Demand and Supply in Differentiated Products Markets

Phil Haile
Yale University

Fall 2013
(please alert me to typos or other errors)

Berry, Levinsohn, and Pakes (1995)

- now-standard empirical model of demand and supply of differentiated products, using only "market level data"
- similar structure to Bresnahan (1981)
  - discrete choice demand
  - multiproduct oligopoly supply
- but...
  - weaker functional form restrictions
  - careful treatment of unobservables (structural errors)
    - demand side unobservables, correlated w/prices
    - cost shocks
  - clear about endogeneity, and use of instruments to address.

Outline

1. BLP Demand Model
2. BLP Supply Model
3. BLP Estimator
4. BLP Estimation Algorithm
5. BLP Results and Simulations
6. Dube-Fox-Su Estimation Algorithm

Demand: Economic Model

Random Utility Discrete Choice

- differentiated goods $j \in \{1, \ldots J\}$
- unit demands
- conditional indirect utilities: u_{ij} ("utility")
- $(u_{i1}, \ldots, u_{iJ}) \sim F_u(\cdot)$
- "outside good" 0, with utility $u_{i0} = 0$
    - without outside good, no aggregate demand elasticity
    - normalization is without loss for each consumer; not without loss if different consumers have different outside options
- consumer i's choice $y_i = \arg\max_j u_{ij}$ (typically unique wp 1).

Choice Probabilities

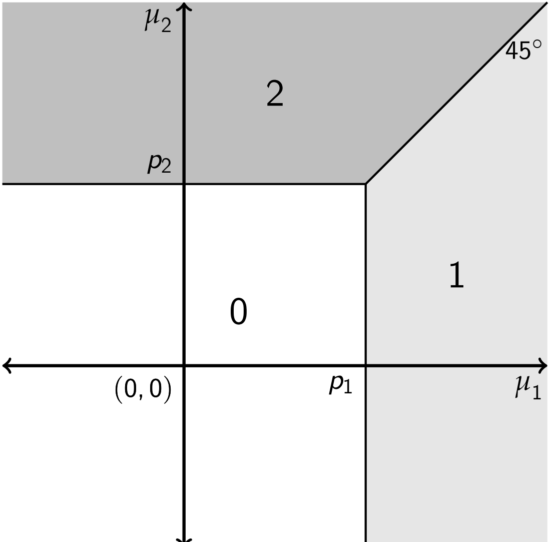
$$\begin{aligned} s_{ij} &= \Pr(y_i = j) \\ &= \int_{\mathcal{A}_j} dF_U(u_{i1}, \ldots, u_{iJ}) \end{aligned}$$

where

$$\mathcal{A}_j = \left\{ (u_{i1}, \ldots, u_{iJ}) \in \mathbb{R}^J : u_{ij} \geq u_{ik} \ \forall k \right\}.$$

,

Example: $J = 2, u_{ij} = \mu_{ij} - p_j$



Econometric Model

Goals

- parsimonious model to generate distribution $F_U(\cdot|)$ of random utilities, depending on product characteristics χ
- allow for rich heterogeneity in preferences
- allow flexibility in substitution patterns model can generate
- be explicit about unobservables
- be explicit about where endogeneity problem arises.

Utility Specification

(slightly simplified)

$$u_{ijt} = x_{jt}\beta_{it} - \alpha p_{jt} + \xi_{jt} + \epsilon_{ijt}$$

- consumer i, good/product j, market t
  (market usually defined by time or location)
- x_{jt}, p_{jt} observable product/market characteristics
- ξ_{jt} unobserved product/market characteristic
- ϵ_{ijt} idiosyncratic taste shock

. . .

Preference Heterogeneity

$$u_{ijt} = x_{jt}\beta_{it} - \alpha p_{jt} + \xi_{jt} + \epsilon_{ijt}$$

sources of heterogeneity: $\epsilon_{ijt}, \beta_{it} = \left(\beta_{it}^1, \ldots \beta_{it}^K\right)$

- $\beta_{it}^k = \beta_0^k + \sigma_k \zeta_{it}^k$ ("random coefficients")
- $\left(\epsilon_{ijt}, \zeta_{it}^k\right)_{j,k}$ i.i.d. across csrs and mts
- typically (but not essential)
  - $\epsilon_{ijt} \sim$ i.i.d. type 1 extreme value (like multinomial logit)
  - $\zeta_{itk} \sim$ i.i.d. standard normal, or reflecting distribution of demographics (e.g., income) in market.

