



Aidan Winblad

Background

Motivation

Results

Summary

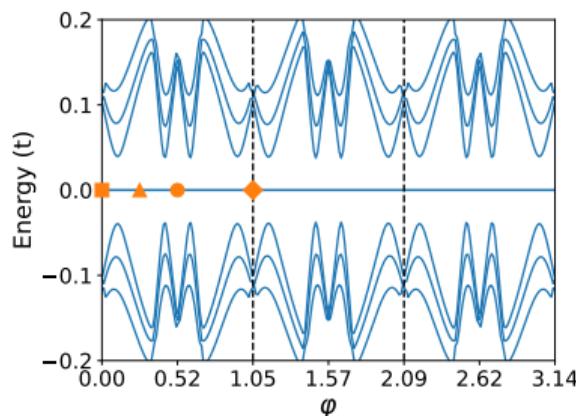
Additional
Research

Motivation

Formulation

Results

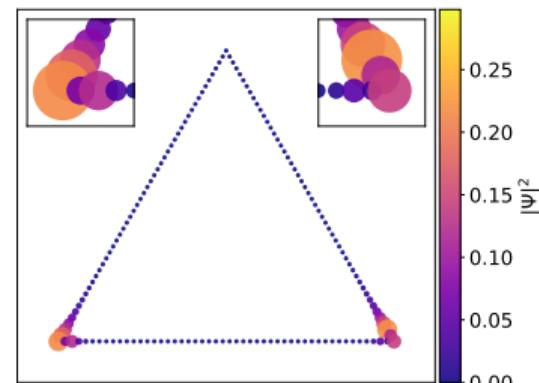
Summary



Aidan Winblad
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Department of Physics
Colorado State University

October 26, 2024





Outline

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Background

Motivation

Results

Summary

Additional
Research

Motivation

Formulation

Results

Summary

■ Background:

- Topology and gauge potentials
- Majorana fermions in particle physics and condensed matter
- Kitaev chain and Majorana number

■ Motivation:

- Braiding in a 2D p -wave SC
- T-junctions to triangular structures for braiding
- Supercurrents as gauge potential

■ Results: Two approaches

- Exactly solvable minimal model
- Hollow triangular islands & bulk-edge correspondence
- Braiding of 4 MF

■ Summary

■ Additional Research



Background

Topology

Motivation

Results

Summary

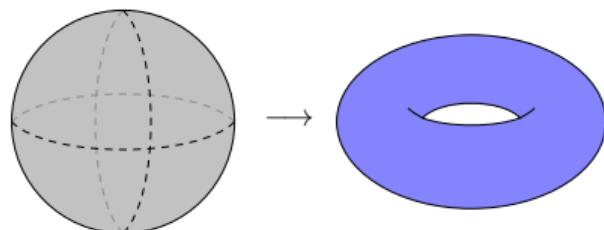
Additional
Research

Motivation

Formulation

Results

Summary





Background

Motivation

Results

Summary

Additional
Research

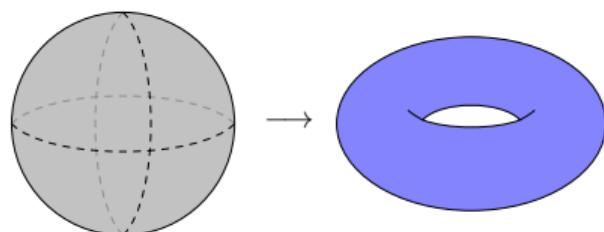
Motivation

Formulation

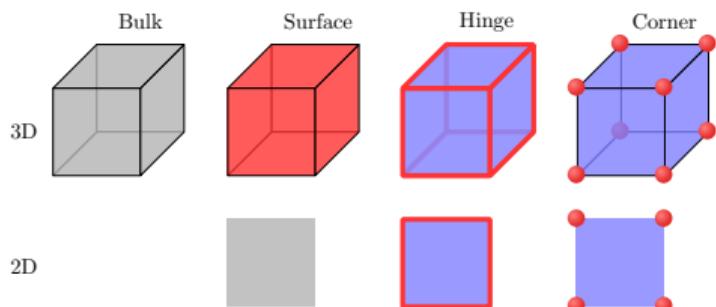
Results

Summary

Topology



Topological States





Gauge Potentials in Hamiltonians

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Minimal Coupling

$$\mathbf{p}_{\text{can}} = \mathbf{p}_{\text{kin}} + q\mathbf{A} \quad (1)$$

$$\mathcal{H} = \frac{1}{2m}(\mathbf{p}_{\text{can}} - q\mathbf{A})^2 + qV \quad (2)$$

Background

Motivation

Results

Summary

Additional
Research

Motivation

Formulation

Results

Summary



Gauge Potentials in Hamiltonians

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Background

Motivation

Results

Summary

Additional
Research

Motivation

Formulation
Results

Summary

Minimal Coupling

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$$\mathcal{H} = \frac{1}{2m}(\mathbf{p}_{\text{can}} - q\mathbf{A})^2 + qV \quad (2)$$

Peierls Phase

$$c_j^\dagger c_l \rightarrow c_j^\dagger c_l \exp \left[\frac{iq}{\hbar} \int_{\mathbf{r}_j}^{\mathbf{r}_l} \mathbf{A} \cdot d\mathbf{l} \right] = e^{i\phi_{jl}} c_j^\dagger c_l \quad (3)$$

$$\mathcal{H} = \sum_{\langle j,l \rangle} t e^{i\phi_{jl}} c_j^\dagger c_l + h.c. \quad (4)$$



Background: MFs in Particle Physics

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Background

Motivation

Results

Summary

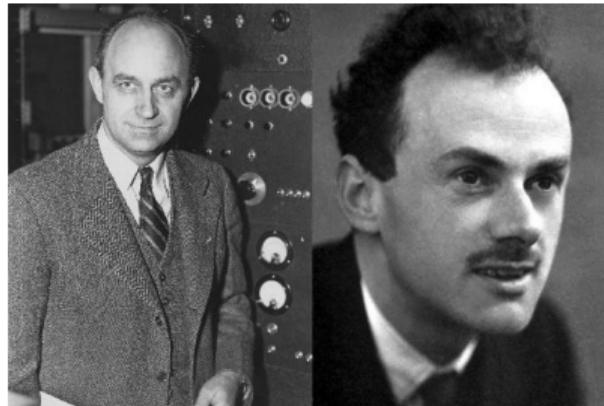
Additional
Research

Motivation

Formulation

Results

Summary



Enrico Fermi

Paul Dirac



Ettore Majorana

- Fermions

- Half-odd-integer spin
- Fermi-Dirac statistics
- Weyl fermions are massless

- Dirac Fermions

- Particle \neq Antiparticle : $c \neq c^\dagger$
- Charged

- Majorana Fermions

- Particle = Antiparticle : $c = c^\dagger$
- Neutral
- Neutrino? Dark Matter?



Background: MFs in Particle Physics

Emergent
topological
phenomena in
low-D systems
induced by gauge
potentials

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Background

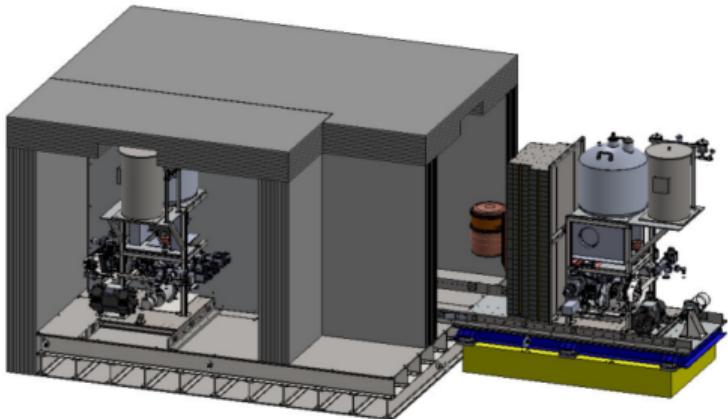
Motivation

Results

Summary

Additional
Research

Motivation
Formulation
Results
Summary



MAJORANA project:
neutrinoless double beta ($0\nu\beta\beta$) decay

- Are neutrinos Majorana fermions?
- If yes, standard model needs revision
- Negative results for Majorana particles



Background: MFs in Condensed Matter

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Background

Motivation

Results

Summary

Additional
Research

Motivation

Formulation

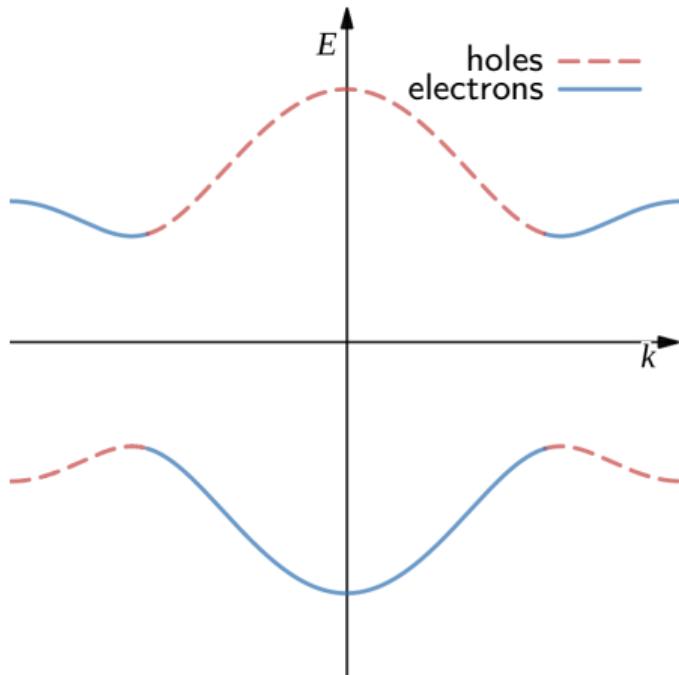
Results

Summary

- Superconductors
 - Cooper pairs
 - Electron-phonon interaction pairs two electrons with opposite spin and momenta.
 - Bogoliubov quasiparticles
 - Excitation from ground state, pairs an electron to a hole.

$$H_{BdG} = \begin{bmatrix} \epsilon(k) & \Delta(k) \\ \Delta^*(k) & -\epsilon(-k) \end{bmatrix}$$

- Zero-energy excitations may be Majorana fermions.
- If so, they come in pairs.





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Background

Motivation

Results

Summary

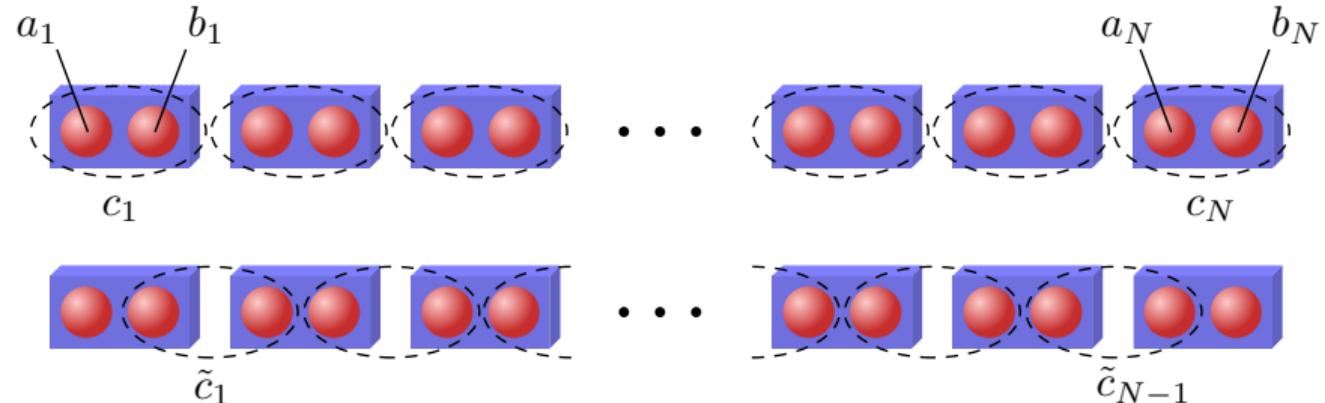
Additional
Research

Motivation

Formulation

Results

Summary



Complex fermion in Majorana fermion basis

$$c_j = \frac{1}{2}(a_j + ib_j). \quad (5)$$



Background: MFs in Condensed Matter

Hamiltonian for a 1D tight-binding chain with spinless p -wave superconductivity

$$\mathcal{H}_{chain} = -\mu \sum_j^N c_j^\dagger c_j - \sum_j^{N-1} t c_j^\dagger c_{j+1} + |\Delta| c_j c_{j+1} + h.c. \quad (6)$$

Hamiltonian in Majorana fermion basis

$$\mathcal{H}_{chain} = \frac{i}{2} \sum_j -\mu a_j b_j + (t + |\Delta|) b_j a_{j+1} + (-t + |\Delta|) a_j b_{j+1}. \quad (7)$$

