Econ 220A Problem Set 2

Demand Estimation and Merger Simulation

File: ps2-notebook.jl
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Description: estimating model for Problem Set #2 of Econ 220A (UC Berkeley, 2022)

The two original files are:

- PSET1-2019-2.pdf (problem questions)
- ps1data.csv (data)

and can be found in this GitHub repo.

```
Enter cell code...
```

Notes:

- Current Status: Starting Problem 2
- Citations:
  - [0] Julia: A Fresh Approach to Numerical Computing. Jeff Bezanson, Alan Edelman, Stefan Karpinski, Viral B. Shah. (2017) SIAM Review, 59: 65–98. doi: 10.1137/141000671.
  - [1] Steven T. Berry, “Estimating Discrete-Choice Models of Product Differentiation,” The RAND Journal of Economics, 1994, 242–62.
  - [2] Aviv Nevo, “A Practitioner’s Guide to Estimation of Random-Coefficients Logit Models of Demand,” Journal of Economics & Management Strategy 9, no. 4 (Winter 2000): 513–48, https://doi.org/10.1162/105864000567954.
  - [3] Daniel A Ackerberg and Gregory S Crawford, “Estimating Price Elasticities in Differentiated Product Demand Models with Endogenous Characteristics,” Working Paper, 2009.
  - [4] Ackerberg and Crawford, “Estimating Price Elasticities in Differentiated Product Demand Models with Endogenous Characteristics.” (2009) Working Paper
  - [5] Kenneth E Train, Discrete Choice Methods with Simulation, 2nd ed., 2009.

Packages

```
begin
    # For Stats stuff
    using Statistics, StatsBase, StatsPlots, StatsFuns, GLM
    using CovarianceMatrices, StatsModels, FixedEffectModels
    using FiniteDifferences, NLsolve
    # For dataframes
    using DataFrames, CSV, DataFramesMeta
    using ShiftedArrays
    using OrderedCollections
    # Random Variable distributions
    using Random, Distributions
    Random.seed!(0);
    # Output
    using Latexify, PlutoUI, Luxor
    using Luxor: text, arrow
    # Optimizing functions
    # using Optim, NLSolversBase
    # using LinearAlgebra: diag
    # using JuMP, KNITRO
end;
```

```
(process:14272): GLib-GIO-WARNING **: 15:12:40.793: Unexpectedly, UWP app `KDEe.V.Okular_22.800.1121.0_x64__7vt06qxq7ptv8' (AUMId `KDEe.V.Okular_7vt06qxq7ptv8!KDEe.V.Okular') supports 5 extensions but has no verbs
```

```
Enter cell code...
```

Setup

```
begin
    # Load data
    df = DataFrame(CSV.File(string(dirname(@__FILE__), "/ps1data.csv")));
    # Add δ (mean utility levels)
    add_logit_depvar!(df)
    # Add within group-market shares s̄ⱼₕ
    df = add_ingroup_share!(df)
end;
```

QUESTION 1: LOGIT

(a) Logit using OLS

Own and cross-price elasticities η_{jk} are calculated below for the simple Conditional Logit model. Cross-price elasticities are estimated only compared to product 3. These elasticities are estimated from observed market price p_j, estimated market shares $\hat{s}_j$, and estimated marginal utility of income $\hat{\alpha}$ using the following formula from Nevo 2001 (pg. 522):

$$\hat{\eta}_{jk} = \frac{\partial s_j}{\partial p_k}\frac{p_k}{s_j} = \begin{cases} -\hat{\alpha}p_k(1-\hat{s}_j) & \text{if } j = k, \\ \hat{\alpha}p_k\hat{s}_k & \text{otherwise.} \end{cases}$$

Average (across markets) Elasticities for OLS

η_{jj} = own-price market share elasticity

η_{j3} = cross-price market share elasticity for product 3's price

prodid	ηjj	ηj3
1	−1.07	0.183
2	−1.03	0.183
3	−1.09	NA
4	−1.08	0.183
5	−1.11	0.183

(b) Logit with IV (cost shifters)

First stage regression: prices on cost-shifters and other explanatory variables

We will use the instrumented price to estimate α in the regression of mean utility δ on prices and product characteristics.

```
reg1b_fs =                          Linear Model
           ============================================================================
           Number of obs:                  500  Degrees of freedom:                   9
           R2:                           0.684  R2 Adjusted:                      0.679
           F-Stat:                     131.151  p-value:                          0.000
           ============================================================================
           pjn         |    Estimate Std.Error    t value Pr(>|t|) Lower 95%  Upper 95%
           ----------------------------------------------------------------------------
           w1          |      1.6339 0.0964691    16.9371    0.000   1.44436    1.82345
           w2          |     1.54632 0.0595915    25.9487    0.000   1.42924    1.66341
           d1          |   -0.517228  0.294143   -1.75842    0.079  -1.09516  0.0607067
           d2          |    0.281673  0.354217   0.795199    0.427 -0.414296   0.977642
           d4          | -0.00835596  0.296359 -0.0281954    0.978 -0.590643   0.573931
           d5          |   -0.555189  0.272751   -2.03551    0.042  -1.09109 -0.0192852
           x1          |    0.683061  0.113085    6.04022    0.000   0.46087   0.905252
           x2          |    0.440479 0.0967653    4.55203    0.000  0.250354   0.630604
           (Intercept) |     3.63797  0.796898    4.56516    0.000   2.07222    5.20372
           ============================================================================
```

IV regression: average utility on product characteristics using cost-shifters as instruments for price