Endogeneity

$$u_{ijt} = x_{jt}\beta_{it} - \alpha p_{jt} + \xi_{jt} + \epsilon_{ijt}$$

- exogenous characteristics: $x_{jt} \perp\!\!\!\perp \xi_{jt}$
- endogenous: p_{jt}
  - typically firms would know $(\xi_{1t}, \ldots, \xi_{Jt})$ when setting prices (or whatever firms choose)
  - $\implies p_{jt}$ will depend on the whole vector $\xi_t = (\xi_{1t}, \ldots, \xi_{Jt})$
  - we will need to distinguish the true effects of prices from the effects of ξ_t.

Utility Specification, Rewritten

Rewrite

$$\begin{aligned} u_{ijt} &= x_{jt}\beta_{it} - \alpha p_{jt} + \xi_{jt} + \epsilon_{ijt} \\ &= \delta_{jt} + \nu_{ijt} \end{aligned}$$

where

$$\begin{aligned} \delta_{jt} &= x_{jt}\beta_0 - \alpha p_{jt} + \xi_{jt} \\ \nu_{ijt} &= \sum_k x_{jt}^k \sigma^k \zeta_{it}^k + \epsilon_{ijt}. \end{aligned}$$

Market Shares

- recall $\delta_{jt} = x_{jt}\beta_0 - \alpha p_{jt} + \xi_{jt}$
- let $\delta_t = (\delta_{1t}, \ldots, \delta_{Jt})$
- continuum of consumers in each market (measure M_t)
- market shares (= choice probabilities)

$$sh_{jt} = \Pr(y_{it} = j) = \int_{\mathcal{A}_j(\delta_t)} dF_\nu(\nu_{i1}, \ldots, \nu_{iJ})$$

where

$$\mathcal{A}_j(\delta_t) = \left\{(\nu_1, \ldots, \nu_J) \in \mathbb{R}^J : \delta_{jt} + \nu_{jt} \geq \delta_{kt} + \nu_{kt} \ \forall k\right\}.$$

Why Random Coefficients?

Without random coefficients:

$$\begin{aligned} u_{ijt} &= \underbrace{x_{jt}\beta_0 - \alpha p_{jt} + \xi_{jt}}_{} + \epsilon_{ijt} \\ &= \qquad \delta_{jt} \qquad + \epsilon_{ijt} \end{aligned}$$

If ϵ_{ijt} i.i.d., then products differ (*ex ante*, or in expectation) only in δ_{jt} : i.e., in "mean utility"

- choice probabilities (market shares) depend only on the mean utilities, not "closeness" of products in characteristics space
- price elasticities (own and cross) depend only on mean utilities too

Hypothetical Example

	Price	Mkt Share
Mercedes S	\$105,00	0.05%
VW Beetle	\$25,000	0.05%
BMW 5	\$50,000	1.5%
Ford F-150 Pickup	\$23,000	1.5%

If assume $u_{ijt} = \delta_{jt} + \epsilon_{ijt}$, with $\epsilon_{ijt} \sim i.i.d.$, then

- must have $\delta_M = \delta_{VW}$, $\delta_{BMW} = \delta_F$
- price elasticity same for
  - Mercedes and VW
  - BMW and Ford pickup
- cross elasticities same for
  - Mercedes wrt price of BMW or Ford
  - BMW wrt price of VW or Mercedes
- if remove Mercedes from market, equal gains in mkt shares to BMW and Ford pickup!

Does this matter?

Yes!

- main point of demand estimation is to learn elasticities
- these determine responses to counterfactual changes in market
- these used with a model of the supply side to infer firm markups, market power, implications of mergers, entry incentives, etc.

Random Coefficients

- recognize that tastes and goods are heterogeneous
- consumers with taste for one German luxury car will probably like other German luxury cars too
- random coefficients on product characteristics can capture this
  - large $\beta_i^k \iff$ strong taste for characteristic x^k
  - i's first choice likely to have high value of x^k
  - i's second choice too!
    (note: cross elasticities always about 1st vs. 2nd choices).

Demand

- recall market shares

$$sh_{jt} = \Pr(y_{it} = j) = \int_{\mathcal{A}_j(\delta_t)} dF_\nu(\nu_{i1}, \ldots, \nu_{iJ})$$

- $F_\nu(\cdot)$ is really $F_\nu(\cdot | x_t, \sigma)$ where
  - $x_t = (x_{1t}, \ldots, x_{Jt}) \in \mathbb{R}^{K \times J}$
  - $\sigma = (\sigma^1, \ldots, \sigma^K)$
- market shares determined by functions

$$s_j(\delta_t, x_t, \sigma)$$

- quantities demanded $q_{jt} = M_t \times s_j(\delta_t, x_t, \sigma)$.

