

# Astrostatistics

Monday, 03 February 2020

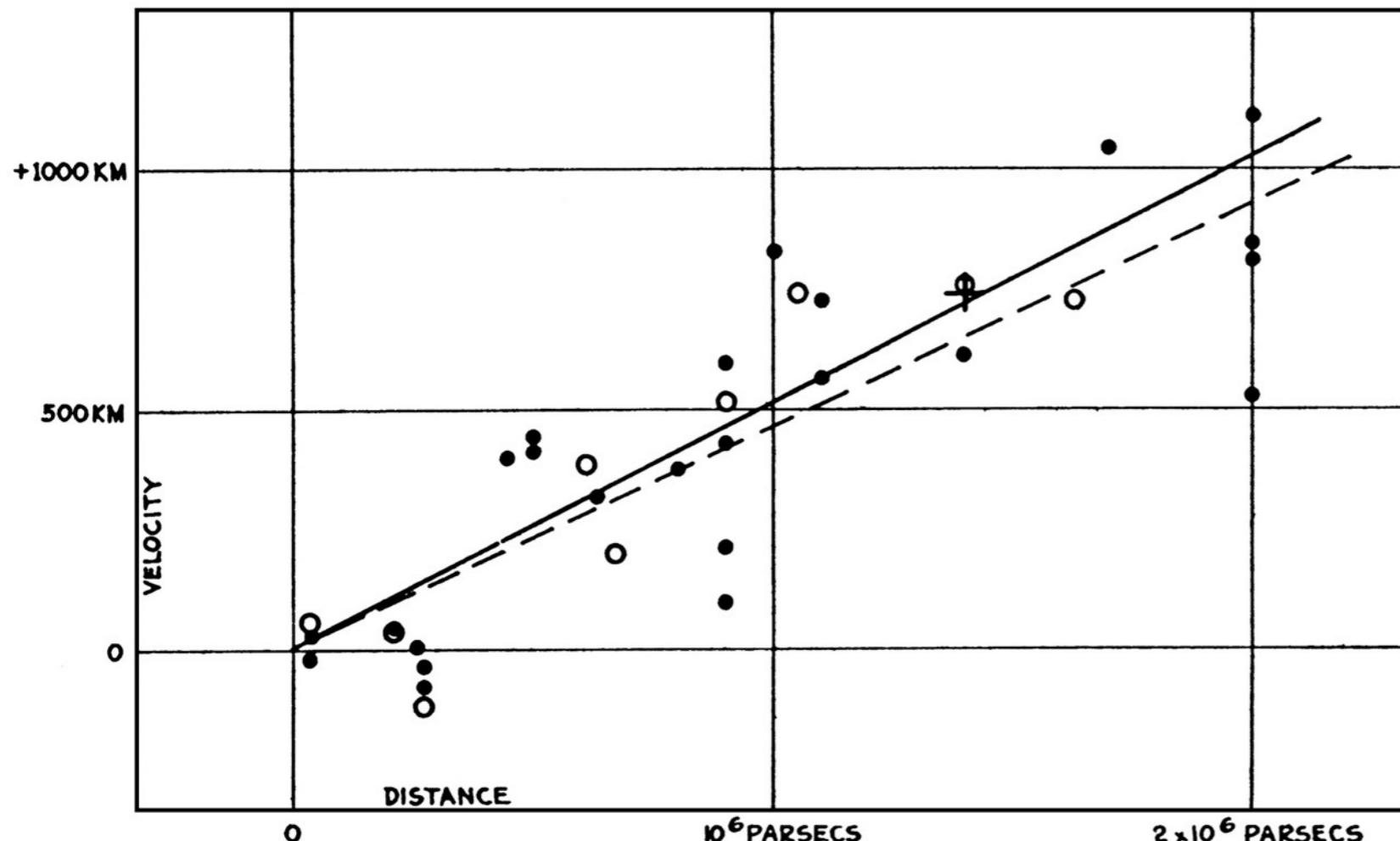
- Statistics Foundations
  - Ivezic Ch 4 “Classical Statistical Inference” & Ch 5 “Bayesian Statistical Inference”
  - F&B Ch 3 “Statistical Inference”
- Review (on your own) properties of multivariate Gaussian random variables and densities  
(see `multivariate_gaussian_notes.pdf` on website)
- and also other standard distributions (F&B Ch 4 and Ivezic Ch 3)

# Determining Astronomical Distances using Standard Candles

1. Estimate or model Luminosity  $L$  of a Class of Astronomical Objects
2. Measure the apparent brightness or flux  $F$
3. Derive the distance  $D$  to Object using Inverse Square Law:  $F = L / (4\pi D^2)$
4. Optical Astronomer's units:  $\mu = m - M$

$m$  = apparent magnitude [log apparent brightness flux],  
 $M$  = absolute magnitude [log Luminosity],  
 $\mu$  = distance modulus [log distance].

# The Expanding Universe: Galaxies are moving apart! Hubble's Law (1929)



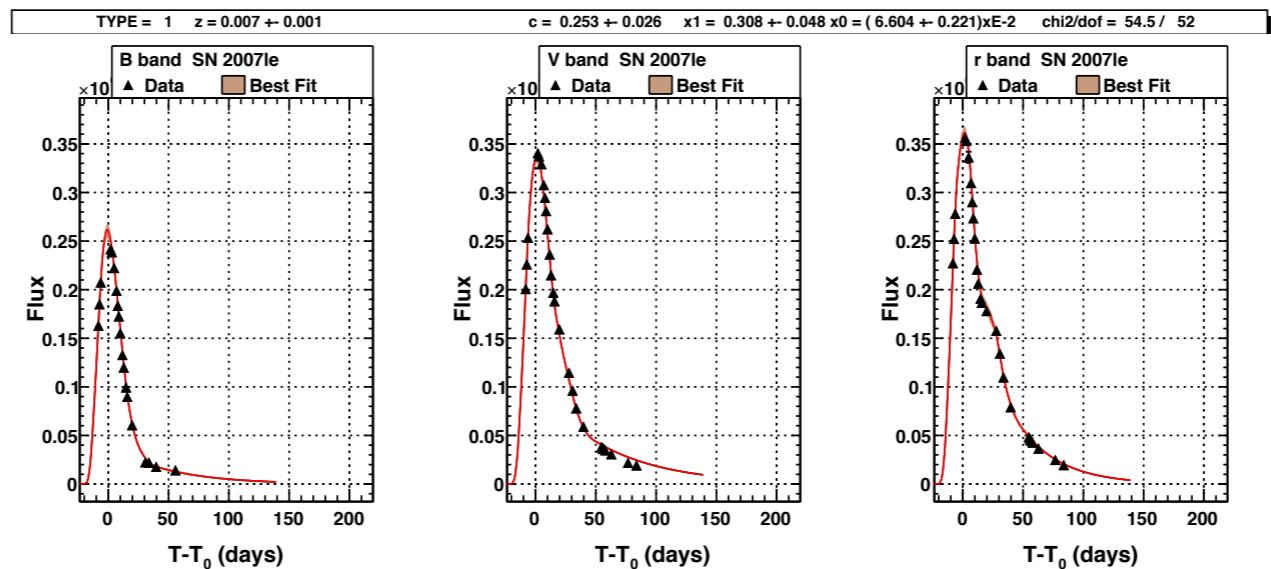
Einstein & Hubble

Distance  $\propto$  Velocity (Redshift)

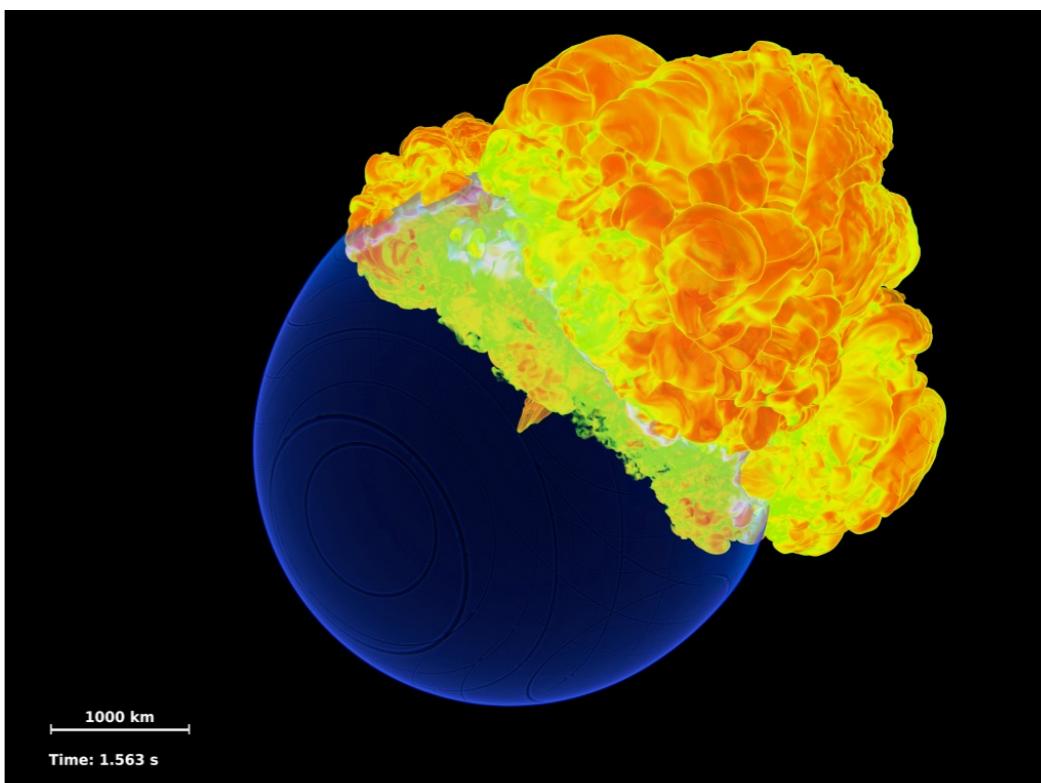
# Type Ia Supernovae (SN Ia) are Almost Standard Candles



Credit: High-Z Supernova Search Team, HST, NASA

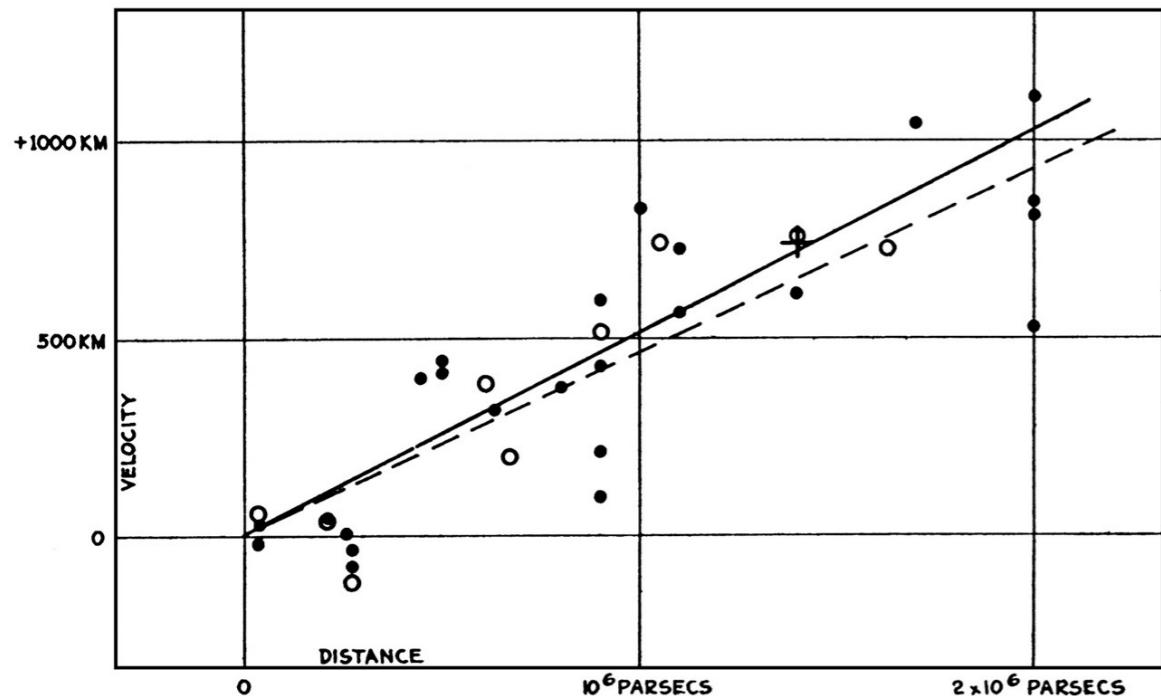


- Progenitor: C/O White Dwarf Star accreting mass leads to instability
- Thermonuclear Explosion: Deflagration/ Detonation
- Nickel to Cobalt to Iron Decay + radiative transfer powers the light curve