$t = |\Delta| = 0$ and $\mu < 0$, trivial phase

$$\mathcal{H} = -\frac{i\mu}{2} \sum_j a_j b_j. \quad (8)$$

$t = |\Delta| > 0$ and $\mu = 0$, non-trivial (topological) phase

$$\mathcal{H} = it \sum_j b_j a_{j+1}. \quad (9)$$



Background: MFs in Condensed Matter

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potentials

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Background

Motivation

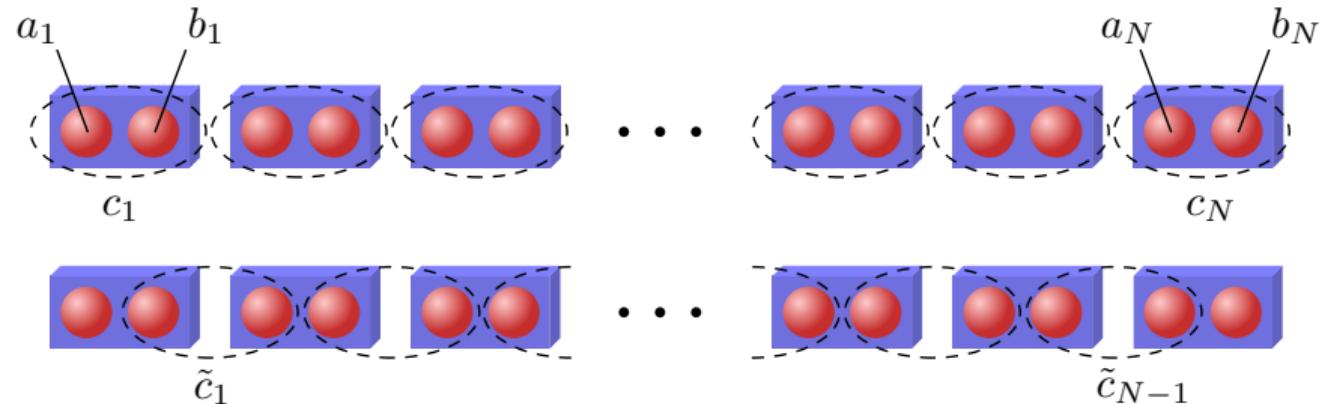
Results

Summary

Additional
Research

Motivation
Formulation

Results
Summary



Intersite fermion representation

$$\tilde{c}_j = \frac{1}{2}(a_{j+1} + ib_j). \quad (10)$$

The highly non-local fermion state

$$f = \frac{1}{2}(a_1 + ib_N), \quad (11)$$

corresponds to zero energy. This is still true for $|\mu| < 2t$.



Background: Majorana Number & Bulk-Edge Correspondence

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potentials

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Background

Motivation

Results

Summary

Additional
Research

Motivation

Formulation

Results

Summary

Majorana number a.k.a Winding Number for 1D superconductors
Write Hamiltonian in Majorana Basis

$$A = -iU\mathcal{H}U^\dagger \quad (12)$$

Majorana number in general

$$\mathcal{M} = \text{sgn}[\text{Pf}(A)] \quad (13)$$



Background: Majorana Number & Bulk-Edge Correspondence

Emergent topological phenomena in low-D systems induced by gauge potentials

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Background

Motivation

Results

Summary

Additional Research

Motivation

Formulation

Results

Summary

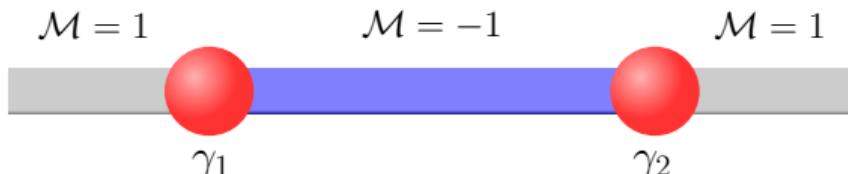
Majorana number a.k.a Winding Number for 1D superconductors
Write Hamiltonian in Majorana Basis

$$A = -iU\mathcal{H}U^\dagger \quad (12)$$

Majorana number in general

$$\mathcal{M} = \text{sgn}[\text{Pf}(A)] \quad (13)$$

- If $|\mu| < 2t$, $\mathcal{M} = -1$, non-trivial topology
- If $|\mu| > 2t$, $\mathcal{M} = 1$, trivial topology





Background: MFs in Condensed Matter

Emergent topological phenomena in low-D systems induced by gauge potentials

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Background

Motivation

Results

Summary

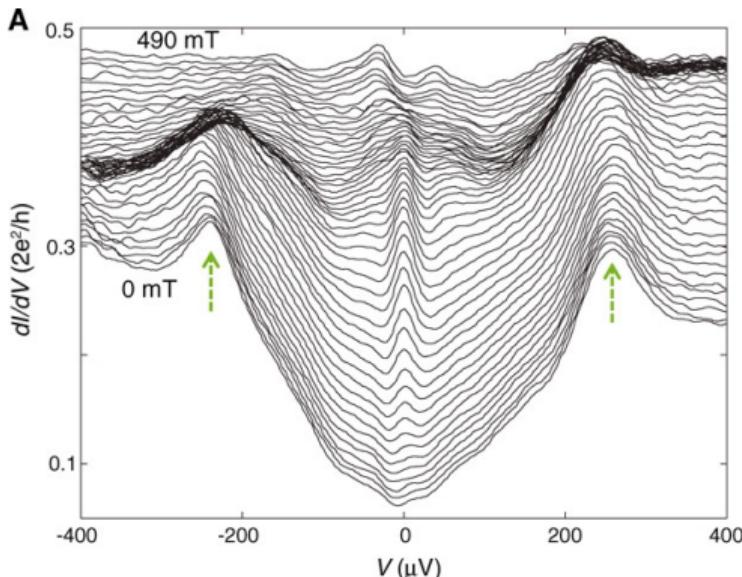
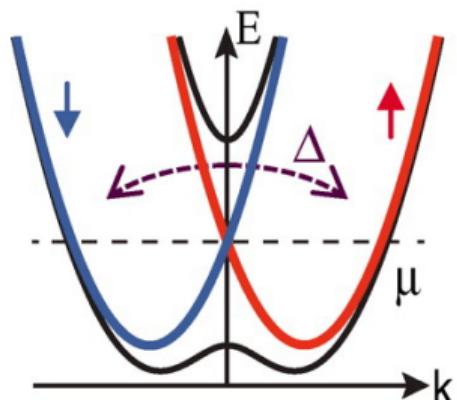
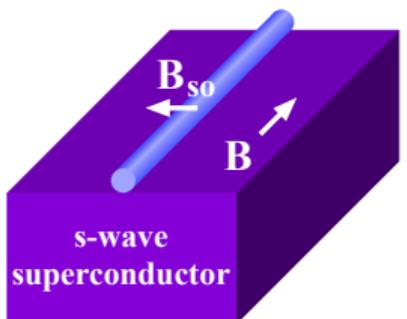
Additional Research

Motivation

Formulation

Results

Summary



Mourik et al., *Science* **336**, 1003 (2012).



Background: MFs in Condensed Matter

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Background

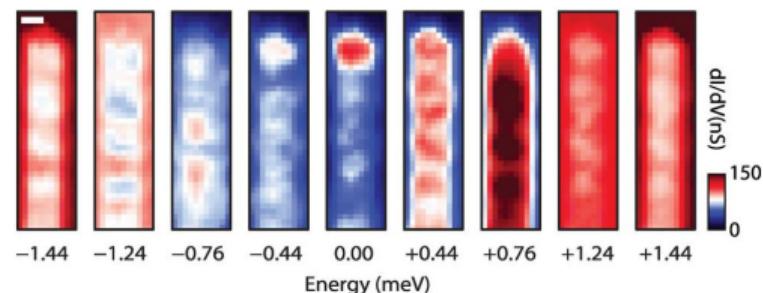
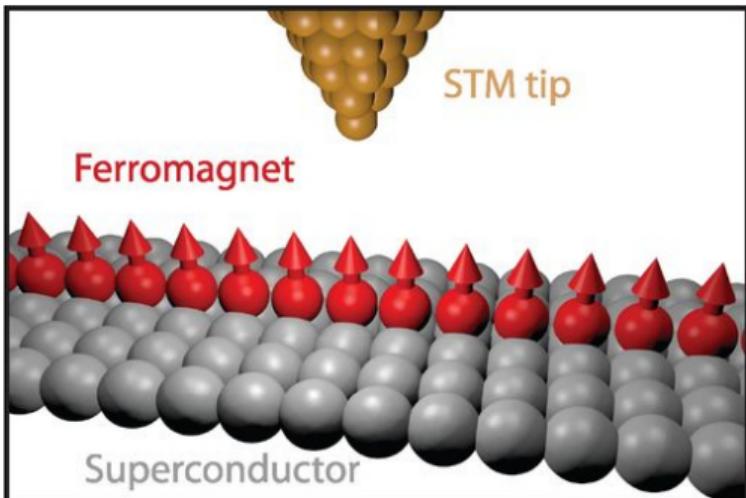
Motivation

Results

Summary

Additional
Research

Motivation
Formulation
Results
Summary



Nadj-Perge et al., *Science* **346**, 602 (2014).



Background

Motivation

Results

Summary

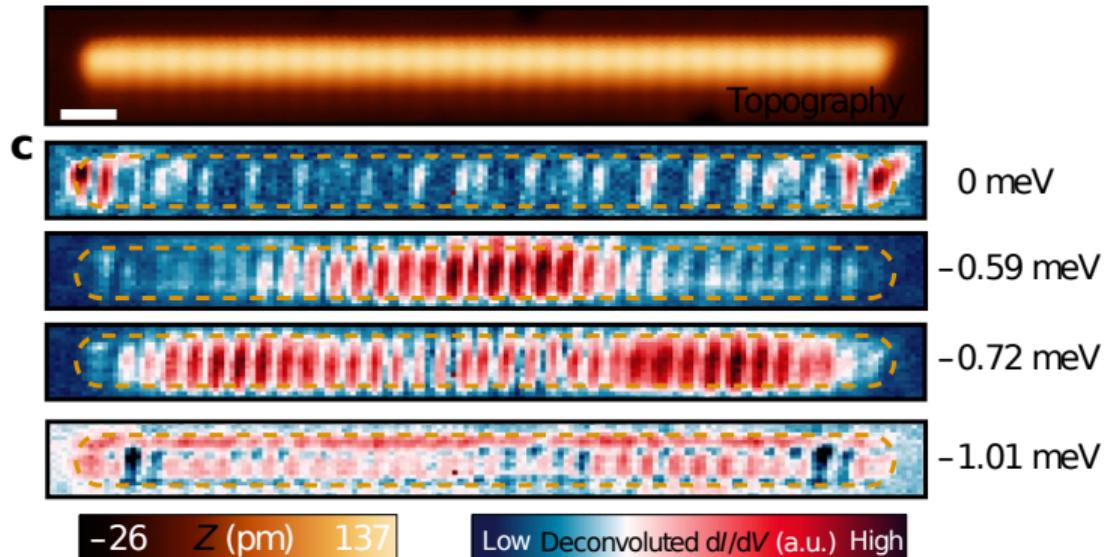
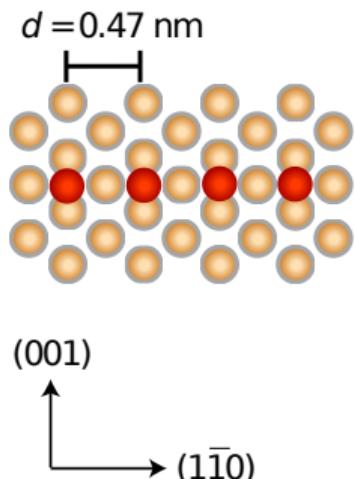
Additional
Research

Motivation
Formulation

Results
Summary

Background: MFs in Condensed Matter

a



Mn atoms (red spheres) on top of superconducting Nb (brown spheres).

Schneider et al., *Nature Nanotechnology* **17**, 384 (2022).



Background

Motivation

Results

Summary

Additional Research

Motivation Formulation Results Summary

Motivation: Braiding in a 2D p -wave SC

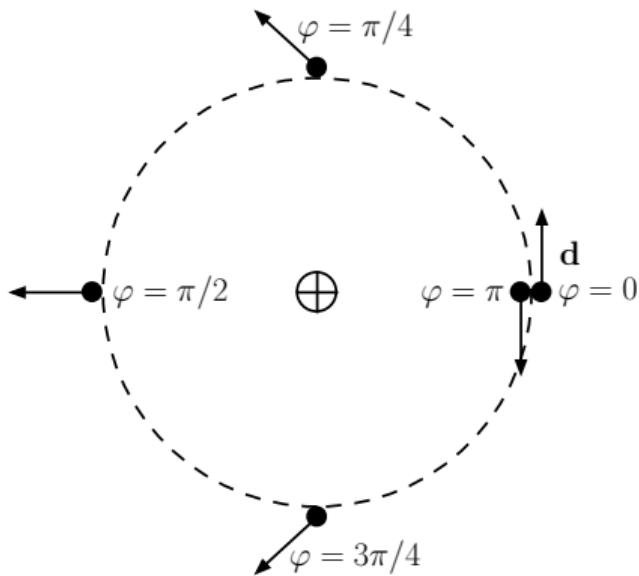
- p -wave superconductors can exhibit half-quantum vortices.

