Week 2: Empirical Models of Competition with Differentiated Products

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Competition with Differentiated Products

Why We Care

- Many important markets are composed of differentiated products
- This means that sellers have some residual market power, potentially even when there are many firms
- The nature of differentiation matters for consumer demand and firm strategy
- Important for:
  - Competition analysis
  - New product introduction
  - Merger Analysis
  - Much more
- Examples

Bresnahan (1987)

Competition with Differentiated Products

- Develop model to test competition in the automobile sector between 1954-1956
- One of first papers to empirically model differentiated products in a structural setup:
  - Borenstein and Shephard (1996) good example of paper with non-differentiated products. Slides / paper online.
  - Substantial structure on differentiation collapsing attributes into one 'quality' dimension
  - Static competition, no endogenous attributes
  - Very specific, structured firm competition environment
- Empirical Context:
  - 1955 autombile market anomaly relative to other years
  - 45% greater sales than in 2 surrounding years
  - Lower quality adjusted prices
  - Consistent with a supply-side shock

Overview of Contribution

Bresnahan (1987)

- Rigorous empirical explanation for otherwise puzzling market evolution
- Explicit test of Bertrand-Nash and Collusive behavior:
  - Test relies on fact that prices for products 'near' each other respond differently under collusion vs. Bertrand
  - Leverages multi-product within firm pricing as source of identification
  - Conduct hypothesis have direct implications for price and quantity comparitive statics with respect to demand elasticities (without knowing MC!)
- Patterns suggest collusion (tacit) in 1954 and 1956, and Bertrand-Nash in 1955
- No discussion of repeated game literature, though it applies to tacit collusion explicitly

Facts about Automobile Markets

Bresnahan (1987)

TABLE I

Year	(1) Auto Production[a]	(2) Real Auto Price-CPI[b]	(3) % Change Auto Price-Cagan[c]	(4) Auto Sales[d]	(5) Auto Quantity Index[e]
1953	6.13	1.01	NA	14.5	86.8
1954	5.51	0.99	NA	13.9	84.9
1955	7.94	0.95	−2.5	18.4	117.2
1956	5.80	0.97	6.3	15.7	97.9
1957	6.12	0.98	6.1	16.2	100.0

Notes: [a] Millions of units over the model year. [Source: *Automotive News.*]
[b] (CPI New automobile component)/CPI. [Source: *Handbook of Labor Statistics.*]
[c] Adjusted for quality change. [See Cagan (1971), especially pp. 232–3.]
[d] Auto output in constant dollars, *QIV* of previous year through *QIII* of named year, in billions of 1957 dollars. [Source: *National Income and Product Accounts.*]
[e] (4)/(2), normalized so 1957 = 100.

Facts about Automobile Markets

Bresnahan (1987)

TABLE II

Year	(6) *Per Capita Disposable Personal Income*[f]	(7) *Interest Rate*[g]	(8) *Durables Expenditures (Non-Auto)*[h]	(9) *Automakers Profits*[i]
1953	1623	1.9	14.5	2.58
1954	1609	0.9	14.5	2.25
1955	1659	1.7	16.1	3.91
1956	1717	2.6	17.1	2.21
1957	1732	3.2	17.0	2.38

Notes: [f] Billions of 1957 dollars, *QIV* of previous year through *QIII* of named year. [Source: *National Income and Product Accounts.*]

[g] Three-month *T*-bill rate. [Source: *Statistical Abstract.*]

[h] Durables component of consumer expenditures minus component for automobiles and parts, billions of 1957 dollars. [Source: *National Income and Product Accounts.*]

Demand

Bresnahan (1987)

- Continuum of potential buyers with unit demand with differences in taste for quality x denoted by v_i
- Strong assumptions about demand to make model tractable

$$\begin{aligned} U(x, Y, v) &= vx + Y - P \quad \text{if buy} \\ U(x, Y, v) &= v\gamma + Y - E \quad \text{if no buy} \end{aligned}$$

- Y-P is money not spent on autos
- **Assumption:** v is uniformly distributed with density δ on $[0, V_{max}]$
- What do you think?