We can see from the IV table that first stage F statistics is 433, well above the rule of thumb of an F statistic of >10. So these instruments are jointly significant in predicting prices. For reference, the F statistic from the regression of just prices on the cost shifters is above 300.

```
                                  IV Model
===========================================================================
Number of obs:                  500  Degrees of freedom:                  8
R2:                           0.013  R2 Adjusted:                    -0.001
F-Stat:                      28.333  p-value:                         0.000
F-Stat (First Stage):       433.228  p-value (First Stage):           0.000
===========================================================================
δjn         |   Estimate Std.Error   t value Pr(>|t|) Lower 95%  Upper 95%
---------------------------------------------------------------------------
d1          |  -0.498625  0.238978  -2.08648    0.037 -0.968169 -0.0290803
d2          |   0.231501    0.2912   0.79499    0.427 -0.340648   0.803651
d4          | -0.0496052   0.22854 -0.217053    0.828  -0.49864   0.399429
d5          |  -0.589717  0.245661  -2.40053    0.017  -1.07239  -0.107043
x1          |   0.864431 0.0829039   10.4269    0.000  0.701541    1.02732
x2          |   0.519021 0.0850551   6.10217    0.000  0.351905   0.686137
pjn         |  -0.265398 0.0310851  -8.53779    0.000 -0.326473  -0.204322
(Intercept) |  -0.468533  0.758258 -0.617907    0.537  -1.95836    1.02129
===========================================================================
```

(c) Logit with IV (average characteristics)

First stage regression: prices on average other-product characteristics and other explanatory variables

```
reg1c_fs =                         Linear Model
           ========================================================================
           Number of obs:                      500  Degrees of freedom:           9
           R2:                               0.097  R2 Adjusted:              0.083
           F-Stat:                         5.44718  p-value:                  0.000
           ========================================================================
           pjn         |  Estimate Std.Error   t value Pr(>|t|) Lower 95% Upper 95%
           ------------------------------------------------------------------------
           x1_avg      | -0.477415  0.358638  -1.33119    0.184  -1.18207   0.22724
           x2_avg      |    0.1484  0.306298  0.484496    0.628 -0.453417  0.750217
           d1          | -0.894878  0.502565  -1.78062    0.076  -1.88232  0.092565
           d2          | 0.0553549  0.508936  0.108766    0.913 -0.944605   1.05532
           d4          | -0.209187  0.448348 -0.466573    0.641   -1.0901   0.67173
           d5          |  -0.55424  0.475481  -1.16564    0.244  -1.48847  0.379989
           x1          |  0.987913  0.176346   5.60214    0.000  0.641427    1.3344
           x2          |  0.489964  0.152646    3.2098    0.001  0.190044  0.789884
           (Intercept) |   18.7926   2.63945   7.11987    0.000   13.6065   23.9786
           ========================================================================
```

IV regression: average utility on product characteristics with average other-product characteristics as instruments for price

Below, we can see that the first-stage F statistic is well below 10 (1.13). This indicates we have weak instruments.

```
                                IV Model
========================================================================
Number of obs:                      500   Degrees of freedom:          8
R2:                              -0.048   R2 Adjusted:            -0.063
F-Stat:                         10.0198   p-value:                 0.000
F-Stat (First Stage):           1.12731   p-value (First Stage):   0.324
========================================================================
δjn         |   Estimate Std.Error   t value Pr(>|t|) Lower 95% Upper 95%
------------------------------------------------------------------------
d1          |  -0.530788  0.394708  -1.34476    0.179  -1.30631  0.244733
d2          |   0.233012  0.308155  0.756151    0.450  -0.37245  0.838473
d4          | -0.0572594  0.243578 -0.235076    0.814 -0.535841  0.421322
d5          |  -0.609464  0.316091  -1.92813    0.054  -1.23052 0.0115912
x1          |   0.901139  0.396671   2.27176    0.024  0.121762   1.68052
x2          |   0.537076   0.19117   2.80942    0.005  0.161466  0.912686
pjn         |   -0.30232  0.387704  -0.77977    0.436  -1.06408   0.45944
(Intercept) |   0.124691   6.22254 0.0200386    0.984  -12.1013   12.3507
========================================================================
```

(d) Logit Conclusion

From Nevo's exposition, it seems obvious that we should favor the average-of-other-product-characteristics instrument. The issue with the cost-shifter instrument is that cost-shifters are often the same across brands. We need cost shifters that are different arcross brands, otherwise our predicted prices will all be the same in the first stage. However, examining the data, our cost shifters (presumably fuel cost and wages) are different across markets *and* brands! I suspect this is just because we are using simulated data and it was constructed to be have more identifying power than the average characteristics IV. We can also see that we might have a weak instruments problem if we use the average characteristics as IVs.

So, I prefer the brand-market-specific cost shifters (part b) to use as IVs and present the elasticity results below.

Average Elasticities (IV - cost shifters)

η_{jj} = own-price market share elasticity
η_{j3} = cross-price market share elasticity for product 3's price

prodid	ηjj	ηj3
1	-5.05	0.859
2	-4.9	0.859
3	-5.08	NA
4	-5.05	0.859
5	-5.23	0.859

Question 1 Functions

`add_logit_depvar!`

Create the logit dependent variable: $\delta_{j,n}$ = log(sjn) - log(sj0); sj0 = outside share (1-Σsjn)

`eq6`

Logit share -probability function; Nevo Eq. (6)

add_predicted_shares!

Estimates shares using linear paremeters (Nevo Eq 6) using regress results.

add_price_elasticities!

Estimate own-price and cross-price elasticities of the predicted market shares for Conditional Logit; add columns.

Only adds cross-price elasticities relative to product 3 (ηjjn and ηj3n). Uses elasticity equation from Nevo (2000) pg 522. Need to use the predicted market share ($\hat{s}_{jn} = X\beta$)

```
"""Estimate own-price and cross-price elasticities of the predicted market shares for
Conditional Logit; add columns.

Only adds cross-price elasticities relative to product 3 (ηjjn and ηj3n).
Uses elasticity equation from Nevo (2000) pg 522.
Need to use the predicted market share (ŝⱼₙ = Xβ)
"""
function add_price_elasticities!(reg, df)
    sort!(df, [:market, :prodid])
    # Get price coefficient
    α = -get_coef(reg, "pjn")
    df.ηjjn = -α * df.pjn .* (1 .- df.sjn_hat)
    df.ηj3n = @chain df begin
        filter(:prodid => ==(3), _)
        combine(:market, [:pjn,:sjn_hat] => ((p,s) -> α*p.*s) => :ηj3n)
        leftjoin(df[df.prodid .!=3,[:market,:prodid]], _, on = :market)
        leftjoin(df, _, on = [:market, :prodid])
        sort([:market, :prodid])
        _[!,:ηj3n]
    end
    add_marginal_costs_NL!(df)
    return df
end
```

add_avg_characteristics!