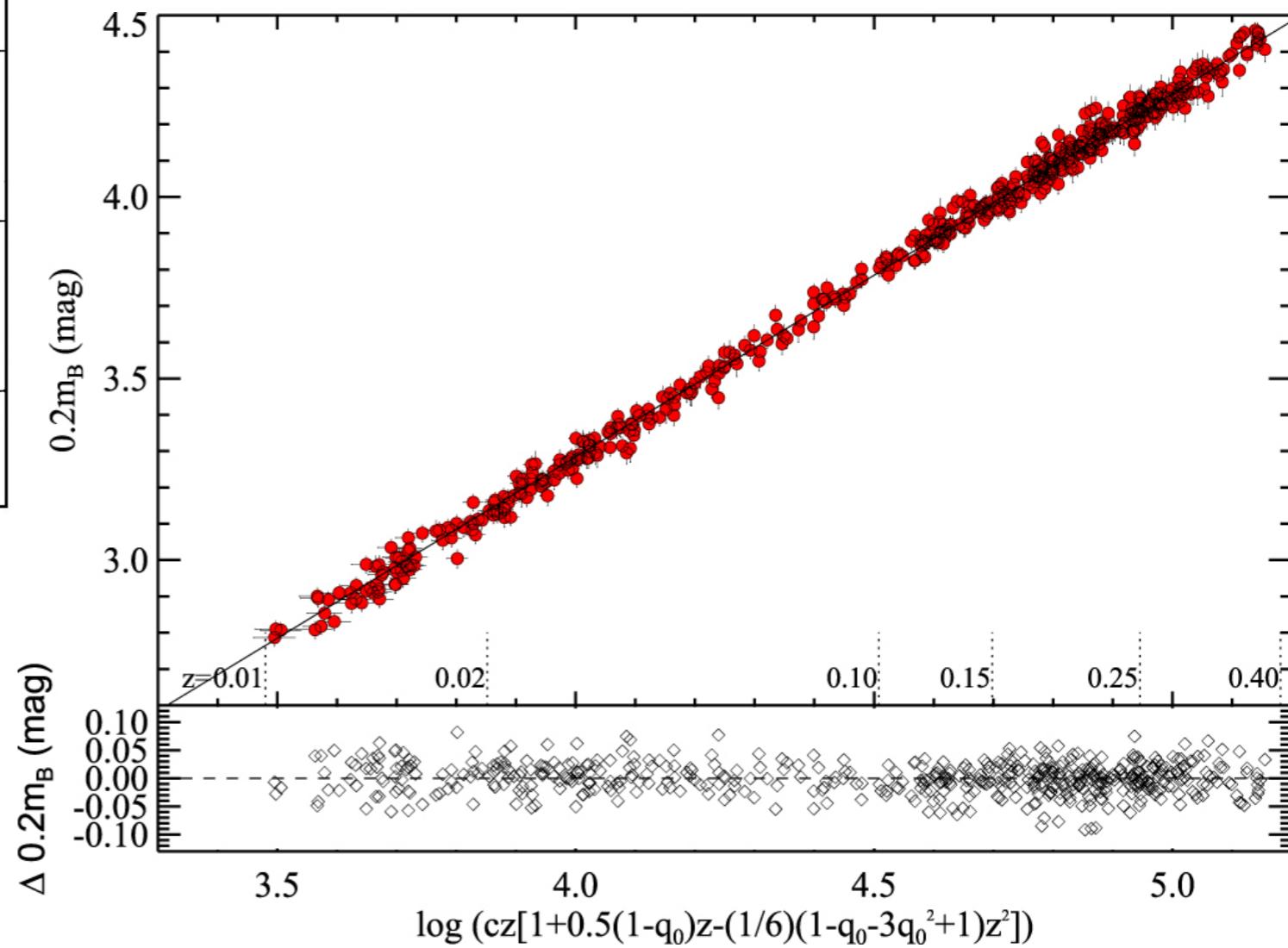


# Hubble Constant

$$\text{velocity} = H_0 \times \text{distance}$$



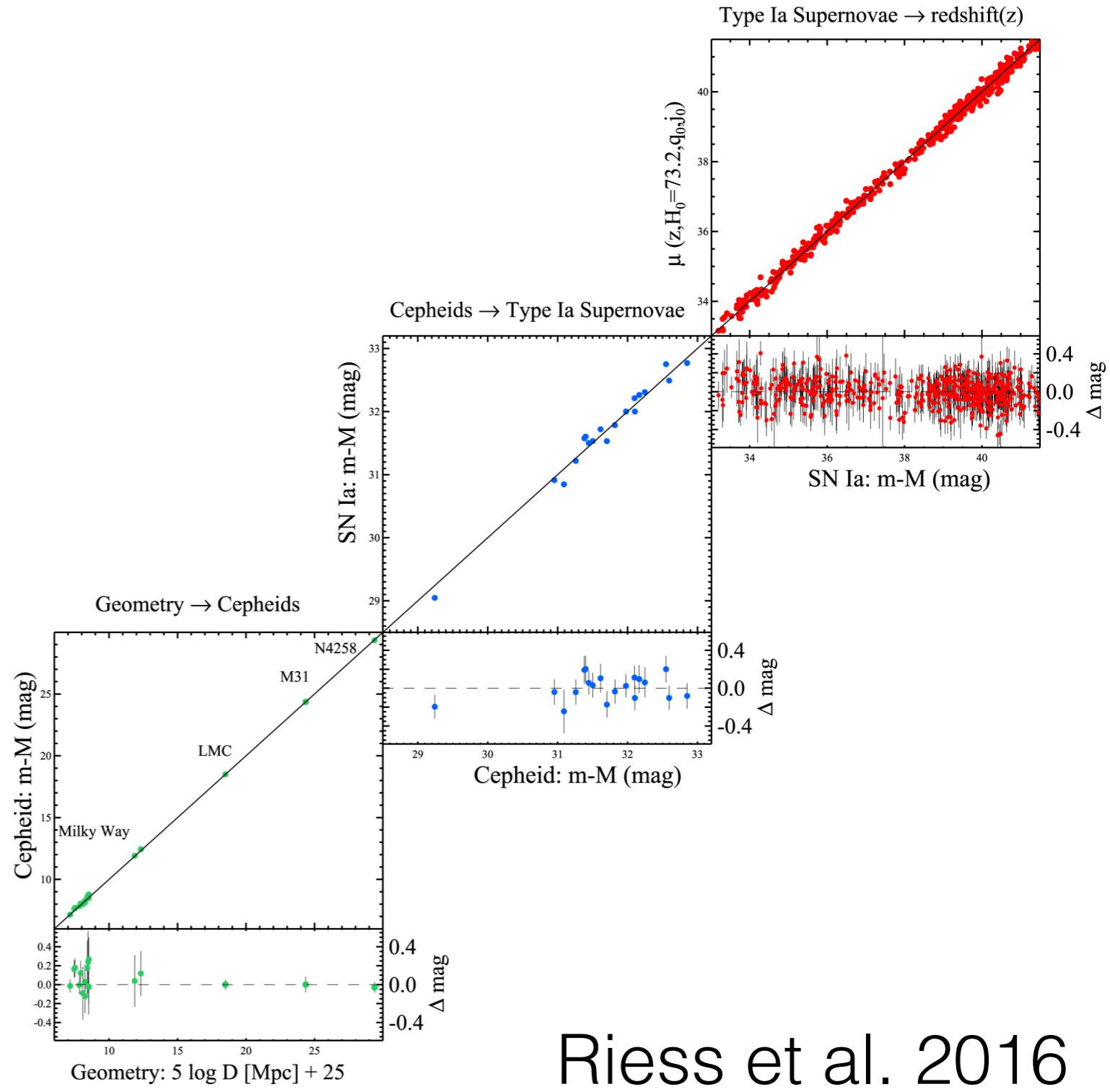
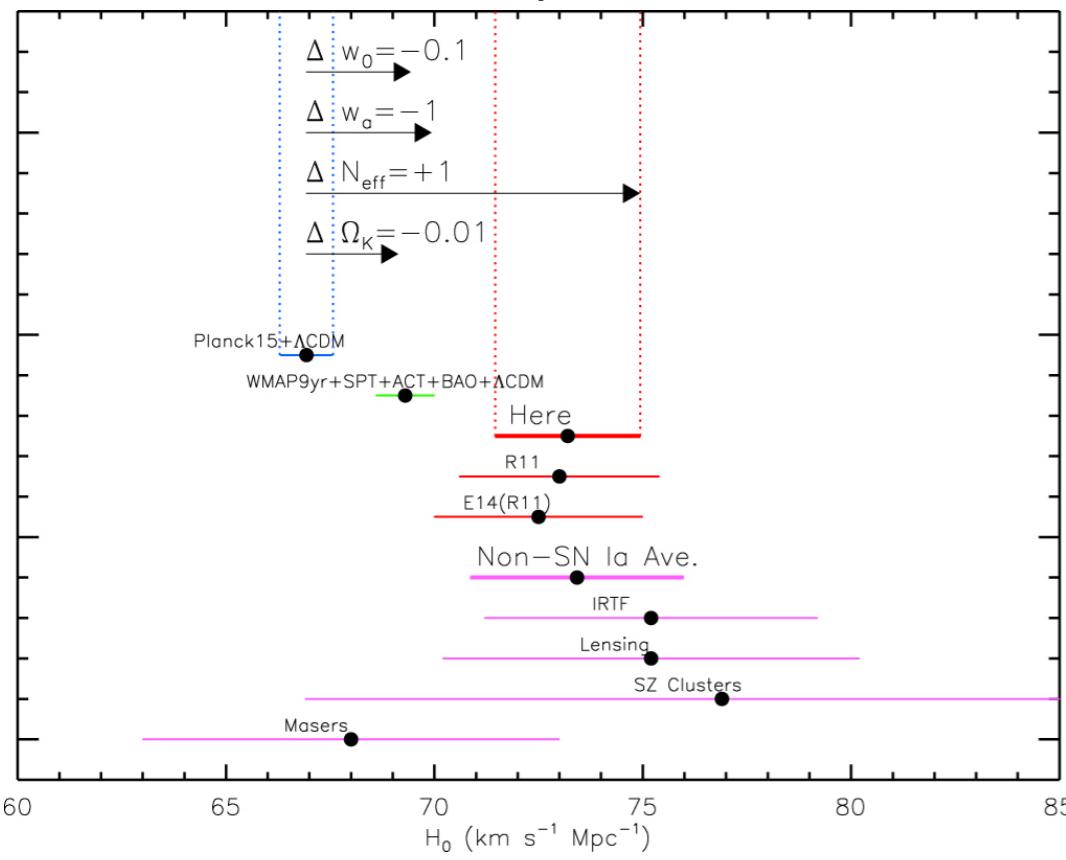
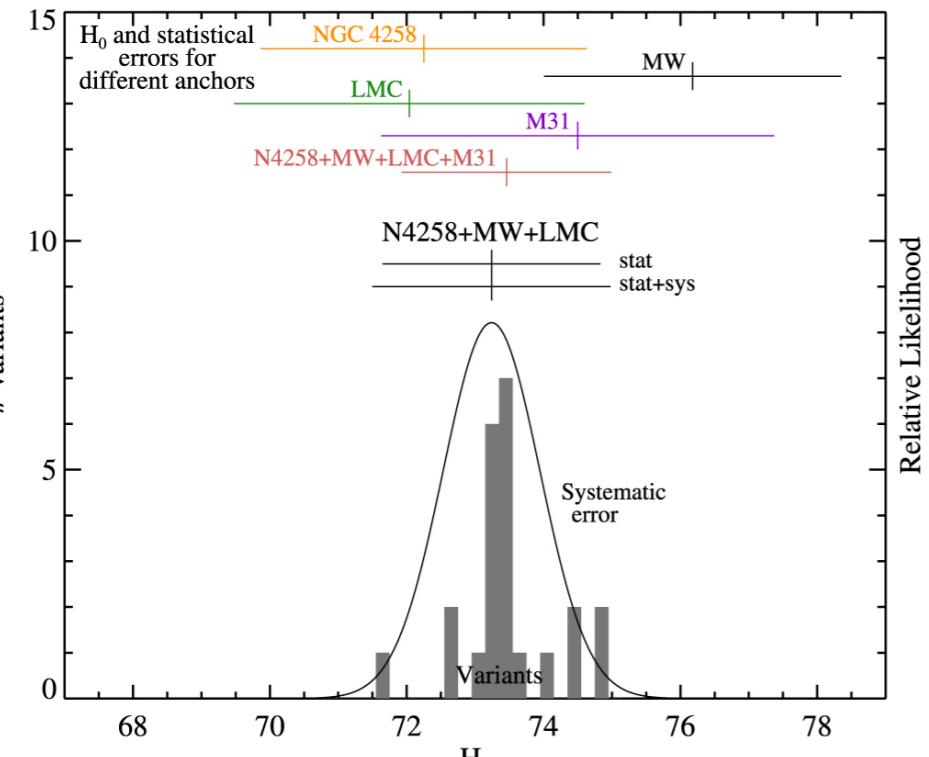
Hubble (1929)



Riess et al. 2016

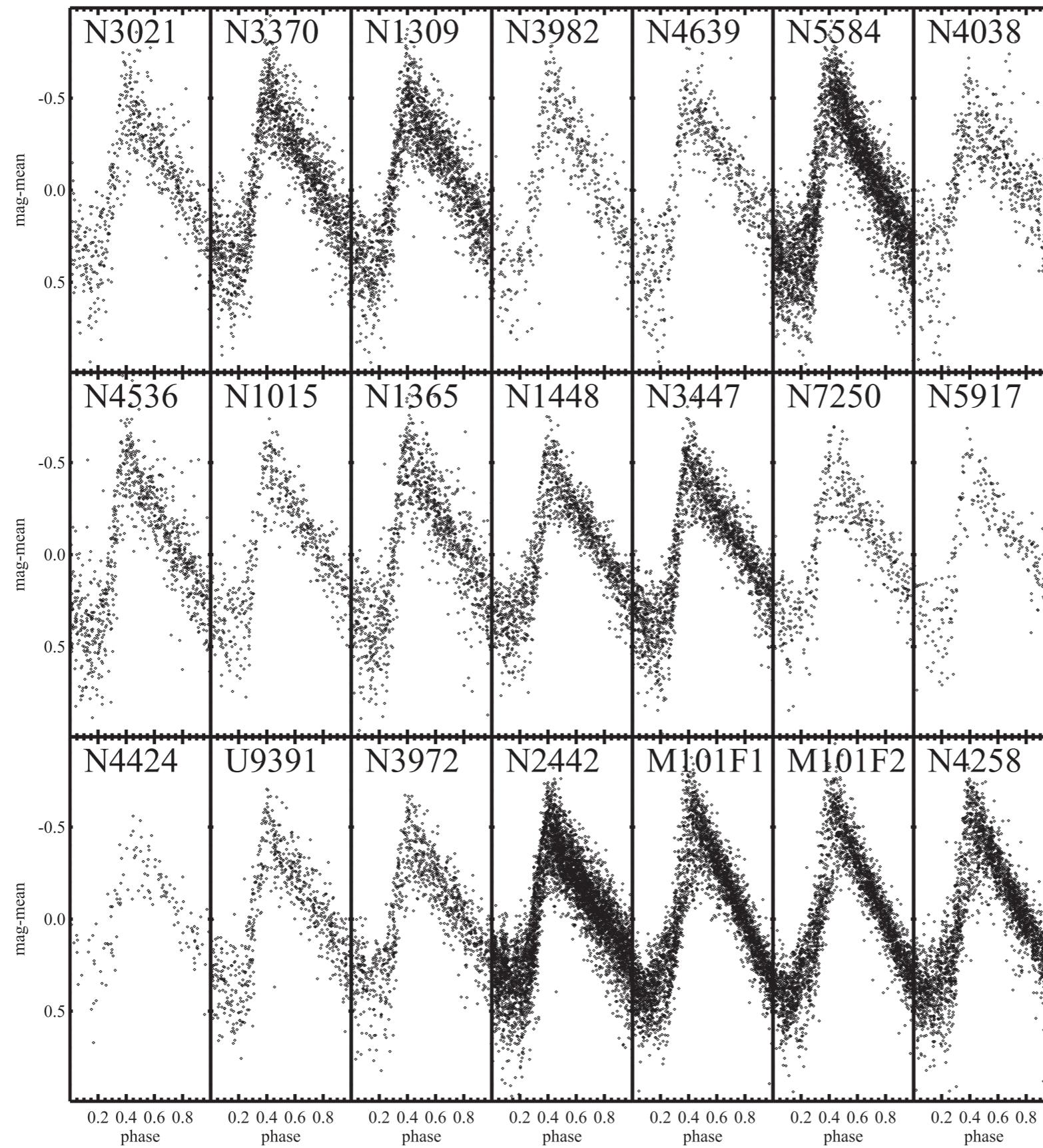
# Hubble Constant

velocity =  $H_0 \times$  distance



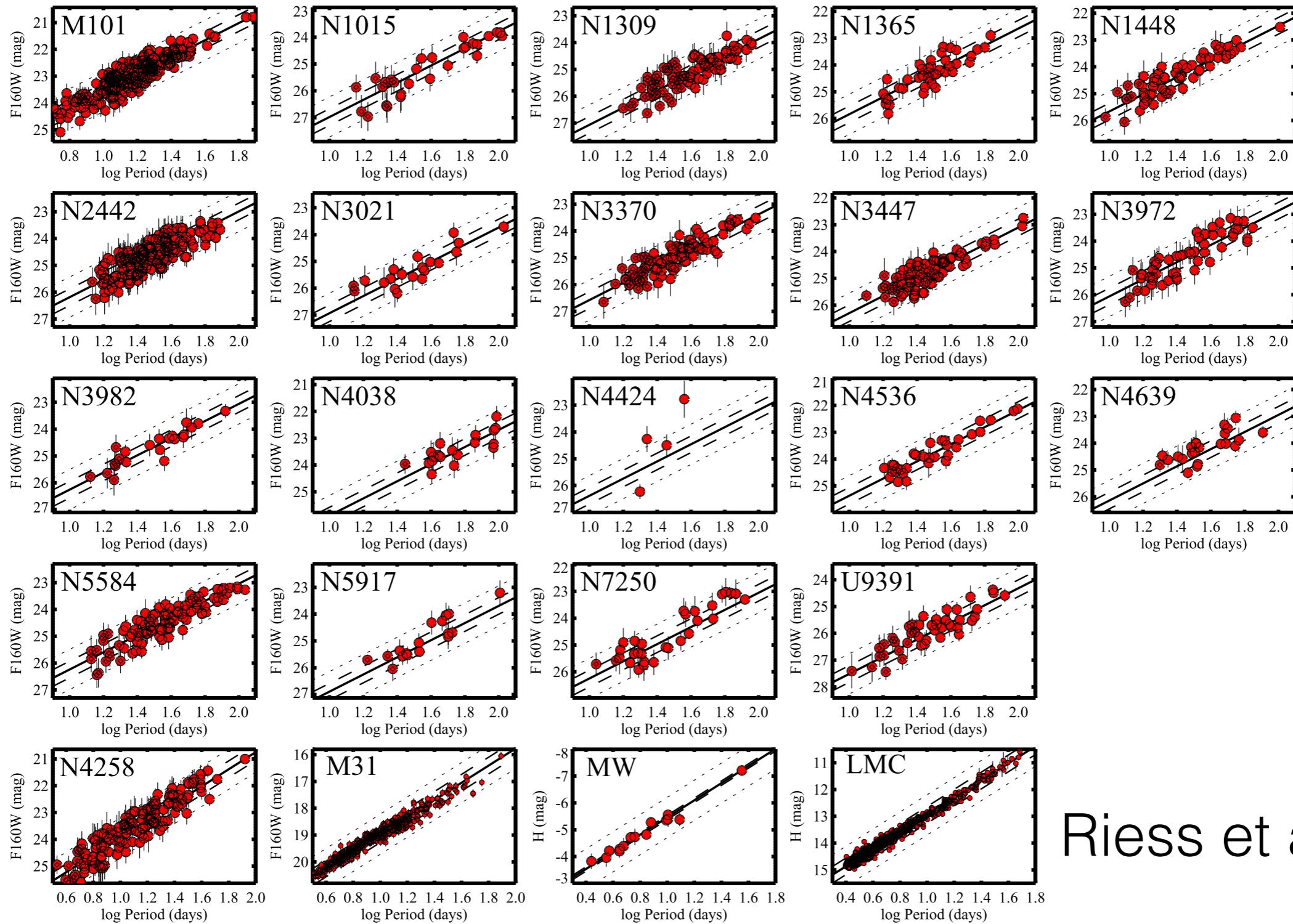
Riess et al. 2016

# Example: Cepheid Light Curves (Time Series)



Riess et al. 2016

# Example: Leavitt's Law: Period-Luminosity Relation

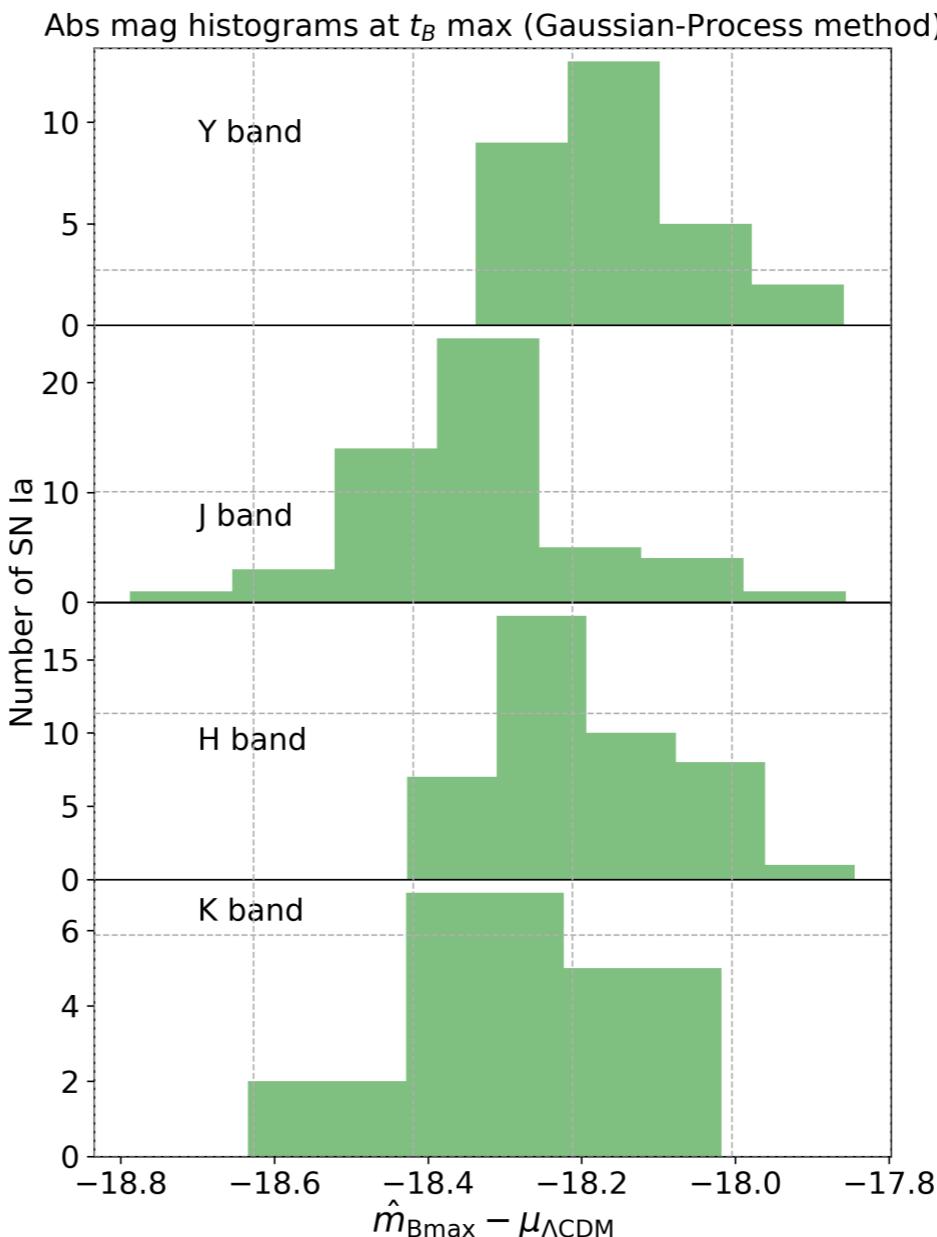


Riess et al. 2016

**Figure 6.** Near-infrared Cepheid  $P$ - $L$  relations. The Cepheid magnitudes are shown for the 19 SN hosts and the four distance-scale anchors. Magnitudes labeled as  $F160W$  are all from the same instrument and camera, WFC3  $F160W$ . The uniformity of the photometry and metallicity reduces systematic errors along the distance ladder. A single slope is shown to illustrate the relations, but we also allow for a break (two slopes) as well as limited period ranges.