Oligopoly Supply

Basics of model

- multi-product firms (e.g., car manufacturer)
- complete information (about each other and demand, including ξ_t)
- products and non-price characteristics given (as in Bresnahan)
- simultaneous price setting (not essential, but need a model)
- Nash equilibrium (assume existence)
- (most of this can be modified; e.g. quantity competition also fine, and could allow some elements of x to be endogenous).

Firm Cost Functions

- additive in costs from each product j
- $C_j\left(q_{jt}, \mathrm{w}_{jt}, \omega_{jt}, \gamma\right)$
  - q_{jt} demanded quantities
  - w_{jt} observable cost shifters
  - ω_{jt} unobserved cost shifters (may be correlated with ξ_{jt})
  - γ, parameters
  - usually, $C_j(\cdot) = C(\cdot)$
  - ignore fixed costs.

Equilibrium Conditions

With Single-Product Firms

$$\pi_{jt} = p_{jt} M_t s_j\left(\delta_t, x_t, \sigma\right) - C_j\left(M_t s_j\left(\delta_t, x_t, \sigma\right), \mathrm{w}_{jt}, \omega_{jt}, \gamma\right)$$

FOC wrt to p_{jt},

$$p_{jt} = mc_{jt} - s_j\left(\delta_t, x_t, \sigma\right)\left(\frac{\partial s_j}{\partial p_{jt}}\right)^{-1}$$

(inverse elasticity pricing against "residual demand curve" $M_t s_j\left(\delta_t, x_t, \sigma\right)$

Since $\frac{\partial s_j}{\partial p_j} = \frac{\partial s_j}{\partial \delta_{jt}} \frac{\partial \delta_{jt}}{\partial p_{jt}} = -\alpha \frac{\partial s_j}{\partial \delta_{jt}}$

$$p_{jt} = mc_{jt} - \frac{s_j\left(\delta_t, x_t, \sigma\right)}{\alpha}\left(\frac{\partial s_j}{\partial \delta_{jt}}\right)^{-1}.$$

Equilibrium Conditions

With multi-product firms:

$$\Pi_{ft} = \sum_{j \in J_f} \pi_{jt} = \sum_{j \in J_f} p_{jt} M_t s_j \left(\delta_t, x_t, \sigma\right) - C_j \left(M_t s_j \left(\delta_t, x_t, \sigma\right), \mathrm{w}_{jt}, \omega_{jt}, \gamma\right)$$

where f is the index of a firm, and J_f is the set of Firm j's products

FOC

$$p_{jt} = mc_{jt} - \left(\frac{\partial s_j}{\partial p_{jt}}\right)^{-1} \left[s_j \left(\delta_t, x_t, \sigma\right) + \sum_{j \in J_f \setminus \{j\}} \frac{\partial s_k}{\partial p_{jt}} \right]$$

(firm internalizes effects of price change on all of its products – cannibalization, diversion to self).

Estimation: A Partial Sketch

Observables: $\left(x_t, p_t, s_t, \mathrm{w}_t, \text{ and } z_t \leftarrow \mathrm{IV}\right)$ ("market level data")

1. start with demand model alone
2. suppose $F_\nu\left(\cdot|x_t, \sigma\right)$ known (i.e., σ known)
3. for each market t, find $\delta_t \in \mathbb{R}^J$ such that $s_j\left(\delta_t, x_t, \sigma\right) = sh_{jt}\ \forall j$
   - i.e., "invert" market shares to recover "mean utilities" δ_t
   - note: choosing the δ_t to fit shares exactly
4. estimate the equation

$$\delta_{jt} = x_{jt}\beta_0 - \alpha p_{jt} + \xi_{jt}$$

using IV on all market-product observations $\longrightarrow \hat{\beta}_0, \hat{\alpha}, \hat{\xi}_{jt}\ \forall j, t$.

Some Details to Fill In

1. What instruments?
2. Will the "inversion" step actually work?
3. What about σ?
4. details of estimator and algorithm for constructing estimate
5. Add Supply Side
   - additional restrictions (moment conditions) aid estimation of demand
   - estimate parameters γ of marginal cost function (why? may care directly; needed for counterfactuals that change equilibrium quantities unless *mc* is constant).