Calculate average of other products' characteristics; add columns.

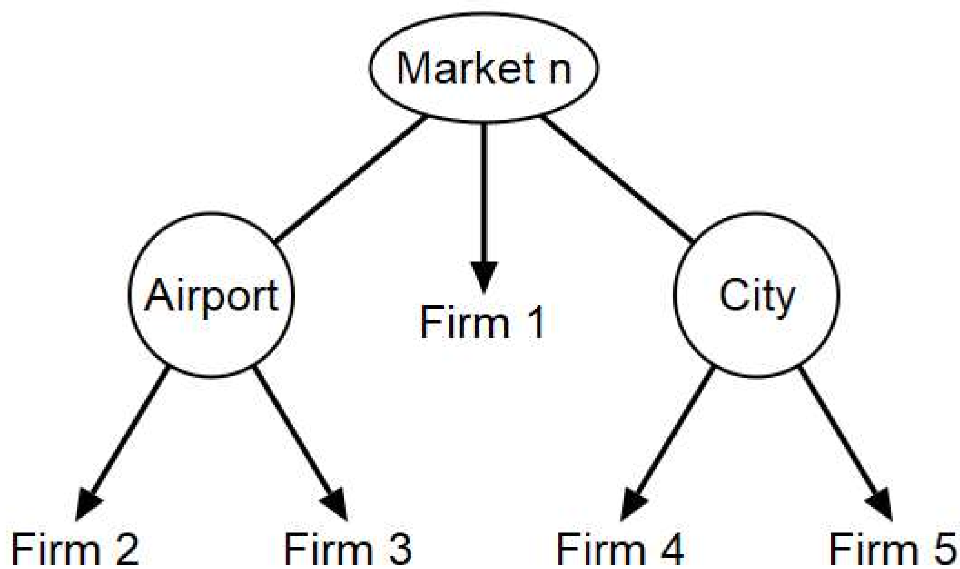
get_coef

Return coefficient from regression for variable var `add_ingroup_share!`

Create the within-nest share variable $\bar{s}_{jh}$

QUESTION 2: NESTED LOGIT

(a) Tree Structure of in-market Substitution



(b) Nested Logit using OLS

Linear estimating equation for Nested Logit from Berry 1994, Eq 28 (pg 253):

$$\ln(s_j) - ln(s_0) = x_j\beta - \alpha p_j + \sigma \ln(\bar{s}_{j|g}) + \xi_j$$

From regressing $[\ln(s_j) - ln(s_0)]$ on $x_j, p_j, \ln(\bar{s}_{j|g})$, we obtain estimates of α, β, σ to use in estimating the predicted market shares, predicted within-nest market shares, and eventually the elasticities.

From this regression, the estimated coefficient of within-group market shares is 0.9021, which is between 0 and 1, so this is at least a plausibl estimate of $\hat{\sigma}$. As Ken Train puts it, "If [the nest correlation coefficient] is between zero and one, the model is consistent with utility maximization for all possible values of the explanatory variables." (pg. 81)

These Nested Logit elasticities are estimated from observed market price p_j, estimated market shares $\hat{s}_j$, estimated within-nest market share $\hat{s}_{j|g}$, estimated marginal utility of income $\hat{\alpha}$, and estimated nest substituion parameter $\hat{\sigma}$ using the following formula from Ackerberg and Crawford 2009 (pg. 15):

$$\hat{\eta}_{jk} = \frac{\partial s_j}{\partial p_k}\frac{p_k}{s_j} = \begin{cases} -\hat{\alpha} p_k(\frac{1}{1-\hat{\sigma}} - \frac{\hat{\sigma}}{1-\hat{\sigma}}\hat{s}_{j|g} - \hat{s}_j) & \text{if } j = k, \\ \hat{\alpha}\hat{s}_k p_k(\frac{\hat{\sigma}}{1-\hat{\sigma}}\frac{\hat{s}_{j|g}}{\hat{s}_j} + 1) & \text{if } j \neq k \text{ but } j \text{ and } k \text{ in same nest,} \\ \hat{\alpha}\hat{s}_k p_k & \text{otherwise} \end{cases}$$

Average (across markets) Nested Logit Elasticities for OLS

η_{jj} = own-price market share elasticity

η_{j3} = cross-price market share elasticity for product 3's price

prodid	ηjj	ηj3
1	−1.39	0.148
2	−7.31	5.96
3	−9.4	NA
4	−7.97	0.148
5	−8.66	0.148

```
begin
    # The group, and in-group share variables are created in question 1 functions
    df2 = copy(df)
    # We keep the same dependent variable ln(sⱼ)-ln(s₀), but we need to call it something else
    # In Nested Logit, ln(sⱼ)-ln(s₀) = δ + σ ln(̄sⱼₕ)
    # Change the name of the dependent variable from δ to ln sⱼs₀ = ln(sⱼ)-ln(s₀)
    rename!(df2, :δjn => :lnsjs0)
    df2b = copy(df2)
    # Regress ln(sⱼ)-ln(s₀) on X and ln(̄sⱼₕ) to estimate α,β,σ that minimizes error term ( BLP Eq 28)
    reg2b = reg(df2b, @formula(lnsjs0 ~ pjn + log(sjhn) + d1 + d2  + d4 + d5 + x1 + x2), Vcov.cluster(:market))
    # Calculate own and cross-price elasticities
    estimate_price_elasticities_NL!(df2b, reg2b)
end
```

(c) Nested Logit with IV (cost shifters)

First stage regression: prices on cost-shifters and other explanatory variables
We will use the instrumented price to estimate α in the regression of $\ln(s_j) - ln(s_0)$ on prices and product characteristics.

We can see from the IV table that first stage F statistics is 508, well above the rule of thumb of an F statistic of >10. So these instruments are jointly significant in predicting prices.

