Class 2

08/30 2021

Identification

What is Identification?

- Many people say "identification" is important for empirical work.
- What does "identification" mean???
  - Informally?
  - Formally?

What is Identification?

- Theory f: Primitives $\rightarrow$ Set of outcomes
- Empirics f^{-1}: Realized outcome (a.k.a.data) $\rightarrow$ Primitives
  - Roughly speaking, identification is about whether f is invertible
- Theorists like unique equilibrium (f is a singleton) c.f. multiple eqm.
- Empirical people like f^{-1} to be a singleton (point identification) c.f. partial ID
  - Relationship between multiple eqm and (point) ID?
  - Tamer (03): multiple eqm, point ID

Motivating Example

Ex.1 SP Auctions, Sealed Bid.

- K (known) symmetric bidders (buyers) with private values.
- Each bidder draws valuation from $F_i^* = F^*$
- Sealed-bid second-price auction (high bid wins, pays second highest bid)
- Researcher observes bids $b_1^t \cdots b_K^t$ for each auction t.
- Primitives: F^*. Data: $(b_1^t \cdots b_K^t)_{t \in \{1, \ldots T\}}$
- Want to know F^*.
- Q: Propose a consistent estimate of $F^*(\tau)$ $\tau \in R$.

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  - Variations: Observe winning bid only, FPA, auction heterogeneity, asymmetric bidders, common values, interdependent values, unobserved heterogeneity, etc.
- Goal of empirical work is to go from data to model.

Definition

Formal Definition

- Let m^* be the profile of true unknown model primitives
- Let M be the set of all possible primitives.
- Let m be generic element of M.
  - In **Ex 1**, $m^* = F^*$, M is the set of all proper c.d.fs, m is some distribution F.

Definition

Formal Definition

- Let $P(m)$ denote the joint distribution of observable variables under the assumption that the data is generated under m.
  - In **Ex 1**, $P(F) = \overbrace{F \times F \times \cdots F}^{K}$ (joint dist. of bids)
- $(m \in M)$ + (assump. about behavior) generate $P(m)$.
  - assumptions about behavior include
    - utility/profit maximazation
    - avoid dominated strategies
    - play rationalizable strategies
    - various notions of Nash
  - In purely statistical models, no need for assump. on behavior.
- Often, should write $P(m, E)$, where E is some kind of equilibrium notion, but typically suppress E.
- As sample size goes to infinity, we will learn $P(m^*)$ for sure.

Definition

Formal Definition

Definition

$m^* \in M$ is identified in M iff $\forall m \in M$ s.t. $m \neq m^*$, $P(m) \neq P(m^*)$.

Definition

$m^* \in M$ is not identified in M iff $\exists m \in M$, $m \neq m^*$, $P(m) = P(m^*)$.

If for some m and m', we have $P(m) = P(m')$, we say that m and m' are **observationally equivalent**.
Identification is about invertibility of $P(\cdot)$ in M.
NB: check dimensionality of M and $P(M)$ to see if there's any hope of identificaiton
NB: m (given some notion of equilibrium) should fully specify the complete data generating process.
NB: if M is a subset of a finite dimensional space, we say that the model M is parameteric. c.f. nonparametric, semiparametric

Example

Ex2 Linear regression with one variable:

$$y = \beta_0^* + \beta_1^* x + \varepsilon,\ \varepsilon \sim N(\mu^*, \sigma^{*2}),\ \varepsilon \perp x;$$

(x, y) are observable, ε is unobservable.

- Purely statistical model, no need for E.

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- What is m, M?
  - $m = (\beta_0, \beta_1, \mu, \sigma^2)$, $M = \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}^+$. M is parametric.

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- What is $P(m)$?

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- What is $P(m)$?
  - Joint distribution of x and y, F_{xy}.

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  - Joint distribution of x and y, F_{xy}.
- Is $m^* = (\beta_0^*, \beta_1^*, \mu^*, \sigma^{*2})$ identified?

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- What is m, M?
  - $m = (\beta_0, \beta_1, \mu, \sigma^2)$, $M = \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}^+$. M is parametric.
- What is $P(m)$?
  - Joint distribution of x and y, F_{xy}.
- Is $m^* = (\beta_0^*, \beta_1^*, \mu^*, \sigma^{*2})$ identified?
  - No. $m^* = (\beta_0^*, \beta_1^*, \mu^*, \sigma^{*2})$, and $m' = (\beta_0^* + \mu^*, \beta_1^*, 0, \sigma^{*2})$ are observationally equivalent.