Instruments

1. Excluded cost shifters
   - classic demand instrument, e.g., wages in supplier's market, material costs, shipping cost to market t, excise taxes/tariffs
2. Proxies for excluded costs shifters
   - typical: price of same good in another market ("Hausman instruments")
3. "BLP instruments": x_{-jt}
   - excluded from equation $\delta_{jt} = \ldots$ $\left(i.e., .E\left[\xi_{jt}|x_t\right] = E\left[\xi_{jt}\right]\right)$
   - affect prices through the equilibrium, (also change the choice set)
4. Characteristics of "nearby" markets
   - e.g., firms may use same price for all towns in a larger region
   - so age/income/education in New Haven's neighboring towns may affect New Haven prices, but perhaps not New Haven preferences (conditional on observed NH observables).

Will the Inversion Step Work?

- given x, σ and any positive shares sh, define $\Phi : \mathbb{R}^J \rightarrow \mathbb{R}^J$ by

$$\Phi(\delta) = \delta + \ln(sh) - \ln(s(\delta, x, \sigma))$$

- Berry (1994) shows (under mild conditions) that Φ is a *contraction*, i.e.,
  - it has a unique fixed point
    - $\implies s(\delta, x, \sigma)$ has an inverse: can write $\delta_t = \delta(sh_t; x_t, \sigma)$
  - convergent algorithm: start with guess δ^0, set $c = 1$
    1. let $\delta^c = \Phi\left(\delta^{c-1}\right)$
    2. repeat to convergence.

Analytical Inversion in Simple Cases

- Multinomial Logit
  - $u_{ijt} = \delta_{jt} + \epsilon_{ijt}$
  - $sh_{jt} = s_j\left(\delta_t, x_t, \sigma\right) = \frac{e^{\delta_j}}{1+\sum_k e^{\delta_k}}$
  - $sh_{0t} = s_j\left(\delta_t, x_t, \sigma\right) = \frac{1}{1+\sum_k e^{\delta_k}}$
  - $\implies \frac{sh_{jt}}{sh_{0t}} = e^{\delta_j}$
  - $\implies \ln\left(sh_{jt}\right) - \ln\left(sh_{0t}\right) = \delta_{jt} = x_{jt}\beta_0 - \alpha p_{jt} + \xi_{jt}$
- Nested Logit (see Berry 94).

What About sigma?

- "Berry inversion" $\implies$ for <u>any</u> market shares s_t and <u>any</u> σ, we can find a vector δ_t that rationalizes the data with the BLP model
- a non-identification result? there is NO information about σ from market shares?

What are we forgetting?

- cross-market and cross-product restrictions
  - one parameter vector σ and one model of δ_{jt} (parameters α, β_0) must "work" across all markets and products
  - residuals $\xi_{jt} = \delta_{jt} - x_{jt}\beta_0 - \alpha p_{jt}$ must be independent of exogenous observables across markets and products.

Identification of sigma

- nonparametric intuition: changes in choice sets
  - two markets with 1 car removed
  - same idea within market given parameterization
- parametric intuition:
  - trial value of $\sigma \implies \delta_t(\sigma)$
  - with trial value of $\beta \implies \xi_t(\sigma, \alpha, \beta)$
  - IV orthogonality condition: $E\left[Z_{jt}\xi_{jt}(\sigma, \alpha, \beta)\right] = 0 \ \forall j, t, Z_{jt}$
- note: obviously need at least as many moments as parameters
  - excluded instruments for price not enough because (α, β) are not all the parameters
  - need excluded instruments "to identify σ" too (x_{-jt} change the choice set)
- formal *nonparametric* identification proof: later.

Intuition for BLP Estimator (demand alone)

idea: method of moments estimator using orthogonality conditions just mentioned

- model moments $\Longrightarrow$ $E\left[Z'_{jt}\xi_{jt}\left(\sigma,\alpha,\beta\right)\right]=0$
- sample analog: $E\left[Z'_{jt}\xi_{jt}\left(\sigma,\alpha,\beta\right)\right]\approx\frac{1}{JT}Z_{jt}\hat{\xi}_{jt}\left(\sigma,\alpha,\beta\right)$
- GMM estimator?
  - $\left(\hat{\sigma},\hat{\alpha},\hat{\beta}\right)$ chosen to set sample analog close to zero (exactly zero if just-identified, but usually more moments than parameters)
  - optimal weighting of observations in the sample analog (heteroskedasticity).