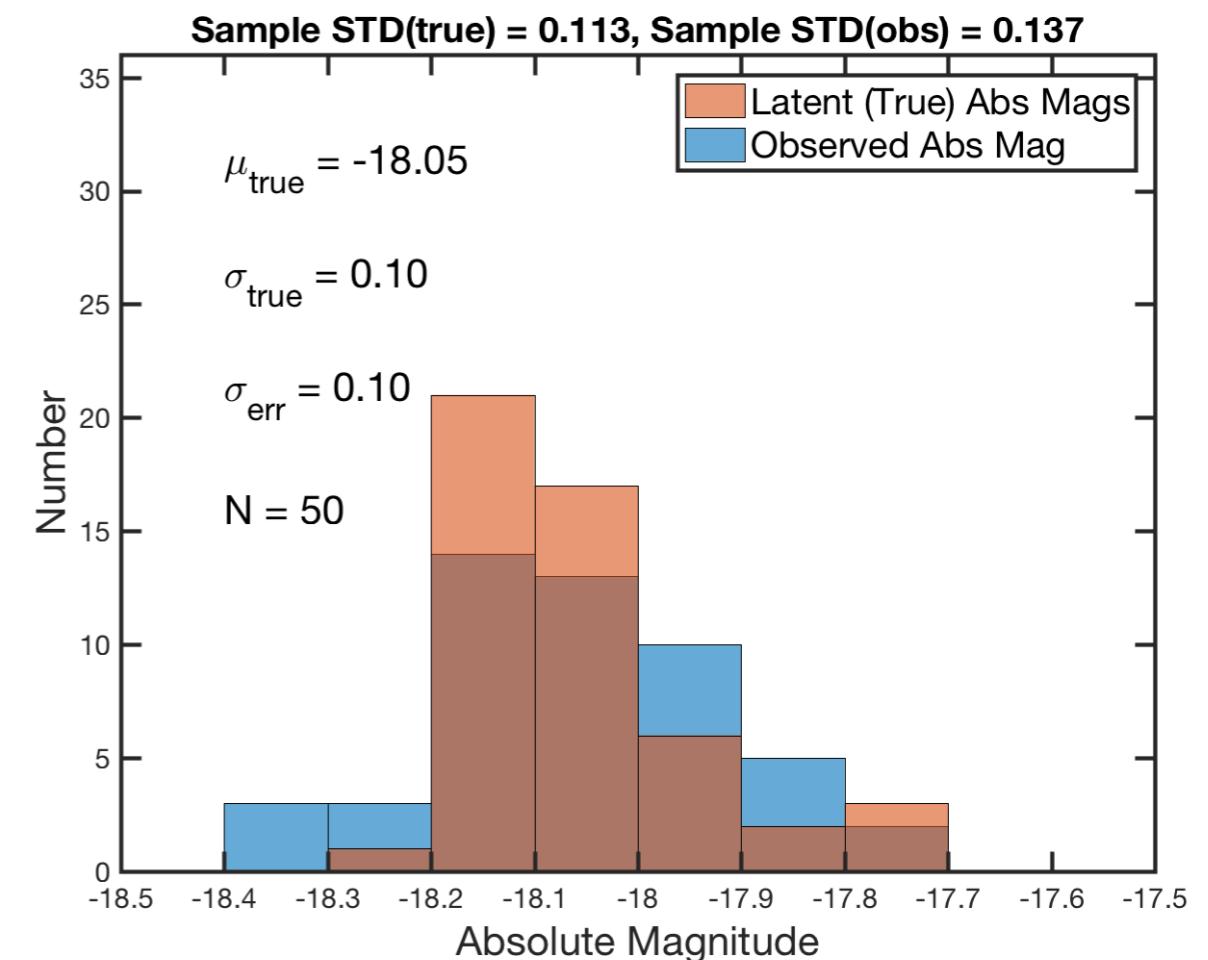
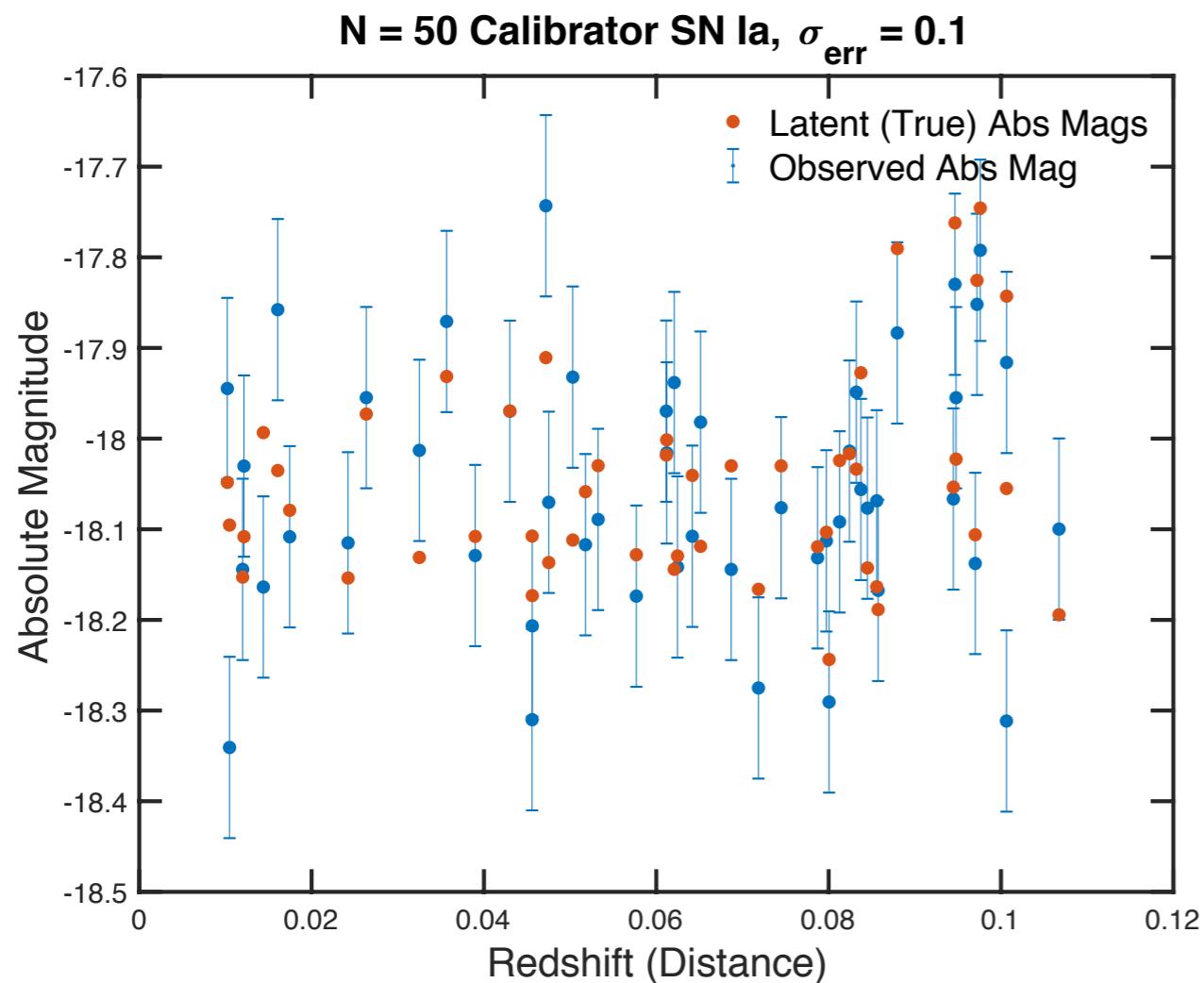
# Calibrating SN Ia Standard Candles (Avelino, Friedman, Mandel et al. 2019)

Determining the  
Distribution  
of Absolute  
Magnitudes



**Figure 4:** Normalized histograms of the absolute magnitudes at phase =  $B_{\max}$ , defined as  $M_{B_{\max},s} \equiv \hat{m}_{B\max,s} - \mu_{\Lambda\text{CDM}}(z_s)$  for the SN Ia sample in the GP method at  $B$  max. The mean, the standard deviation, and the number of supernovae in each histogram are

Want to Calibrate SN Ia (N=50)  
 determine  $M_0$ ,  $\sigma_{\text{int}}$   
 from data with measurement error std dev  $\sigma_{\text{err}} = 0.1$

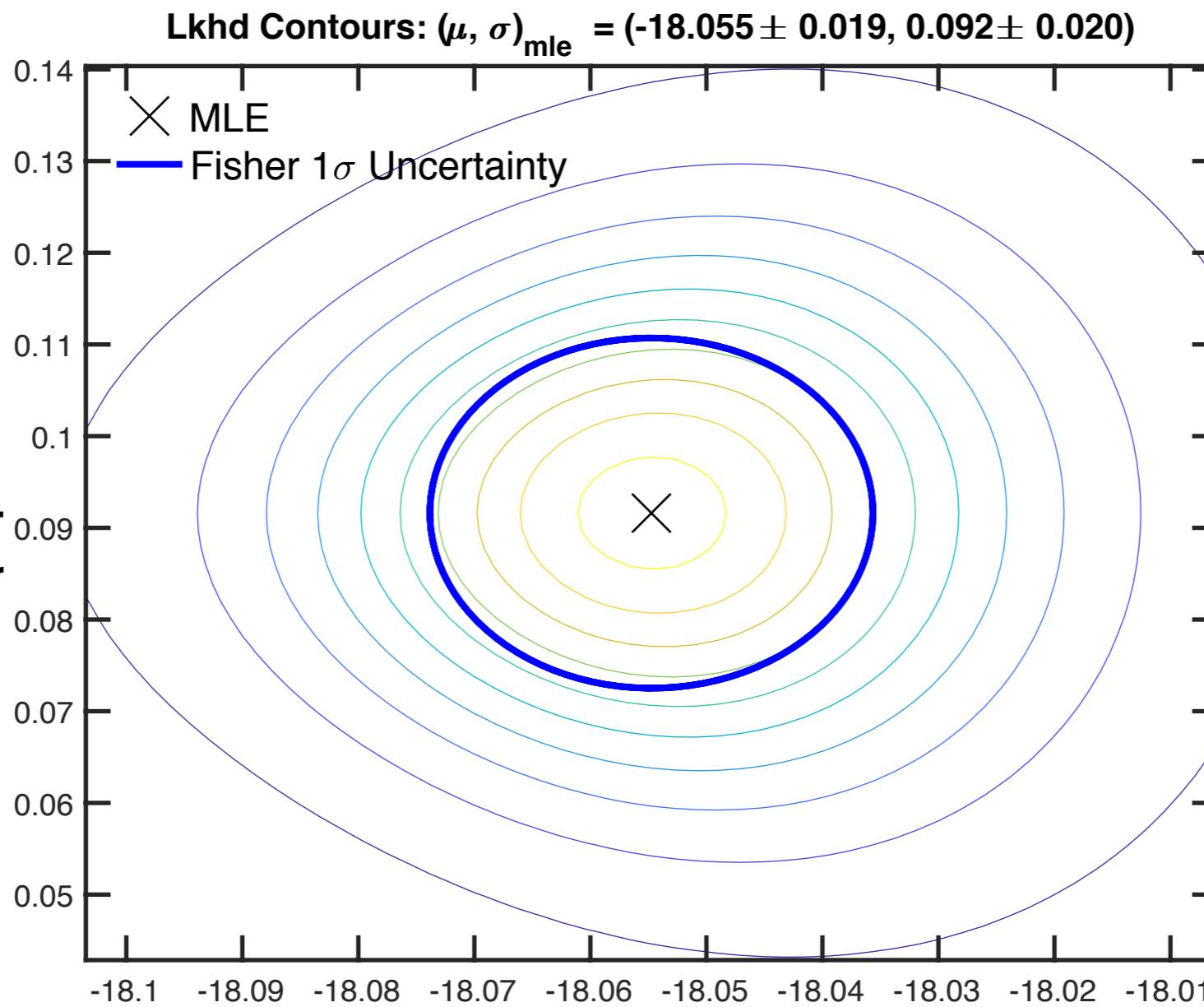


# Maximum Likelihood with measurement error

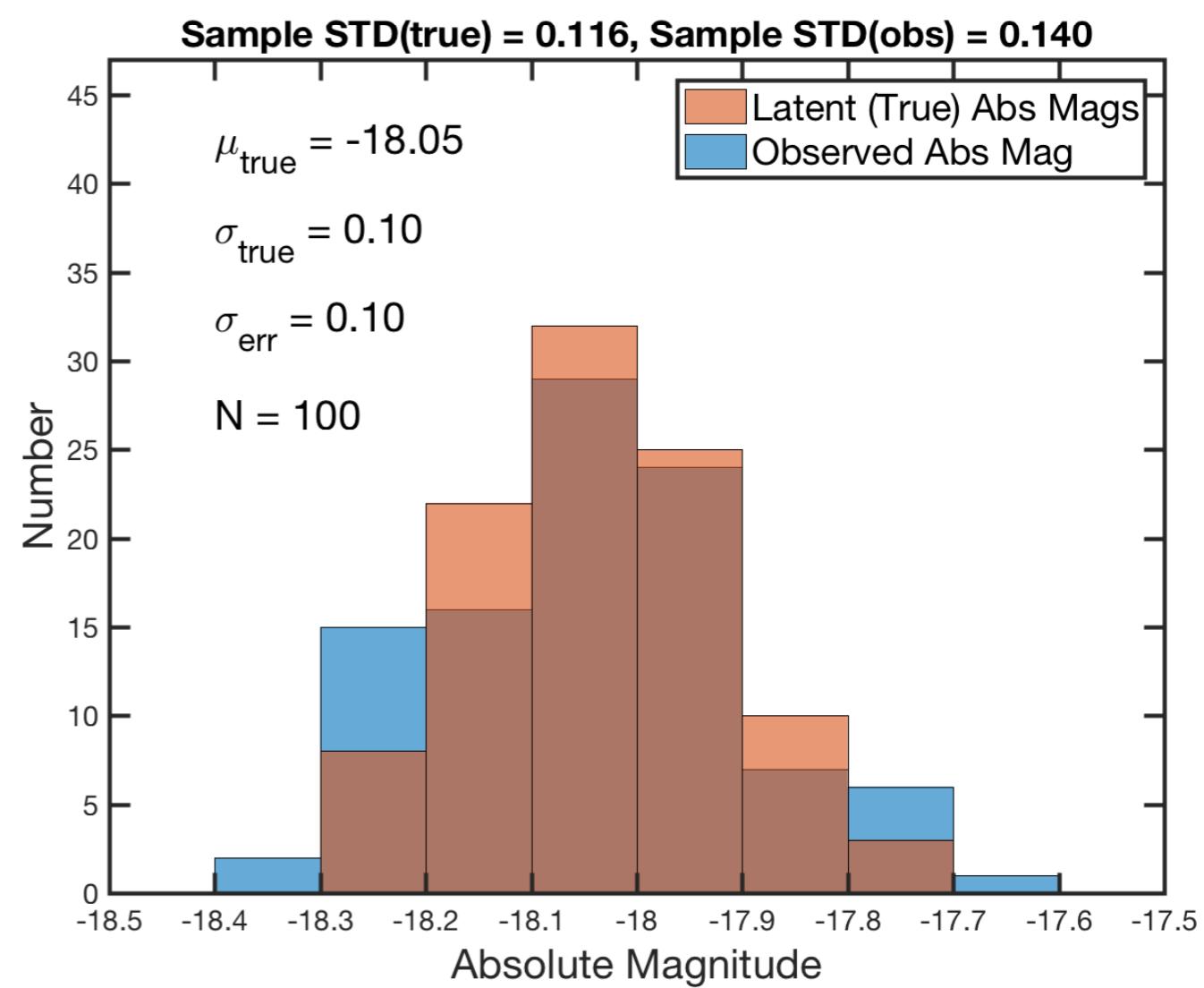
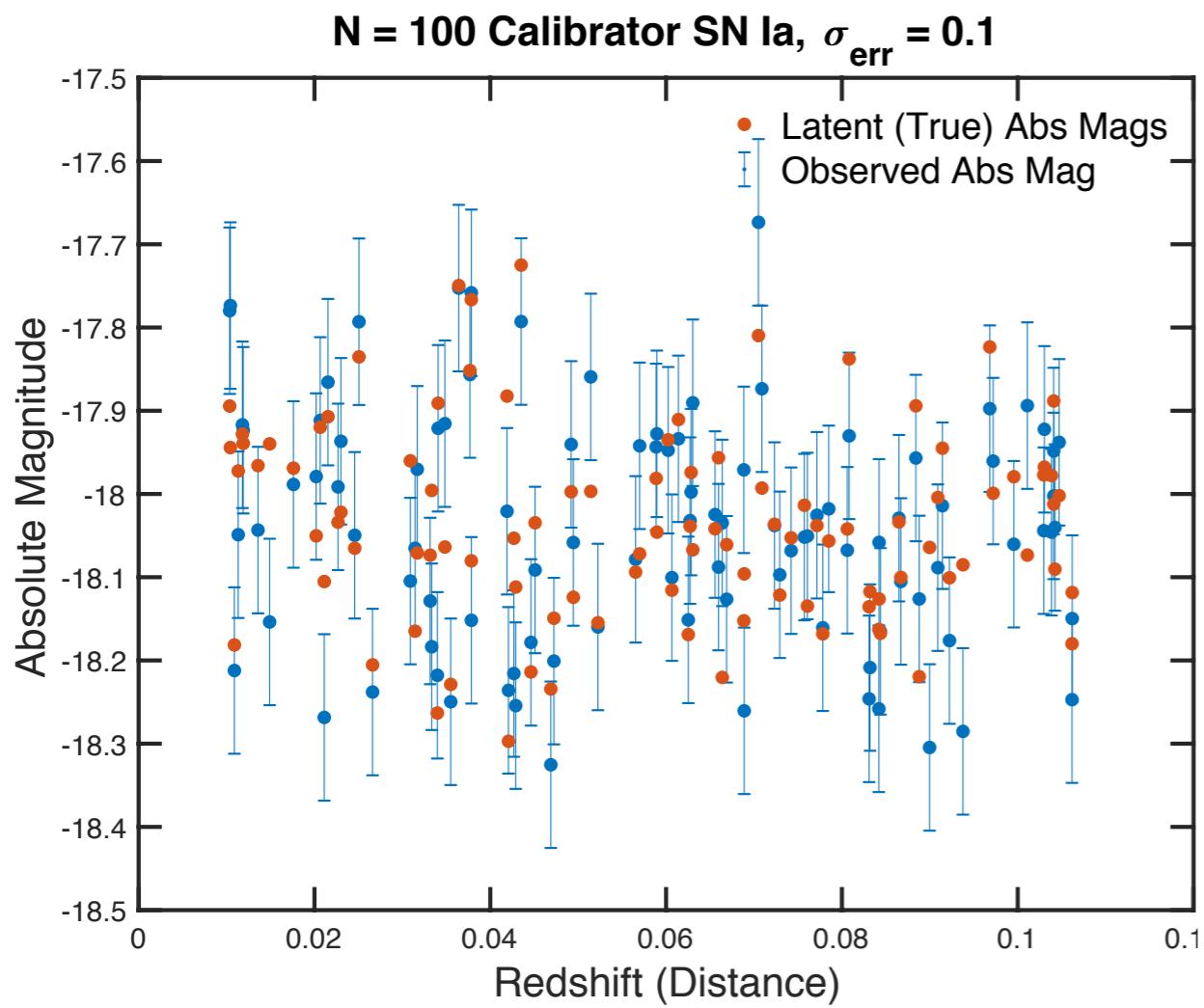
$N = 50$