Demand

Bresnahan (1987)

- Consumer will select product that maximizes $vx_j - P_j$
- Aggregate into market demand by investigating indifferent consumers between two products h and i:

$$\begin{aligned} P_i - x_i v_{hi} &= P_h - x_h v_{hi} \Rightarrow \\ v_{hi} &= \frac{P_i - P_h}{x_i - x_h} \end{aligned}$$

- If product j on other side of i then aggregate demand for i is:

$$q_i = \delta[\frac{P_j - P_i}{x_j - x_i} - \frac{P_i - P_h}{x_i - x_h}]$$

- Closer x imply cross-price derivative closer to own-derivative

Demand

Bresnahan (1987)

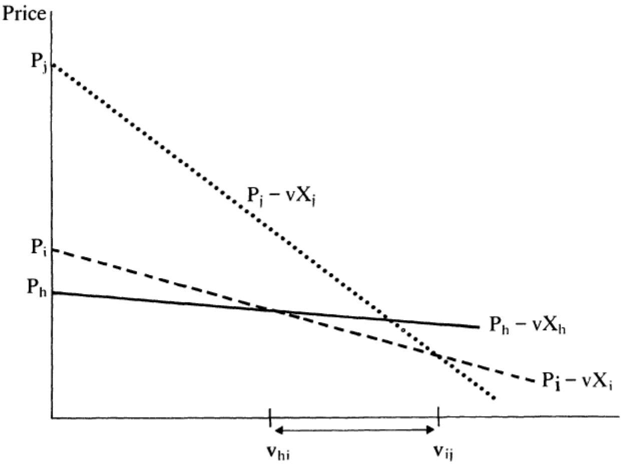


Figure 1

Final Demand Equations

Bresnahan (1987)

- Final demand equations from model:

$$\begin{aligned}
q_1 &= \delta[\frac{P_2 - P_1}{x_2 - x_1} - \frac{P_1 - E}{x_1 - \gamma}] \\
q_i &= \delta[\frac{P_j - P_i}{x_j - x_i} - \frac{P_i - P_h}{x_i - x_h}] \\
q_n &= \delta[v_{max} - \frac{P_n - P_{n-1}}{x_n - x_{n-1}}]
\end{aligned}$$

- E, γ, v_{max}, and δ are parameters to estimate

Supply

Bresnahan (1987)

- Firms are assumed to have cost function:

$$C(x, q) = A(x) + mc(x)q$$

- Assume that marginal cost is increasing and convex, μ parameter to estimate:

$$mc(t) = \mu e^t$$

- Profit function is:

$$\pi_i = P_i q_i - mc(x_i)q_i - A(x_i)$$

Supply: Firm Interactions

Bresnahan (1987)

- Bresnahan investigates two potential assumptions on firm interactions:
  - Collusion: All firms set prices to maximize sum of all profits
  - Nash-Bertrand: Each firm sets prices to maximize own profits
- Since quantities linear in prices, solution to profit maximization is linear in prices
- Only neighboring products have interactions with one another in supply and demand!!
  - Are these desirable substitution patterns?
  - What do you think about model of collusion?

Supply: Firm Interactions

Bresnahan (1987)

- Nash-Bertrand FOC:

$$\pi_i' = q_i + (P_i - mc(x_i))\frac{\partial q_i}{\partial P_i} = 0$$

- Multi-product or collusive FOC for i and $i+1$:

$$\pi_i' = q_i + (P_i - mc(x_i))\frac{\partial q_i}{\partial P_i} + (P_{i+1} - mc(x_{i+1}))\frac{\partial q_{i+1}}{\partial P_i} = 0$$

- If products closely spaced, impact of multi-product behavior is high
  - Examples in reality?
- Simultaneous FOC solution for all products in equilibrium

Supply: Firm Interactions

Bresnahan (1987)

- Let matrix H_{ij} represent cooperation with $H_{ij} = 1$ if products are cooperating and 0 otherwise. Example:

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Given this, the model supply FOC for product i is:

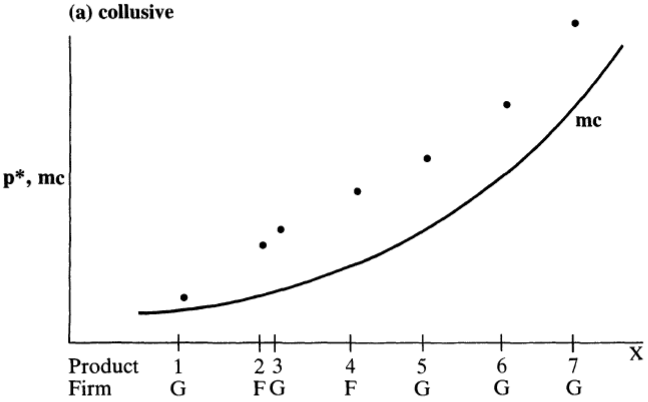
$$\begin{aligned} 0 &= q_i + (P_i - mc(x_i))\frac{\partial q_i}{\partial P_i} + H_{i,i+1}(P_{i+1} - mc(x_{i+1}))\frac{\partial q_{i+1}}{\partial P_i} \\ &+ H_{i,i-1}(P_{i-1} - mc(x_{i-1}))\frac{\partial q_{i-1}}{\partial P_i} = 0 \end{aligned}$$

- Changing $H_{i,i+1}$ from 0 to 1 increases margins

Supply Model Implications

Bresnahan (1987)

- No full lines, model good for 50s.