Some Complications

1. model predictions $s_j(\delta_t, x_t, \sigma)$ matched to shares involve high-dimensional integrals
   - use simulation to approximate
   - $\Longrightarrow$ "method of simulated moments" (Pakes and Pollard, 1989; McFadden, 1989) instead of GMM
2. moment conditions involve $\xi_t(\sigma, \alpha, \beta)$ which has no closed form
   - solve for $\xi_t(\sigma, \alpha, \beta)$ at each trial value of (σ, α, β) $\Longrightarrow$ "nested fixed point" algorithm (BLP)
   - or, optimize over $\sigma, \alpha, \beta, \xi_t$ simultaneously, subject to $\xi_t = \xi_t(\sigma, \alpha, \beta)$ $\Longrightarrow$ constrained optimization algorithm (Dube-Fox-Su, 2012).

Notation for Estimation

- let $\theta = (\theta_1, \theta_2) = ([\alpha, \beta], \sigma)$
- let Z_{jt} denote the exogenous variables $(x_{jt}, \mathrm{w}_{jt}, \tilde{z}_{jt})$ where $\tilde{z}_{jt}$ may be (should be) BLP IV .
- let $\delta_{jt}(\theta_2)$ be shorthand for $\delta_j(s_t; x_t, \sigma)$.

The BLP Estimator

GMM estimator of θ defined as solution to mathematical program:

$$
\begin{aligned}
\min_{\theta} & \quad g(\xi(\theta))' W g(\xi(\theta)) \quad \text{s.t.} \\
g(\xi(\theta)) &= \frac{1}{N} \sum_{\forall j,t} \xi_{jt}(\theta)' z_{jt} \\
\xi_{jt}(\theta) &= \delta_{jt}(\theta_2) - x_{jt}\beta - \alpha p_{jt} \\
s_j(\delta_t, \theta_2) &= \int \frac{\exp[\delta_{jt}(\theta_2) + \nu_{ij}]}{1 + \sum_k \exp[\delta_{jt}(\theta_2) + \nu_{ik}]} f_\nu(\nu | x_t, \theta_2) d\nu \\
\log(sh_{jt}) &= \log(s_j(\delta_t, x_t, \theta_2)) \\
W &= \text{ standard GMM weight matrix.}
\end{aligned}
$$

Simulated Moments: use simulation to approximate $s_j(\delta_t, x_t, \theta_2)$

BLP Estimation Algorithm: Sketch

"Nested Fixed Point" (used for other things too)

- Outer Loop
  - search over trial values of θ to solve: $\min_\theta g(\xi(\theta))' W g(\xi(\theta))$
  - stop when $\|g(\xi(\theta))' W g(\xi(\theta))\| < \tau_{outer}$
- Inner Loop
  - given θ, find solution for $\xi(\theta)$
    - given θ_2, solve for $\delta(\theta_2)$ as fixed point of contraction mapping
    - then $\xi_{jt}(\theta) = \delta(\theta_2) + \alpha p_{jt} - x_{jt}\beta_0$

BLP Estimation Algorithm: More Detail

1. start with initial guess at $\delta_t^0 \; \forall t$
   - e.g., from MNL or nested logit

BLP Estimation Algorithm: More Detail

1. start with initial guess at $\delta_t^0 \; \forall t$
2. take a trial value of the parameters θ
    - selected by non-derivative search algorithm (e.g., Nelder-Meade)

BLP Estimation Algorithm: More Detail

1. start with initial guess at $\delta_t^0 \; \forall t$
2. take a trial value of the parameters θ
3. approximate $s_j(\delta_t, \theta)$ via simulation
    - sample values of ζ_{it}^k from normal distribution
    - take many draws to simulate population of csrs:
      $s_j(x_t, \xi_t, \theta) = \int \frac{\exp[x_{jt}\beta_i + \xi_{jt}]}{1+\sum_{j'} \exp[x_{jt}\beta_i + \xi_{jt}]} f(\beta_i|\theta)$
      $\approx \frac{1}{NS} \sum_{i=1}^{NS} \frac{\exp[x_{jt}\beta_0 + \xi_{jt} + \sum_k x_{jkt}\sigma_k \zeta_{it}^k]}{1+\sum_{j'} \exp[x_{jt}\beta_0 + \xi_{jt} + \sum_k x_{jkt}\sigma_k \zeta_{it}^k]}$
    - more sophisticated sampling possible ("importance sampling").

