Instructions for estimation

The panel regression is

$$e_{i,t} = \frac{1}{b}\left(b_{0i} + h\left(t\right) + \nu_{i,t}\right), \tag{1}$$

where $e_{i,t}$ equals carbon emissions in region i in period t and $h\left(t\right) = c_1 t + c_2 t^2$. The error is

$$\nu_{it} = \left(\rho \sum_{k=0}^{\infty} \rho^k \left(\alpha_{t-k-1} + \theta_{t-k-1}\right) + \alpha_t + \mu_{it}\right)$$

and the primitives α_t and $\mu_{i,t}$ satisfy

$$\alpha_t \sim iid\left(0, \sigma_\alpha^2\right), \ \mu_{i,t} \sim iid\left(0, \sigma_\mu^2\right), \ \mathbf{E}\left(\alpha_t \mu_{i,\tau}\right) = 0 \ \forall t, \tau.$$

(The foundation for this model is:

$$\begin{gathered}\nu_{i,t} = \nu_t + \mu_{i,t} - \tfrac{\sum_j \mu_{jt}}{n}, \text{ with } \nu_t = \rho\nu_{t-1} + \alpha_t + \tfrac{\sum_j \mu_{j,t}}{n} \Rightarrow \\ \nu_{i,t} = \rho\nu_{t-1} + \alpha_t + \mu_{i,t}, \text{ with}\end{gathered} \tag{2}$$

The contemporaneous shock to region i in period t, $\alpha_t + \mu_{i,t}$, consists of an ideosyncratic and a systemic component.)

There is both cross-regional and intertemporal correlation of ν_{it}, so the sytem must be estimated using GLS. The nonnegative integer s denotes the difference between two times in the sample, $s = |t - \tau|$. The elements of the covariance matrix are

$$\begin{gathered}\mathbf{E}\nu_{i,t}\nu_{j,t-s} = \mathbf{E}\nu_{i,t}\nu_{j,t+s} = \tfrac{\rho^s}{1-\rho^2}\sigma_\alpha^2 + \left(\iota\left(i,j\right) + \tfrac{\rho^{s+2}}{n}\tfrac{1}{1-\rho^2}\right)\sigma_\mu^2 \\ \text{with } \iota\left(i,j\right) = \begin{cases} 1 \text{ for } i = j \\ 0 \text{ for } i \neq j \end{cases}.\end{gathered} \tag{3}$$

The error on the regression equation is $\frac{1}{b}\nu_{it}$ so the covariance of this error is

$$\begin{aligned}\tfrac{1}{b^2}\mathbf{E}\nu_{i,t}\nu_{j,t-s} = \tfrac{1}{b^2}\mathbf{E}\nu_{i,t}\nu_{j,t+s} &= \tfrac{1}{b^2}\left(\tfrac{\rho^s}{1-\rho^2}\sigma_\alpha^2 + \left(\iota\left(i,j\right) + \tfrac{\rho^{s+2}}{n}\tfrac{1}{1-\rho^2}\right)\sigma_\mu^2\right) \\ &= \tfrac{\sigma_\alpha^2}{b^2}\left[\tfrac{\rho^s}{1-\rho^2} + \left(\iota\left(i,j\right) + \tfrac{\rho^{s+2}}{n}\tfrac{1}{1-\rho^2}\right)\lambda\right] \\ \text{with } \lambda \equiv \tfrac{\sigma_\mu^2}{\sigma_\alpha^2} &\text{ and } \iota\left(i,j\right) = \begin{cases} 1 \text{ for } i = j \\ 0 \text{ for } i \neq j \end{cases}.\end{aligned} \tag{4}$$

I want to use the data that Andy organized, with observations from 1945 - 2005. There are four regions ($n = 4$): BRIC, EU, US, and Other. With equation 3 it is possible to group the data either by regions (e.g. the first 61 observations consist of the time series of BRIC) or by time (e.g. the first four observations consist of the first observations for the four regions). I don't know which is more convenient.

The parameters of interest are: $\frac{b_{0i}}{b}$ for $i = 1...4$; $0 < \rho < 1$; $\frac{\sigma_\alpha^2}{b^2} > 0$; and $\lambda > 0$. (The time trend parameters c_1 and c_2 are not interesting.)