- Triplet pairing

$$\mathbf{d}(\mathbf{k}) = \Delta e^{i\phi} \langle \cos \alpha, \sin \alpha, 0 \rangle (k_x + ik_y)$$

- The order phase ϕ and angle α of \mathbf{d} rotate by π : $(\phi, \mathbf{d}) \mapsto (\phi + \pi, -\mathbf{d})$.
- The order parameter θ maps to itself, $(0, 2\pi)$, under the simultaneous change of both \mathbf{d} and ϕ : $\theta = \phi + \alpha$.

$$\mathcal{H}_\Delta = \int d^2\mathbf{r} \Delta \left[\Psi^\dagger \left[e^{i\theta} * (\partial_x + i\partial_y) \right] \Psi + h.c. \right]$$



- if overall phase shifts by θ : $\Psi_\alpha \mapsto e^{i\theta/2} \Psi_\alpha$.
- $(u, v) \mapsto (ue^{i\theta/2}, ve^{-i\theta/2})$



Motivation: Braiding in a 2D p -wave SC

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Background

Motivation

Results

Summary

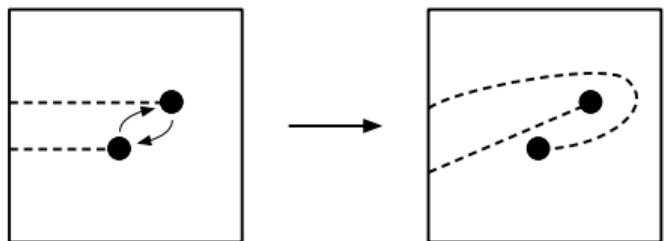
Additional
Research

Motivation

Formulation

Results

Summary

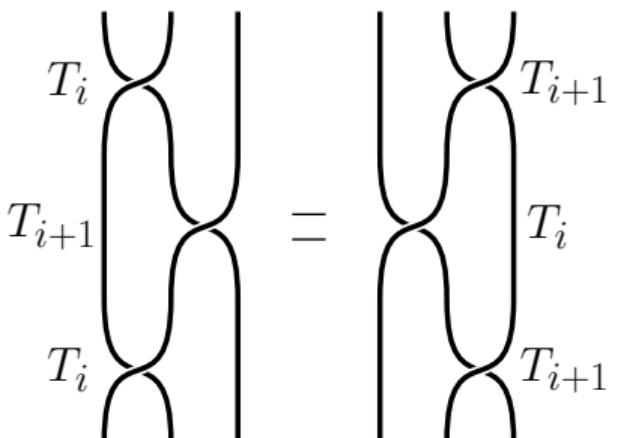


- Interchanging two MFs:

$$\gamma_1 \rightarrow \gamma_2$$

$$\gamma_2 \rightarrow -\gamma_1$$

- Exhibit Non-Abelian Statistics
- $a * b \neq b * a$



$$T_i T_j = T_j T_i$$

$$T_i T_{i+1} T_i = T_{i+1} T_i T_{i+1}$$

Ivanov, PRL **86**, 268 (2001).



Motivation: T-junction as a Quantum Logic Gate

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Background

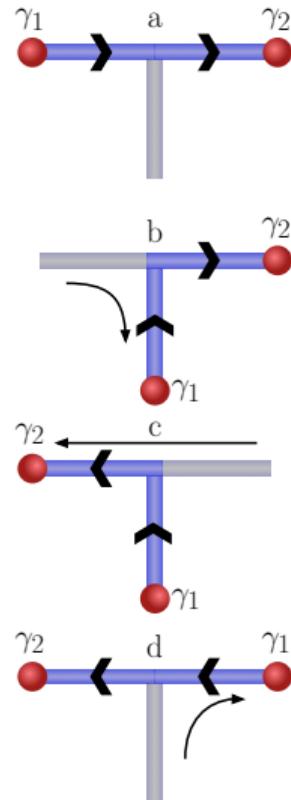
Motivation

Results

Summary

Additional
Research

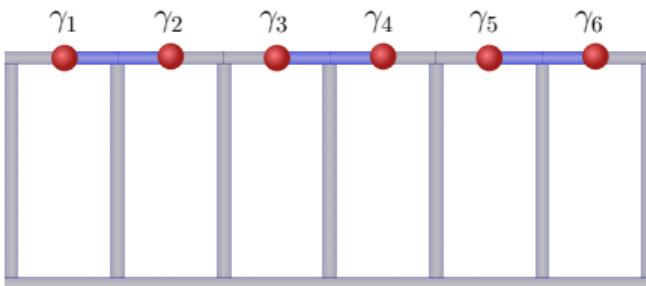
Motivation
Formulation
Results
Summary



$$\mathcal{H}_T = -\mu \sum_j c_j^\dagger c_j - \sum_j t c_j^\dagger c_{j+1} + |\Delta| e^{i\phi} c_j c_{j+1} + h.c. \quad (14)$$

$$c_j = e^{-i(\phi/2)} (\gamma_{j+1,1} + i\gamma_{j,2})/2 \quad (15)$$

- Take pairing term $|\Delta| e^{i\phi} c_j c_{j+1}$ such that the site indices:
- Increase moving \rightarrow / \uparrow in the horizontal/vertical wires: $\phi = 0$,
- Decrease moving \leftarrow / \downarrow in the horizontal/vertical wires: $\phi = \pi$.





Motivation: Triangular Structures for Braiding

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Background

Motivation

Results

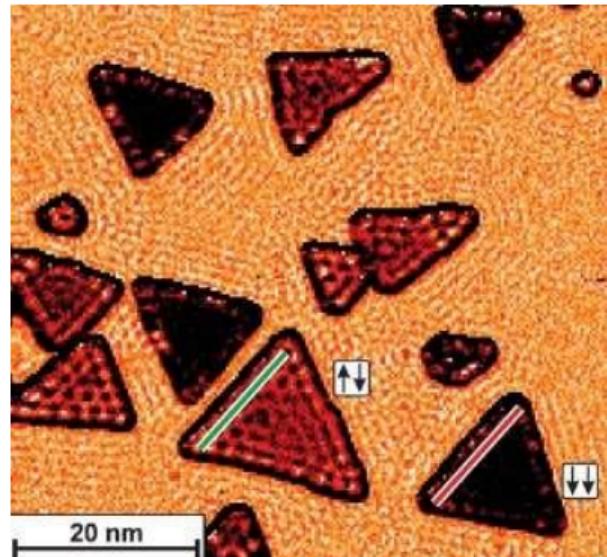
Summary

Additional
Research

Motivation
Formulation

Results
Summary

- Consider triangular islands, topologically similar to T-junctions.
- Islands of three-fold rotational symmetry occur naturally in epitaxial growth on close-packed metal surfaces.
- Make a smooth connection from 1D to 2D superconductors.



Triangular Co islands on Cu(111).
Pietzsch et al., *PRL* **96**, 237203 (2006)



Topological phase transition induced by a supercurrent

Emergent topological phenomena in low-D systems induced by gauge potentials

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Background

Motivation

Results

Summary

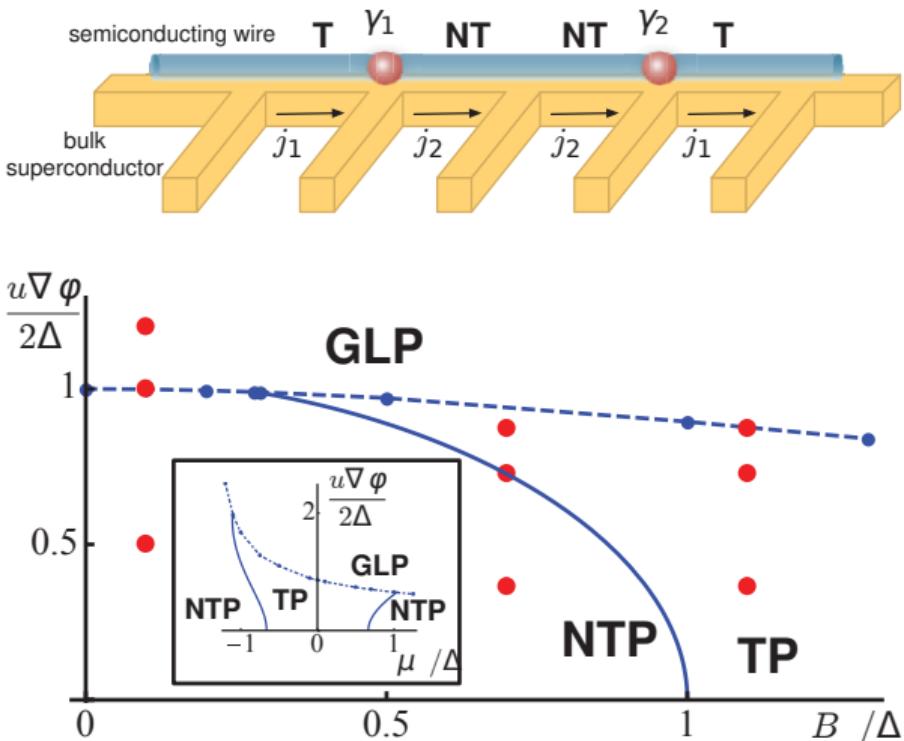
Additional Research

Motivation

Formulation

Results

Summary



Romito et al., PRB 85, 020502(R) (2012).



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Aidan Winblad

Background

Motivation

Results

Summary

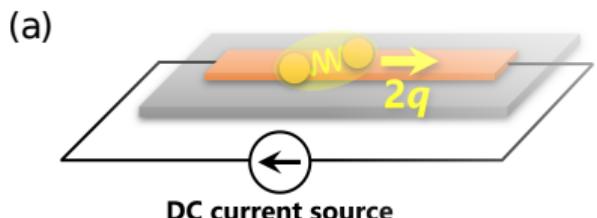
Additional Research

Motivation

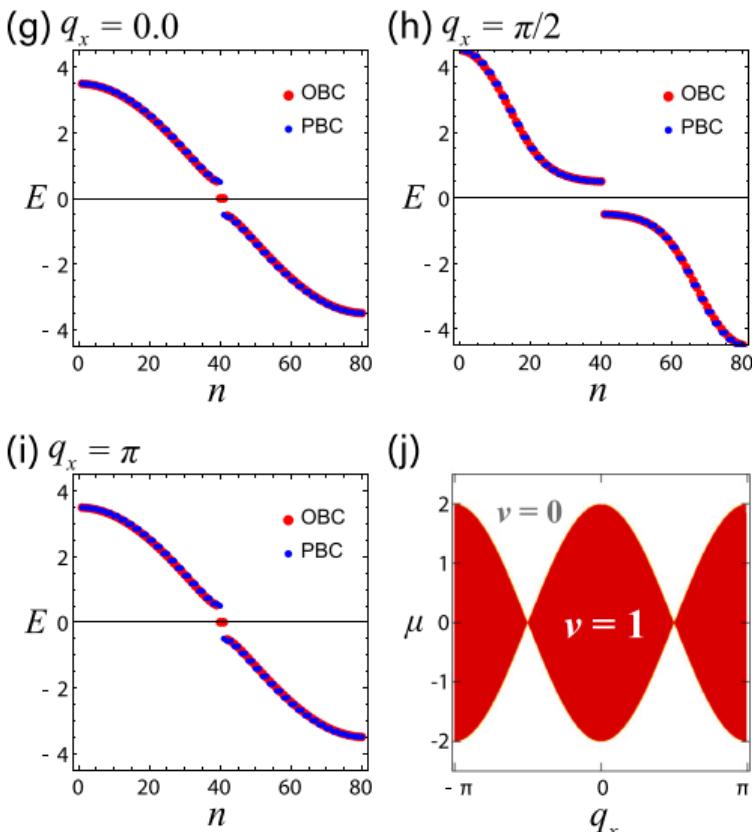
Formulation

Results

Summary



Takasan et al., *PRB* **106**, 014508 (2022).