$\sigma_{\text{int}}$

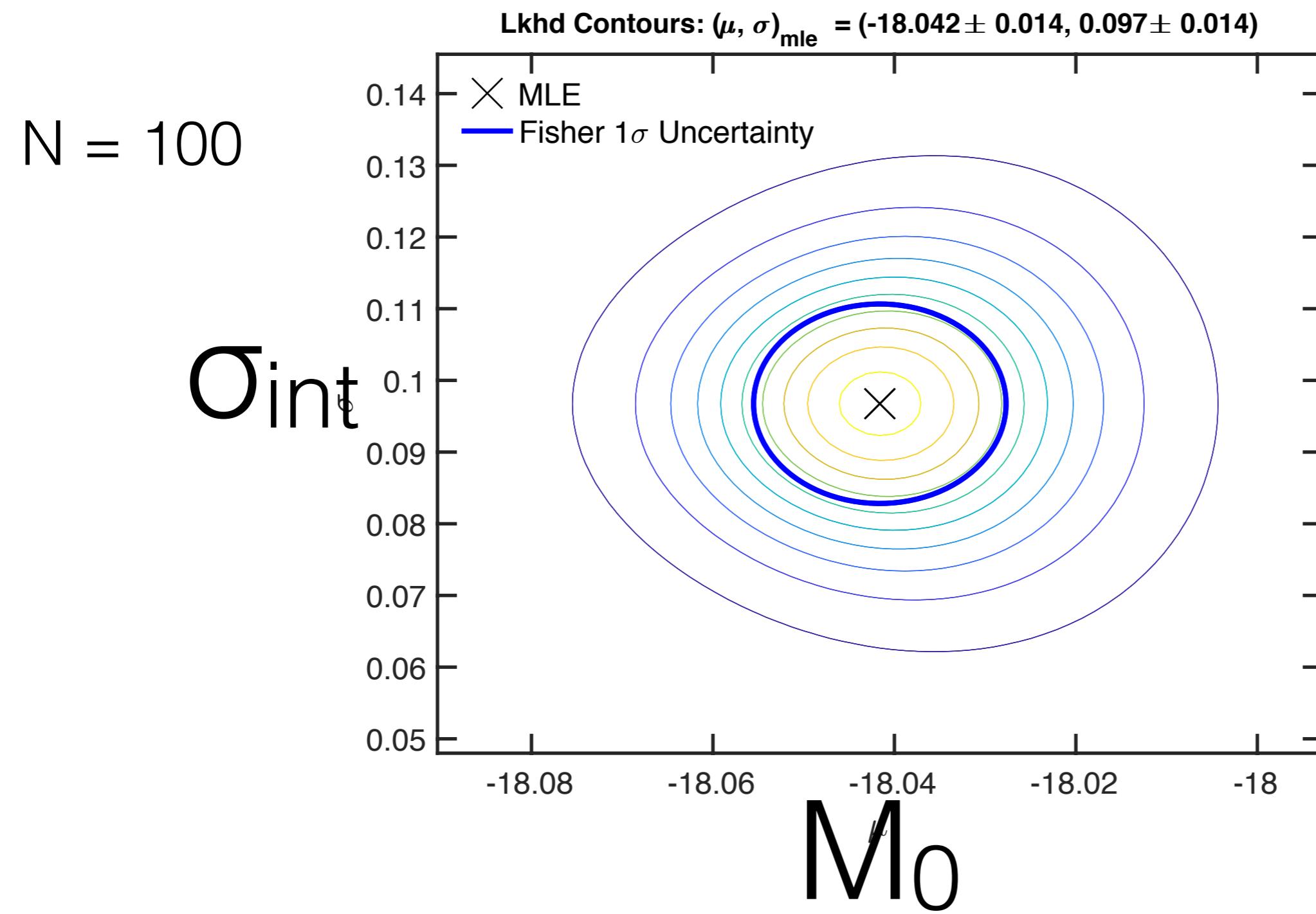
$M_0^\mu$



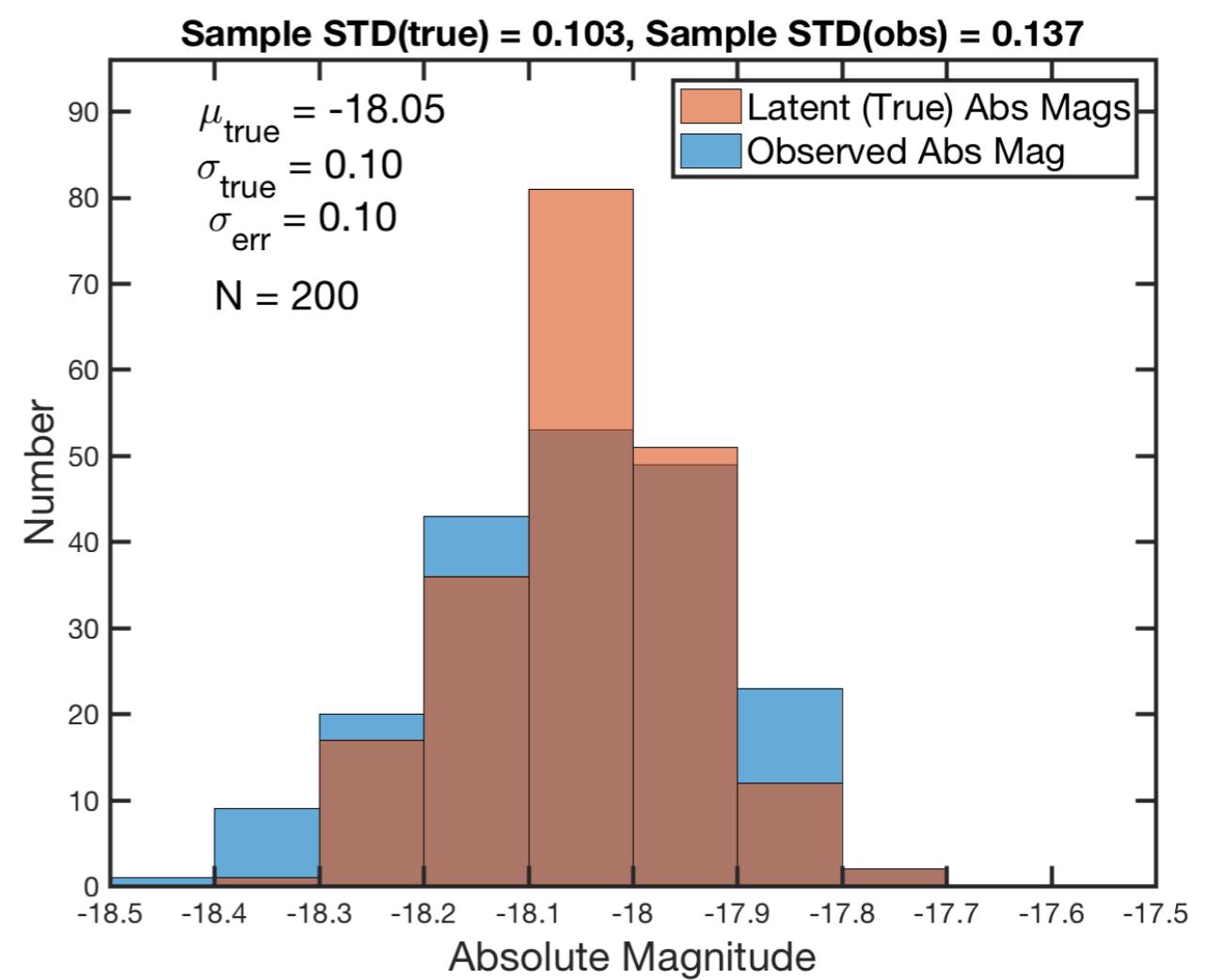
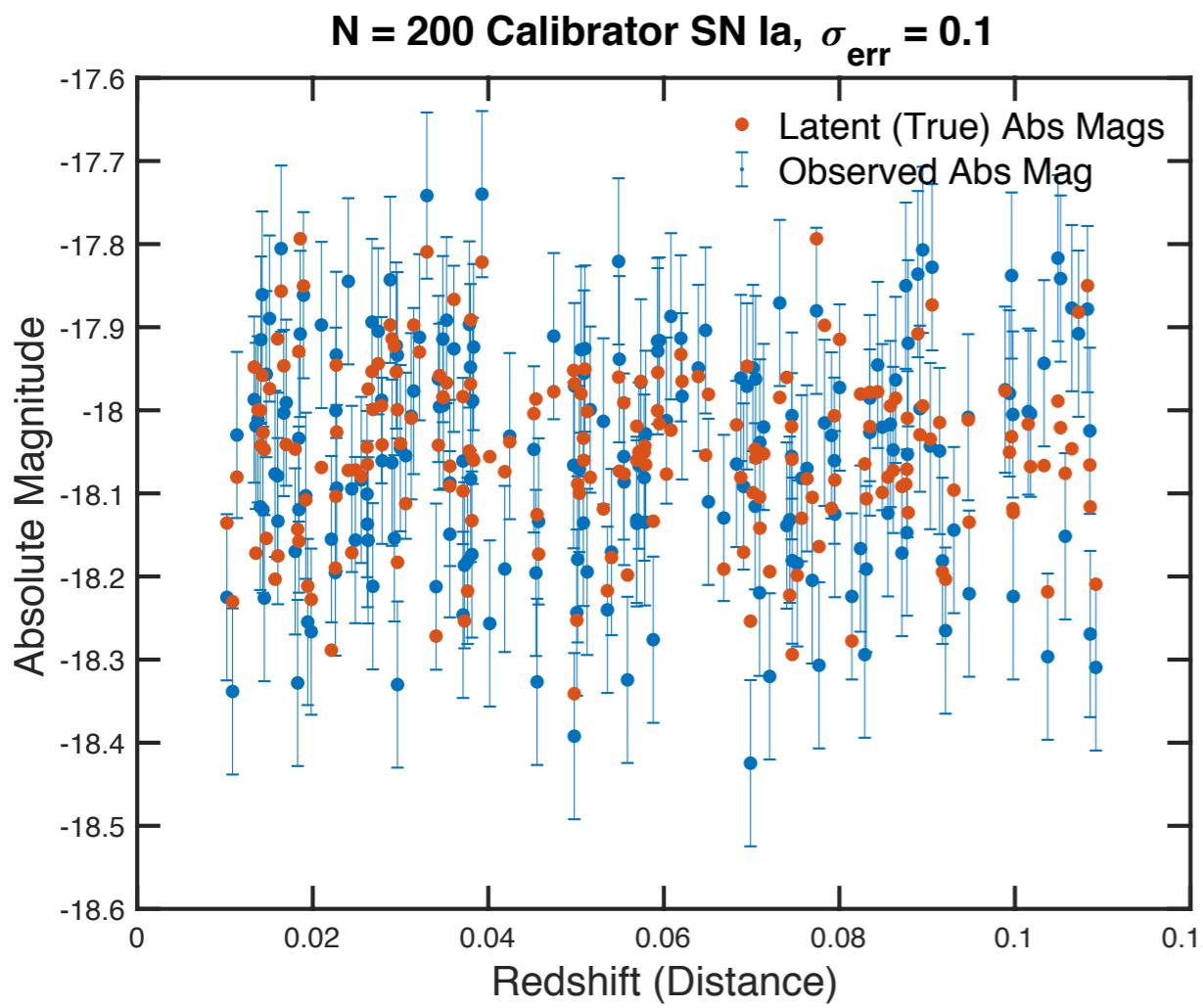
# Want to Calibrate SN Ia (N=100) determine $M_0$ , $\sigma_{\text{int}}$



# Maximum Likelihood with measurement error

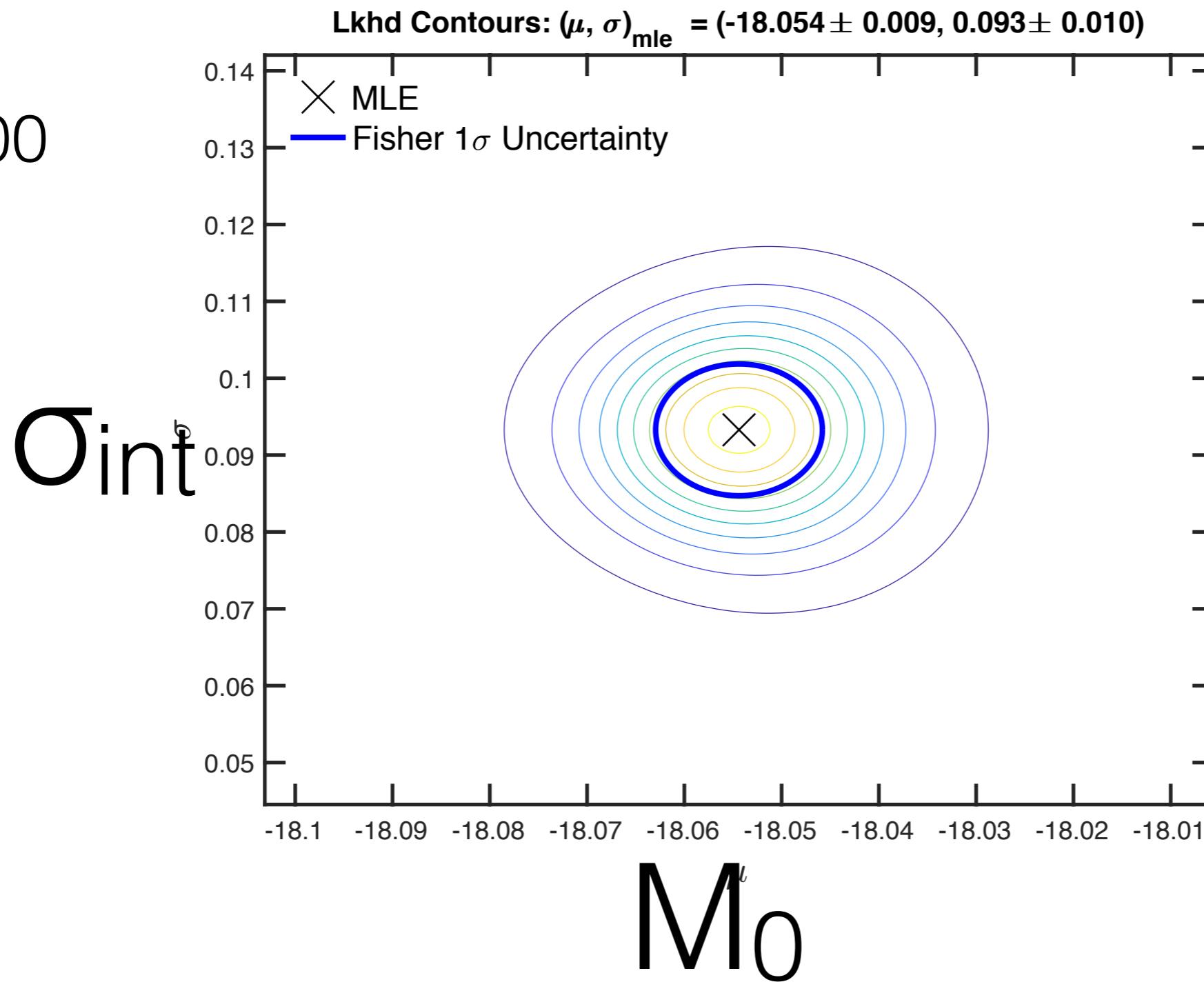


# Want to Calibrate SN Ia (N=200) determine $M_0$ , $\sigma_{\text{int}}$



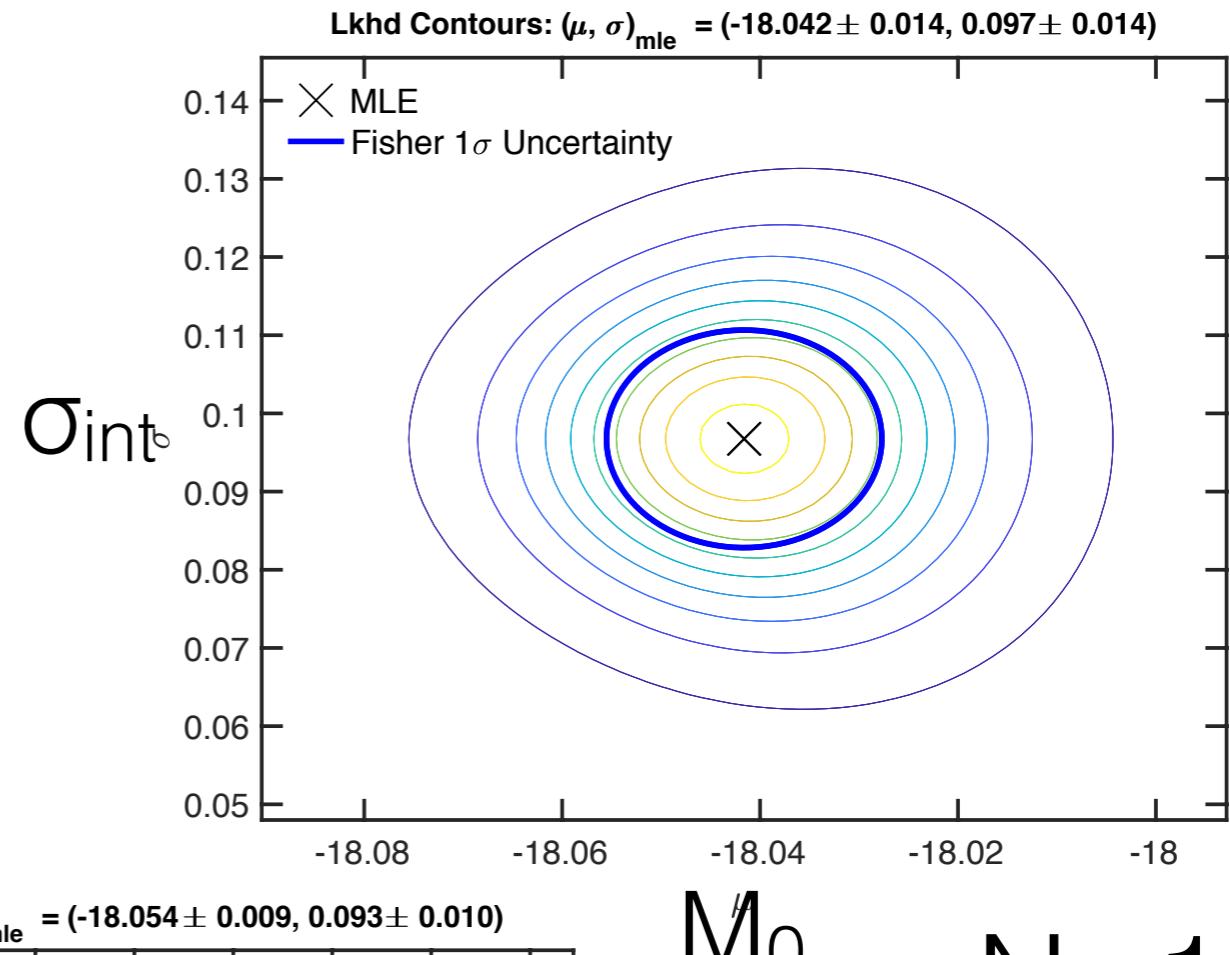
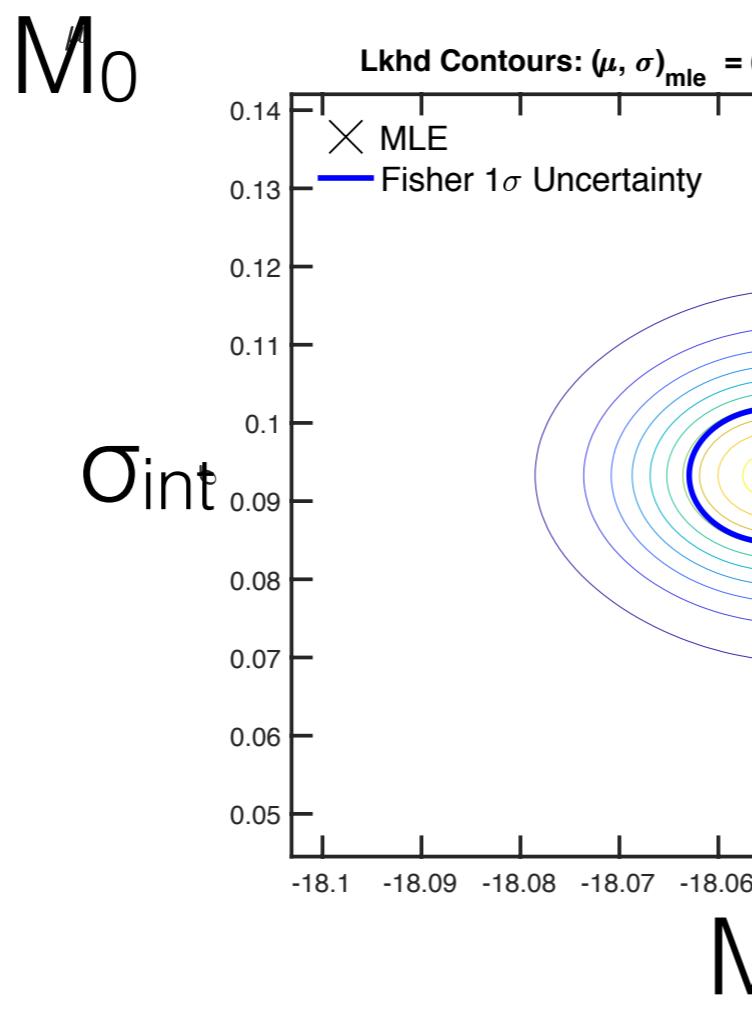
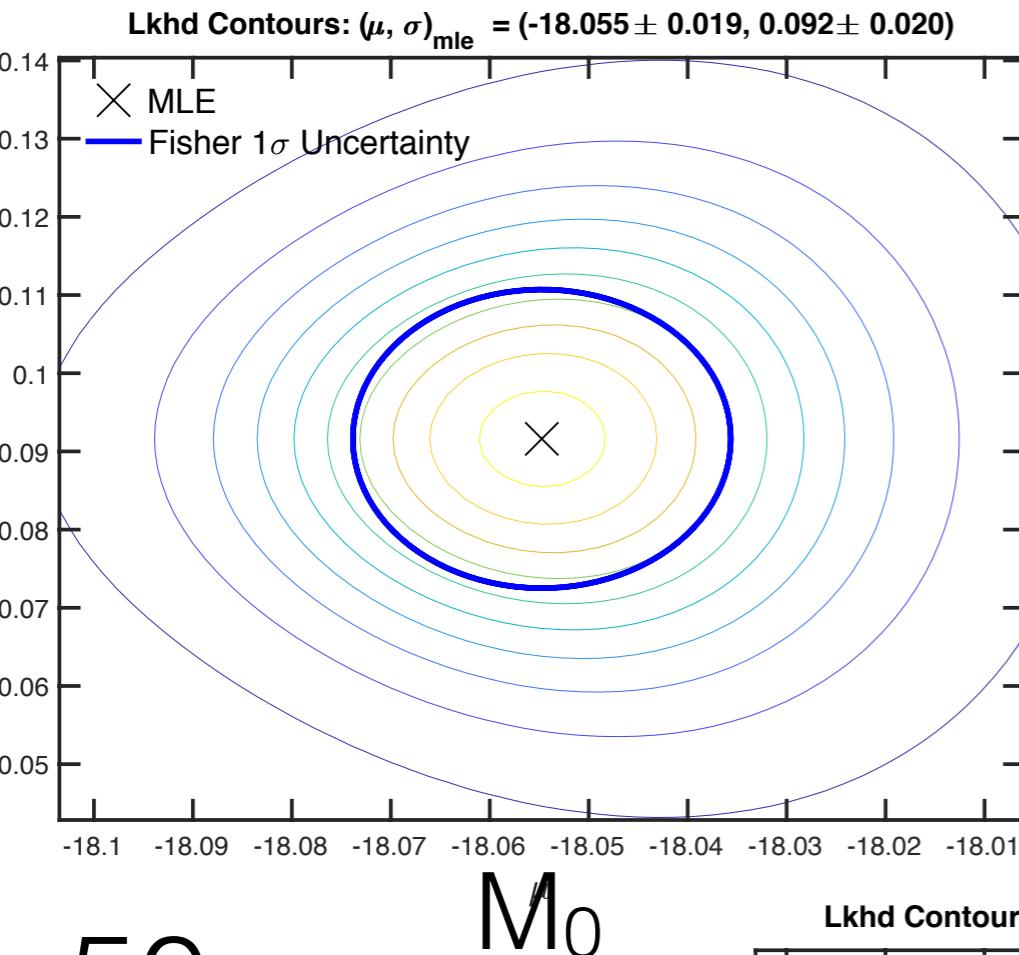
# Maximum Likelihood with measurement error