Example

EX2') Linear regression with one variable:

$$y = \beta_0^* + \beta_1^* x + \varepsilon,\ \varepsilon \sim N(0, \sigma^{*2}),\ \varepsilon \perp x$$

Same as **EX2**, but with $\mu^* = 0$.

- Is $m^* = (\beta_0^*, \beta_1^*, \sigma^{*2})$ identified?

Example

EX2') Linear regression with one variable:

$$y = \beta_0^* + \beta_1^* x + \varepsilon,\ \varepsilon \sim N(0, \sigma^{*2}),\ \varepsilon \perp x$$

Same as **EX2**, but with $\mu^* = 0$.

- Is $m^* = (\beta_0^*, \beta_1^*, \sigma^{*2})$ identified?
- Yes (under weak regularity conditions)

Example

EX2') Linear regression with one variable:

$$y = \beta_0^* + \beta_1^* x + \varepsilon,\ \varepsilon \sim N(0, \sigma^{*2}),\ \varepsilon \perp x$$

Same as **EX2**, but with $\mu^* = 0$.

- Is $m^* = (\beta_0^*, \beta_1^*, \sigma^{*2})$ identified?
- Yes (under weak regularity conditions)
- To show this, let's show that $P(m^*) = P(m') \Rightarrow m^* = m'$.

PF:
Take $m^* = (\beta_0^*, \beta_1^*, \sigma^{2*})$ and consider $F_{y|x}(\cdot; m^*)$:

$$F_{y|x}(\tau; m^*) = \Phi\left(\frac{\tau - \beta_0^* - \beta_1^* x}{\sigma^*}\right).$$

Now take $m^{'} = (\beta_0^{'}, \beta_1^{'}, \sigma^{2'})$ and consider $F_{y|x}(\tau; m')$:

$$F_{y|x}(\tau; m^{'}) = \Phi\left(\frac{\tau - \beta_0^{'} - \beta_1^{'} x}{\sigma^{'}}\right).$$

If $P(m^*) = P(m')$, then $F_{y|x}(\tau; m^*) = F_{y|x}(\tau; m')$ for all τ and a.e. x.
Hence,

$$\frac{\tau - \beta_0^* - \beta_1^* x}{\sigma^*} = \frac{\tau - \beta_0^{'} - \beta_1^{'} x}{\sigma^{'}} \text{ for all } \tau, \text{ a.e. } x$$

Hence, $\beta_0^* = \beta_0^{'}$, $\beta_1^* = \beta_1^{'}$, $\sigma^{*2} = \sigma^{'2}$, i.e., $m' = m^*$.

Proving Identification

- Two popular ways of proving identification.
  1) Take m', and m^*, and proceed as in definition, i.e., prove that $P(m') \neq P(m^*)$ if $m' \neq m^*$.
  1)' Show $P(m^*) = P(m') \Rightarrow m^* = m'$.
- 2)Express m as a function of $P(m)$ for all $m \in M$, as $m = T(P(m))$ for some fn. T (T should not depend on m).
- Why is 2) a proof?
  - Take m' and m^* and assume $P(m^*) = P(m')$.
  - Then $m^* = T(P(m^*)) = T(P(m')) = m'$. That is, we have shown the following: $P(m') = P(m^*) \Rightarrow m' = m^*$.

Example 2

- Let's give alternative pf of **Ex2** using method 2).
- Note that

$$\mathbf{E}[y|x=0] = \mathbf{E}[\beta_0 + \varepsilon|x=0] = \beta_0.$$

- Now consider $\mathbf{E}[y|x=1]$.

$$\mathbf{E}[y|x=1] = \mathbf{E}[\beta_0 + \beta_1 + \varepsilon|x=1] = \beta_0 + \beta_1.$$

- So, can express β_1 as

$$\beta_1 = \mathbf{E}[y|x=1] - \mathbf{E}[y|x=0].$$

- Lastly, $\sigma^2 = Var(y|x=0)$
- Objects such as $\mathbf{E}[y|x=1]$, $\mathbf{E}[y|x=0]$, etc. is a function of $P(m)$.

Example 2

- So what's the intuition???
- As sample size goes to infinity, we will learn $P(m^*)$.
- $\mathbf{E}[y|x=0;m^*]$, $\mathbf{E}[y|x=1;m^*] - \mathbf{E}[y|x=0;m^*]$, and $Var(y|x=0;m^*)$ are all known functions of (can be derived from) $P(m^*)$.
- Hence we will learn these objects too, as sample size goes to infinity.
- AND $\mathbf{E}[y|x=0;m^*] = \beta_0^*$,
  $\mathbf{E}[y|x=1;m^*] - \mathbf{E}[y|x=0;m^*] = \beta_1^*$, and
  $Var(y|x=0;m^*) = \sigma^{2*}$.