Two Proposals

Emergent
topological
phenomena in
low-D systems
induced by gauge
potentials

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Background

Motivation

Results

Summary

Additional
Research

Motivation

Formulation

Results

Summary

- Exactly solvable “Kitaev Triangle”
 - Three fermion sites
 - Three edges controlled by Peierls phases



Two Proposals

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topological
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potentials

Aidan Winblad

Background

Motivation

Results

Summary

Additional
Research

Motivation

Formulation

Results

Summary

- Exactly solvable “Kitaev Triangle”
 - Three fermion sites
 - Three edges controlled by Peierls phases
- Finite-size triangle with hollow interior
 - Under uniform vector potential
 - Bulk-edge correspondence



Kitaev Triangle

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Background

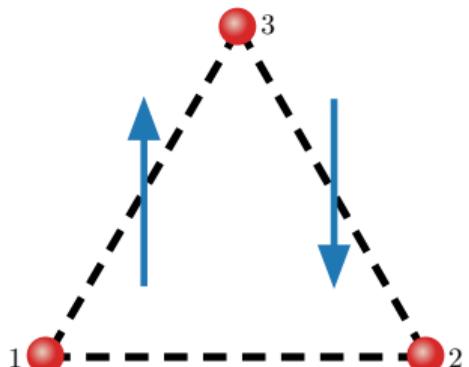
Motivation

Results

Summary

Additional
Research

Motivation
Formulation
Results
Summary



$$\mathbf{A} = [1 - 2\Theta(x)] \frac{2\pi}{3\sqrt{3}} \hat{\mathbf{y}} \quad (16)$$

$$\begin{aligned} c_j^\dagger c_l &\rightarrow c_j^\dagger c_l \exp\left(\frac{ie}{\hbar} \int_{r_j}^{r_l} \mathbf{A} \cdot d\mathbf{l}\right) \\ &\rightarrow c_j^\dagger c_l e^{i\phi_{jl}} \end{aligned} \quad (17)$$

$$\mathcal{H} = \sum_{\langle j, l \rangle} (-te^{i\phi_{jl}} c_j^\dagger c_l + \Delta c_j^\dagger c_l^\dagger + h.c.) - \mu c_j^\dagger c_j \quad (18)$$

In Kitaev limit, $t = \Delta \neq 0$ and $\mu = 0$,

$$(\phi_{12}, \phi_{23}, \phi_{31}) = (0, -\frac{\pi}{3}, -\frac{\pi}{3}) = \boldsymbol{\phi}_1 \quad (19)$$

MFs localized at sites 1 and 2



Background

Motivation

Results

Summary

Additional
Research

Motivation

Formulation

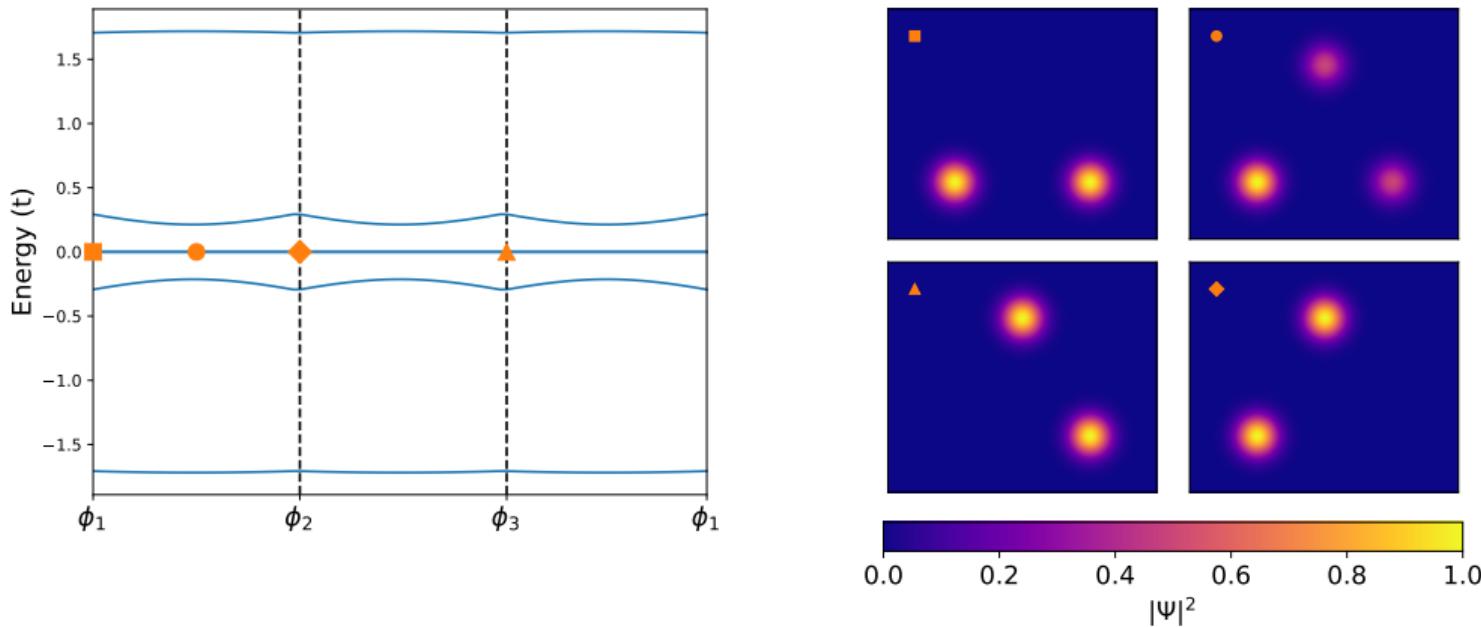
Results

Summary

Kitaev Triangle Braiding

A closed parameter path linearly interpolating between the following sets of ϕ_{jk} :

$$(\phi_{12}, \phi_{23}, \phi_{31}) : \phi_1 \rightarrow \phi_2 \rightarrow \phi_3 \rightarrow \phi_1 \quad (20)$$





Triangular Ribbon and Topological Phases

Emergent topological phenomena in low-D systems induced by gauge potentials

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Background

Motivation

Results

Summary

Additional Research

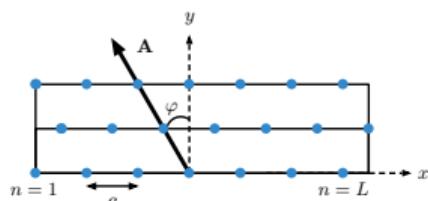
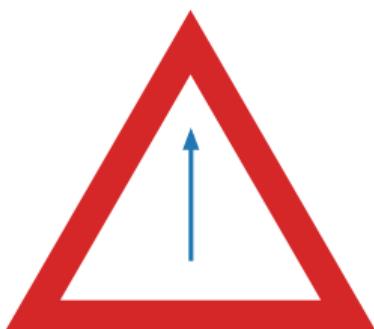
Motivation

Formulation

Results

Summary

$$\phi_{jl} = \frac{e}{\hbar} \int_{\mathbf{r}_j}^{\mathbf{r}_l} \mathbf{A} \cdot d\mathbf{l} = \mathbf{A} \cdot \mathbf{r}_{jl} = -\phi_{lj}$$





Triangular Ribbon and Topological Phases

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Aidan Winblad

Background

Motivation

Results

Summary

Additional Research

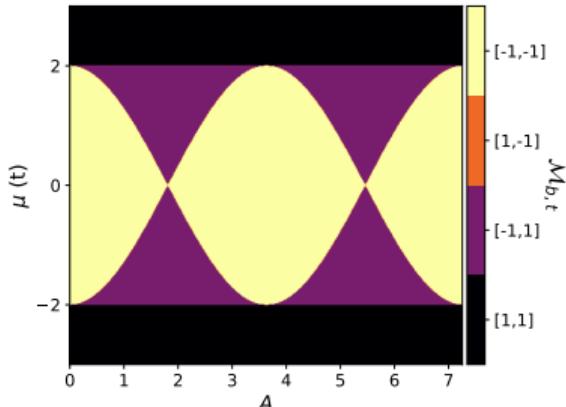
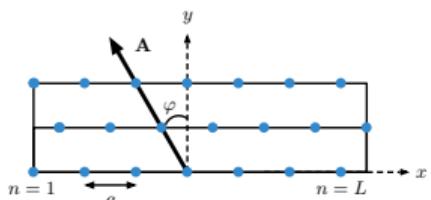
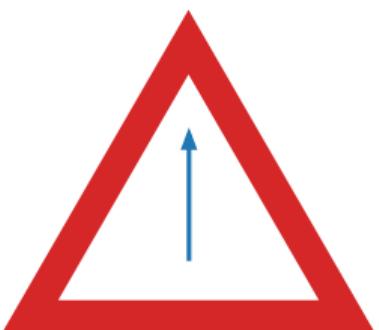
Motivation

Formulation

Results

Summary

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Background

Motivation

Results

Summary

Additional Research

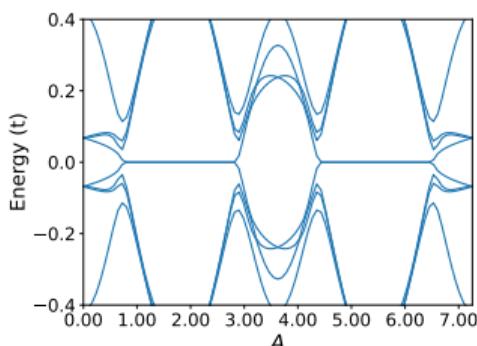
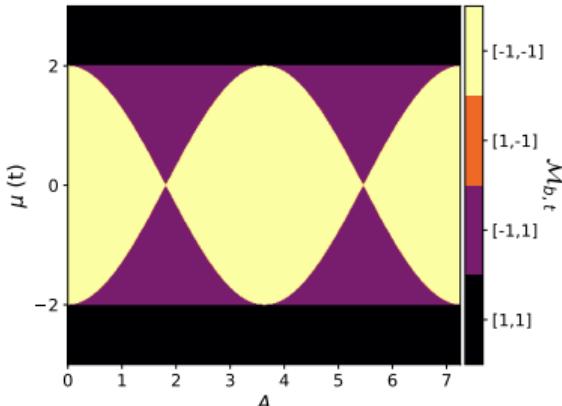
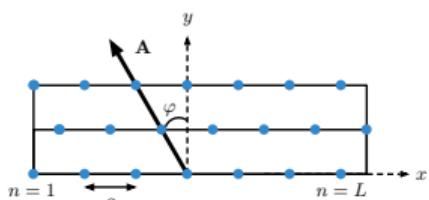
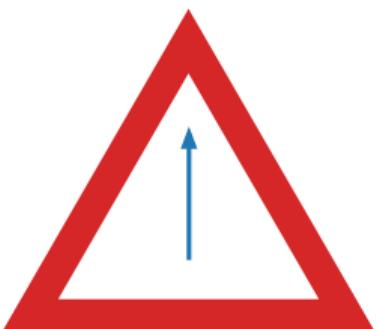
Motivation

Formulation

Results

Summary

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Rotating MFs on a Triangular Chain ($W=1$)

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Background

Motivation

Results

Summary

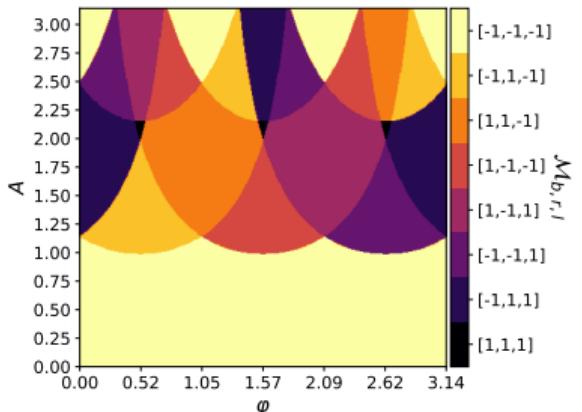
Additional
Research

Motivation

Formulation

Results

Summary





Rotating MFs on a Triangular Chain ($W=1$)

Emergent
topological
phenomena in
low-D systems
induced by gauge
potentials

Aidan Winblad

Background

Motivation

Results

Summary

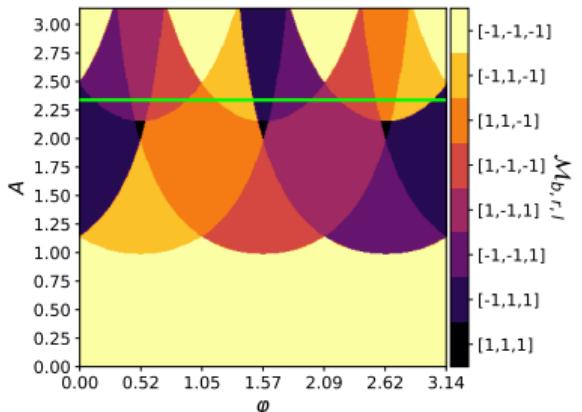
Additional
Research

Motivation

Formulation

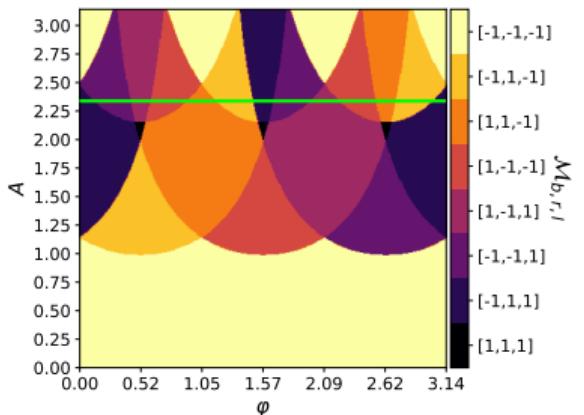
Results

Summary

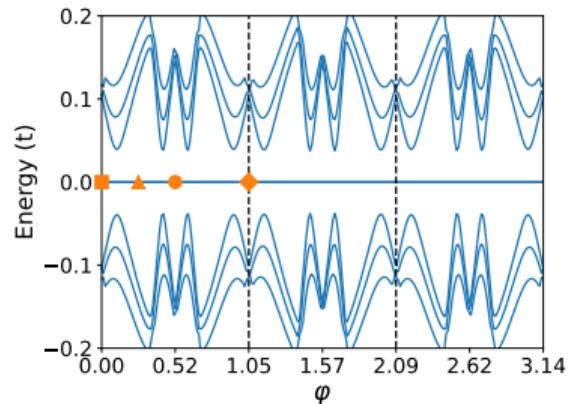




Rotating MFs on a Triangular Chain ($W=1$)



$$L = 50, W = 1, \mu = 1.1, A = 2.35$$





Background

Motivation

Results

Summary

Additional Research

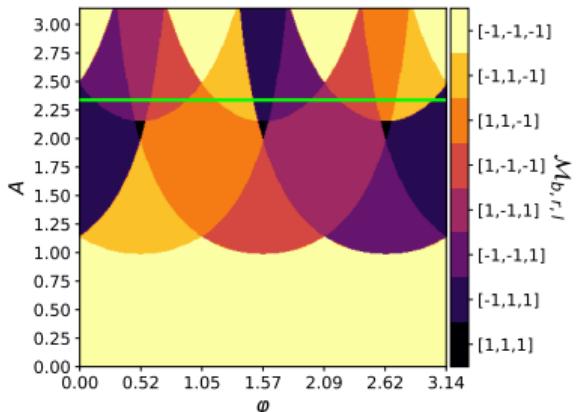
Motivation

Formulation

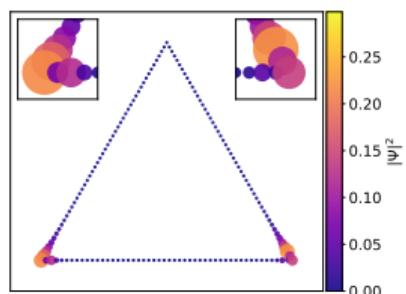
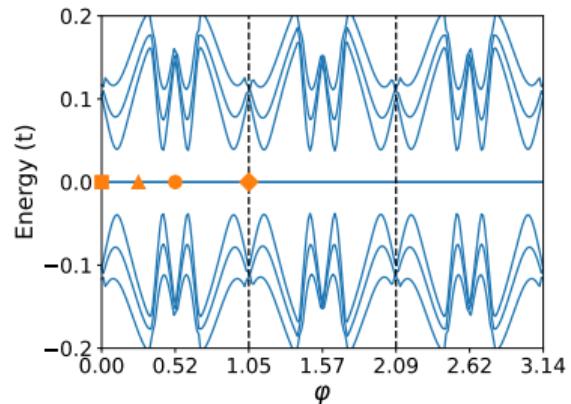
Results