$N = 200$



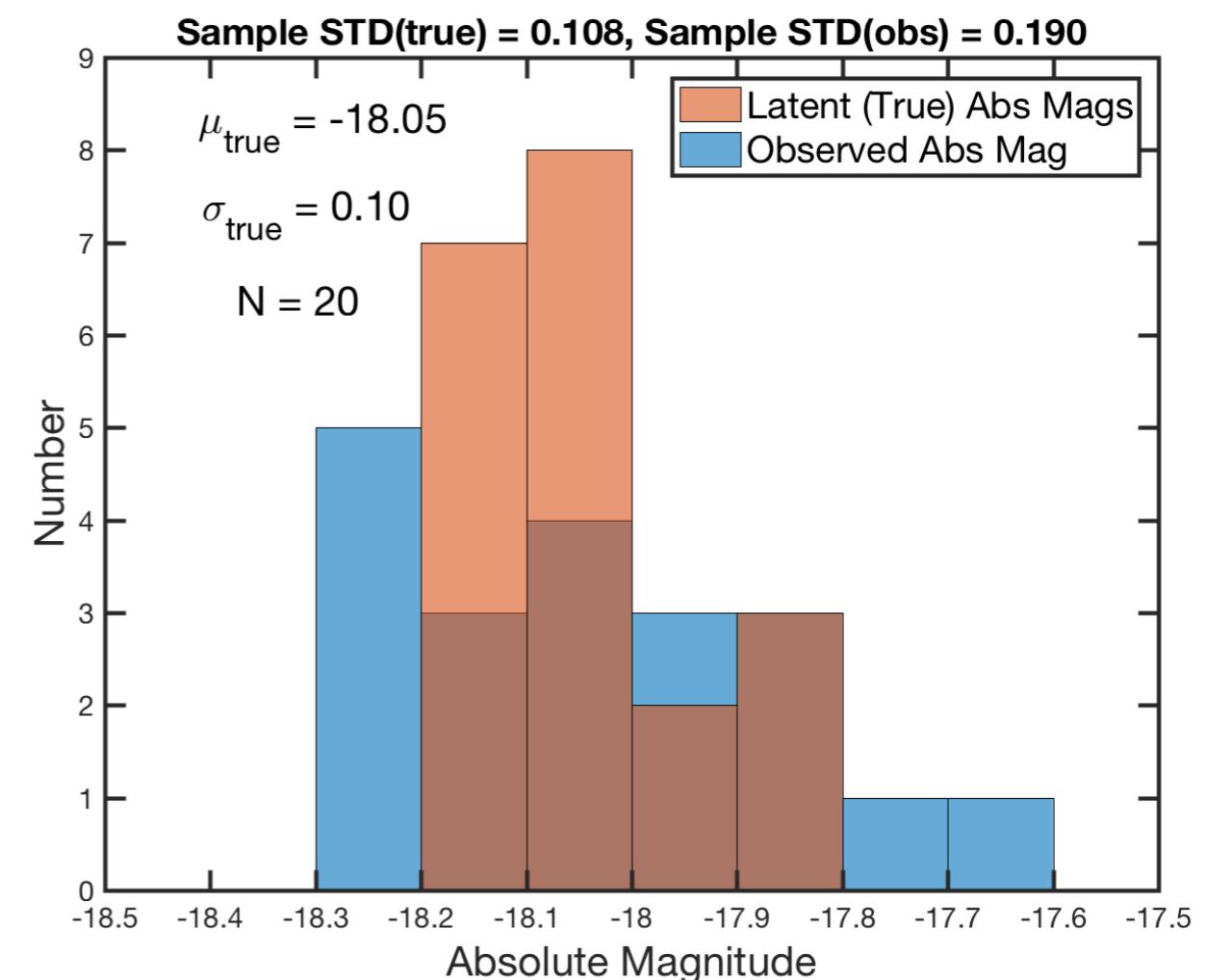
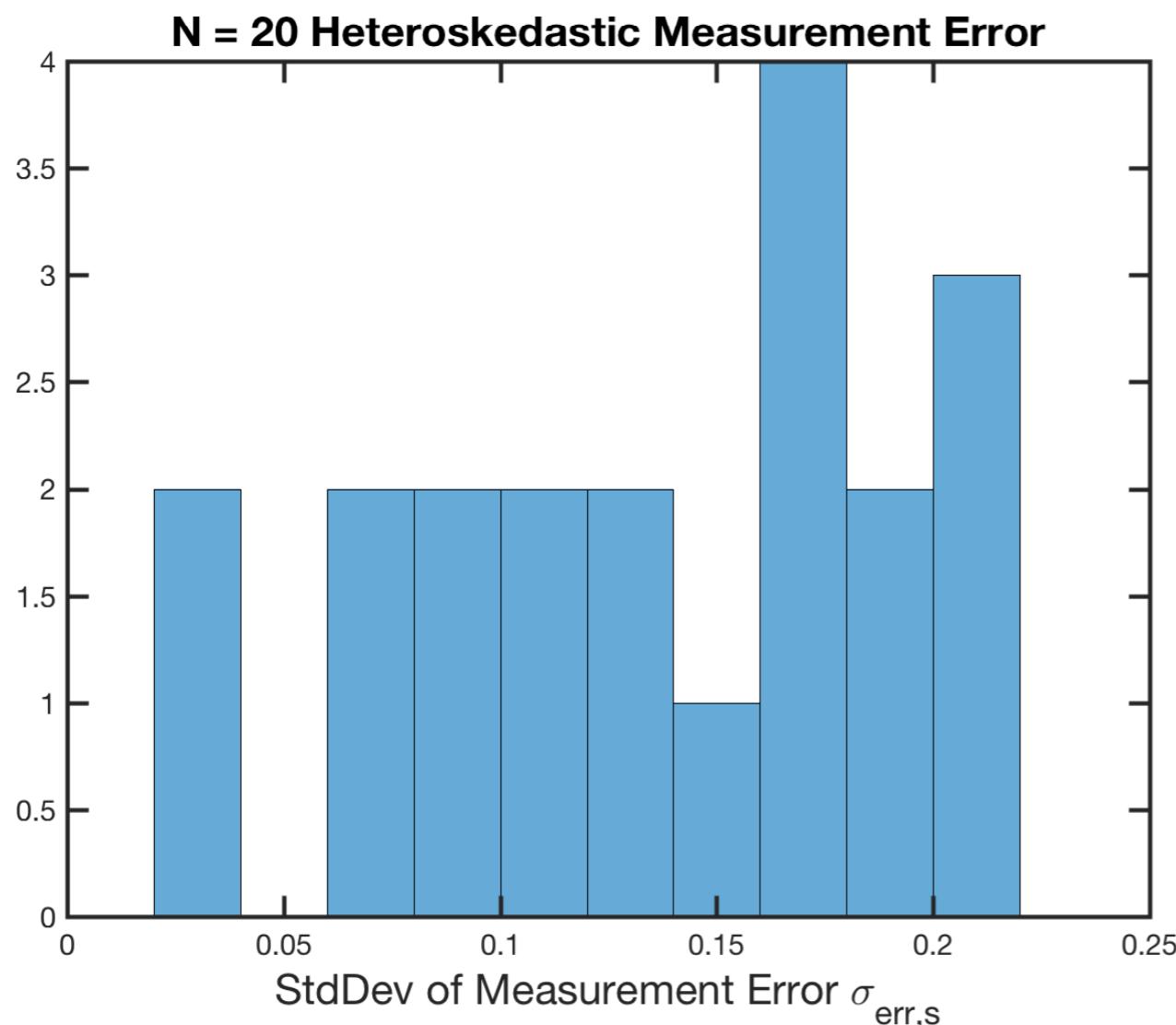
# Constraints vs. sample size

N=50



N=200

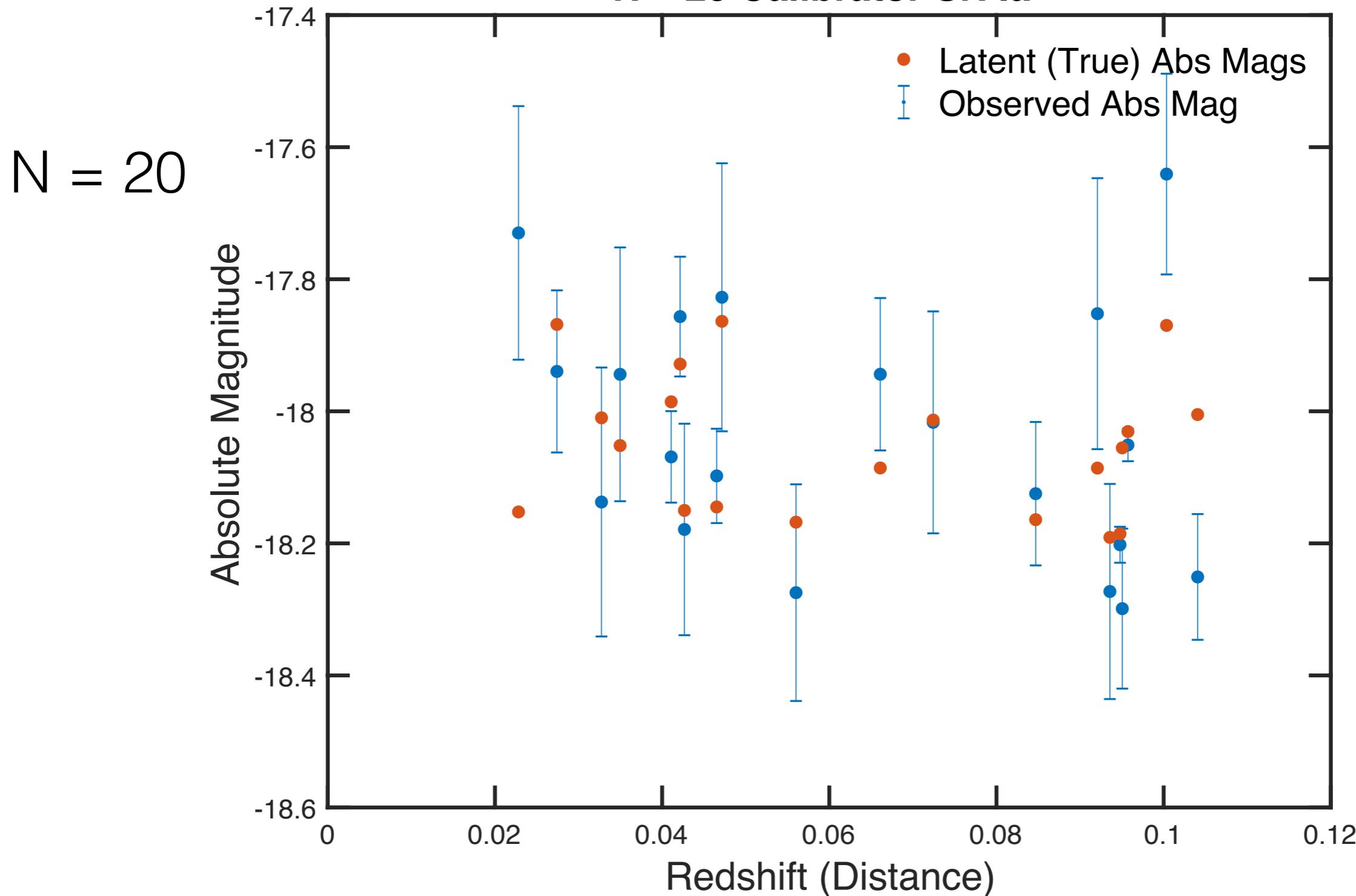
# Maximum Likelihood with heteroskedastic measurement error with std devs $\sigma_{\text{err}} = 0.01$ to $0.21$



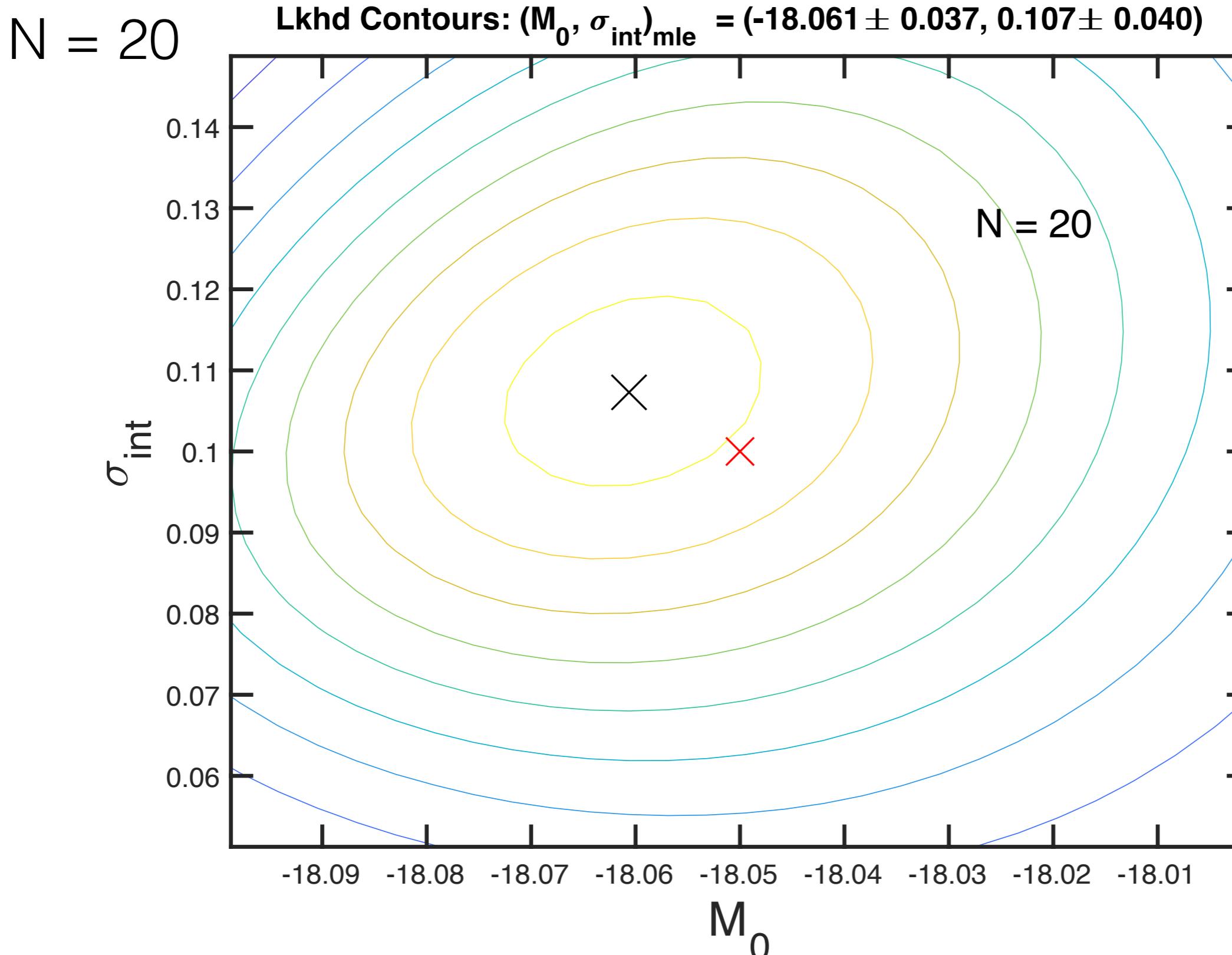
N = 20

Maximum Likelihood with  
heteroskedastic measurement error  
with std devs  $\sigma_{\text{err}} = 0.01$  to  $0.21$