More Definition

Definition
A coordinate m_i^* of $m^*(=(m_i^*, m_{-i}^*))$ is identified in M iff $\forall m = (m_i, m_{-i}) \in M$, $m_i \neq m_i^*$, $P(m) \neq P(m^*)$.

c.f. **EX 2**, β_1^*, σ^{*2} are identified although β_0^* and μ_0^* are not.

Definition
A function $c^* = C(m^*)$ is identified in $C(M)$ iff $\forall c' \in C(M)$, $c' \neq c^*$, $\{P(m)|C(m) = c^*\} \cap \{P(m)|C(m) = c'\} = \phi$.

c.f. **EX 2**, $\beta_0^* + \mu^*$is identified.

Definition
$m^* \in M$ is partially identified in M if there is a subset N of M, and $\forall m' \in N \subset M$, $m' \neq m^*$, $P(m') \neq P(m^*)$.

Example: SPA with only winning bid observed

- Let K^t be the number of bidders in auction t, $(2 \leq K^t \leq \bar{K})$
- Symmetric IPV $(v_1, ...v_{K^t})$ drawn from $F_v^* \times \cdots \times F_v^*$.
- Sealed Bid.
- Observe winning bid y^t (i.e., 2nd highest bid) and K^t in auction t.
- Data: $(\{y^1, \cdots, y^T\}, \{K^1, \cdots, K^T\})$.
- M is the set of all distributions on $\mathbb{R}^1$ (nonparametric).
- $m^* = F_v^*$
- $P(m)$ is joint dist. of (y^t, K^t), $F_{y,K}$.
- Is $F_v^* \in M$ identified?

Example: SPA with only winning bid observed

- Assume $K^1 = K^2 = \cdots = K^T = K$ (if not, what do you do?)
- To show F_v^* is identified, let's show $F_v = T(F_y)$.
- Consider distribution of y given F_v: $F_y(\cdot; F_v)$

$$
\begin{aligned}
F_y(\tau; F_v) &= \Pr(y \leq \tau; F_v) \\
&= \Pr(\text{At least } K-1 \text{ of } \{v_1, ..., v_K\} \leq \tau) \\
&= \overbrace{KF_v(\tau)^{K-1}(1 - F_v(\tau))}^{\text{Exactly } K-1 \text{ bids below } \tau} + \overbrace{F_v(\tau)^K}^{K \text{ bids below } \tau} \\
&= (1-K)F_v(\tau)^K + KF_v(\tau)^{K-1}.
\end{aligned}
$$

- Identification boils down to "Can I uniquely solve for $F_v(\tau)$ as a function of $F_y(\tau; F_v)$?"

SPA, Sealed Bid

- Now, consider the following function on $X \in [0,1]$

  $$a(X) = (1-K)X^K + KX^{K-1}.$$

  Consider the derivative of $a(X)$:

  $$\begin{aligned} \frac{d}{dx}a(X) &= (1-K)KX^{K-1} + K(K-1)X^{K-2} \\ &= \overbrace{(K-1)}^{\text{Positive}}K\overbrace{\left[x^{K-2} - x^{K-1}\right]}^{\text{positive for } x\in(0,1)}. \end{aligned}$$

- $a(X)$ is str. incr. on $X \in [0,1]$.
- Image of $a(X)$ $(X \in [0,1])$ is $[0,1]$.
- We can define $a^{-1}(X)$ on $[0,1]$.

SPA, Sealed Bid

- Recall

$$F_y(\tau; F_v) = (1-K)F_v(\tau)^K + KF_v(\tau)^{K-1}.$$

- Previous argument implies we can solve for $F_v(\tau)$ as a function of $F_y(\tau; F_v)$,

$$F_v(\tau) = a^{-1}(F_y(\tau; F_v)) \text{ for all } \tau.$$

- Notice we have now expressed F_v as $T(P(F_v))$.