Summary

Rotating MFs on a Triangular Chain ($W=1$)



$L = 50, W = 1, \mu = 1.1, A = 2.35$





Rotating MFs on a Triangular Chain ($W=1$)

Emergent topological phenomena in low-D systems induced by gauge potentials

Aidan Winblad

Background

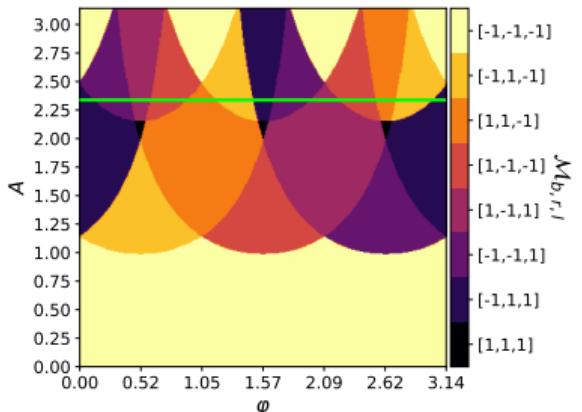
Motivation

Results

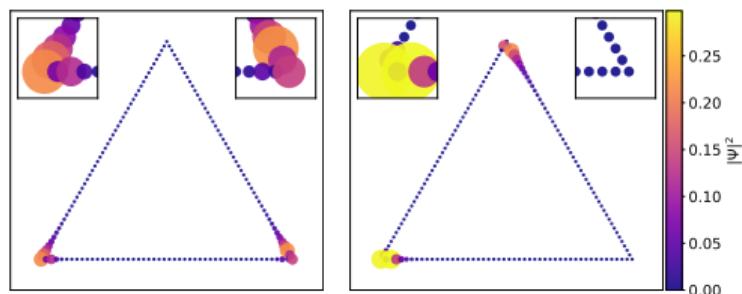
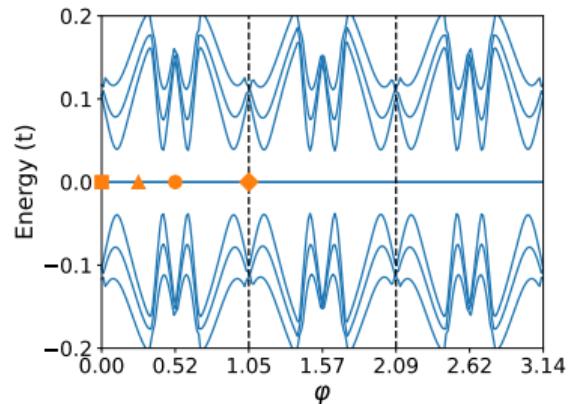
Summary

Additional Research

Motivation
Formulation
Results
Summary



$$L = 50, W = 1, \mu = 1.1, A = 2.35$$





Rotating MFs on a Triangular Chain ($W=1$)

Emergent topological phenomena in low-D systems induced by gauge potentials

Aidan Winblad

Background

Motivation

Results

Summary

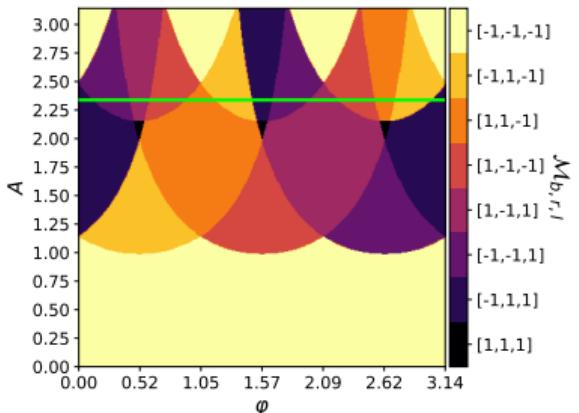
Additional Research

Motivation

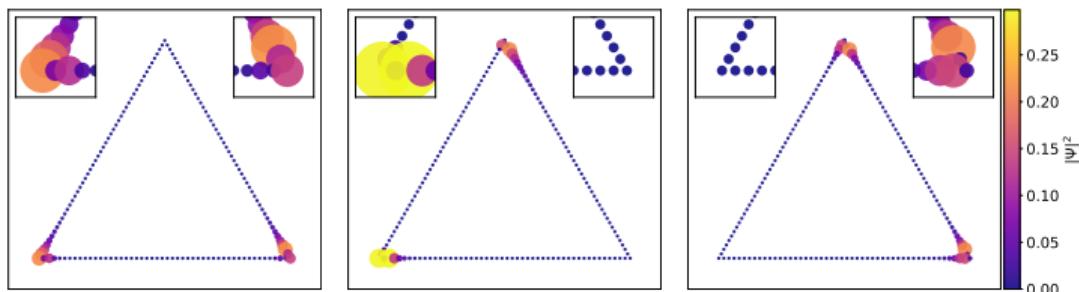
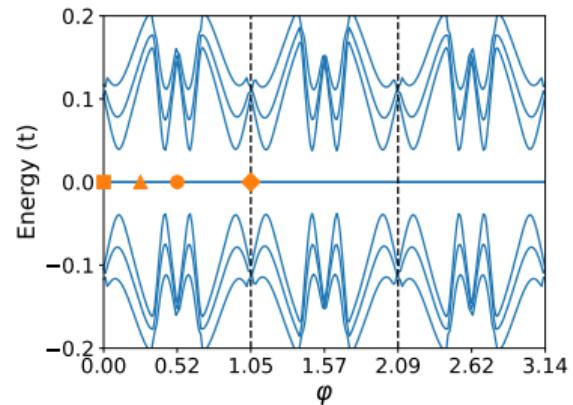
Formulation

Results

Summary



$$L = 50, W = 1, \mu = 1.1, A = 2.35$$





Rotating MFs on a Triangular Chain ($W=1$)

Emergent topological phenomena in low-D systems induced by gauge potentials

Aidan Winblad

Background

Motivation

Results

Summary

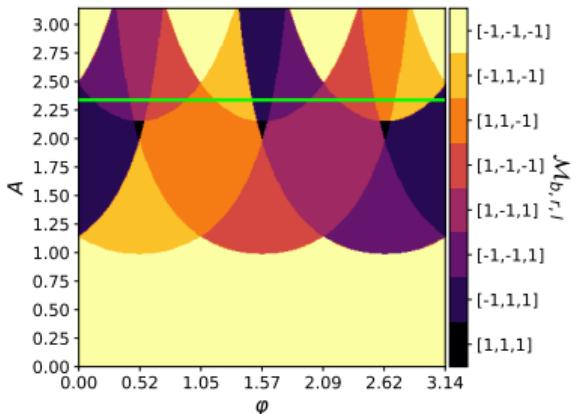
Additional Research

Motivation

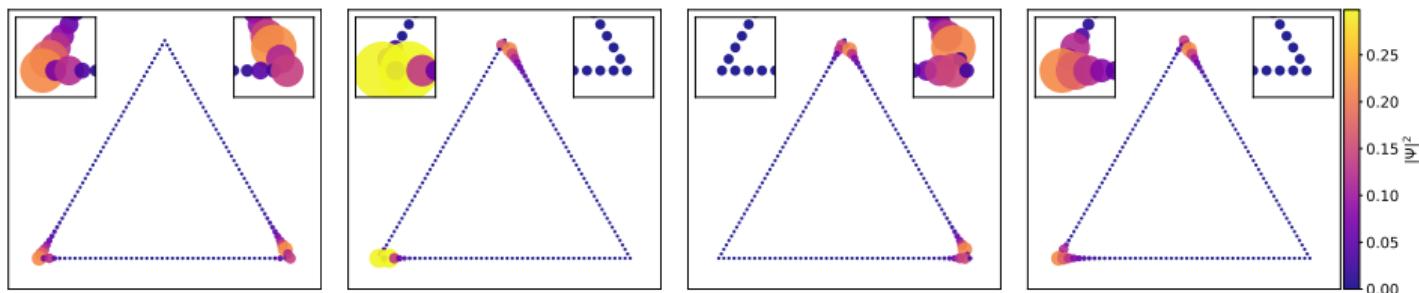
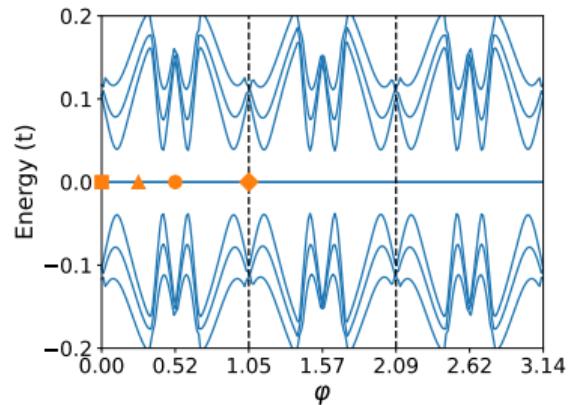
Formulation

Results

Summary



$$L = 50, W = 1, \mu = 1.1, A = 2.35$$





Rotating MFs on a Hollow Triangle ($W=3$)

Emergent
topological
phenomena in
low-D systems
induced by gauge
potentials

Aidan Winblad

Background

Motivation

Results

Summary

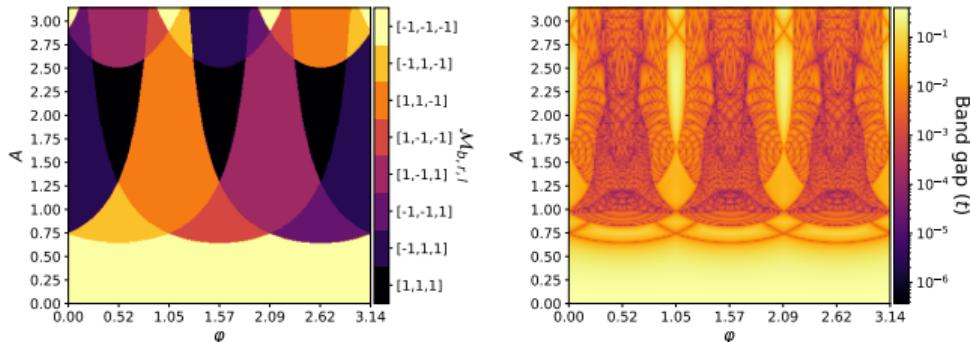
Additional
Research

Motivation

Formulation

Results

Summary





Rotating MFs on a Hollow Triangle ($W=3$)

Emergent
topological
phenomena in
low-D systems
induced by gauge
potentials

Aidan Winblad

Background

Motivation

Results

Summary

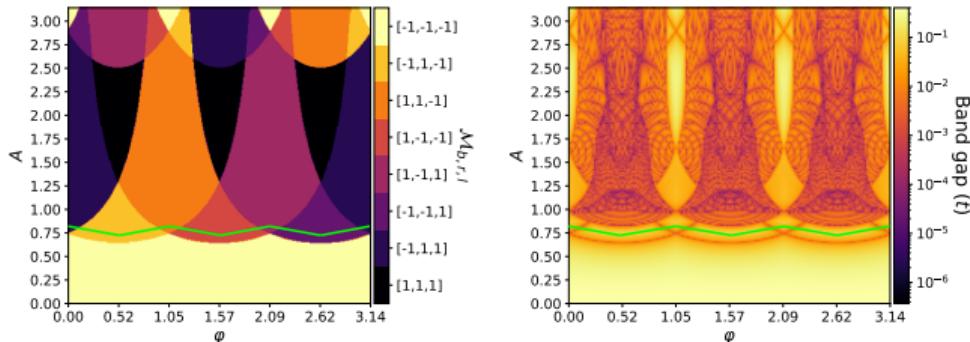
Additional
Research

Motivation

Formulation

Results

Summary





Rotating MFs on a Hollow Triangle ($W=3$)