First-stage regression for prices

Linear Model

Number of obs:	500	Degrees of freedom:	9
R2:	0.684	R2 Adjusted:	0.679
F-Stat:	131.151	p-value:	0.000

pjn	Estimate	Std.Error	t value	Pr(>|t|)	Lower 95%	Upper 95%
w1	1.6339	0.0964691	16.9371	0.000	1.44436	1.82345
w2	1.54632	0.0595915	25.9487	0.000	1.42924	1.66341
d1	-0.517228	0.294143	-1.75842	0.079	-1.09516	0.0607067
d2	0.281673	0.354217	0.795199	0.427	-0.414296	0.977642
d4	-0.00835596	0.296359	-0.0281954	0.978	-0.590643	0.573931
d5	-0.555189	0.272751	-2.03551	0.042	-1.09109	-0.0192852
x1	0.683061	0.113085	6.04022	0.000	0.46087	0.905252
x2	0.440479	0.0967653	4.55203	0.000	0.250354	0.630604
(Intercept)	3.63797	0.796898	4.56516	0.000	2.07222	5.20372

First-stage regression for $ln(s_{j|h})$

Linear Model

Number of obs:	500	Degrees of freedom:	9
R2:	0.230	R2 Adjusted:	0.218
F-Stat:	56.7979	p-value:	0.000

log(sjhn)	Estimate	Std.Error	t value	Pr(>|t|)	Lower 95%	Upper 95%
w1	-0.167178	0.0504994	-3.31048	0.001	-0.266399	-0.067956
w2	-0.129353	0.0331769	-3.89889	0.000	-0.194539	-0.0641669
d1	1.20931	0.124895	9.68257	0.000	0.963913	1.4547
d2	0.237421	0.211844	1.12074	0.263	-0.178811	0.653654
d4	0.221782	0.171925	1.28999	0.198	-0.116017	0.559582
d5	-0.112399	0.193098	-0.582081	0.561	-0.491799	0.267002
x1	0.278537	0.0545233	5.10859	0.000	0.171409	0.385665
x2	0.121207	0.0503144	2.40898	0.016	0.0223485	0.220065
(Intercept)	-1.76656	0.39858	-4.43213	0.000	-2.54969	-0.983425

IV regression: $\ln(s_j) - ln(s_0)$ on product characteristics using cost-shifters as instruments for price

We can see from the IV table that first stage F statistics is 0.13, well below the rule of thumb of an F statistic of >10. So these are weak instruments!

From this regression, the estimated coefficient of within-group market shares is 2.159, which is greater than one, so this might not be plausible for $\hat{\sigma}$. As Ken Train puts it, "For [nest correlation coefficient values] greater than one, the model is consistent with utility-maximizing behavior for some range of the explanatory variables but not for all values." (pg. 81)

```
                              IV Model
=======================================================================
Number of obs:                     500  Degrees of freedom:           9
R2:                             -0.143  R2 Adjusted:             -0.161
F-Stat:                        15.9792  p-value:                  0.000
F-Stat (First Stage):         0.127692  p-value (First Stage):    0.613
=======================================================================
lnsjs0         |  Estimate Std.Error   t value Pr(>|t|) Lower 95% Upper 95%
-----------------------------------------------------------------------
d1             |  -2.99636   3.75011 -0.799005    0.425  -10.3646   4.37189
d2             | -0.327123  0.909174 -0.359802    0.719  -2.11348   1.45923
d4             |  -0.51753   0.77463   -0.6681    0.504  -2.03953   1.00447
d5             | -0.231473  0.613381 -0.377373    0.706  -1.43665  0.973702
x1             |  0.130871   1.08502  0.120616    0.904  -2.00099   2.26273
x2             |  0.166885  0.552564  0.302018    0.763 -0.918797   1.25257
pjn            | -0.070634  0.293082 -0.241004    0.810 -0.646484  0.505216
log(sjhn)      |   2.15897   3.20638  0.673336    0.501  -4.14095    8.4589
(Intercept)    |   2.62591   4.60238  0.570555    0.569  -6.41688   11.6687
=======================================================================
```

(d) Nested Logit with IV (average characteristics)

First stage regression: prices on average in-nest other-product characteristics and other explanatory variables

```
                              Linear Model
=========================================================================
Number of obs:                       500  Degrees of freedom:            9
R2:                                0.104  R2 Adjusted:               0.090
F-Stat:                          6.33109  p-value:                   0.000
=========================================================================
pjn            |   Estimate Std.Error   t value Pr(>|t|)  Lower 95% Upper 95%
-------------------------------------------------------------------------
x1_avg         |  -0.336537  0.179102  -1.87902    0.061  -0.688438 0.0153646
x2_avg         |   0.269047  0.169829   1.58423    0.114 -0.0646337  0.602728
d1             |   -2.51246   1.15899   -2.1678    0.031   -4.78965 -0.235263
d2             |  0.0901175  0.503239  0.179075    0.858   -0.89865   1.07889
d4             | -0.0623615  0.452005 -0.137967    0.890  -0.950464  0.825741
d5             |  -0.450004  0.479367 -0.938747    0.348   -1.39187  0.491859
x1             |   0.990229  0.171138   5.78616    0.000   0.653977   1.32648
x2             |   0.519141  0.152338   3.40782    0.001   0.219826  0.818455
(Intercept)    |    17.7013   1.54268   11.4744    0.000    14.6703   20.7324
=========================================================================
```

First stage regression: $ln(s_{j|h})$ on average in-nest other-product characteristics and other explanatory variables

```
                         Linear Model
=============================================================================
Number of obs:                    500   Degrees of freedom:                 9
R2:                             0.230   R2 Adjusted:                    0.218
F-Stat:                        73.835   p-value:                        0.000
=============================================================================
log(sjhn)    |   Estimate Std.Error   t value Pr(>|t|) Lower 95% Upper 95%
-----------------------------------------------------------------------------
x1_avg       |  -0.302731 0.0578921  -5.22922    0.000 -0.416477 -0.188984
x2_avg       | 0.00125162 0.0671514 0.0186388    0.985 -0.130688  0.133191
d1           |  -0.521784  0.371215  -1.40561    0.160  -1.25115  0.207582
d2           |   0.269099  0.214266   1.25591    0.210 -0.151893   0.69009
d4           |   0.315782  0.175374   1.80063    0.072 -0.028793  0.660358
d5           | -0.0995225  0.195169  -0.50993    0.610 -0.482992  0.283947
x1           |   0.254231 0.0500036   5.08426    0.000  0.155984  0.352478
x2           |   0.143274 0.0512666   2.79468    0.005 0.0425446  0.244003
(Intercept)  |   -1.17571  0.314126  -3.74279    0.000   -1.7929  -0.55851
=============================================================================
```

IV regression: $\ln(s_j) - ln(s_0)$ on product characteristics with average in-nest other-product characteristics as instruments for price

Below, we can see that the first-stage F statistic is well below 10 (1.08). This indicates we have weak instruments.

