
% Polynomial basis functions.  Using 2nd order polynomial
%
% inputs
% X     n x 1   points for X
% Y     n x 1   points for Y
%
% outputs
% B     n x 6   array of basis functions: 1, X, Y, X^2, X*Y, Y^2

Yb = Y.*ones(size(X));
B = [ones(size(X)) X Yb X.^2 X.*Yb Yb.^2];

end
```

b

```
function b = PolyGetCoef(X,Y,Z)
% b = PolyGetCoef(X,Y,Z)
%   Fits the polynomial from PolyBasis to the function(s) in column(s) of
%   Z.
%
% inputs
% X     n x 1   points for X
% Y     n x 1   points for Y
% Z     n x 1   valies for function at (X,Y)
%
```

```
% outputs
% b      6 x 1     basis  coefficients

b = PolyBasis(X,Y) \ Z;

end
```

c

```
%% N = 5

% Create Representation Grid
xmin = .1; xmax = 2;
ymin = .1; ymax = 2;
n = 5;
xgrid = linspace(xmin,xmax,n);
ygrid = linspace(ymin,ymax,n);
[XXmesh,YYmesh] = meshgrid(xgrid,ygrid);
XX = XXmesh(:);
YY = YYmesh(:);
```

d

```
%True Values
Ztrue = log(XX + YY);
```

e

```
%Calculate Approximation Coefficients
b = PolyGetCoef(XX,YY,Ztrue);
```

f

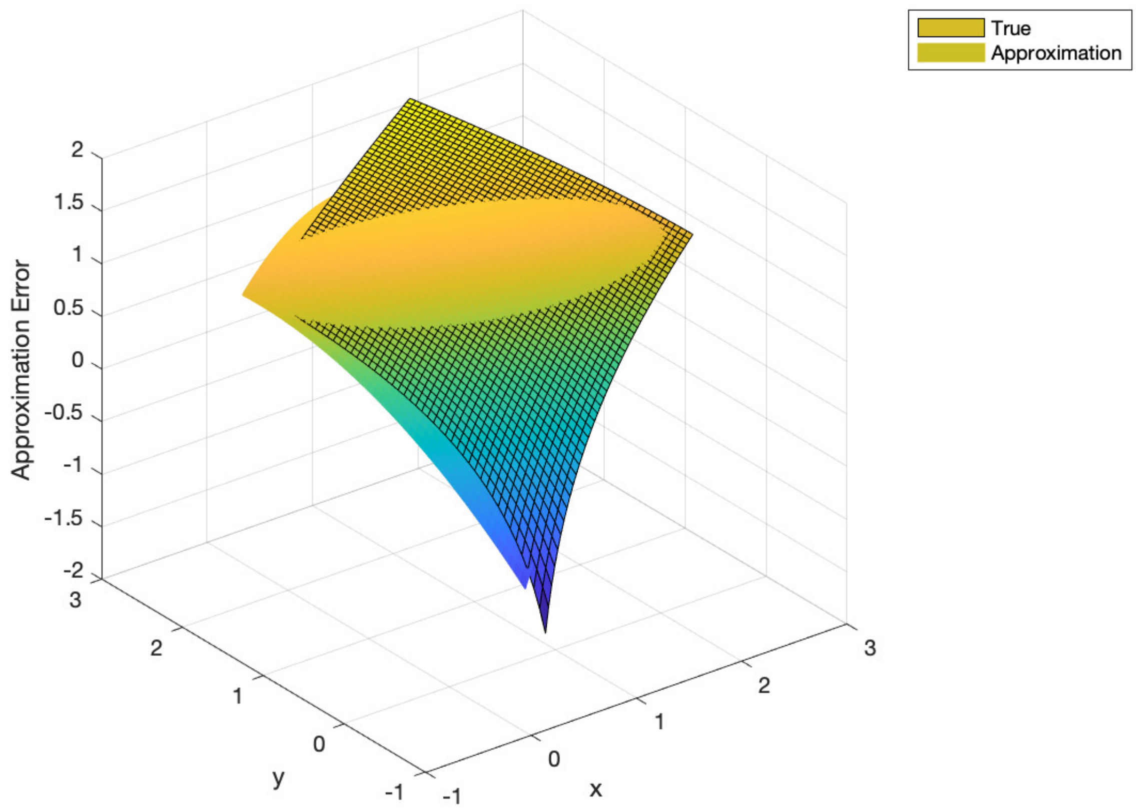
```
% Create Largest Grid
xmax = 2.5;
ymax = 2.5;
n = 49;
xgrid = linspace(xmin,xmax,n);
ygrid = linspace(ymin,ymax,n);
[XXXmesh,YYYmesh] = meshgrid(xgrid,ygrid);
XXX = XXXmesh(:);
YYY = YYYmesh(:);
```

g

```
%Calculate true value
Zeval_true = log(XXX+YYY);