BLP Estimation Algorithm: More Detail

1. start with initial guess at δ_t^0 $\forall t$
2. take a trial value of the parameters θ
3. approximate $s_j(\delta_t, \theta)$ by simulation
4. solve fixed point problem by iterating on the contraction $\delta_{jt}^{h+1} = \delta_{jt}^{h} + \log(sh_{jt}) - \log(s_j(\delta^h, x_t, \theta_2))$ $\forall j$ to find the δ_t that satisfies all share equations
    - convergence tolerance τ_{inner}
    - tempting to make this loose, at least in beginning, but this can lead to poor performance (see Dube, Fox, and Su (2012))

BLP Estimation Algorithm: More Detail

1. start with initial guess at $\delta_t^0 \ \forall t$
2. take a trial value of the parameters θ
3. approximate $s_j(\delta_t, \theta)$ via simulation
4. solve fixed point problem by iterating on the contraction $\delta_{jt}^{h+1} = \delta_{jt}^{h} + \log(sh_{jt}) - \log(s_j(\delta^h, x_t, \theta_2)) \ \forall j \rightarrow \delta_{jt}(\theta_2)$ for all j, t
5. $\delta_{jt}(\theta_2) = x_{jt}\beta_0 - \alpha p_{jt} + \xi_{jt} \implies \hat{\hat{\xi}}_t(\theta)$
6. approximate $g(\xi(\theta))$ by $\frac{1}{JT} Z_{jt}\hat{\hat{\xi}}_{jt}(\theta)$
7. plug into GMM objective function
8. iterate (from 2) to convergence

Standard errors: standard MSM (e.g., Pakes-Pollard).

Nonlinear vs. Linear Parameters

Note:

- $\theta_1 = (\alpha, \beta_0)$ enter objective function linearly
- so given θ_2 and W, have closed-form expression for optimal θ_1
- so outer loop search only involves $\theta_2 = \sigma$.

The NFP Algorithm

Challenges

- contraction rate ("inner loop") is often SLOW
- "outer loop" is hard
    - existence of inner loop often yields highly non-convex optimization problem, Nelder-Meade etc. can fail
    - user-defined tolerances important: can get wrong answer without realizing it
    - NFP approach potentially inefficient: forces constraints on market shares and linear parameters hold exactly at every guess of θ_2 when we care only about their holding at the optimum.

Constrained Optimization ("MPEC") Algorithm

(mathematical programming with equilibrium constraints)

- Dube, Fox, and Su (2012): alternative algorithm for BLP estimator
- general idea: same estimator and same standard error formula, but
    - instead of inner and outer loops, *minimize GMM objective function over parameters, subject to constraint that the inner loop fixed point equations hold*
    - use constrained optimization routines that work well for "sufficiently nice" constrained optimization problems
    - use some tricks to make the BLP problem nice enough.

BLP by MPEC: Simple Version

- constrained optimization formulation of BLP estimator

$$\begin{aligned}
\min_{\theta,\xi} \quad & \xi' Z' \; W \; Z' \xi \quad \text{s.t.} \\
\log(sh_{jt}) \quad &= \quad \log(s_j(x_t, \xi_t, \theta)) \quad \forall j, t_i \\
\text{where } s_j(x_t, \xi_t, \theta) \quad &\equiv \quad \frac{1}{NS} \sum_{i=1}^{NS} \frac{\exp[x_{jt}\beta_0 + \xi_{jt} + \sum_k x_{jkt}\zeta_{it}^k]}{1 + \sum_{j'} \exp[x_{jt}\beta_0 + \xi_{jt} + \sum_k x_{jkt}\zeta_{it}^k]}
\end{aligned}$$

- Note: ξ_{jt} treated as parameters (a lot of them!) whose values must satisfy constraints that define the fixed point.

BLP by MPEC: Better Version

- introduce superfluous parameters g

$$
\begin{aligned}
\min_{\theta,\xi,g} \quad & g' \; W \; g \quad \text{s.t.} \\
g \; &= \; Z'\xi \\
\log(sh_{jt}) \; &= \; \log(s_j(x_t, \xi_t, \theta)) \quad \forall j, t_i \\
\text{where } s_j(x_t, \xi_t, \theta) \; &\equiv \; \frac{1}{NS} \sum_{i=1}^{NS} \frac{\exp[x_{jt}\beta_0 + \xi_{jt} + \sum_k x_{jkt}\zeta_{it}^k]}{1 + \sum_{j'} \exp[x_{jt}\beta_0 + \xi_{jt} + \sum_k x_{jkt}\zeta_{it}^k]}
\end{aligned}
$$

- even more parameters now, but Hessian now may more *sparse* (see Dube, Fox, Su, 2012)
  - (parameter ξ_{jt} has no effect on constraints outside market t).