From this regression, the estimated coefficient of within-group market shares is -0.2499, which is negative, so this doesn't seem plausible for $\hat{\sigma}$. As Ken Train puts it, "A negative value of [the nest correlation coefficient] is inconsistent with utility maximization and implies that improving the attributes of an alternative (such as lowering its price) can decrease the probability of the alternative being chosen." (pg. 81)

```
                            IV Model
============================================================================
Number of obs:                    500  Degrees of freedom:                 9
R2:                            -0.699  R2 Adjusted:                   -0.727
F-Stat:                       7.23026  p-value:                        0.000
F-Stat (First Stage):         1.08012  p-value (First Stage):          0.142
============================================================================
lnsjs0       |  Estimate Std.Error   t value Pr(>|t|) Lower 95% Upper 95%
----------------------------------------------------------------------------
d1           |  0.364873   1.24505  0.293058    0.770  -2.08142   2.81116
d2           |  0.269198  0.306536  0.878194    0.380 -0.333086  0.871482
d4           |  0.141248  0.398117  0.354789    0.723 -0.640975   0.92347
d5           | -0.278542  0.410988 -0.677736    0.498  -1.08605  0.528971
x1           |  0.293801  0.376588  0.780165    0.436 -0.446122   1.03372
x2           |  0.237357  0.213773   1.11032    0.267 -0.182666   0.65738
pjn          |  0.371415  0.467033  0.795265    0.427 -0.546215   1.28905
log(sjhn)    |  -0.24991  0.652537 -0.382983    0.702  -1.53202    1.0322
(Intercept)  |  -11.4204   9.08536  -1.25702    0.209  -29.2714   6.43053
============================================================================
```

(e) Nested Logit with IV (both)

Instrumental Variables (IV) with with both cost-shifters and within-group average of rivals' characteristics as instruments.

First stage regression: prices on average in-nest other-product characteristics *and* cost shifters and other explanatory variables

```
                              Linear Model
========================================================================
Number of obs:                     500  Degrees of freedom:           11
R2:                              0.689  R2 Adjusted:               0.683
F-Stat:                        105.882  p-value:                   0.000
========================================================================
pjn         |  Estimate Std.Error  t value Pr(>|t|)  Lower 95% Upper 95%
------------------------------------------------------------------------
w1          |   1.63585 0.0969408  16.8747    0.000    1.44538   1.82632
w2          |   1.53616 0.0610593  25.1585    0.000    1.41619   1.65613
x1_avg      | -0.260432 0.0945956 -2.75311    0.006  -0.446296 -0.074568
x2_avg      |  0.167622 0.0956205  1.75299    0.080 -0.0202558    0.3555
d1          |  -1.83848  0.622837 -2.95178    0.003   -3.06224  -0.61471
d2          |  0.312041  0.348024  0.89661    0.370  -0.371765  0.995848
d4          | 0.0928261  0.299166 0.310283    0.756  -0.494984  0.680636
d5          | -0.500703  0.269309 -1.85921    0.064   -1.02985 0.0284433
x1          |  0.682317  0.110604  6.16903    0.000      0.465  0.899634
x2          |  0.464412 0.0958394  4.84573    0.000   0.276104   0.65272
(Intercept) |   4.99268  0.803794   6.2114    0.000    3.41337     6.572
========================================================================
```

First stage regression: $ln(s_{j|h})$ on average in-nest other-product characteristics *and* cost shifters and other explanatory variables

```
                                  Linear Model
================================================================================
Number of obs:                         500  Degrees of freedom:               11
R2:                                  0.271  R2 Adjusted:                   0.256
F-Stat:                            47.4058  p-value:                       0.000
================================================================================
log(sjhn)   |     Estimate Std.Error      t value Pr(>|t|)  Lower 95%  Upper 95%
--------------------------------------------------------------------------------
w1          |    -0.163522 0.0492218     -3.32215    0.001  -0.260235   -0.06681
w2          |    -0.136973 0.0317995      -4.3074    0.000  -0.199453 -0.0744925
x1_avg      |    -0.309167 0.0558584     -5.53482    0.000  -0.418919  -0.199414
x2_avg      |    0.0104489 0.0671054     0.155709    0.876  -0.121402     0.1423
d1          |    -0.577689  0.365715     -1.57961    0.115   -1.29626   0.140878
d2          |     0.250765  0.205834      1.21829    0.224  -0.153664   0.655194
d4          |     0.303615  0.166381      1.82482    0.069 -0.0232952   0.630525
d5          |   -0.0929105  0.188355    -0.493273    0.622  -0.462996   0.277175
x1          |     0.283081  0.050537      5.60146    0.000   0.183784   0.382377
x2          |      0.14706 0.0523488      2.80924    0.005  0.0442041   0.249917
(Intercept) | -0.000390313  0.395784 -0.000986178    0.999  -0.778037   0.777256
================================================================================
```

IV regression: $\ln(s_j) - ln(s_0)$ on product characteristics with average in-nest other-product characteristics *and* cost shifters as instruments for price

Below, we can see that the first-stage F statistic is well below 10 (2.25). This indicates we have weak instruments.