**N = 20 Calibrator SN Ia**

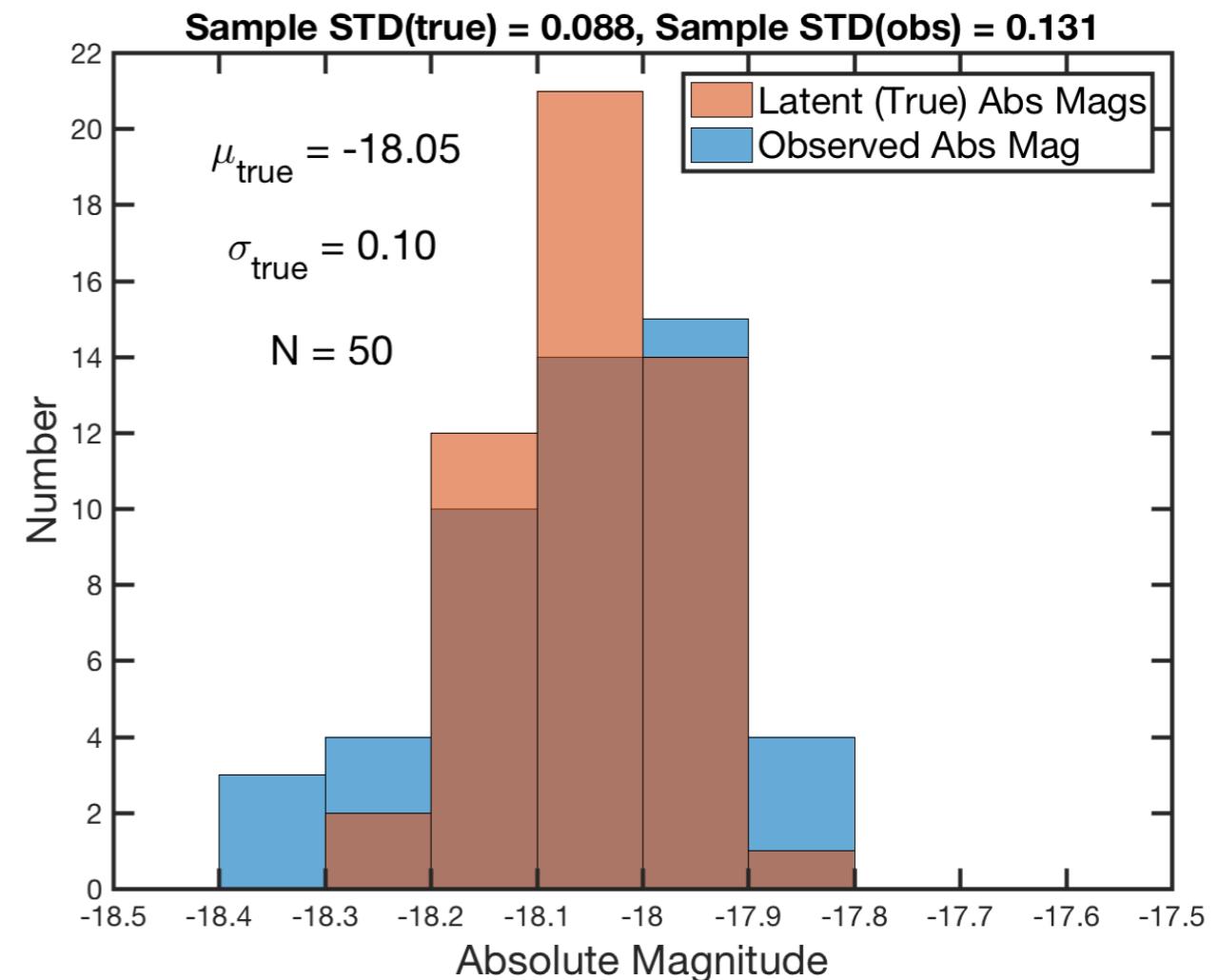
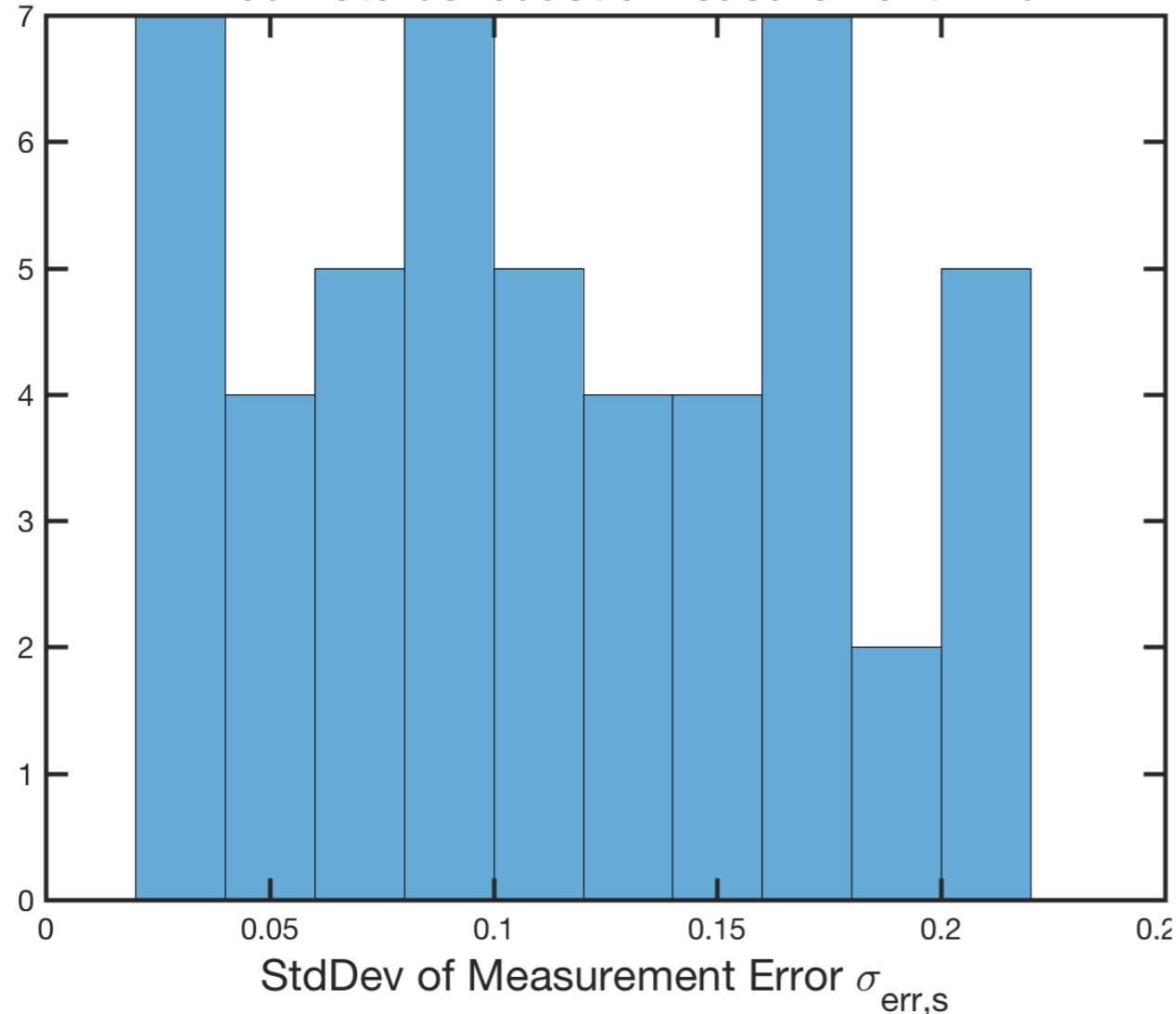


Maximum Likelihood with  
heteroskedastic measurement error  
with std devs  $\sigma_{\text{err}} = 0.01$  to  $0.21$



# Maximum Likelihood with heteroskedastic measurement error with std devs $\sigma_{\text{err}} = 0.01$ to $0.21$

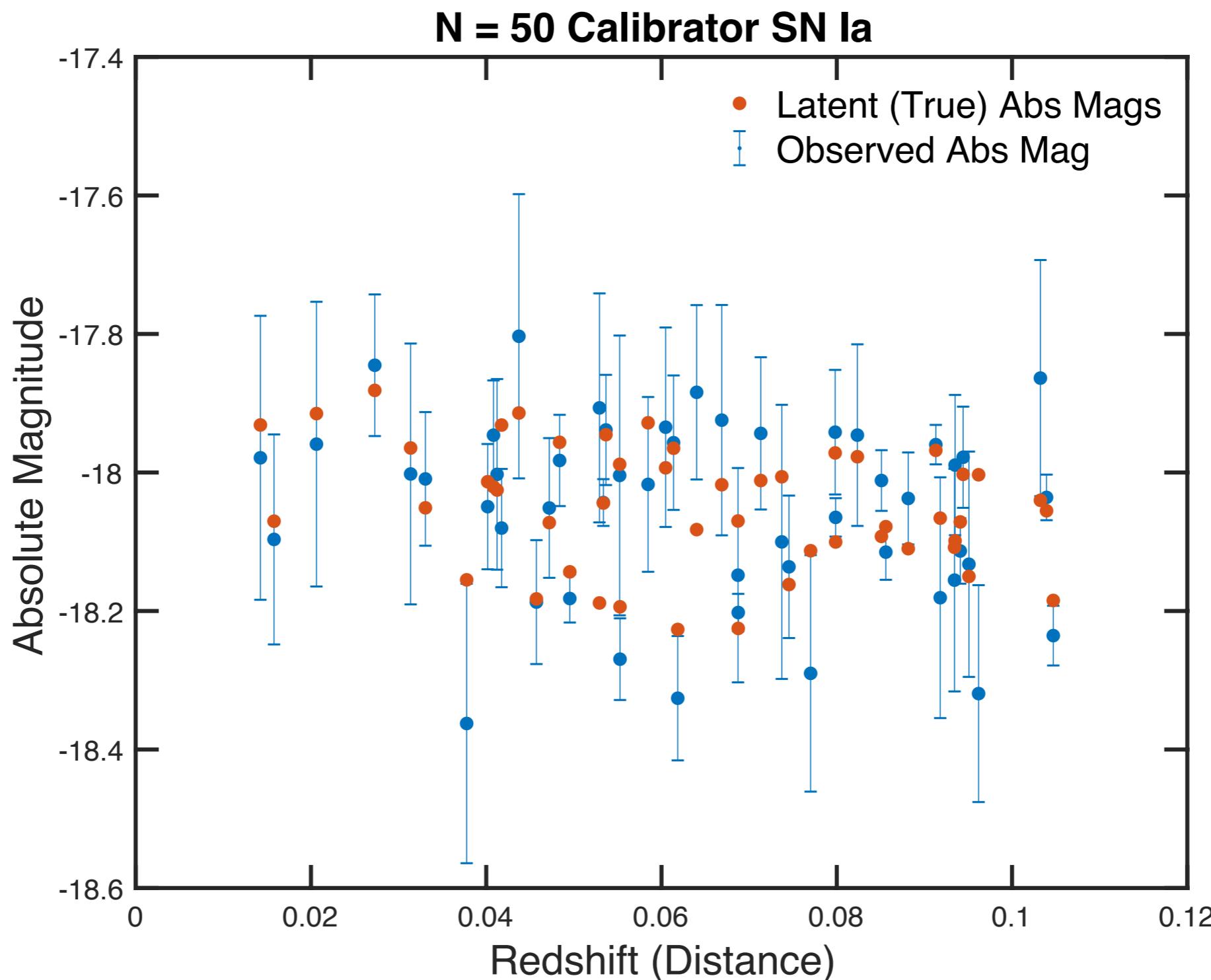
N = 50 Heteroskedastic Measurement Error



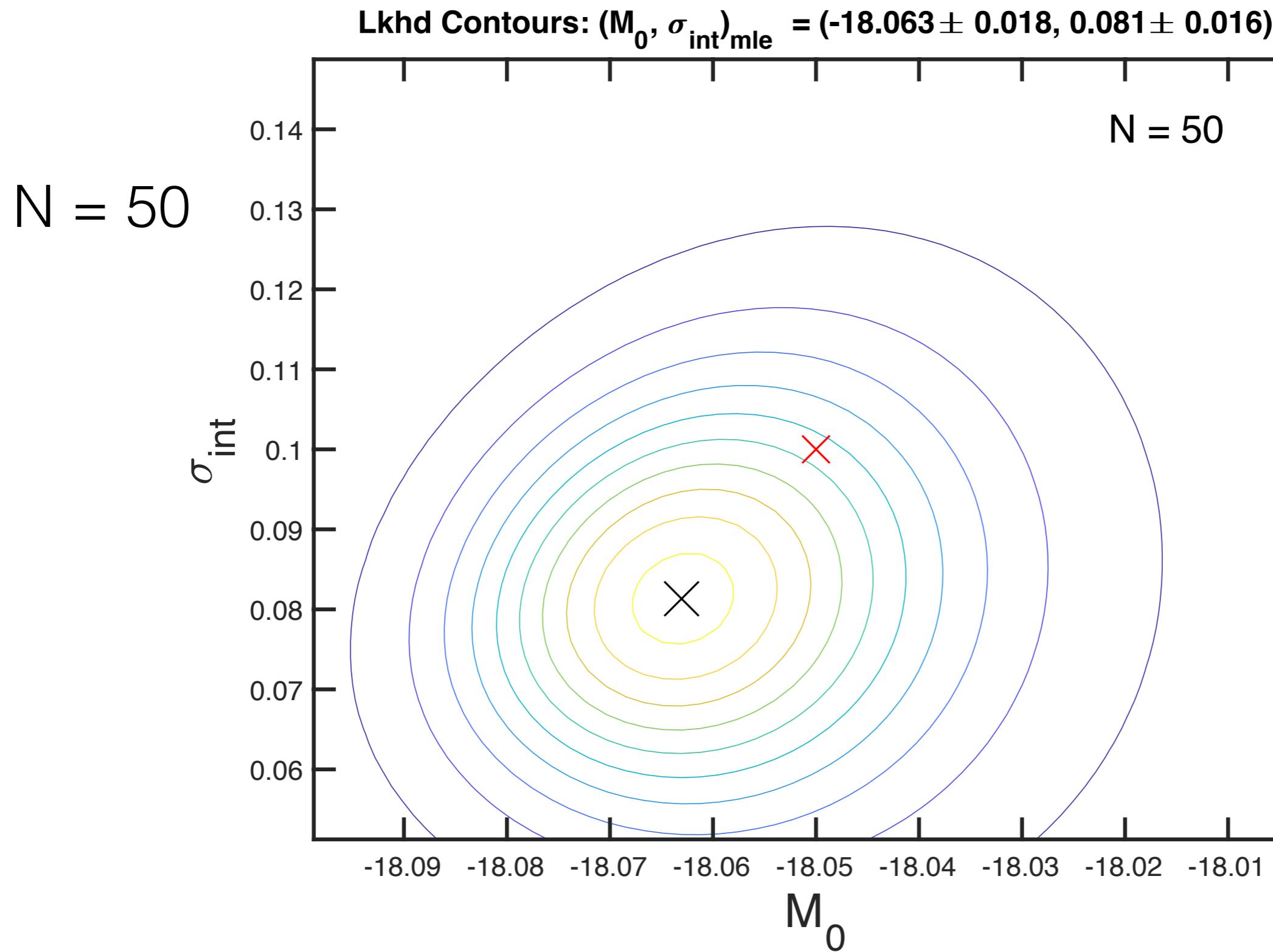
N = 50

Maximum Likelihood with  
heteroskedastic measurement error  
with std devs  $\sigma_{\text{err}} = 0.01$  to  $0.21$

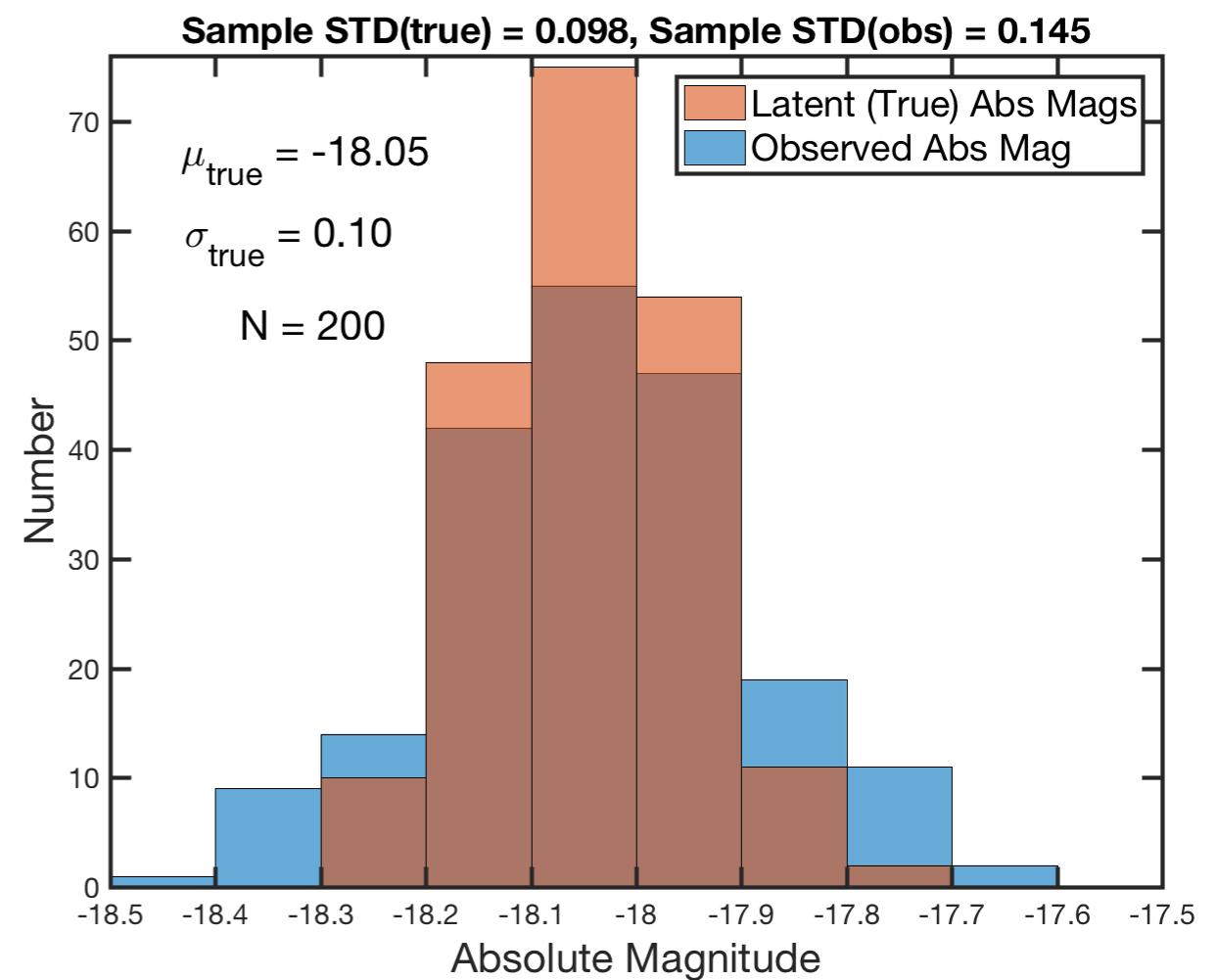
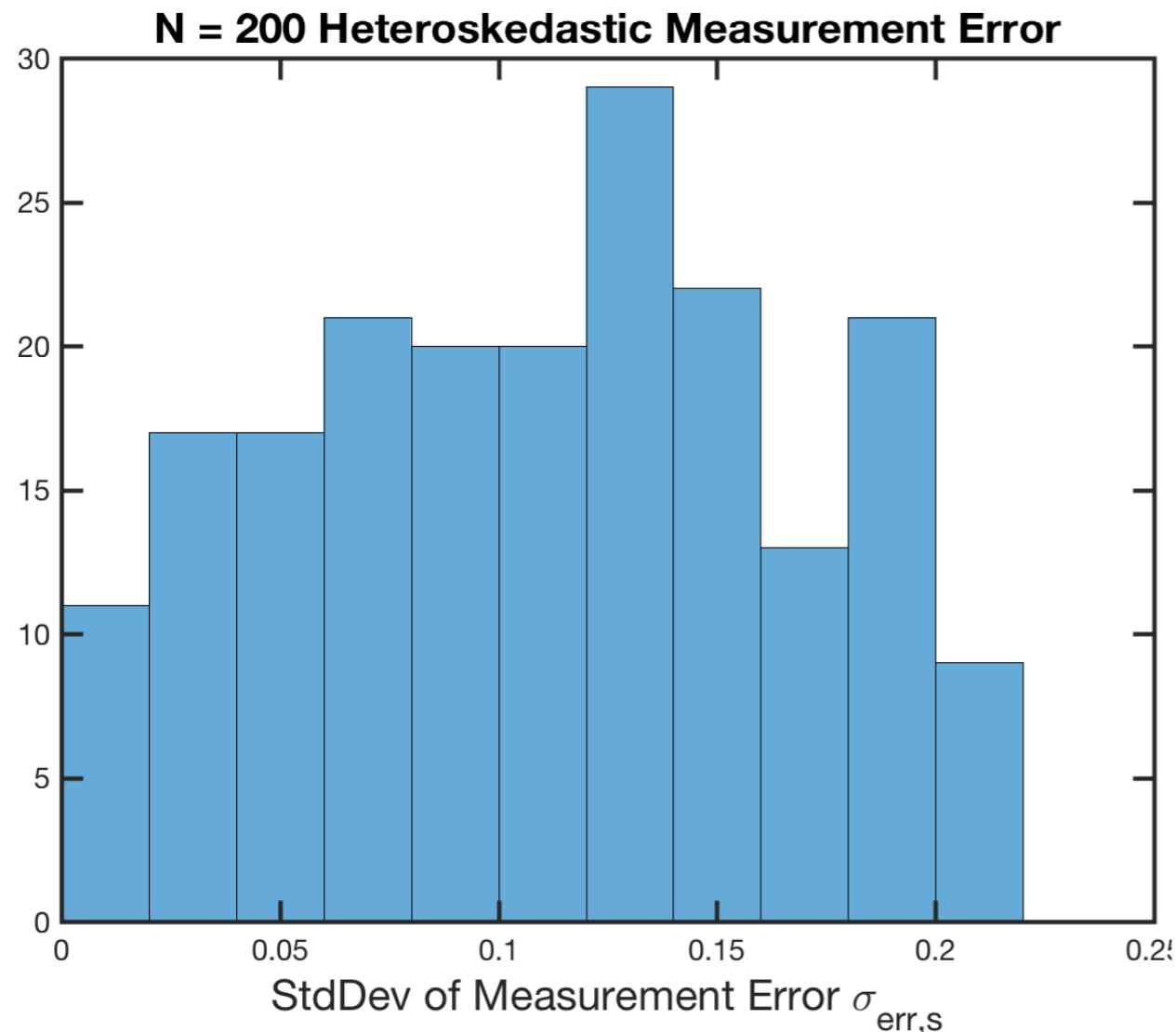
$N = 50$