MPEC Algorithm

1. single-step optimization (no inner/outer loop)
2. "canned" optimization algorithms from engineering
    - AMPL (free language used to define problem)
    - NEOS server with many optimizers (for free use by internet)
    - some (KNITRO) also usable from Matlab
    - Matlab fmincon may work for nice problems
3. advantages
    - no user-defined tolerances etc. to mess up
    - solvers optimized for constrained optimization problems, so may be faster and more reliable
        - "automatic differentiation" of Lagrangian objective function in AMPL
        - constraints imposed at solution, not at every trial value of parameters
        - AMPL automatically detects "sparsity patterns"
    - can be faster than NFP.

MPEC Algorithm

Disadvantages

1. often (usually?) need to code 1st and 2nd derivatives, and may need to work hard to make problem "nice" (tricks in the formulation of the problem to induce sparsity); this can take a long time and introduces problem-specific coding and, therefore, the likelihood of coding error; so speed advantage and avoidance of dependence on user-defined tolerances may be less compelling;
2. optimization problem has much higher dimension (new "parameters"); need sparsity to get speed gains (or for optimization to work at all), and this vanishes when J large and T small.

Code on the Internet

- BLP: from Aviv Nevo (Northwestern)
  - includes some fake data, but fake instruments not very good
  - may not be updated with latest insights about convergence tolerances (inner loop tolerance must be very small)
  - see also his "Practitioners Guide…" in *JEMS*
- BLP and MPEC: from Dubé-Fox-Su (e.g., Su at U. Chicago).

Adding the Supply Side

- suppose

$$mc_{jt}\left(\mathrm{w}_{jt},\omega_{jt},\gamma\right)=\mathrm{w}_{jt}\gamma+\omega_{jt}$$

- recall firm FOC: $p_{jt} - \mathrm{w}_{jt}\gamma + \omega_{jt} - \frac{s_j(\delta_t, x_t, \sigma)}{\alpha}\left(\frac{\partial s_j}{\partial \delta_{jt}}\right)^{-1} = 0$
- so for any $(\sigma, \alpha, \beta, \gamma)$, we have an implied ω_{jt}
- additional moments for estimation:

$$E\left[Z_{jt}'\omega_{jt}\left(\sigma,\alpha,\beta,\gamma\right)\right]=0$$

- note
  - supply moments depend on demand parameters too
  - in practice, these often important to getting precise estimates of demand.

TABLE 1

DESCRIPTIVE STATISTICS

Year	No. of Models	Quantity	Price	Domestic	Japan	European	HP/Wt	Size	Air	MPG	MP$
1971	92	86.892	7.868	0.866	0.057	0.077	0.490	1.496	0.000	1.662	1.850
1972	89	91.763	7.979	0.892	0.042	0.066	0.391	1.510	0.014	1.619	1.875
1973	86	92.785	7.535	0.932	0.040	0.028	0.364	1.529	0.022	1.589	1.819
1974	72	105.119	7.506	0.887	0.050	0.064	0.347	1.510	0.026	1.568	1.453
1975	93	84.775	7.821	0.853	0.083	0.064	0.337	1.479	0.054	1.584	1.503
1976	99	93.382	7.787	0.876	0.081	0.043	0.338	1.508	0.059	1.759	1.696
1977	95	97.727	7.651	0.837	0.112	0.051	0.340	1.467	0.032	1.947	1.835
1978	95	99.444	7.645	0.855	0.107	0.039	0.346	1.405	0.034	1.982	1.929
1979	102	82.742	7.599	0.803	0.158	0.038	0.348	1.343	0.047	2.061	1.657
1980	103	71.567	7.718	0.773	0.191	0.036	0.350	1.296	0.078	2.215	1.466
1981	116	62.030	8.349	0.741	0.213	0.046	0.349	1.286	0.094	2.363	1.559
1982	110	61.893	8.831	0.714	0.235	0.051	0.347	1.277	0.134	2.440	1.817
1983	115	67.878	8.821	0.734	0.215	0.051	0.351	1.276	0.126	2.601	2.087
1984	113	85.933	8.870	0.783	0.179	0.038	0.361	1.293	0.129	2.469	2.117
1985	136	78.143	8.938	0.761	0.191	0.048	0.372	1.265	0.140	2.261	2.024
1986	130	83.756	9.382	0.733	0.216	0.050	0.379	1.249	0.176	2.416	2.856
1987	143	67.667	9.965	0.702	0.245	0.052	0.395	1.246	0.229	2.327	2.789
1988	150	67.078	10.069	0.717	0.237	0.045	0.396	1.251	0.237	2.334	2.919
1989	147	62.914	10.321	0.690	0.261	0.049	0.406	1.259	0.289	2.310	2.806
1990	131	66.377	10.337	0.682	0.276	0.043	0.419	1.270	0.308	2.270	2.852
All	2217	78.804	8.604	0.790	0.161	0.049	0.372	1.357	0.116	2.099	2.086