Supply Model Implications

Bresnahan (1987)

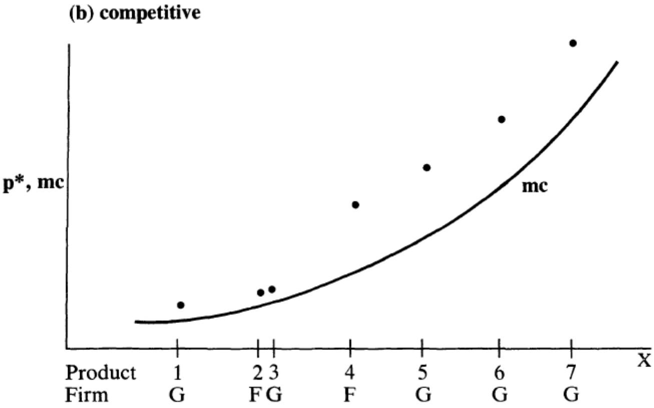


Figure 2(b)

Equilibrium

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- Equilibrium solves the demand equations and supply equations simultaneously:

$$p = p^*(x, H, \gamma, V_{max}, \delta, \mu)$$
$$q = q^*(x, H, \gamma, V_{max}, \delta, \mu)$$

- In equilibrium cars spaced further apart in the product space are impacted less by change in firm behavior regimes.
- Prices are mapped to quality in equilibrium model: substitution patterns!
- Argue that options are discrete enough with differences in product space, which is good for model
  - Large cars further apart, made mostly by GM
  - Small cars a lot of different firms, close together

Econometric Model

Bresnahan (1987)

- **Assumption:** Proxies for quality from hedonic models like Rosen (1974) depend on both tastes and technology:

$$x(c) = \sqrt{\beta_0 + \Sigma_j z_j \beta_j}$$

- Functional form is arbitrary
- Steps to compute predicted prices and quantities conditional on $\beta, \gamma, \mu, \delta, V_{max}$ and Z:
  - Product assigned quality depending on β and z
  - Products ordered highest to lowest
  - Everything used to solve for P^* and Q^* and plugged plugged into likelihood function

Econometric Model

Bresnahan (1987)

- Data: quantity aggregation problem
  - Definition of different products central to study
  - Autombile is model only if distinct in *attributes* implying 80-85 models per year
  - Level of disaggregation of model finer than some data: aggregate predicted values
  - Have to deal with heeroskedasticity

- Likelihood function with assumed independent normal errors on price and quantity:

$$\prod_{i=1}^{N_p} \frac{1}{\sqrt{2\pi^2 p}} \exp\left[-\frac{(P_i - P_i^*)^2}{2\sigma^2 p}\right]$$

$$* \prod_{j=1}^{N_q} \frac{1}{\sqrt{2\pi k_j \sigma^2 q}} \exp\left[-\frac{(q_j - \sum q^* k, kI_j)^2}{2k_j \sigma^2 q}\right]$$

Econometric Model

Bresnahan (1987)

- Alternative models to test hypotheses
- Hedonic price model follows Rosen (1974) but firm dummies for oligopoly:
  - Prices and quantities set in a recursive structure
  - Should hold if prices set in some non-maximizing way
  - Really different supply side might matter

$$
\begin{aligned}
P_i^* &= exp[\alpha_0 + \Sigma_j \alpha_j z_{ij} \\
q_j^* &= exp[\lambda_0 + \lambda_1 (P_j - P_j^*)]
\end{aligned}
$$

- Also look at identity matrix product model

Empirical Results

Bresnahan (1987)

- Cox Hypothesis Tests of different supply models:
  - Confronted with data, and alternative non-nested hypothesis
  - Likelihood ratio of two hypothesis is central test statistic
  - Computed assuming hypothesis true, test statistic asymptotically normal
  - Possible for two models to be rejected against each other
- Intuition for hypothesis test: If the residuals under null H_0 can be explained by statistically significant extent by H_1, then H_0 is rejected
- Collage of evidence

Empirical Results

Bresnahan (1987)

TABLE III
COX TEST STATISTICS

Hypotheses	*C*	*N–C*	'*p*'	*H*
a—1954				
Collusion	—	0.8951	0.9464	−1.934
Nash-Competition	−2.325	—	−0.8878	−2.819
"Products"	−3.978	3.029	—	−1.604
Hedonic	−12.37	−10.94	−13.02	—
b—1955				
Collusion	—	−10.36	−9.884	−13.36
Nash-Competition	−1.594	—	1.260	0.6341
"Products"	−0.7598	−4.379	—	−1.527
Hedonic	−3.353	−8.221	−5.950	—
c—1956				
Collusion	—	1.227	0.8263	1.629
Nash-Competition	−2.426	—	−4.586	0.8314
"Products"	−3.153	0.9951	—	4.731
Hedonic	−5.437	−9.671	−11.58	—