# Maximum Likelihood with heteroskedastic measurement error with std devs $\sigma_{\text{err}} = 0.01$ to $0.21$

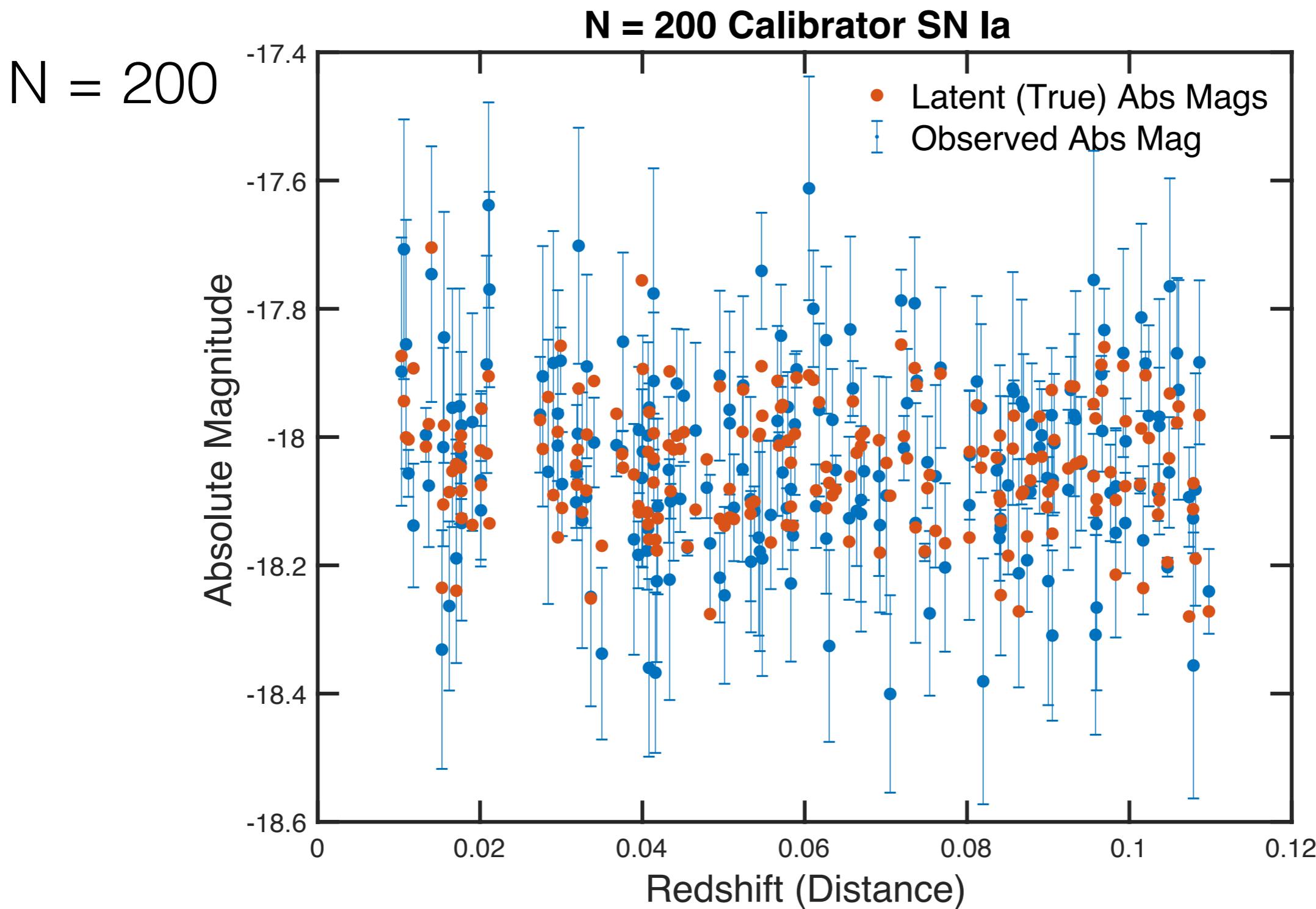


# Maximum Likelihood with heteroskedastic measurement error with std devs $\sigma_{\text{err}} = 0.01$ to $0.21$



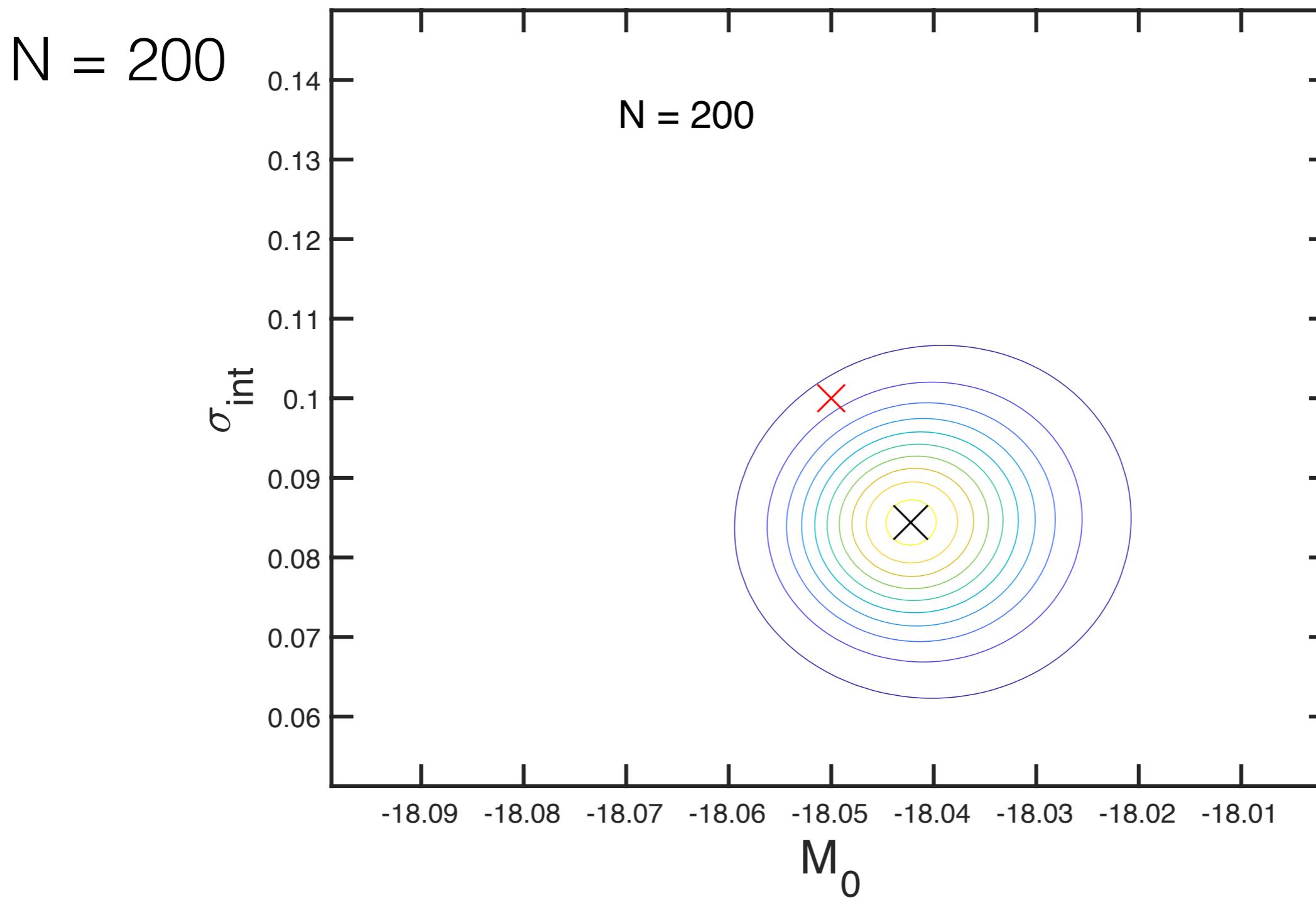
N = 200

# Maximum Likelihood with heteroskedastic measurement error with std devs $\sigma_{\text{err}}$ = 0.01 to 0.21

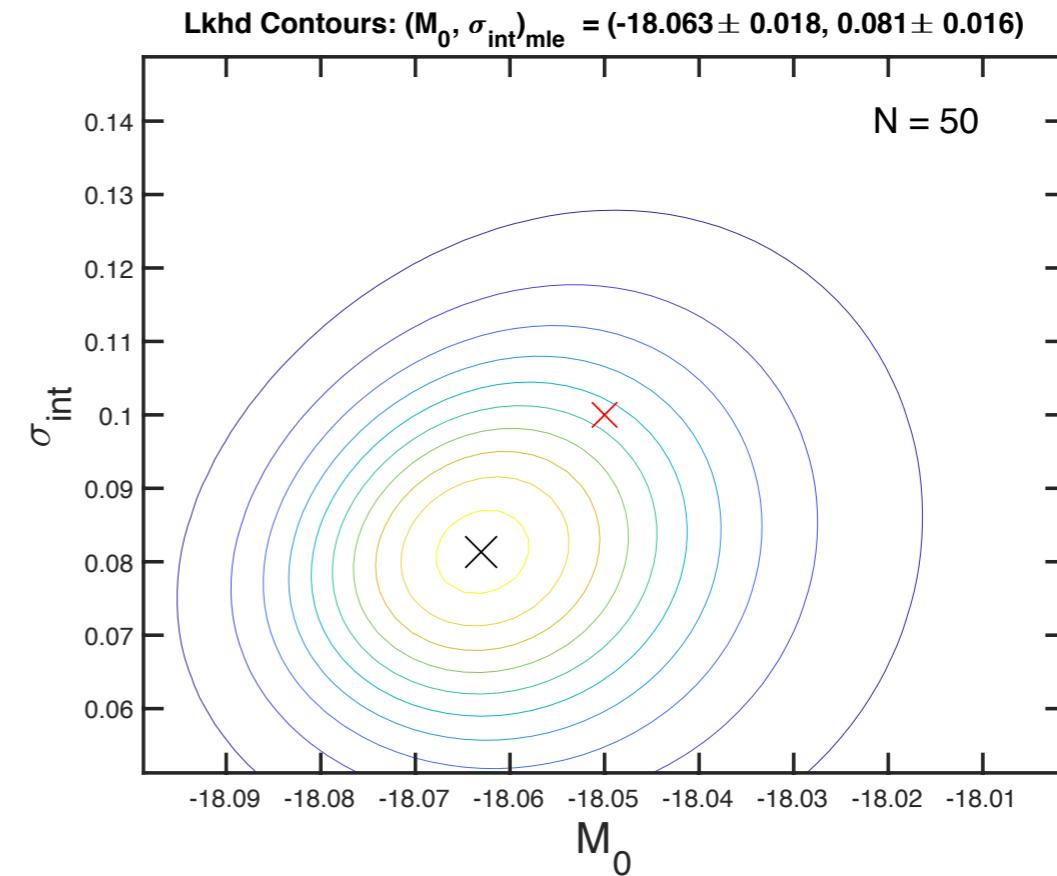
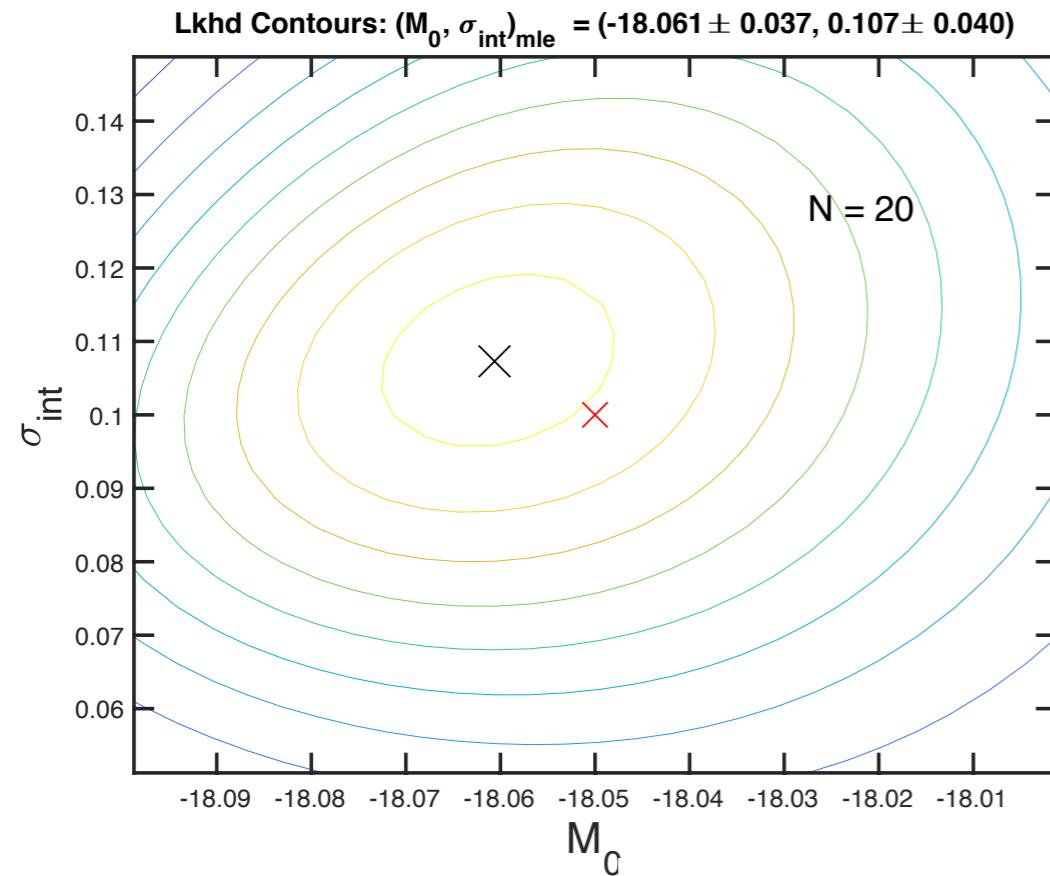


# Maximum Likelihood with heteroskedastic measurement error with std devs $\sigma_{\text{err}} = 0.01$ to $0.21$

Lkhd Contours:  $(M_0, \sigma_{\text{int}})_{\text{mle}} = (-18.042 \pm 0.009, 0.084 \pm 0.008)$