Aidan Winblad

Background

Motivation

Results

Summary

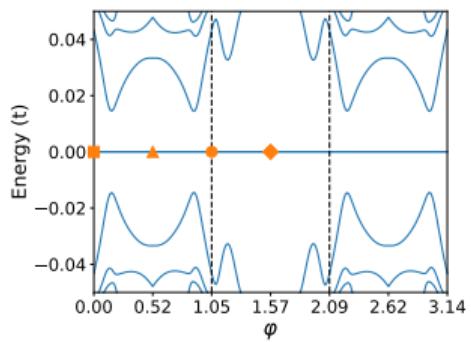
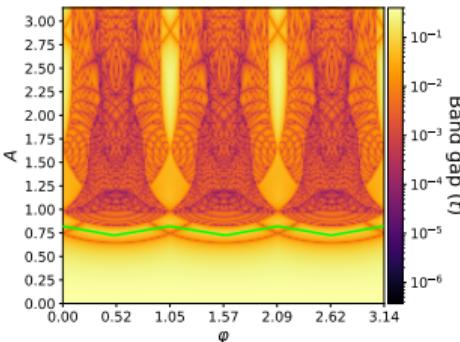
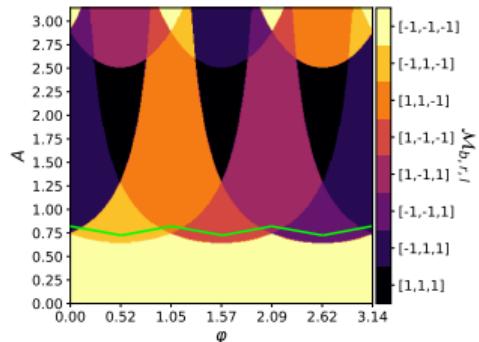
Additional
Research

Motivation

Formulation

Results

Summary



$$L = 80, W = 3, \mu = 1.6, (A, \varphi) = (0.83, 0) \rightarrow (0.77, \frac{\pi}{6}) \rightarrow (0.83, \frac{\pi}{3}) \rightarrow (0.77, \frac{\pi}{2}) \dots$$



Rotating MFs on a Hollow Triangle ($W=3$)

Aidan Winblad

Background

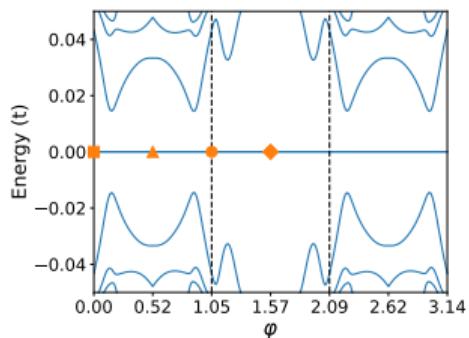
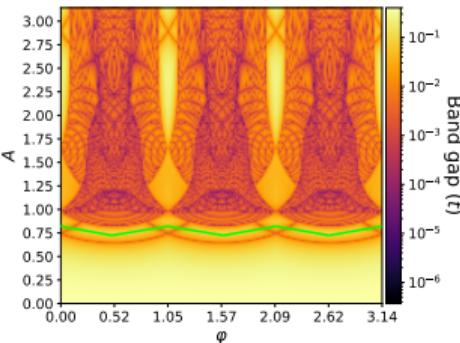
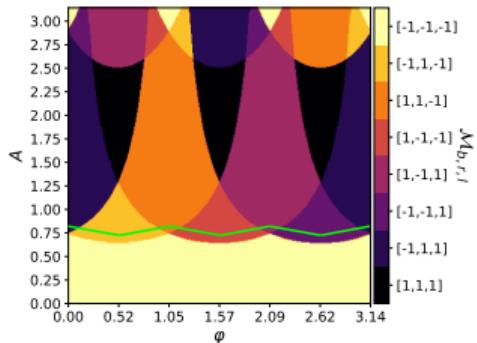
Motivation

Results

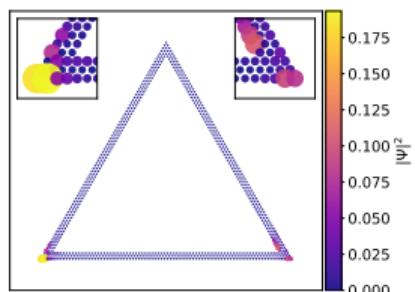
Summary

Additional
Research

Motivation
Formulation
Results
Summary



$$L = 80, W = 3, \mu = 1.6, (A, \varphi) = (0.83, 0) \rightarrow (0.77, \frac{\pi}{6}) \rightarrow (0.83, \frac{\pi}{3}) \rightarrow (0.77, \frac{\pi}{2}) \dots$$





Rotating MFs on a Hollow Triangle ($W=3$)

Emergent topological phenomena in low-D systems induced by gauge potentials

Aidan Winblad

Background

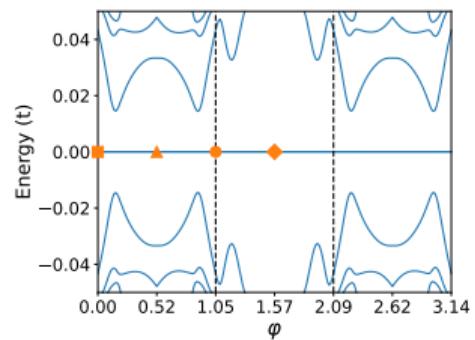
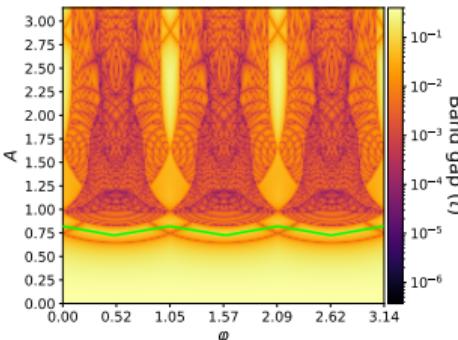
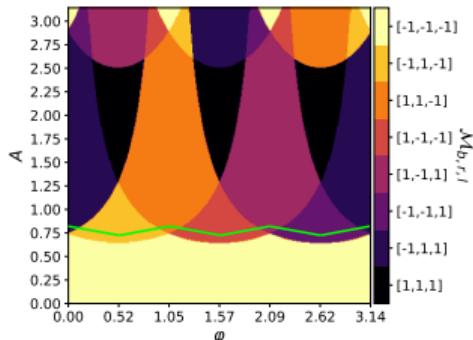
Motivation

Results

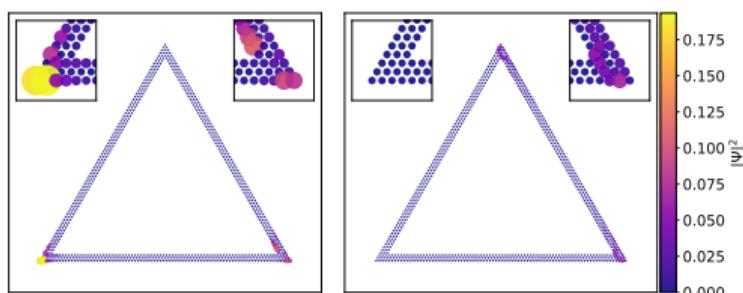
Summary

Additional Research

Motivation
Formulation
Results
Summary



$$L = 80, W = 3, \mu = 1.6, (A, \varphi) = (0.83, 0) \rightarrow (0.77, \frac{\pi}{6}) \rightarrow (0.83, \frac{\pi}{3}) \rightarrow (0.77, \frac{\pi}{2}) \dots$$





Rotating MFs on a Hollow Triangle ($W=3$)

Aidan Winblad

Background

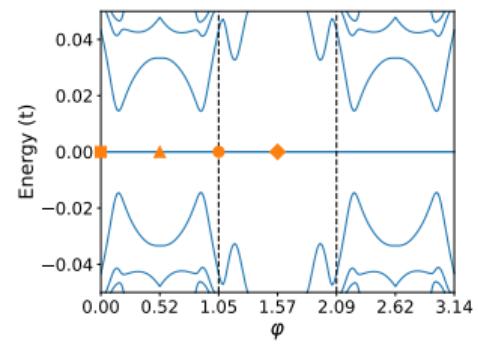
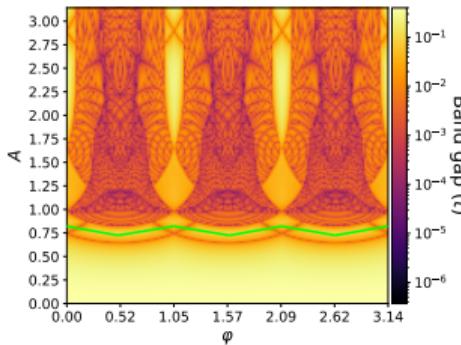
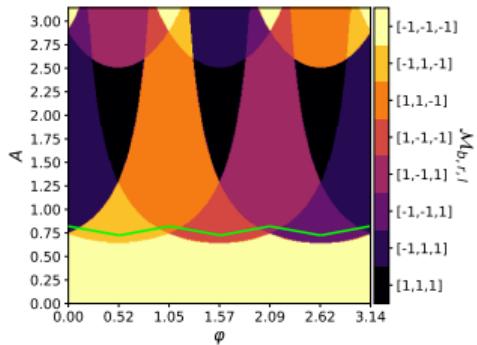
Motivation

Results

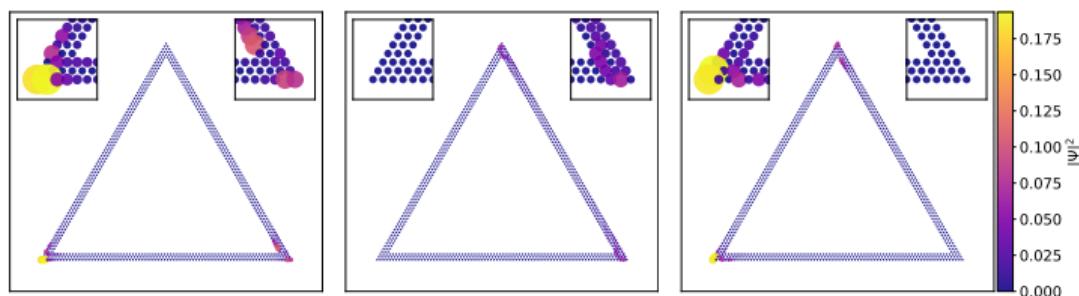
Summary

Additional
Research

Motivation
Formulation
Results
Summary



$$L = 80, W = 3, \mu = 1.6, (A, \varphi) = (0.83, 0) \rightarrow (0.77, \frac{\pi}{6}) \rightarrow (0.83, \frac{\pi}{3}) \rightarrow (0.77, \frac{\pi}{2}) \dots$$





Rotating MFs on a Hollow Triangle ($W=3$)

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Background

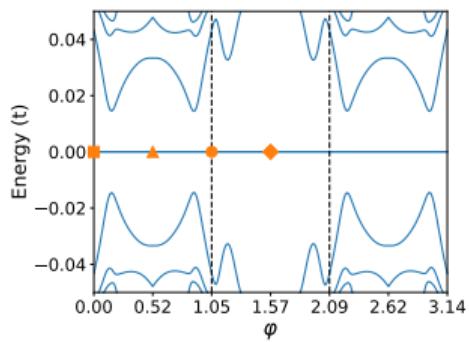
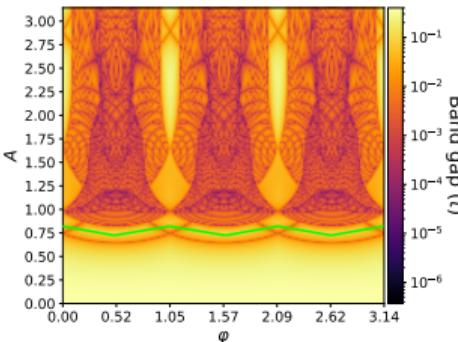
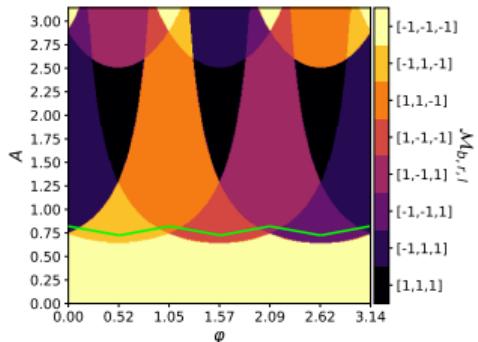
Motivation

Results

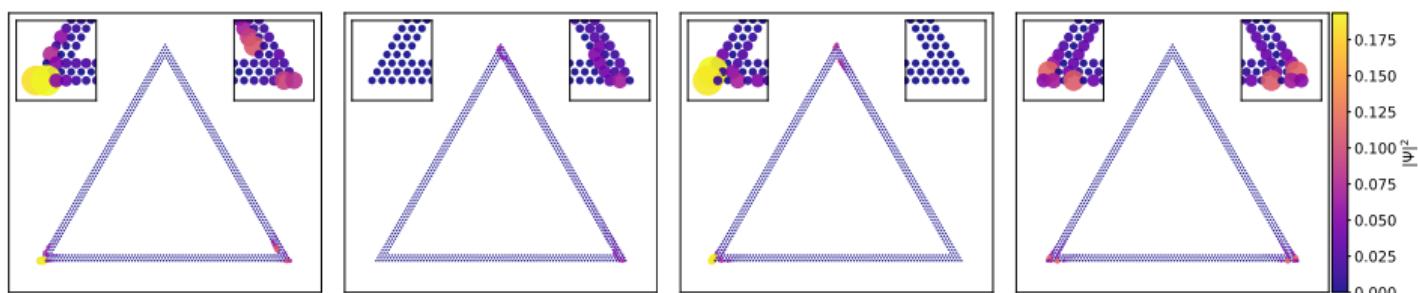
Summary

Additional
Research

Motivation
Formulation
Results
Summary



$$L = 80, W = 3, \mu = 1.6, (A, \varphi) = (0.83, 0) \rightarrow (0.77, \frac{\pi}{6}) \rightarrow (0.83, \frac{\pi}{3}) \rightarrow (0.77, \frac{\pi}{2}) \dots$$