Note: The entry in each cell of the last nine columns is the sales weighted mean.

TABLE II

THE RANGE OF CONTINUOUS DEMAND CHARACTERISTICS (AND ASSOCIATED MODELS)

Variable	Percentile 0	25	50	75	100
Price	90 Yugo 3.393	79 Mercury Capri 6.711	87 Buick Skylark 8.728	71 Ford T-Bird 13.074	89 Porsche 911 Cabriolet 68.597
Sales	73 Toyota 1600CR .049	72 Porsche Rdstr 15.479	77 Plym. Arrow 47.345	82 Buick LeSabre 109.002	71 Chevy Impala 577.313
HP/Wt.	85 Plym. Gran Fury 0.170	85 Suburu DH 0.337	86 Plym. Caravelle 0.375	89 Toyota Camry 0.428	89 Porsche 911 Turbo 0.948
Size	73 Honda Civic 0.756	77 Renault GTL 1.131	89 Hyundai Sonata 1.270	81 Pontiac F-Bird 1.453	73 Imperial 1.888
MP$	74 Cad. Eldorado 8.46	78 Buick Skyhawk 15.57	82 Mazda 626 20.10	84 Pontiac 2000 24.86	89 Geo Metro 64.37
MPG	74 Cad. Eldorado 9	79 BMW 528i 17	81 Dodge Challenger 20	75 Suburu DL 25	89 Geo Metro 53

Notes: The top entry for each cell gives the model name and the number directly below it gives the value of the variable for this model.

TABLE VII

SUBSTITUTION TO THE OUTSIDE GOOD

Model	Given a price increase, the percentage who substitute to the outside good (as a percentage of all who substitute away.) Logit	BLP
Mazda 323	90.870	27.123
Nissan Sentra	90.843	26.133
Ford Escort	90.592	27.996
Chevy Cavalier	90.585	26.389
Honda Accord	90.458	21.839
Ford Taurus	90.566	25.214
Buick Century	90.777	25.402
Nissan Maxima	90.790	21.738
Acura Legend	90.838	20.786
Lincoln Town Car	90.739	20.309
Cadillac Seville	90.860	16.734
Lexus LS400	90.851	10.090
BMW 735*i*	90.883	10.101

TABLE VIII

A SAMPLE FROM 1990 OF ESTIMATED PRICE-MARGINAL COST MARKUPS AND VARIABLE PROFITS: BASED ON TABLE 6 (CRTS) ESTIMATES

	Price	Markup Over MC $(p - MC)$	Variable Profits (in $'000's) $q * (p - MC)$
Mazda 323	$5,049	$ 801	$18,407
Nissan Sentra	$5,661	$ 880	$43,554
Ford Escort	$5,663	$1,077	$311,068
Chevy Cavalier	$5,797	$1,302	$384,263
Honda Accord	$9,292	$1,992	$830,842
Ford Taurus	$9,671	$2,577	$807,212
Buick Century	$10,138	$2,420	$271,446
Nissan Maxima	$13,695	$2,881	$288,291
Acura Legend	$18,944	$4,671	$250,695
Lincoln Town Car	$21,412	$5,596	$832,082
Cadillac Seville	$24,353	$7,500	$249,195
Lexus LS400	$27,544	$9,030	$371,123
BMW 735*i*	$37,490	$10,975	$114,802

Summary

- BLP model/estimator attractive for differentiated products
- allows rich heterogeneity, endogeneity
- generalization of Bresnahan supply side: infer markups from firm optimality conditions
- widely used in IO, but also many other fields of economics
    - economics is about choices
    - choices are always differentiated, usually in unobservable ways
    - supply is almost always imperfectly competitive
    - many variations of the model in the literature: basic ideas of the approach are very robust.