From this regression, the estimated coefficient of within-group market shares is 0.4888, which is between 0 and 1, so this is at least a plausible estimate of $\hat{\sigma}$. As Ken Train puts it, "If [the nest correlation coefficient] is between zero and one, the model is consistent with utility maximization for all possible values of the explanatory variables." (pg. 81)

```
                                IV Model
=========================================================================
Number of obs:                      500  Degrees of freedom:            9
R2:                               0.346  R2 Adjusted:               0.335
F-Stat:                         41.9491  p-value:                   0.000
F-Stat (First Stage):           2.25473  p-value (First Stage):     0.029
=========================================================================
lnsjs0       |  Estimate Std.Error   t value Pr(>|t|) Lower 95% Upper 95%
-------------------------------------------------------------------------
d1           |  -1.06198  0.304511   -3.4875    0.001  -1.66029 -0.463676
d2           |  0.104939  0.191203  0.548836    0.583 -0.270738  0.480616
d4           | -0.155039  0.172605 -0.898231    0.370 -0.494173  0.184096
d5           | -0.507332   0.17485  -2.90153    0.004 -0.850879 -0.163786
x1           |  0.695977 0.0955135   7.28669    0.000  0.508311  0.883643
x2           |  0.438129 0.0709838   6.17223    0.000  0.298659  0.577598
pjn          | -0.218906 0.0281033  -7.78932    0.000 -0.274123 -0.163688
log(sjhn)    |  0.488759  0.172111   2.83979    0.005  0.150595  0.826924
(Intercept)  |  0.193437   0.74691  0.258983    0.796   -1.2741   1.66097
=========================================================================
```

```
• Enter cell code...
```

```
• Enter cell code...
```

(f) Nested Logit Conclusion

From the 2SLS results in the above parts, we can see that the marginal utiltiy of income ($-\hat{\alpha}$) in part (d) is negative. This seems unlikely and makes me think that the average in-nest other-product characteristics are poor instruments for price. This makes me doubt if they should be used at all as instruments. However, the F-stat for the first stage of part (d) is much larger than for part (c), but still much less than 10.

The F-stat for the combined 4 instruments (part e, both average characteristics and cost shifters), is larger than for just the average characteristics IV, and $-\hat{\alpha}$ from the 4-instrument case is significantly negative. Also, of the IV specifications, the combined IV is the only estimation that has an estimate for $\hat{\sigma}$ between 0 and 1. So even though it appears there are not any reliable IV results from the Nested Logit model, if I had to pick, I would choose the combined 4-instrument IV.

Average Elasticities (IV - both cost-shifters and average characteristics)

prodid	ηjj	ηj3
1	−4.45	0.459
2	−6.58	2.47
3	−7.11	NA
4	−6.62	0.459
5	−7.08	0.459

Note that the cross-price elasticity w.r.t. product 3 is much higher for the product that shares the nest (product 2). This makes sense because the model is assuming that product 2 and 3 are good substitutes, but we should keep in mind that this elasticity is artificially higher because of model assumptions, not necessarily because products 2 and 3 are empirically the closest substitutes.

Question 2 Functions

`add_mean_utility_NL!`

Create the new observed δ analog from BLP Eq. 27: $\delta_j(s,\sigma) = \ln(s_j) - \sigma \ln(\bar{s}_{jh}) - \ln(s_0)$

`add_mean_utility_NL!`

Create the new observed δ analog from BLP Eq. 27: $\delta_j(s,\sigma) = \ln(s_j) - \sigma \ln(\bar{s}_{jh}) - \ln(s_0)$

Create the new predicted δ from below BLP Eq. 27: $\delta = x_j\beta - \alpha p_j$

`add_group_denominator_NL!`

Estimate Nested Logit in-group share denominator D_h (BLP Eq 23). $D_h = \sum(j \in J_h) \exp(\delta_j / (1-\sigma))$ requires δ_hat = X'β in df

add_market_demoninator_NL!

Estimate Nested Logit group share denominator $\sum_h D_h$ (BLP Eq 24). `D_sum` $= \sum_h D_h$ in each market.

Requires `δ_hat` $= X'\beta$ in df

Note: h=0 is outside group, and $D_0 = 1$ (since δ_0 is normalized to 0, and e^0=1)

add_ingroup_share_NL!

Estimate Nested Logit predicted in-group share (BLP Eq 23). $s_j|_h = \exp(\delta_j / (1\text{-}\sigma)) / D_h$. Requires δ_hat = $X'\beta$ in df

add_market_share_NL!

Estimate Nested Logit predicted market share (BLP Eq 25).

`sⱼ = exp(δⱼ / (1-σ)) / (Dₕ^σ * D_sum).`

Requires `δ_hat = X'β` in df

add_product3_price_and_shares!

Expand prices and market shares for product 3 to all rows to help with elasticity computation

Cross- and own-price derivatives from [4], pg 15: (suppressing the market subscript)

```
∂sⱼ/∂pₖ = -α sⱼ (1/(1-σ) - σ/(1-σ)*sⱼ|ₕ - sⱼ)  if j = k (same product) \
        =   α sₖ (σ/(1-σ)*sⱼ|ₕ + sⱼ)            if j & k in same nest \
        =   α sₖ sⱼ                             otherwise \
    ηⱼₖ = ∂sⱼ/∂pₖ * pₖ / sⱼ
        = -α pₖ (1/(1-σ) - σ/(1-σ)*sⱼ|ₕ - sⱼ)      if j = k (same product)
        =   α sₖ (σ/(1-σ) * sⱼ|ₕ * pₖ / sⱼ + pₖ)    if j & k in same nest
        =   α sₖ pₖ                                 otherwise
```

Need to use the predicted market share $\hat{s}_{jn}$(δ_hat) BLP Eq 25

`cross_price_elasticity_NL`

Return Nested Logit cross-price elasticity w.r.t. product 3; checks if in same nest as product 3. Uses price derivatives from [4], pg 15. Need to use the predicted market share $\hat{s}_{jn}$(δ_hat) BLP Eq 25.

`own_price_elasticity_NL`

Return Nested Logit own-price elasticity

`add_price_elasticities_NL!`

Estimate own-price and cross-price elasticities of the predicted market shares for Nested Logit; add columns. Only adds cross-price elasticities relative to product 3. ηjjn and ηj3n Requires that p3 (price of product 3) and s3 (market share of product 3) be added as new columns.

```
report_avg_price_elasticities (generic function with 1 method)
```

`estimate_price_elasticities_NL!`

Return Nested Logit price elasticities using dataframe and BLP Eq 28 regression results.