Braiding Two of Four MFs

Aidan Winblad

Background

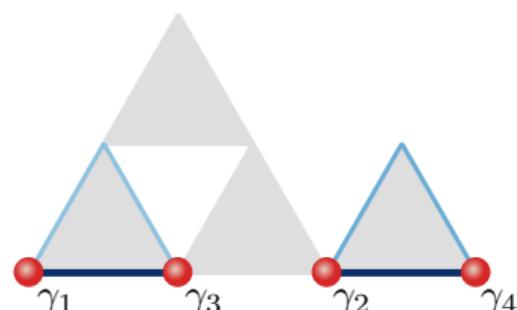
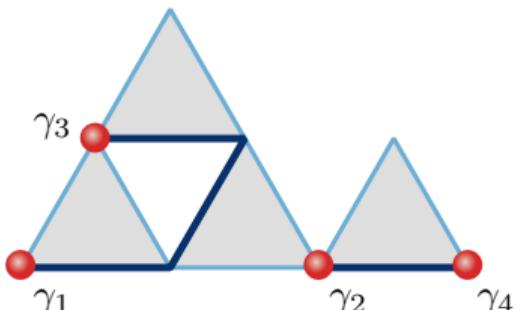
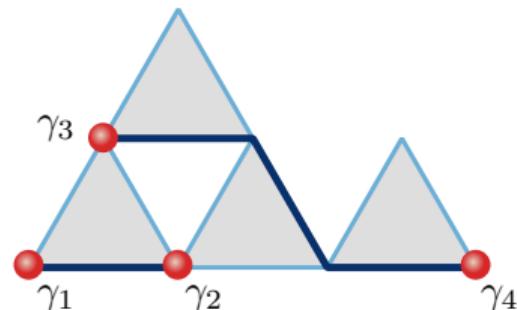
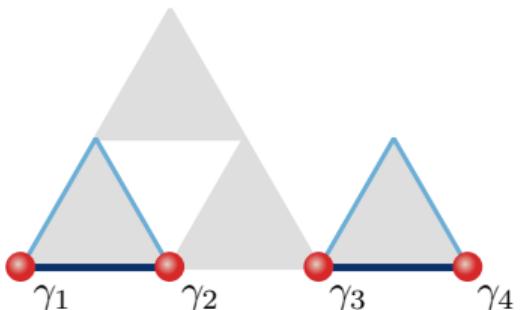
Motivation

Results

Summary

Additional
Research

Motivation
Formulation
Results
Summary





Summary

Aidan Winblad

Background

Motivation

Results

Summary

Additional
Research

Motivation

Formulation

Results

Summary

- Introduction of Peierls phase allows for a minimal Kitaev triangle, reducing fermionic sites down to 3.
- Vector potential field and its rotation allows additional tunability of topology.
- MFs can be hosted and braided on a network of triangular islands.



Motivation: Theory and Experiment

Emergent topological phenomena in low-D systems induced by gauge potentials

Aidan Winblad

Background

Motivation

Results

Summary

Additional Research

Motivation

Formulation

Results

Summary

PRL 112, 156801 (2014)

PHYSICAL REVIEW LETTERS

week ending
18 APRIL 2014

PHYSICAL REVIEW LETTERS 121, 036801 (2018)



Floquet Fractional Chern Insulators

Adolfo G. Grushin,^{1,2} Álvaro Gómez-León,² and Titus Neupert³

¹Max-Planck-Institut für Physik komplexer Systeme, 01177 Dresden, Germany

²Instituto de Ciencia de Materiales de Madrid, CSIC, Cantoblanco, E-28049 Madrid, Spain

³Princeton Center for Theoretical Science, Princeton University, Princeton, New Jersey 08544, USA
(Received 25 November 2013; published 18 April 2014)

We show theoretically that periodically driven systems with short range Hubbard interactions offer a feasible platform to experimentally realize fractional Chern insulator states. We exemplify the procedure for both the driven honeycomb and the square lattice, where we derive the effective steady state band structure of the driven system by using the Floquet theory and subsequently study the interacting system with exact numerical diagonalization. The fractional Chern insulator state equivalent to the $1/3$ Laughlin state appears at $7/12$ total filling ($1/6$ filling of the upper band). The state also features spontaneous ferromagnetism and is thus an example of the spontaneous breaking of a continuous symmetry along with a topological phase transition. We discuss light-driven graphene and shaken optical lattices as possible experimental systems that can realize such a state.

Many-Body Dynamics and Gap Opening in Interacting Periodically Driven Systems

Ervan Kandekari and Mark S. Rudner

Niels Bohr International Academy and Center for Quantum Devices, Copenhagen University,
Blegdamsvej 17, 2100 Copenhagen, Denmark

(Received 27 October 2017; published 16 July 2018)

We study transient dynamics in a two-dimensional system of interacting Dirac fermions subject to a quenched drive with circularly polarized light. In the absence of interactions, the drive opens a gap at the Dirac point in the quasimomentum spectrum, inducing nontrivial band topology. We investigate the dynamics of the gap opening process, taking into account the essential role of electron-electron interactions. Crucially, scattering due to interactions (1) induces dephasing, which erases memory of the system's prequench state and yields the intrinsic timescale for gap emergence, and (2) provides a mechanism for the system to absorb energy of the drive, leading to heating which must be mitigated to ensure the success of Floquet band engineering. We characterize the gap opening process via the system's generalized spectral function and correlators probed by photoemission experiments, and we identify a parameter regime at moderate driving frequencies where a hierarchy of timescales allows a well-defined Floquet gap to be produced and studied before the deleterious effects of heating set in.

Band structure engineering and non-equilibrium dynamics in Floquet topological insulators

Mark S. Rudner¹ and Netanel H. Lindner²

Abstract Non-equilibrium topological phenomena can be induced in quantum many-body systems using time-periodic fields (for example, by laser or microwave illumination). This Review begins with the key principles underlying Floquet band engineering, wherein such fields are used to change the topological properties of a system's single-particle spectrum. In contrast to equilibrium systems, non-trivial band structure topology in a driven many-body system does not guarantee that robust topological behaviour will be observed. In particular, periodically driven many-body systems tend to absorb energy from their driving fields and thereby tend to heat up. We survey various strategies for overcoming this challenge of heating and for obtaining new topological phenomena in this non-equilibrium setting. We describe how drive-induced topological edge states can be probed in the regime of mesoscopic transport, and three routes for observing topological phenomena beyond the mesoscopic regime: long-lived transient dynamics and prethermalization, disorder-induced many-body localization, and engineered couplings to external baths. We discuss the types of phenomena that can be explored in each of the regimes covered, and their experimental realizations in solid-state, cold atomic, and photonic systems.

Floquet–Bloch manipulation of the Dirac gap in a topological antiferromagnet

Received: 23 October 2023

Accepted: 13 December 2024

Published online: 21 January 2025

Check for updates

Nina Bieliński^{1,2}, Rajas Chari³, Julian May-Mann⁴, Soyeun Kim^{1,2,5},

Jack Zwettler^{1,2}, Yujun Deng⁴, Anuva Aishwarya⁶,

Subhajit Roychowdhury^{1,6}, Chandra Shekhar⁷, Makoto Hashimoto⁸,

Donghui Lu⁹, Jiaqiang Yan⁹, Claudia Felser¹⁰, Vidya Madhavan^{11,12},

Zhi-Xun Shen^{1,4,6,9}, Taylor L. Hughes¹ & Fahad Mahmood^{1,13}

Floquet–Bloch manipulation, achieved by driving a material periodically with a laser pulse, is a method that enables the engineering of electronic and magnetic phases in solids by effectively modifying the structure of their electronic bands. However, the application of Floquet–Bloch manipulation in topological magnetic systems, particularly those with inherent disorder, remains largely unexplored. Here we realize Floquet–Bloch manipulation of the Dirac surface-state mass of the topological antiferromagnet MnBi₂Te₄. Using time- and angle-resolved photoemission spectroscopy, we show that opposite helicities of mid-infrared circularly polarized light result in substantially different Dirac mass gaps in the antiferromagnetic phase, despite the equilibrium Dirac cone being massless. We explain our findings in terms of a Dirac fermion with a random mass. Our results underscore Floquet–Bloch manipulation as a powerful tool for controlling topology, even in the presence of disorder, and for uncovering properties of materials that may elude conventional probes.



Motivation: Experimental Findings

Emergent
topological
phenomena in
low-D systems
induced by gauge
potentials

Aidan Winblad

Background

Motivation

Results

Summary

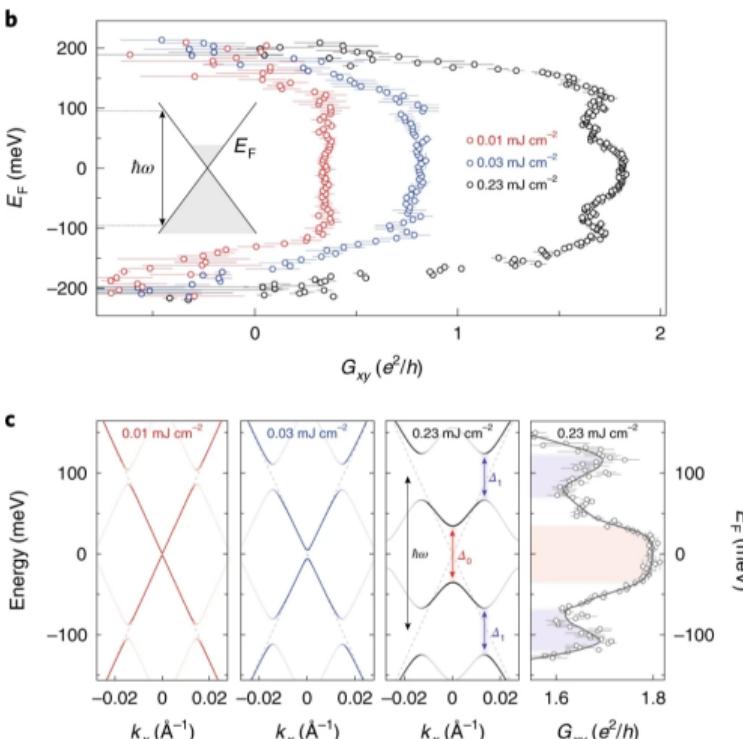
Additional
Research

Motivation

Formulation

Results

Summary



McIver et. al., *Nature Phys.* **16**, 38 (2020)



Formulation: Inhomogeneous laser light on 2D systems

Emergent
topological
phenomena in
low-D systems
induced by gauge
potentials

Aidan Winblad

Background

Motivation

Results

Summary

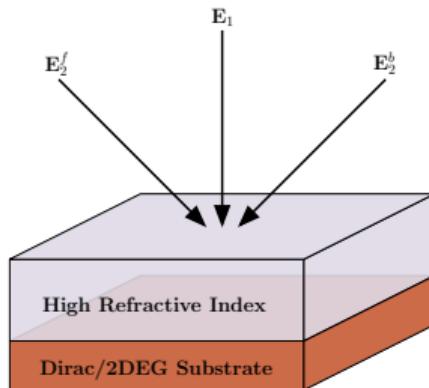
Additional
Research

Motivation

Formulation

Results

Summary



Generalized electric field at substrate surface

$$\mathbf{E}_1 = E \cos(\omega t) \hat{\mathbf{x}},$$

$$\mathbf{E}_2 = \mathbf{E}_2^f + \mathbf{E}_2^b = -E \cos(Kx) \sin(b\omega t) \hat{\mathbf{y}}$$

where

$$K = \frac{\omega \sin(\theta_i)}{v_p} \tag{21}$$



Formulation: Dirac Systems

Emergent
topological
phenomena in
low-D systems
induced by gauge
potentials

Aidan Winblad

Background

Motivation

Results

Summary

Additional
Research

Motivation

Formulation

Results

Summary

$$\mathcal{H}(t) = v_F \boldsymbol{\sigma} \cdot (\mathbf{p} + e\mathbf{A}(t)) \quad (22)$$

$$\begin{aligned}\mathbf{E}_1 &= E \cos(\omega t) \hat{\mathbf{x}}, \\ \mathbf{E}_2 &= E \sin(Kx) \sin(2\omega t) \hat{\mathbf{y}}\end{aligned} \quad (23)$$

Perform Fourier time-transform, HF expansion, and the limit $Kx \ll 1$

$$\mathcal{H}_{\text{eff}}^D = v_F \sigma_x p_x + v_F \sigma_y \left(C p_y + e B^D x \right), \quad (24)$$

where $C = 1 - \left(\frac{v_F e E}{\hbar \omega^2} \right)^2$ and

$$B^D = \frac{K v_F^2 e^2 E^3}{4 \hbar^2 \omega^5}, \quad (25)$$

$$(26)$$



Formulation: 2DEG Systems

Aidan Winblad

Background

Motivation

Results

Summary

Additional
Research

Motivation

Formulation

Results

Summary

$$\mathcal{H}(t) = \frac{1}{2m^*}(\mathbf{p} + e\mathbf{A}(t))^2 \quad (27)$$

$$\begin{aligned}\mathbf{E}_1 &= E \cos(\omega t) \hat{\mathbf{x}}, \\ \mathbf{E}_2 &= -E \cos(Kx) \sin(\omega t) \hat{\mathbf{y}}\end{aligned} \quad (28)$$