SPA, Sealed Bid

- Proof of identification through showing $m = T(P(m))$ leads to a way of estimation as well.
- In the previous example, note that $F_y(\tau) = \Pr(y \leq \tau)$ can be consistently estimated by

$$\begin{aligned} \widehat{\Pr(y \leq \tau)} &= \frac{1}{T}\sum_{t=1}^{T} 1_{\{(y_t \leq \tau)\}} \\ &= \frac{(\#\text{ of auctions with winning bid} \leq \tau)}{(\#\text{ of auctions})} \end{aligned}$$

- This means that $F_v(\tau)$ can be consistently estimated by

$$\widehat{F_v(\tau)} = a^{-1}\left(\frac{1}{T}\sum_{t=1}^{T} 1_{\{(y_t \leq \tau)\}}\right).$$

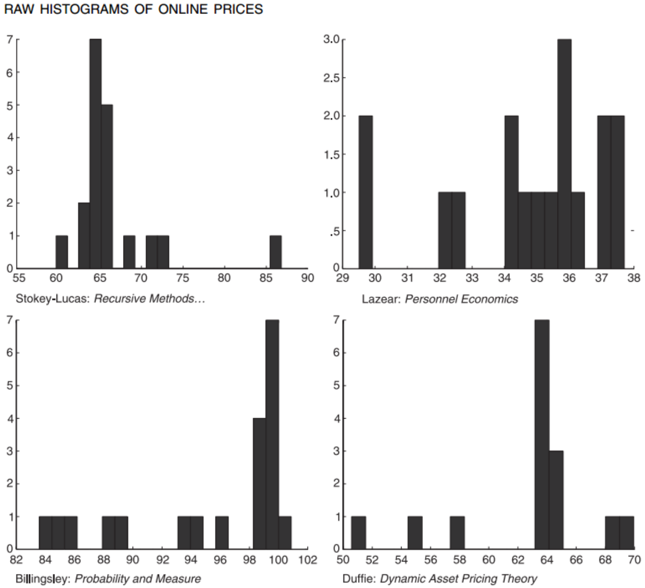
- In general, if $m = T(P(m))$, then $\widehat{m} = T(\widehat{P(m)})$.

Exmaple: Search (Hong and Shum Rand '06)

- Hong and Shum (06) consider both simultaneous and sequential search.
- Here, we consider simultaneous search.
- Based on Burdett and Judd (1983), model of price dispersion with firm mixed strategies. (Do you know this model?)

Hong and Shum, Setting

- Setting: online prices of books from various vendors



Model

- Consumer i has search cost C_i.
- Decides how many times to search ex-ante (i.e., simultaneous).
- Primitive: $F_C(\cdot)$. Assume F_C has support on $[\underline{C}, \overline{C}]$, $\underline{C} > 0$.
  - We take as given that $\underline{C}$ and $\overline{C}$ are known.
- Consumer's utility is $\overset{\text{value of good}}{v_i} - \overset{\text{search cost}}{(n-1)C_i} - \overset{\text{price}}{p_i}$
  - n is # of search
- For now, assume away the decision of whether or not to buy (say lower bound of v_i is higher than upper bound of p_i)
- Consider identification of F_C.
- Given distribution of prices F_P (known to consumer), consumer samples $n(C_i)$ prices from the distribution.

- Consumer i's problem

$$n(C_i) = \arg\min_{n \geq 1} (n-1) C_i + \int_{\underline{p}}^{\overline{p}} pn(1 - F_P(p))^{n-1} f_P(p) dp$$

- What is $\int_{\underline{p}}^{\overline{p}} pn(1 - F_P(p))^{n-1} f_P(p) dp$?
- $\int_{\underline{p}}^{\overline{p}} pn(1 - F_P(p))^{n-1} f_P(p) dp = \mathbf{E}[$lowest price out of n draws$]$

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- $n(C_i)$ is nonincreasing in C_i (why?)

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- Incremental cost: C_i

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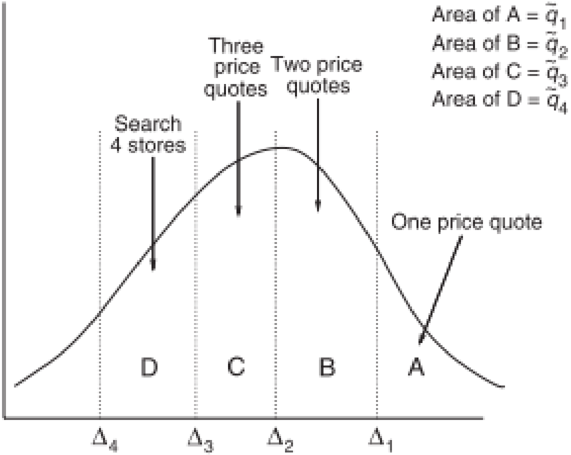
- $\int_{\underline{p}}^{\overline{p}} pn(1 - F_P(p))^{n-1} f_P(p)dp = \mathbf{E}[\text{lowest price out of } n \text{ draws}]$
- $n(C_i)$ is nonincreasing in C_i (why?)
- Incremental cost: C_i
- Incremental Benefit: $\mathbf{E}[\text{lowest price out of } n+1 \text{ draws}] - \mathbf{E}[\text{lowest price out of } n \text{ draws}]$