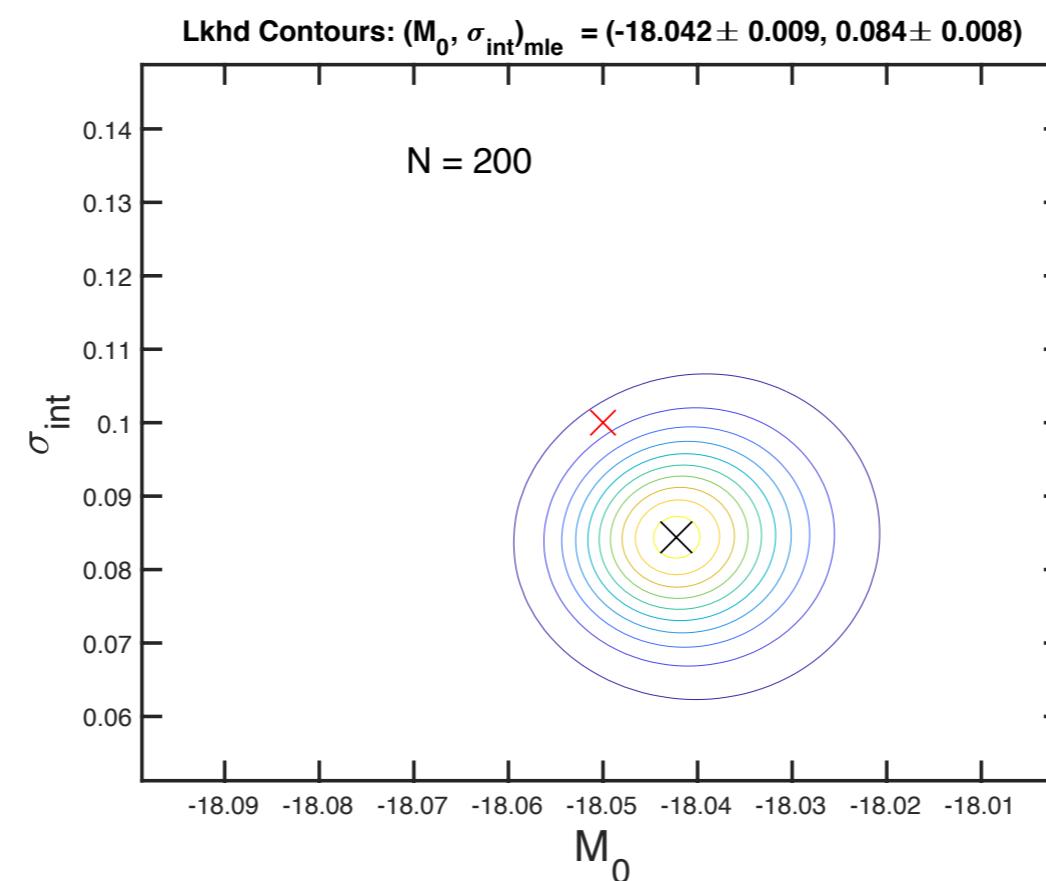


# Constraints vs. sample size



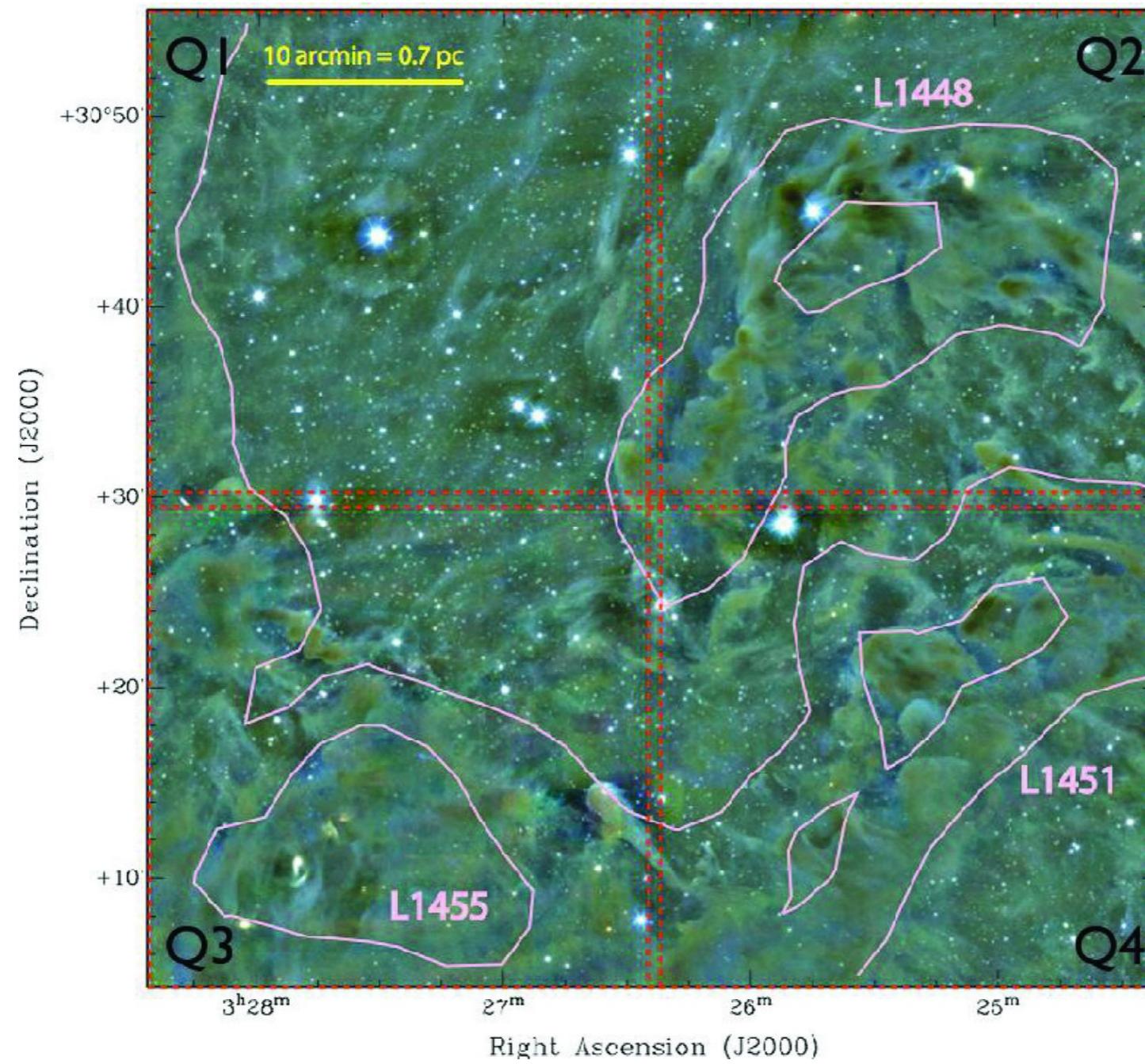
$N=20$

$N=200$

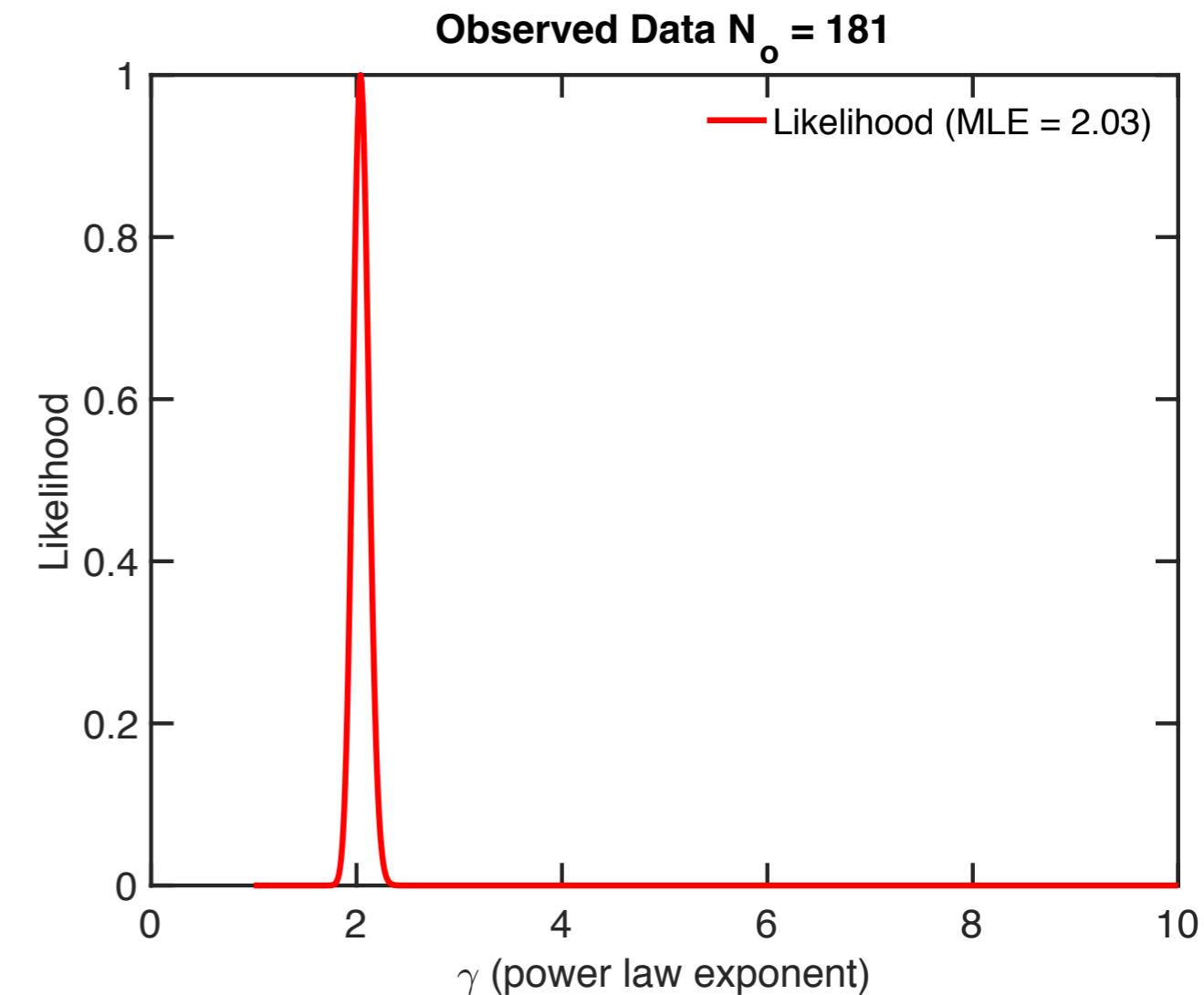
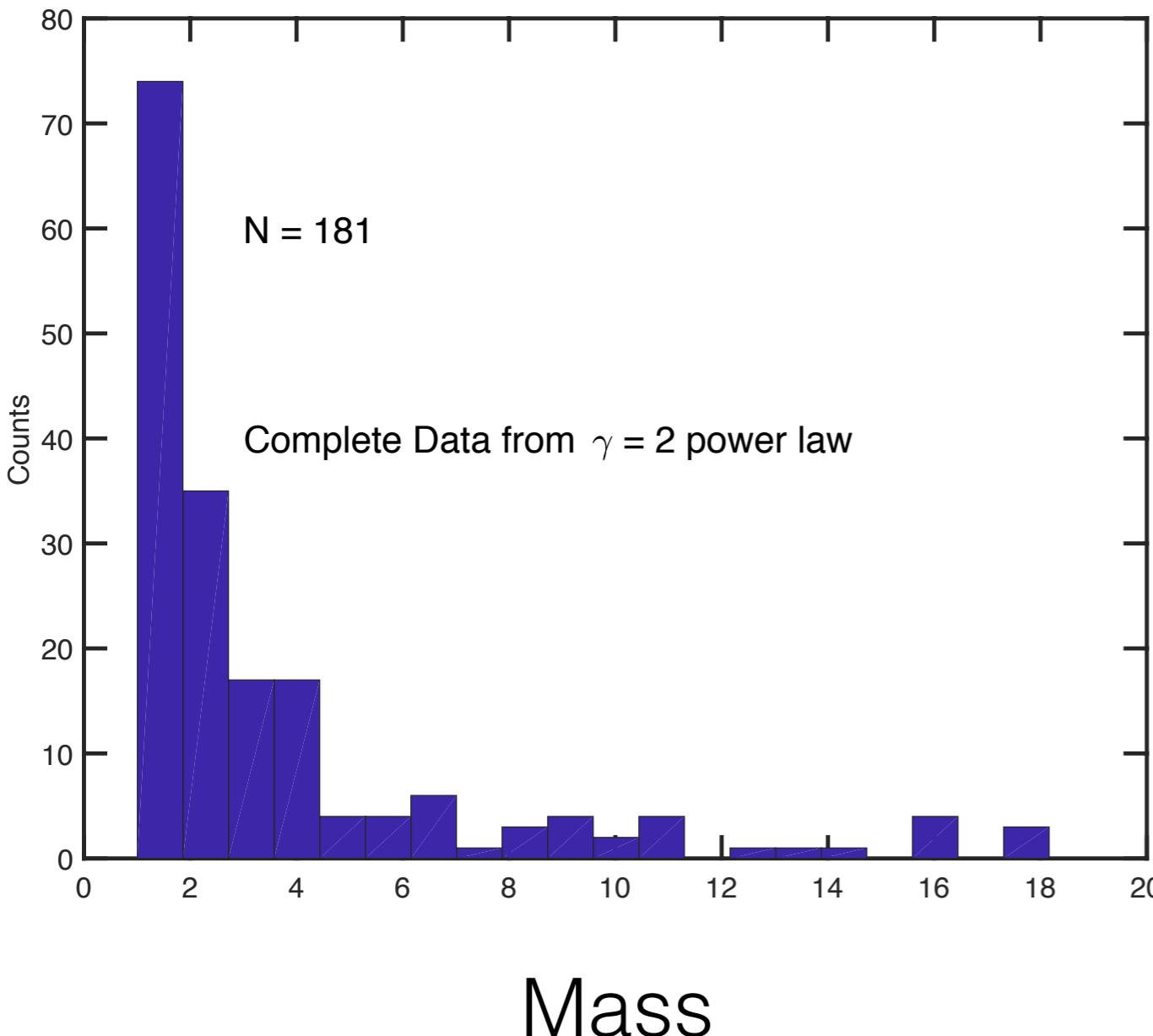


$N=50$

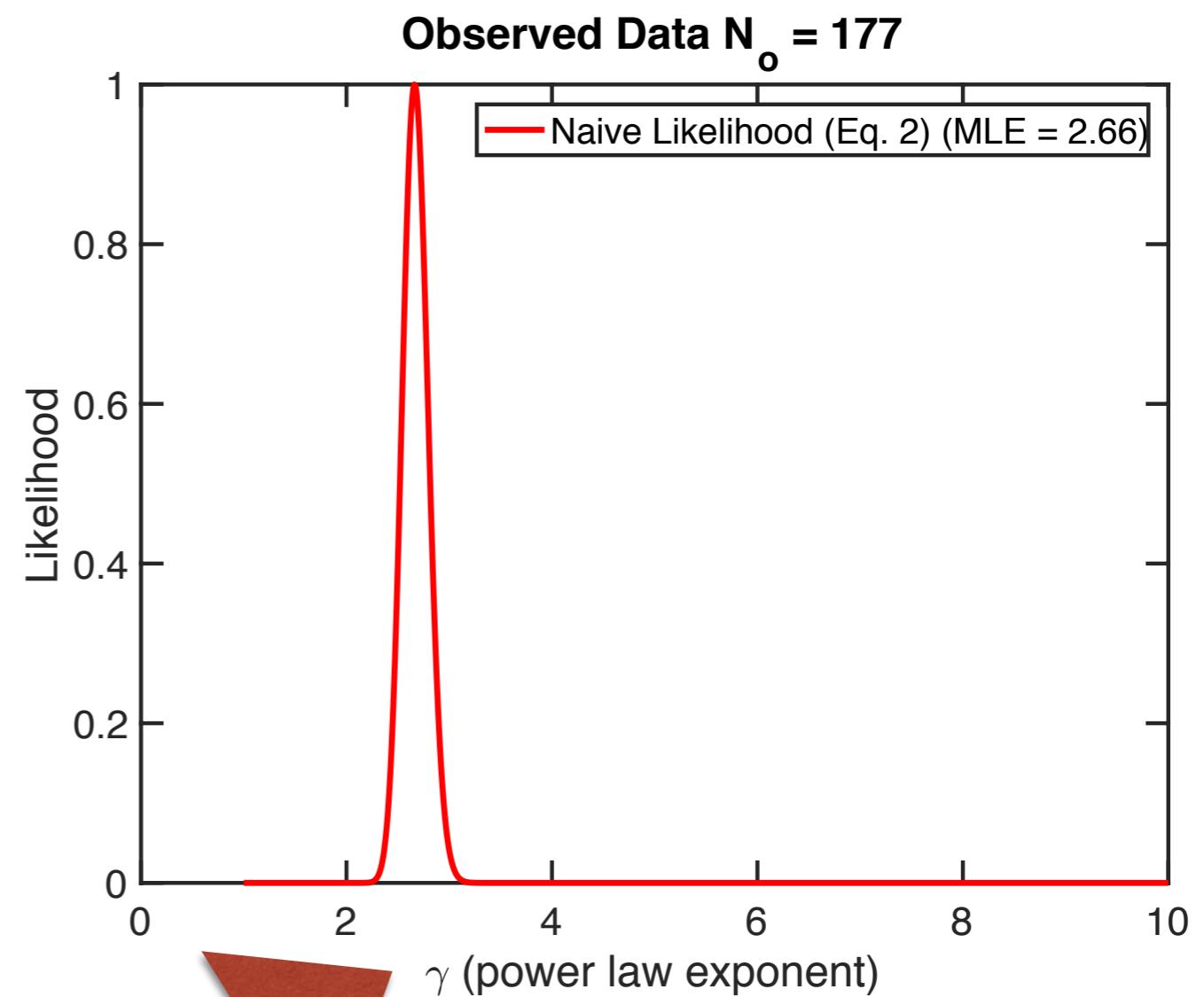
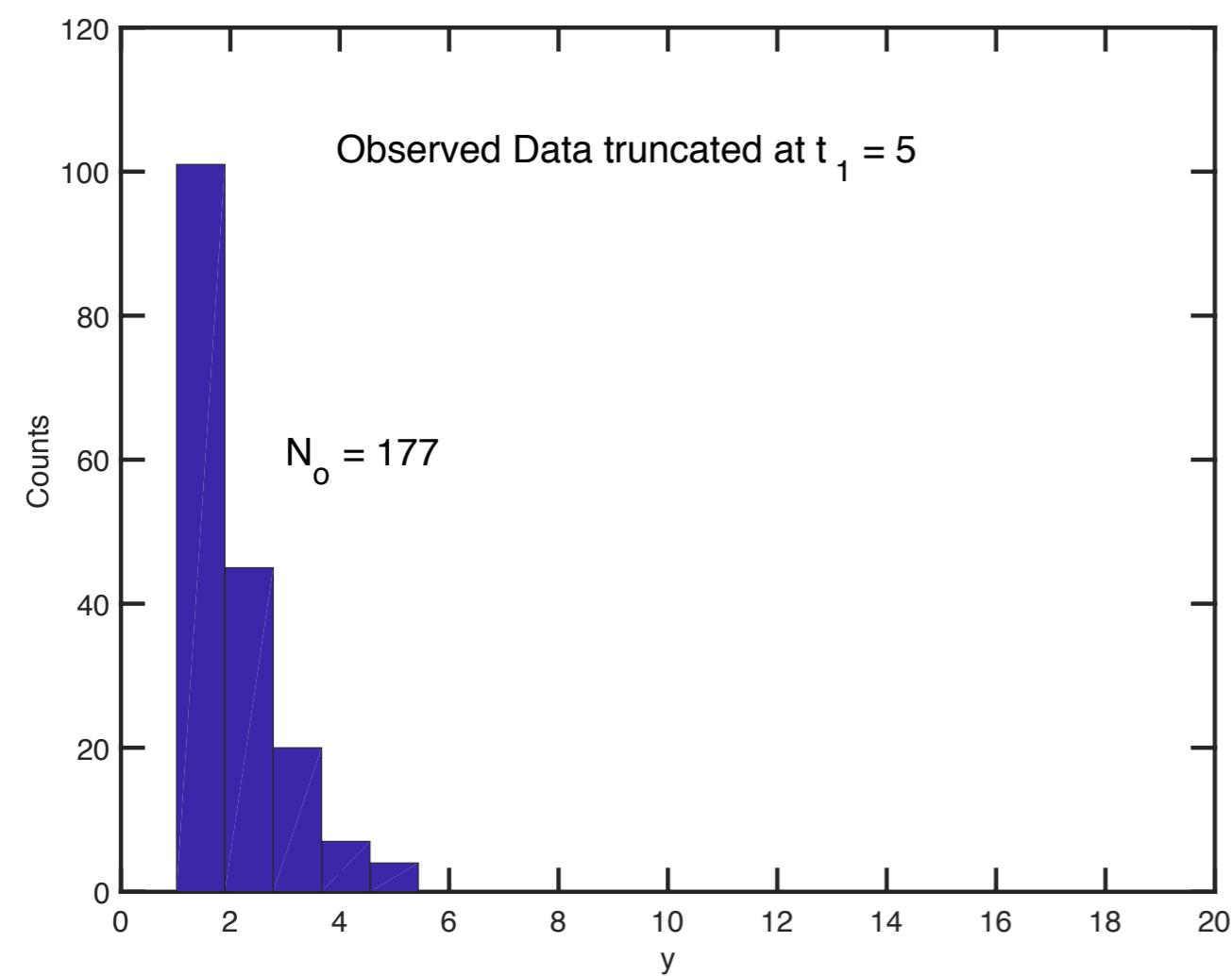
# Star Formation in Perseus



# Inference of Stellar Mass Function

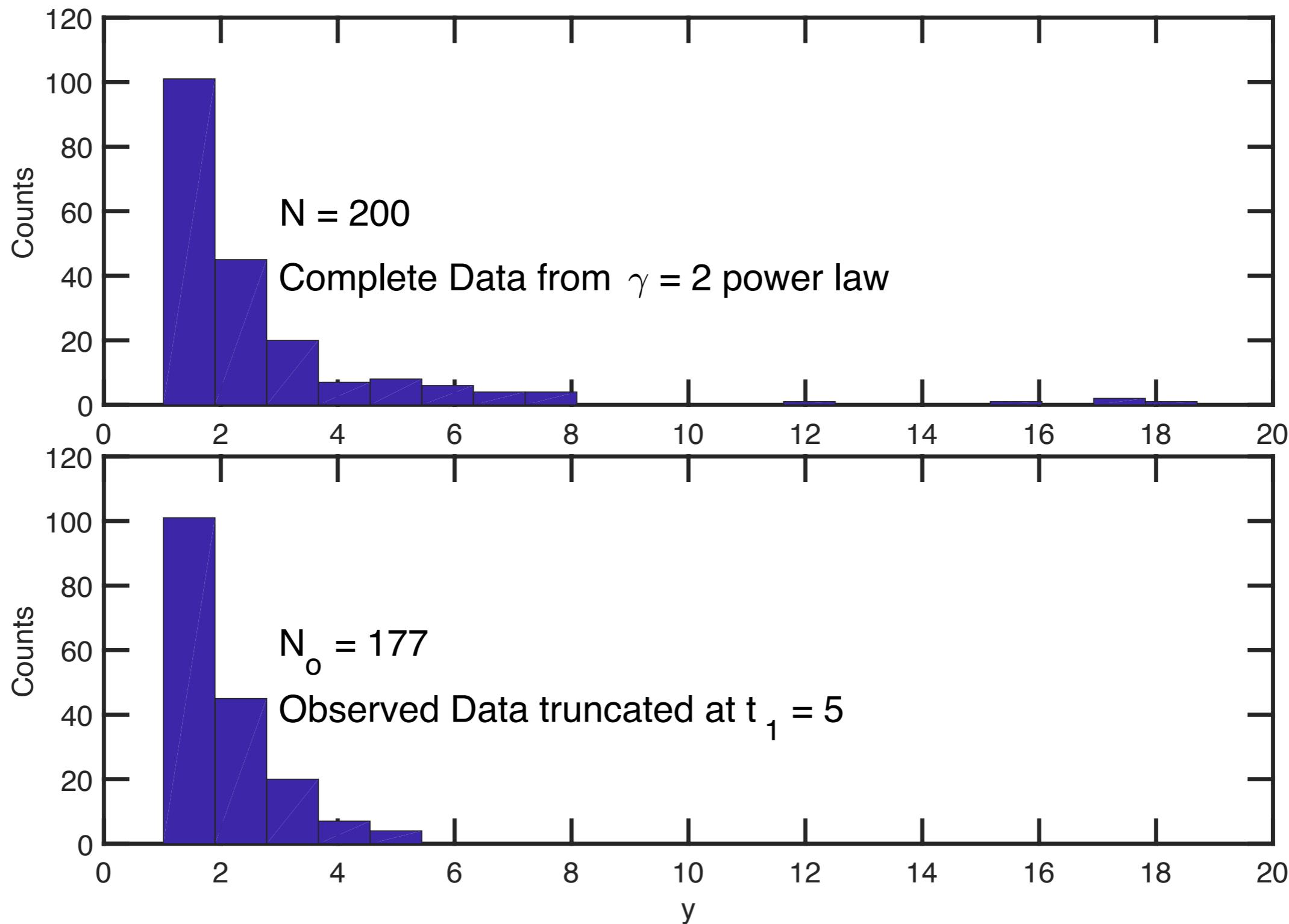
$$P(M) \propto M^{-\gamma}$$


# Selection Effect (Truncation)



MLE Biased!

# Stellar Mass Function Inference with Selection Effect: Complete Data vs. Truncated Data



# Stellar Mass Function Inference with Selection Effect: Modified Likelihood Function