`add_avg_characteristics_NL!`

"Add 'average' in-nest in-market other-product characteristics. Because there is at most 2 products in each market-nest, this 'average' is just the product characteristic value of the other product in the nest.

`add_marginal_costs_NL!`

Add estimated marginal costs to dataframe. Requires the own-price elasticity :ηjjn column.

QUESTION 3: MARGINAL COSTS

```
Enter cell code...
```

From Berry (1994), we know that in the oligopoly setting, where firms choose their price assuming prices of other products remain constant, that the estimable part of marginal cost is

$$\widetilde{MC}_j = p_j \left(1 + \frac{1}{\eta_{jj}}\right), \quad \eta_{jj} = \frac{\partial s_j}{\partial p_j} \frac{p_j}{s_j}$$

So using our demand-side estimates of η_{jj} for the four Nested Logit models above, I have estimated the marginal costs for each product in each market. The proportion (across observations) of marginal costs that are negative for each product in each model is below.

prodid	MC<o_b	MC>p_b	MC<o_c	MC>p_c	MC<o_d	MC>p_d	MC<o_e	MC>p_e
1	0.06	0	0.03	0	0	1	0	0
2	0.11	0	0.62	0.38	0	1	0	0
3	0.1	0	0.11	0.89	0	1	0	0
4	0.07	0	0.79	0.21	0	1	0	0
5	0.05	0	0.12	0.88	0	1	0	0

In the table above, "MC<0_x" columns show the share of observations from estimate (x) that were negative, and "MC>p_x" columns show the share of observations from estimate (x) that were above the price of the product. We see that marginal costs from parts (d) and (e) seem to be the most reasonable based on few or no observations that have negative marginal costs. But part (d) also has marginal cost estimates that are above price because the own-price elasticity estimates were positive (this results from the estimate of α from that part being negative). So part (e) seems to be the most reasonable estimate because it has both few negative marginal costs and no marginal costs above price. Part (b) also seems reasonable but likely suffers from endogeneity of prices and within-nest share.

One missing part from these analyses is standard errors, which would tell us if these MC estimates were significantly negative or above price. If we look at the OLS Nested Logit results from part (b), we can see the average marginal cost estimate for the observations that have a negative estimate is only -$1.97, compared to an average positive marginal cost of $14 (See below table). It is possible that many of the observations that have negative marginal costs are not significantly different from a small positive number.

Because the formula for marginal costs is $\widetilde{MC}_j = p_j\left(1+\frac{1}{\eta_{jj}}\right)$, a mechanical reason that marginal costs would be negative is that the own-price elasticity is between 0 and -1 (inelastic). If $\eta_{jj} \in (-1, 0)$ then $(1+\frac{1}{\eta_{jj}}) < 0$ and marginal cost would be negative. This seems to be implied by the model: if I face a very inelastic demand, then the models says I should be charging a markup even larger than the market price. This is probably due to the assumption that the firms are independent oligopolies and take each other's prices as given, when really there may be some form of collusion or are price takers perhaps.

Also note that IV models (c) and (d) both produce estimates of σ that are outside the (0,1) range, thus are not consistent with utility maximization. These σ estimates impact the marginal cost estimation via the calculation of own-price elasticity.

Average marginal costs from estimate (b), separately for positive and negative marginal costs

MC > 0	Average Price
0	-1.97
1	14

`get_share_neg`

Return vector of shares of marginal cost estimates that are negative, by product.

`get_share_above_price`

Return vector of shares of marginal cost estimates that are above price, by product.

QUESTION 4: PRICING EQUATIONS

(a) Logit price increases when firms 2 and 3 merge

prodid	Mean Price Increase (ppt)
2	4.46
3	5.58

(b) Nested logit price increases when firms 2 and 3 merge

prodid	Mean Price Increase (ppt)
2	14.9
3	16.3

(c) Differences and discussion

In both logit and nested logit, the price increase for product 3 is larger than the price increase for product 2. However, the nested logit price increases are much larger than the logit price increases (about 3 times larger). In logit, all firms are equal competitors, but in nested logit, firms in the same nest are much closer substitues (and thus much more competitive rivals). In the logit model then, when products 2 and 3 are merged into the same firm, consumers can more easily substitute to any other goods. But in nested logit, products outside the nest are less substitutable, so if products 2 and 3 can merge into the same firm, cross-price elasticities with goods outside the nest are less, and the firm could push both prices higher and keep a larger market share than under the logit model.

Both specifications have problems, but I am tempted to think the nested logit results are more realistic as long as the nests I defined ex ante reflect the way the firms think of their close rivals. If they also think of products 1, 4, and 5 as less substitutable, then the would be more likely to raise p2 and p3 prices because they have a captive market. However, these results strongly rely on the ex ante specification of the nests.

Question 4 Functions

`p2p3`

Return new p2 or p3 if product id is 2 or 3, respectively. Otherwise, return old price p.

`replace_p2p3`

Replace observed prices for products 2 and 3 in single-market dataframe (5 rows) with input p2 and p3

`δ`

Estimate Logit mean utility as a function of new p2, p3, given data and IV estimates

s

Estimate market shares s2,s3 based on p2, p3 -> δ(p2,p3) type = "logit", "nested logit" Need to set df.type

s3 (generic function with 1 method)

```
begin
    s2(p, df) = s(df, p...)[1]
    s3(p, df) = s(df, p...)[2]
end
```

∂s3∂p3 (generic function with 1 method)

```
begin
    """Estimate price derivatives of market shares at p2,p3"""
    mygrad(f,x,df) = FiniteDifferences.grad(central_fdm(5, 1), x -> f(x, df), x)
    ∂s2∂p2(p, df) = mygrad(s2, p, df)[1][1];  ∂s2∂p3(p, df) = mygrad(s2, p, df)[1][2]
    ∂s3∂p2(p, df) = mygrad(s3, p, df)[1][1];  ∂s3∂p3(p, df) = mygrad(s3, p, df)[1][2]
end
```

price_eqations!