Perform Fourier time-transform, HF expansion, apply periodic potential $V(x)$, and the limit $Kx \ll 1$

$$\mathcal{H}_{\text{eff}}^{\text{2DEG}} = \frac{1}{2m^*} \left[p_x^2 + \left(p_y - eB^{\text{2DEG}}x \right)^2 \right], \quad (29)$$

where

$$B^{\text{2DEG}} = \frac{K^2 e E^2}{m^* \omega^3}, \quad (30)$$

$$(31)$$



Results: Dirac Effective Magnetic Field and Quasienergies

Aidan Winblad

Background

Motivation

Results

Summary

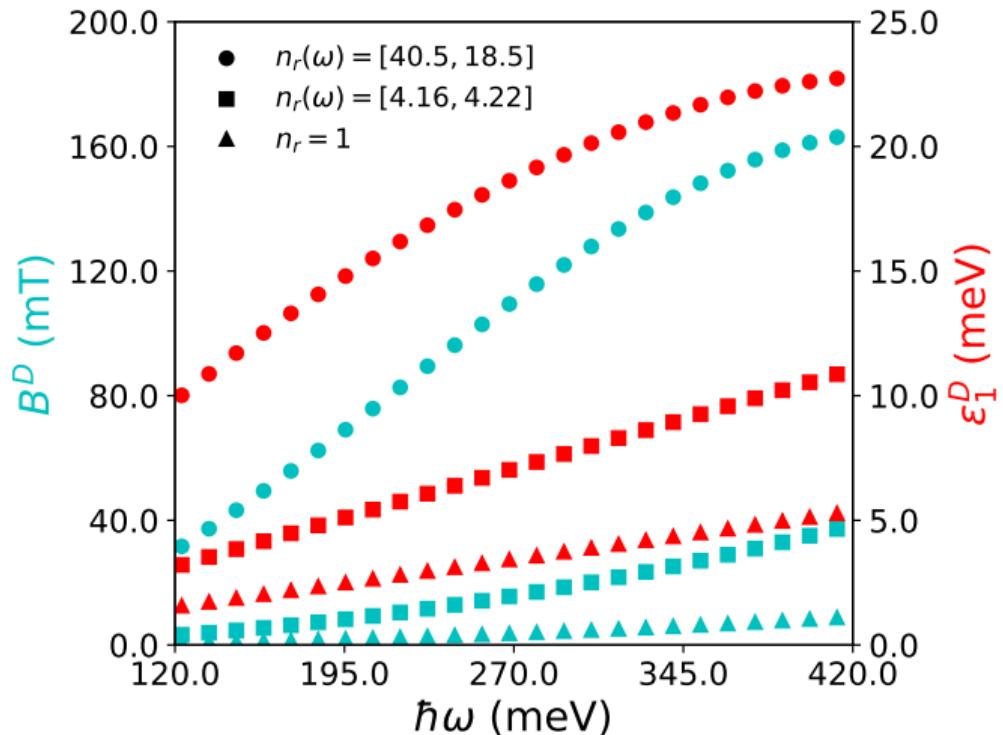
Additional
Research

Motivation

Formulation

Results

Summary





Results: 2DEG Effective Magnetic Field and Quasienergies

Aidan Winblad

Background

Motivation

Results

Summary

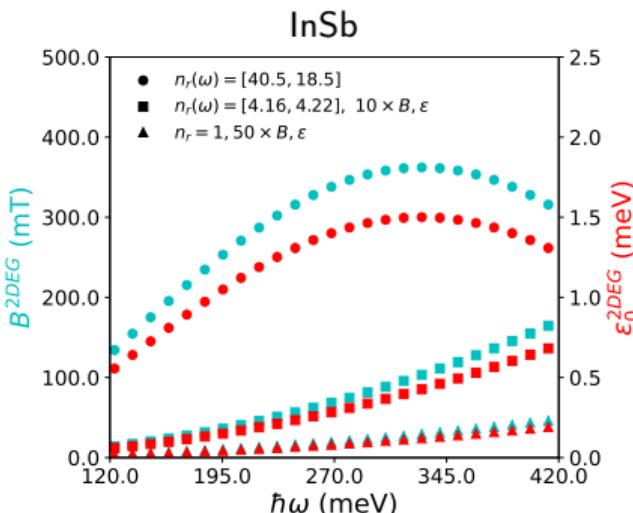
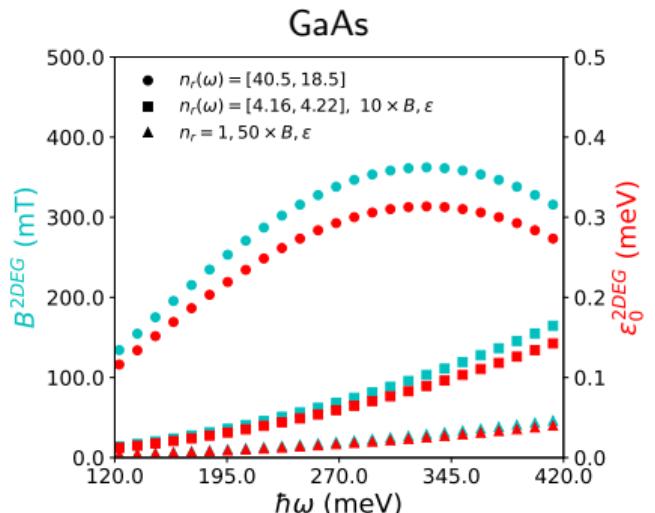
Additional
Research

Motivation

Formulation

Results

Summary





Summary

Aidan Winblad

Background

Motivation

Results

Summary

Additional
Research

Motivation

Formulation

Results

Summary

- Oblique incident laser light induces QHE in Dirac and 2DEG systems.
- Showed a non-equilibrium system exhibits equilibrium results.
- Effective magnetic field can be enhanced by several parameters.

Acknowledgments

Aidan Winblad

Background

Motivation

Results

Summary

Additional
Research

Motivation

Formulation

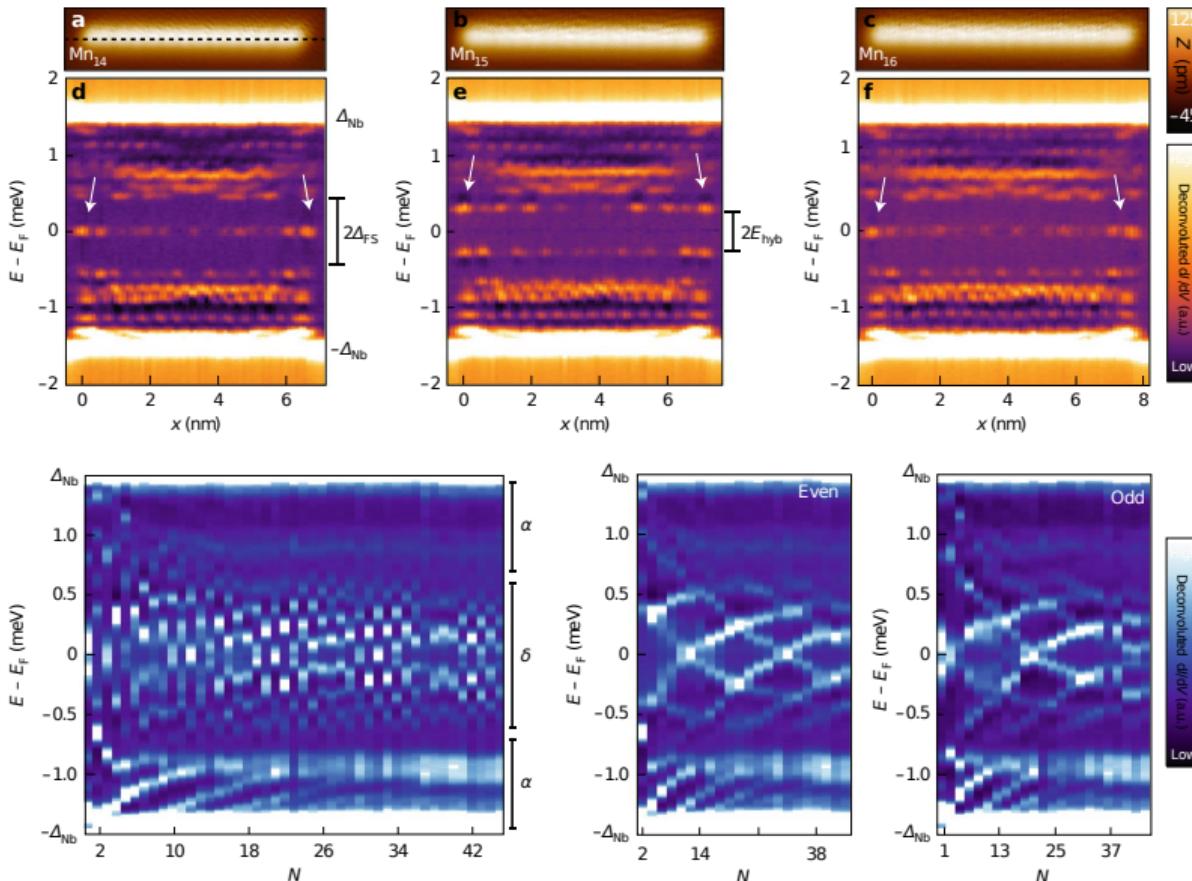
Results

Summary



- Advisor: Dr. Hua Chen
- Committee: Dr. Martin Gelfand, Dr. Richard Eykholt, Dr. Olivier Pinaud
- Friends and Family
- CSU Mental Health Services

Additional results from Schneider et al.



Majorana fermion notation and coupling isolation

Aidan Winblad

The complex fermion operator can be written as a superposition of two Majorana fermions $c_j = \frac{1}{2}(a_j + ib_j)$. Due to the nature of Majorana fermions, $a_j^\dagger = a_j$, the creation operator is $c_j^\dagger = \frac{1}{2}(a_j - ib_j)$.

$$H = -\frac{i\mu}{4} \sum_j (a_j b_j - b_j a_j) - \frac{i}{4} \sum_{<j,l>} [(t \sin \phi - \Delta \sin \theta) a_l a_j + (t \sin \phi + \Delta \sin \theta) b_l b_j + (t \cos \phi + \Delta \cos \theta) a_l b_j - (t \cos \phi - \Delta \cos \theta) b_l a_j].$$

$$(t \sin \phi_{j,l} - \Delta \sin \theta_{j,l}) a_l a_j, \quad (32)$$

$$(t \sin \phi_{j,l} + \Delta \sin \theta_{j,l}) b_l b_j, \quad (33)$$

$$(t \cos \phi_{j,l} + \Delta \cos \theta_{j,l}) a_l b_j, \quad (34)$$

$$(t \cos \phi_{j,l} - \Delta \cos \theta_{j,l}) b_l a_j \quad (35)$$

Formulation: Dirac Systems

Aidan Winblad

$$\mathcal{H}(t) = v_F \boldsymbol{\sigma} \cdot (\mathbf{p} + e\mathbf{A}(t)) \quad (36)$$

$$\begin{aligned} \mathbf{E}_1 &= E \cos(\omega t) \hat{\mathbf{x}}, \\ \mathbf{E}_2 &= E \sin(Kx) \sin(2\omega t) \hat{\mathbf{y}} \end{aligned} \quad (37)$$

Perform Fourier time-transform, HF expansion, and the limit $Kx \ll 1$

$$\mathcal{H}_{\text{eff}}^D = v_F \sigma_x p_x + v_F \sigma_y \left(C p_y + e B^D x \right), \quad (38)$$

where $C = 1 - \left(\frac{v_F e E}{\hbar \omega^2} \right)^2$ and

$$B^D = \frac{K v_F^2 e^2 E^3}{4 \hbar^2 \omega^5}, \quad (39)$$

$$\epsilon_n^D = \pm v_F^2 \sqrt{\frac{n K e^3 E^3}{2 \hbar \omega^5}} \quad (40)$$

Formulation: 2DEG Systems

Aidan Winblad

$$\mathcal{H}(t) = \frac{1}{2m^*}(\mathbf{p} + e\mathbf{A}(t))^2 \quad (41)$$

$$\begin{aligned}\mathbf{E}_1 &= E \cos(\omega t) \hat{\mathbf{x}}, \\ \mathbf{E}_2 &= -E \cos(Kx) \sin(\omega t) \hat{\mathbf{y}}\end{aligned} \quad (42)$$

Perform Fourier time-transform, HF expansion, apply periodic potential $V(x)$, and the limit $Kx \ll 1$

$$\mathcal{H}_{\text{eff}}^{\text{2DEG}} = \frac{1}{2m^*} \left[p_x^2 + \left(p_y - eB^{\text{2DEG}}x \right)^2 \right], \quad (43)$$

where

$$B^{\text{2DEG}} = \frac{K^2 e E^2}{m^* \omega^3}, \quad (44)$$

$$\epsilon_n^{\text{2DEG}} = \frac{\hbar K^2 e^2 E^2}{m^{*2} \omega^3} \left(n + \frac{1}{2} \right) \quad (45)$$