%Calculate approximation
Zeval_approx = PolyBasis(XXX,YYY)*b;
```

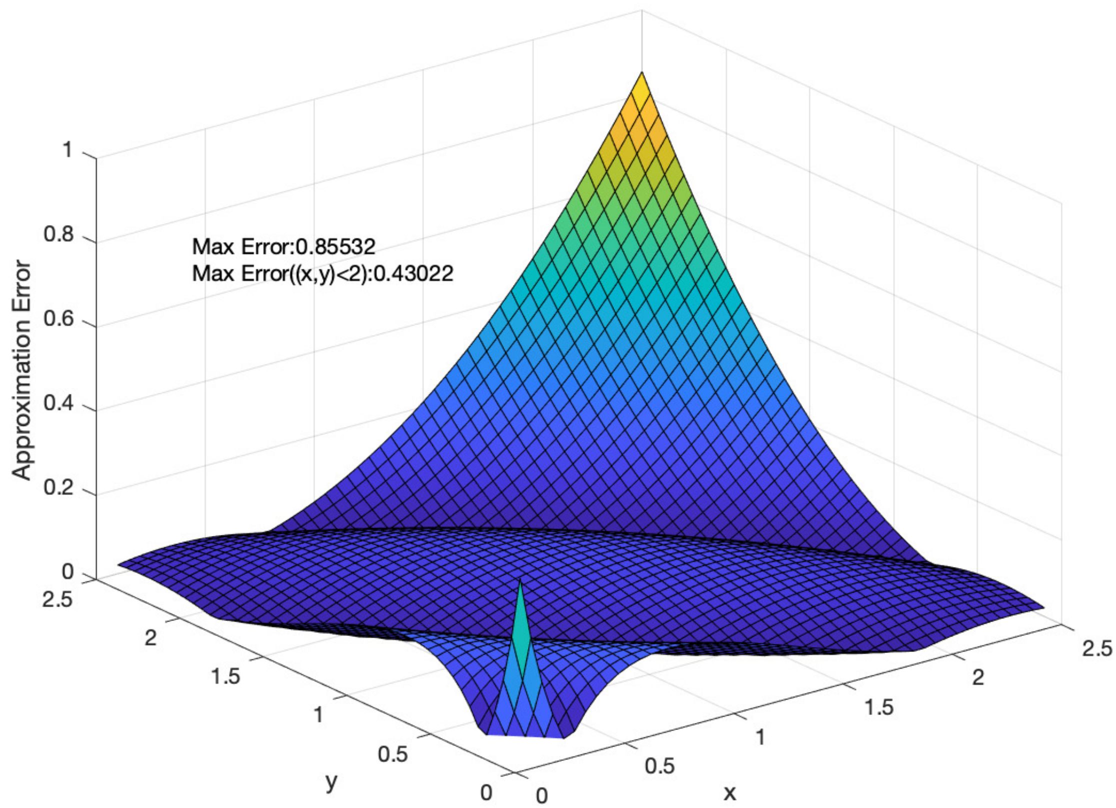
h



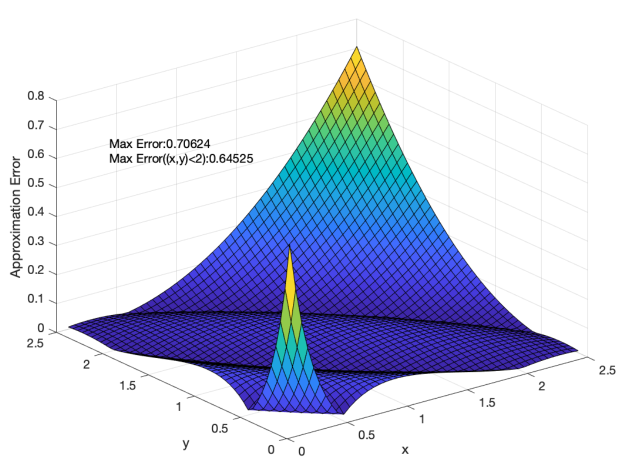
i

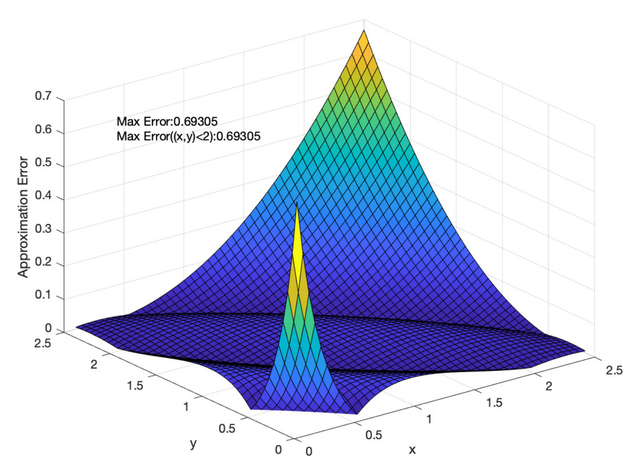
```
%Find Errors
abs_errors5 = abs(Zeval_true-Zeval_approx);
max_error5 = max(abs_errors5);
max_errors5_restrict = max(abs_errors5(XXX<=2 & YYY<=2));
```

j


Max Error:0.85532
Max Error((x,y)<2):0.43022
Approximation Error
x
y

k


Max Error:0.70624
Max Error((x,y)<2):0.64525
Approximation Error
x
y

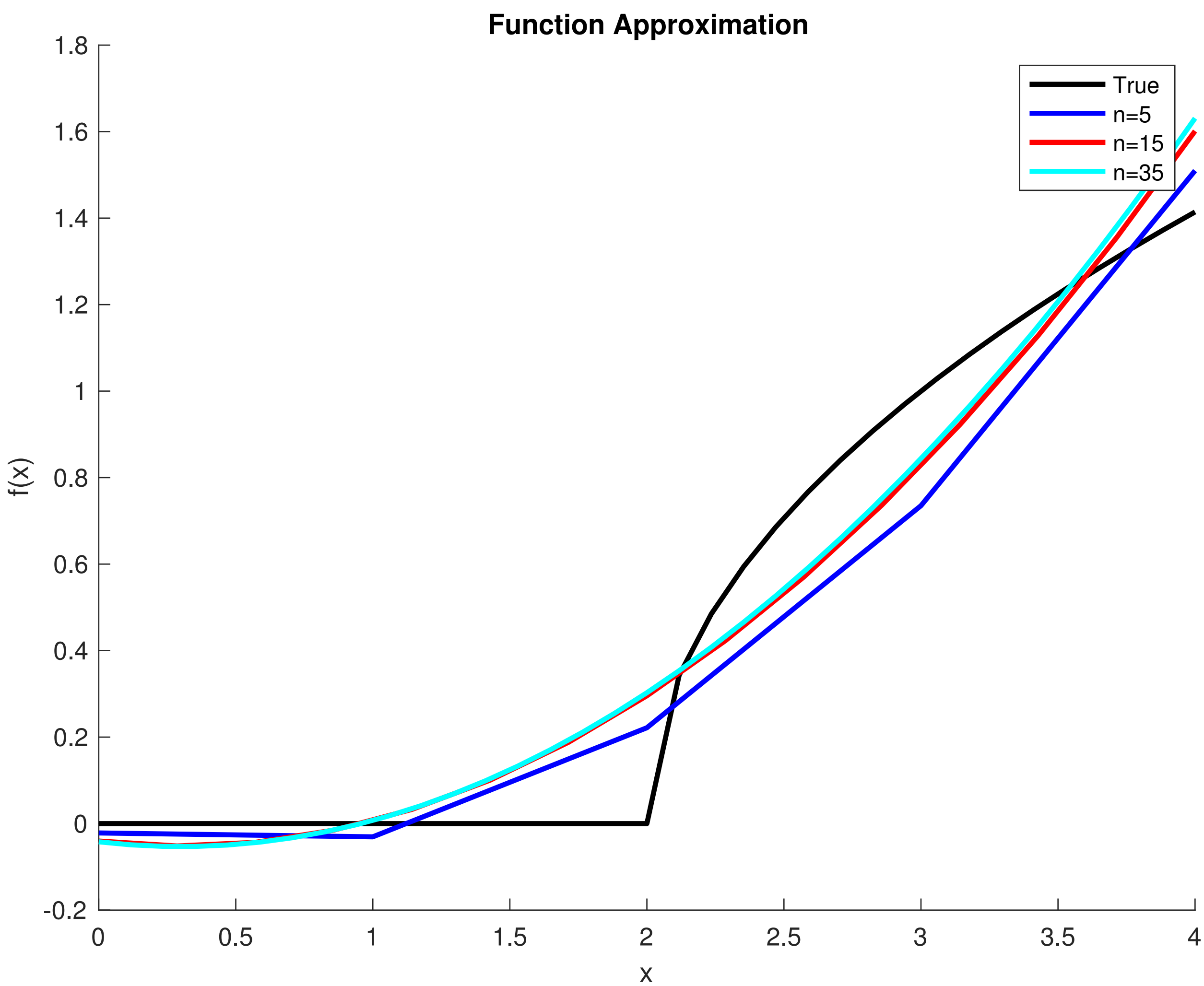

Max Error:0.69305
Max Error((x,y)<2):0.69305
Approximation Error
x
y

l

These exercises have taught me three central lessons. First, as the number of approximation points increases(e.g. 5, 15 , 35) the absolute errors between true as estimated values decreases. Second, estimating values outside the approximation points (e.g. (2,2.5]) results in larger approximation errors than those inside the approximation region. Third, the approximation errors the result from this polynomial basis function follow a similar periodic pattern across the three different approximation grids. That is, the errors are not clustered in one region within the approximation points.

m

The figure below plots the true function $f(x) = \sqrt{x-2}[x >= 2]$ evaluated at the 35 evenly spaced grid points between $x = 0$ and $x = 4$, along with the polynomial approximations $f^n(x)$ approximated using n evenly space grid points between 0 and 4, where $n = 5, 15, 35$, evaluated at the same 35 grid points as the true function.



The following figure plots the absolute value of the approximation error of the different polynomial approximations $f^n(x)$, where $n = 5, 15, 35$. Additionally, the figure includes in the color coded text the value of the absolute maximum approximation error. Interestingly, the maximum approximation error grows with the n, the number of grid points used to estimate the polynomial approximation.