- **1954 and 1956:** Product and hedonic rejected, Nash rejected against coll. and hedonic, collusive not rej.
- **1955:** Collusion rejected always, Nash never rejected
- Coincidence with product expansion is interesting

Empirical Results: Overview

Bresnahan (1987)

- When non-rejected hypotheses are examined over time, the rest of the parameters are quite consistent!
- When you look at one hypotheis over time (e.g. collusion) parameters are not consistent in manner you expect. Other parameters have to fit conduct changes.
- β are generally consistent with what we would expect for different attributes in terms of sign
- Discussion of how competition model and significant expansion in quantity in 1955 are independent events. Not sure I completely understand this.

Empirical Results: Overview

Bresnahan (1987)

- Two central parameters V_{max} and δ show diverse tastes for automobile quality
  - 20 million wide and 0.4 deep which is pretty diffuse. Estimating a rectangle.
  - Used cars are not that close substitutes for lowest quality new car
- Counterfactual shows 25% quantity gain in 1955 under collusion, rather than 40% actual in data. This is shift in demand vs. shift in competitive model.
- Robustness: quality shocks and durability

Empirical Results

Bresnahan (1987)

TABLE IV
PARAMETER ESTIMATES 1954–56, MAINTAINED SPECIFICATION

Parameters	*1954*[a]	*1955*[b]	*1956*[a]
Physical Characteristics			
Quality Proxies			
Constant	47.91	48.28	50.87
	(32.8)	(43.2)	(29.4)
Weight #/1000	0.3805	0.5946	0.5694
	(0.332)	(0.145)	(0.374)
Length "/1000	0.1819	0.1461	0.1507
	(0.128)	(0.059)	(0.146)
Horsepower/100	2.665	3.350	3.248
	(0.692)	(0.535)	(0.620)
Cylinders	−0.7387	−0.9375	−0.9639
	(0.205)	(0.115)	(0.186)
Hardtop Dummy	0.9445	0.4531	0.4311
	(0.379)	(0.312)	(0.401)
Demand/Supply			
μ—*Marginal Cost*	0.1753	0.1747	0.1880
	(0.024)	(0.020)	(0.035)
γ—*Lower Endpoint*	4.593	3.911	4.441
	(1.49)	(2.08)	(1.46)
V_{max}—*Upper Endpoint*	$1.92E+7$	$2.41E+7$	$2.83E+7$
	$(8.44E+6)$	$(9.21E+6)$	$(7.98E+6)$
δ—*Taste Density*	0.4108	0.4024	0.4075
	(0.138)	(0.184)	(0.159)

Empirical Results

Bresnahan (1987)

TABLE V(i)

PARAMETER ESTIMATES 1954–56, COLLUSIVE SPECIFICATION

Parameters	*1954*	*1955*	*1956*
Constant	47.91	−23.37	50.87
	(32.8)	(24.5)	(29.4)
Weight	0.3805	0.0103	0.5694
	(0.332)	($5.43E-2$)	(0.374)
Length	0.1819	$-2.88E-3$	0.1507
	(0.128)	(0.102)	(0.146)
Horsepower	2.665	0.1165	3.248
	(0.692)	(0.106)	(0.620)
Cylinders	−0.7387	−1.309	−0.9639
	(0.205)	(1.52)	(0.186)
Hardtop	0.9445	1.468	0.4311
	(0.379)	(1.08)	(0.401)
μ	0.1753	1.344	0.1880
	(0.024)	(0.151)	(0.035)
γ	4.593	1.604	4.441
	(1.49)	(4.83)	(1.46)
V_{max}	$1.92E+7$	$1.46E+8$	$2.83E+7$
	($8.44E+6$)	($6.74E+6$)	($7.98E+6$)
δ	0.4108	$5.75E-2$	0.4075
	(0.138)	($8.28E-2$)	(0.159)

Note: Figures in parentheses are asymptotic standard errors.

Final Thoughts and Discussion

Bresnahan (1987)

- Structural model and trade-offs between assumptions and credibility:
  - Data discussion not really big part of paper. Why? Probably because it's not very good.
  - Product differentiation is very stylized
  - Assumed supply side
- Maybe trust the collusive vs. competitive test, but would we be comfortable doing much more with this model?
  - Wouldn't want to do welfare
  - Wouldn't want to do new products
- No discussion of endogeneity as all product quality is subsumed in x. Correlation between errors and prices likely, not discussed here.
- Great example of clear structural work with product differentiation and supply side without BLP additions