- Define $\Delta_n = \mathbf{E}[p_{1:n} - p_{1:n+1}]$ where $p_{1:n}$ is the lowest realization from n independent samples from F_P.
  - Δ_n is a function of $P(m)$. Hence Δ_n are all identified.
  - Δ_n: incremental value of an extra search.
  - If $C_i > \Delta_n$, then (incremental cost of search) $>$ (incremental value of search)
  - If $C_i < \Delta_n$, then (incremental cost of search) $<$ (incremental value of search)
  - Δ_n corresponds to the threshold search cost that determines whether consumer searches n or $n+1$ times.
  - Can show $\Delta_{n+1} \leq \Delta_n$ (decreasing marginal benefit), and $\Delta_n \to 0$ as $n \to \infty$.
- Define $q_1 = 1 - F_C(\Delta_1)$: mass of consumers who only take one price draw
- Define $q_K = F_C(\Delta_{K-1}) - F_C(\Delta_K)$ for $K = 2 \cdots \overline{K}$.
  - q_K is the mass of consumers who take K draws.
  - Note that $\underline{C} > 0$ implies $\overline{K}$ is finite.

Graphically...



- We now show that $q_K = F_C(\Delta_{K-1}) - F_C(\Delta_K)$ are identified.
- To do so, use firms' eqm condition in a mixed-strategy price dispersion model based on Burdett and Judd.
- Firm profit from charging price p:

$$\Pi(p) \propto \overbrace{(p-r)}^{\text{Margin}} \sum_{K=1}^{\overline{K}} q_K K \times \overbrace{(1-F_P(p))^{K-1}}^{\text{Prob. } p \text{ is lower than } K-1 \text{ other draws}} .$$

- q_K: mass of consumers who take K price quotes.
- $q_K K$: probability that a given price quote is received by a person who takes K price quotes.
- Needs $\Pi(p) = \Pi(p')$ for all p, p' (b/c mixed strategy).
- In particular, $\Pi(p) = \Pi(p^M)$ where p^M is monopoly price (why?).
  - Observe that p^M should be the upper support of F_P.

- Let $p_1 < p_2 < \cdots < p_T = p^M$ be any price in the support of F_P.

$$(p^M - r)q_1 = (p_i - r)\sum_{K=1}^{\overline{K}} q_K K(1 - F_P(p_i))^{K-1} \text{ for all } i.$$

- As long as $T > \overline{K}$, there is generically a unique solution $(r, q_1, \cdots q_{\overline{K}})$ that solves equation above.
- Hence $(q_1, \cdots q_{\overline{K}})$ are identified.
- This means that F_C is identified on $\overline{K}$ points $(\Delta_1 \cdots \Delta_{\overline{K}})$
  - $q_1 = 1 - F_C(\Delta_1)$, and $q_K = F_C(\Delta_{K-1}) - F_C(\Delta_K)$ for $K = 2 \cdots \overline{K}$.
- F_C is not identified on all other points (partial identification)
- Q: propose an estimator for $F_C(\Delta_1) \cdots F_C(\Delta_{\overline{K}})$.

TABLE 2 **Search-Cost Distribution Estimates for Nonsequential-Search Model**

Product	K^a	M^b	$\tilde{q}_1^c$	$\tilde{q}_2$	$\tilde{q}_3$	Selling Cost r	MEL Value
Parameter estimates and standard errors: nonsequential-search model							
Stokey-Lucas	3	5	.480 (.170)	.288 (.433)		49.52 (12.45)	102.62
Lazear	4	5	.364 (.926)	.351 (.660)	.135 (.692)	27.76 (8.50)	84.70
Billingsley	3	5	.633 (.944)	.309 (.310)		69.73 (68.12)	199.70
Duffie	3	5	.627 (1.248)	.314 (.195)		35.48 (96.30)	109.13

Search-cost distribution estimates

	Δ_1	$F_c(\Delta_1)$	Δ_2	$F_c(\Delta_2)$	Δ_3	$F_c(\Delta_3)$
Stokey-Lucas	2.32	.520	.68	.232		
Lazear	1.31	.636	.83	.285	.57	.150
Billingsley	2.90	.367	2.00	.058		
Duffie	2.41	.373	1.42	.059		