Residuals function to find root – find p2,p3 that sets F[1] = 0 = F[2]

p_soln

Return vector of 5 product prices after merger of firms 2 and 3

percent_inc

Return percentage point increase from old to new

δNL

Estimate Nested Logit δ as a function of new p2, p3, given data and IV estimates. Removing the σ·log(s_{jhn}) term from the regression prediction to get δ instead of ln(sj/s0)

merger_NL

Estimate Nested Logit market shares of goods 2 and 3 for a single-market dataframe, as a function of new p2, p3

QUESTION 5: RANDOM COEFFICIENTS MODEL

Not finished: After some tweaking with Nevo's Matlab code, I was able to get results with his simulated data files. I was also able to load our data into similar matricies and get the first few iterations to run. Getting everything into conformable matricies was a helpful first step, so I am reporting the code / steps below. After the first iterations, the gradient returns NaN values, and the line search algorithm is unable to handle the issue. I think the next step would be to give the gradient function an "undesirable" value in the case it encounters NaN, so it will try a different direction. I hope to return to this estimation in the future to work out the kinks, but for now, I have to stop short of getting results for our dataset.

```
Enter cell code...
```

Need to have MATLAB installed on computer. See https://juliahub.com/ui/Packages/MATLAB/6HSOn/0.7.3 for more instructions
(1) Install MATLAB
(2) Start a Command Prompt as an Administrator and enter `matlab /regserver`.
(3) From Julia run: Pkg.add("MATLAB")

```
using MATLAB , SparseArrays
```

```
cd("NevoCode")
```

```
tomat (generic function with 1 method)
```

```
# Convert a vector to matrix
tomat(v) = reshape(v, length(v), 1)
```

```
df5 = copy(df);
```

```
# Sort: relations to Nevo: `:market` ~ date-city, `:prodid` ~ brand
sort!(df5, [:market,:prodid]);
```

`id` - (2256x1) -> (500x1) an id variable in the format bbbbccyyq, where bbbb is a unique 4 digit identifier for each brand (the first digit is company and last 3 are brand, i.e., 1006 is K Raisin Bran and 3006 is Post Raisin Bran), cc is a city code, yy is year (=88 for all observations is this data set) and q is quarter. All the other variables are sorted by date city brand.

```
id = @chain df5 begin
    @transform(:id = parse.(Int, string.(:prodid).*lpad.(:market,3,"0")))
    _.id
    reshape(_, length(_), 1)
    Matrix{Float64}(_)
end;
```

`id_demo` - (94x1) -> (100x1) an id variable for the random draws and the demographic variables, of the format ccyyq. Since these variables do not vary by brand they are not repeated. The first observation here corresponds to the first market, the second to the next and so forth.

```
id_demo = @chain df5.market unique;
```

`s_jt` - (2265x1) -> (500x1) the market shares of brand j in market t. Each row corresponds to the equivalent row in id.

```
s_jt = df5.sjn;
```

`x1` - (2256x25) -> (500x6) the variables that enter the linear part of the estimation. Here this consists of a price variable (first column) and 24 brand dummy variables. Each row corresponds to the equivalent row in id. This matrix is saved as a sparse matrix.

```
x1 = @chain df5 begin
    @select(:pjn, :d1,:d2,:d3,:d4,:d5)
    sparse(Matrix(_))
end ;
```

`x2` - (2256x4) -> (500x4) the variables that enter the non-linear part of the estimation. Here this consists of a constant, price, sugar content and a mushy dummy, respectively. Each row corresponds to the equivalent row in id. For us, this means constant, price, x1, x2 (product characteristics)

```
#= =#
x2 = @chain df5 begin
    @transform(:one = 1)
    @select(:one, :pjn, :x1,:x2)
    Matrix(_)
end;
```

`v` - (94x80) -> (100x80) N(0,1) random draws given for the estimation. For each market 80 iid normal draws are provided. They correspond to 20 "individuals", where for each individual there is a different draw for each column of x2. The ordering is given by id_demo.

```
v =  rand(Normal(), 100, 80);
```

`demogr` - (94x80) -> (100x80) draws of demographic variables from the CPS for 20 individuals in each market. The first 20 columns give the income, the next 20 columns the income squared, columns 41 through 60 are age and 61 through 80 are a child dummy variable (=1 if age <= 16). Each of the variables has been demeaned (i.e. the mean of each set of 20 columns over the 94 rows is 0). The ordering is given by id_demo.

Because we don't have data on the demographic distributions of each market (or all the markets), I'm going to pretend Nevo's demographics match our markets. This requires me to sample 6 rows at random and append them to get upto 100 markets. I could also remove the demographics completely and still estimate the model.

```
I realize that repeating 6 rows creates identical copies of 6 markets. I wonder if t
his could create the NaN issue that the gradient function was running into?
```

```
begin
    idx = sample(1:94, 6)
    demogr = @chain read_matfile("ps2.mat")["demogr"] begin
        jmatrix
        vcat(_, _[idx, :])
    end
end;
```

`iv` - (2256x21) -> (500x5) The file iv.mat contains the variable iv which consists of an id column (see the id variable above) and 20 columns of IV's for the price variable. The variable is sorted in the same order as the variables in the ps2.mat.

I need to combine the id column with our 4 IV's from the preferred Nested Logit model above.

```
iv = @chain df2e begin
    @transform(:id = id[:,1])
    @select(:id, :w1, :w2, :x1_avg, :x2_avg)
    Matrix{Float64}(_)
end;
```

Write the matricies to matlab files to open in matlab and run with `rc_dc_acw.m`

```
write_matfile("ps2.mat";
    id = tomat(id),
    id_demo = tomat(id_demo),
    s_jt = tomat(s_jt),
    x1 = x1,
    x2 = x2,
    v = v,
    demogr = demogr)
```

```
write_matfile("iv.mat"; iv = iv)
```

Notes

2SLS Weight matrix for logit and nested logit (BLP):

Nevo Footnote 22: That is, $\Phi = Z'Z$, which is the "optimal" weight matrix under the assumption of homoskedastic errors.

```
# Create table of contents on right side of screen
#TableOfContents()
```