Problem Set 1: Due Oct 7, 11:59pm PST.

Question 1

Type I Extreme Value distribution has the following CDF:

$$F(\tau) = \exp(-\exp(-(\tau + \alpha))),$$

where α is a parameter of the distribution.

Let $\varepsilon_1, \cdots \varepsilon_J$ be distributed type I EV, with parameters $\alpha_1, \cdots \alpha_J$. Assume that $\varepsilon_1 \cdots \varepsilon_J$ are also independent.

(a) Show that $u = \max\{\varepsilon_1, \cdots \varepsilon_J\}$ is also distributed Type I Extreme Value.

(b) Consider the following discrete choice model:

$$\begin{aligned} U_j &= u_j + \varepsilon_j \\ Y &= \arg\max_{j \in J}\{U_j\}, \end{aligned}$$

where $(u_1, \cdots u_J)$ is a J-vector and ε_j is distributed type I EV with parameter α_j. Show the following

$$\Pr(Y = j) = \frac{\exp(u_j - \alpha_j)}{\sum_i^J \exp(u_i - \alpha_i)}.$$

Question 2

Consider the following model

$$Y = \max\left\{0, \frac{u_1(X)}{u_2(X)} + \frac{1}{u_2(X)}\varepsilon\right\}, X \perp \varepsilon,$$

where (Y, X) are observed, ε is not observed. **Assume further that** $\Pr(Y = 0|X) < 1$**, for all** X and $u_2(X) > 0$ for all X**.** What we want to identify are $u_1(\cdot)$, $u_2(\cdot)$ and the distribution of ε, F_ε.

(a) Show that the primitives are not identified.

(b) Suppose we make the following normalization, $u_2(\mathbf{x}_0) = 1$, at some $\mathbf{x}_0$ that is **known**. Show that $u_2(\cdot)$ is identified.

(c) Continue to assume $u_2(\mathbf{x}_0) = 1$, at some $\mathbf{x}_0$ that is **known**. Assume also that median of ε is zero, $Med(\varepsilon) = 0$. Assume also that $\Pr(Y = 0|\mathbf{x}_0) < 1/2$. Show that $u_1(\cdot)$ is identified.

Question 3

Consider a static entry model. Assume that in each market t, there are two potential entrants, $i = 1, 2$ (you can think of Walmart and Kmart, for example). The profit from entry in market t is given as follows:

$$\pi = \beta_i Z_t - \alpha_i \mathbf{1}_{\{\text{competitor}\}} + \epsilon_{i,t},$$

where $Z_t \in \mathbb{R}^L$ is a vector of market characteristics, $\mathbf{1}_{\{\text{competitor}\}}$ is an indicator function for whether or not there is a competitor, and $\epsilon_{i,t} \in \mathbb{R}$ is an idiosyncratic shock distributed independently across i and t. If firm i is a monopolist, the profit is $\beta_i Z_t + \epsilon_{i,t}$. If the firm is a duopolist, the profit is $\beta_i Z_t - \alpha_i + \epsilon_{i,t}$. Profit from staying out of the market is normalized to 0. Assume that you know the distribution of $\epsilon_{i,t}$ (you can assume that it is uniform [-1,0]). The primitives of the model are $\{\alpha_i, \beta_i\}_{i=1,2}$. The researcher has access to data $\{\chi_{1,t}, \chi_{2,t}, Z_t\}_{t=1}^T$, where $\chi_{i,t} \in \{0, 1\}$ is an indicator variable that corresponds to whether or not firm i is in market t. Each firm makes an entry decision in each market.

(1) Assume that firms observe their own realization $\epsilon_{i,t}$ as well as their opponent's real-

ization $\epsilon_{-i,t}$ (The researcher does not observe the realizations however). Firm i's strategy is $\sigma_i(Z_t, \epsilon_{i,t}, \epsilon_{-i,t}) : \mathbb{R}^L \times \mathbb{R} \times \mathbb{R} \to \{0, 1\}$. Assume that firms are playing Nash equilibrium in each market. Are the primitives of the model identified? Discuss.

(2) Propose an estimator of $\{\alpha_i, \beta_i\}$ that is consistent.

(3) Suppose now that firms only observe their own realization of $\epsilon_{i,t}$, so that their strategy is $\sigma_i(Z_t, \epsilon_{i,t}) : \mathbb{R}^L \times \mathbb{R} \to \{0, 1\}$. Are the primitives of the model identified? If so, propose an estimator.

Question 4

Recall the function Q we defined in class that maps $(J-1) \times 1$ vector of utilities into $(J-1) \times 1$ vector of probabilities. Recall that we defined r as a mapping from $(J-1) \times 1$ vector of utilities to $(J-1) \times 1$ vector implicitly using Q.

a) Show that the $\sum_j \left| \frac{\partial}{\partial u_j} r_k \right| < 1$ for all k.

b) Make sure you understand the proof that Q is onto and one-to-one.

Question 5

(a) Tell me if there are any suggestions/comments about the class.

(b) Tell me briefly what you are thinking of doing in terms of your class project.