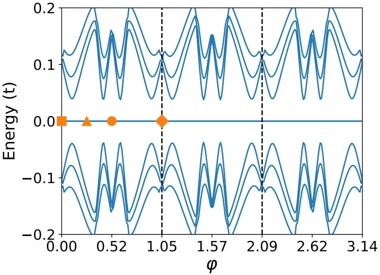
Emergent Topological Phenomena in Low-D Systems Induced by Gauge Potentials

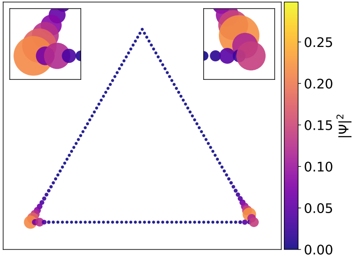


Aidan Winblad
Hua Chen

Department of Physics

Colorado State University

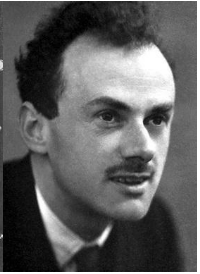
October 26, 2024



Outline

- Background:
  - Majorana fermions in particle physics and condensed matter
  - Quantum information storage
- Motivation:
  - Braiding in a 2D p-wave SC
  - T-junctions
  - Triangular structures for braiding
- Formulation: Two Approaches
  - Topological phase diagram for linear vector potential on Kitaev chain
    Bulk-edge correspondence for a double chain model
  - Vector potential on a triangular island in the Kitaev limit
- Results:
  - MCMs on 3 triangular structures
- Summary
  - Conclusions
  - Additional projects

Background: MFs in Particle Physics

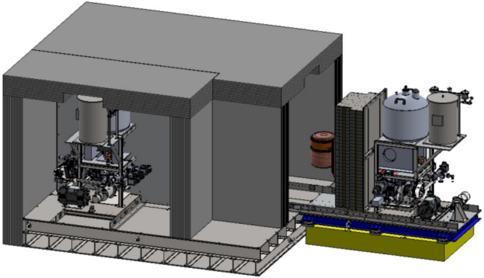



Enrico Fermi   Paul Dirac

Ettore Majorana

- Fermions
  - Half-odd-integer spin
  - Fermi-Dirac statistics
  - Weyl fermions are massless
- Dirac Fermions
  - Particle $\neq$ Antiparticle : $c \neq c^\dagger$
  - Charged
- Majorana Fermions
  - Particle $=$ Antiparticle : $c = c^\dagger$
  - Neutral
  - Neutrino? Dark Matter?

Background: MFs in Particle Physics



MAJORANA project:
neutrinoless double beta ($0\nu\beta\beta$) decay

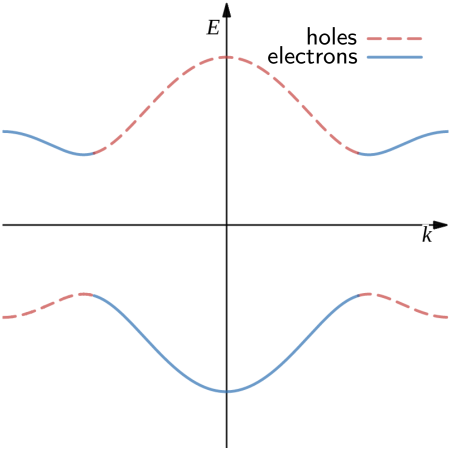
- Are neutrinos Majorana fermions?
- If yes, standard model needs revision
- Negative results for Majorana particles

Background: MFs in Condensed Matter

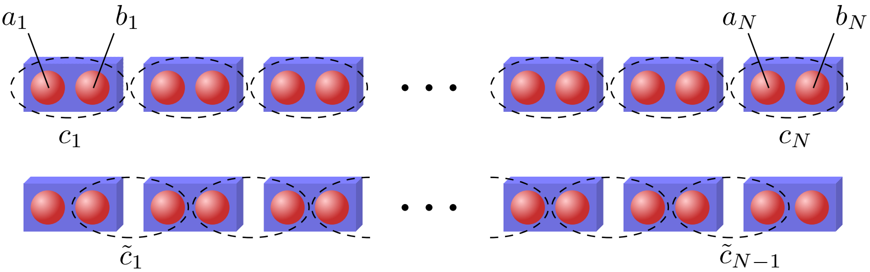
- Superconductors
  - Cooper pairs
    - Electron-phonon interaction pairs two electrons with opposite spin and momenta.
  - Bogoliubov quasiparticles
    - Excitation from ground state, pairs an electron to a hole.

$$H_{BdG} = \begin{bmatrix} \epsilon(k) & \Delta(k) \\ \Delta^*(k) & -\epsilon(-k) \end{bmatrix}$$

- Zero-energy excitations may be Majorana fermions.
- If so, they come in pairs.



Background: MFs in Condensed Matter



Complex fermion in Majorana fermion basis

$$c_j = \frac{1}{2}(a_j + ib_j). \tag{1}$$

Background: MFs in Condensed Matter

Hamiltonian for a 1D tight-binding chain with spinless *p*-wave superconductivity

$$\mathcal{H}_{chain} = -\mu \sum_{j}^{N} c_j^\dagger c_j - \sum_{j}^{N-1} t c_j^\dagger c_{j+1} + |\Delta| c_j c_{j+1} + h.c. \tag{2}$$

Hamiltonian in Majorana fermion basis

$$\mathcal{H}_{chain} = \frac{i}{2} \sum_{j} -\mu a_j b_j + (t + |\Delta|) b_j a_{j+1} + (-t + |\Delta|) a_j b_{j+1}. \tag{3}$$

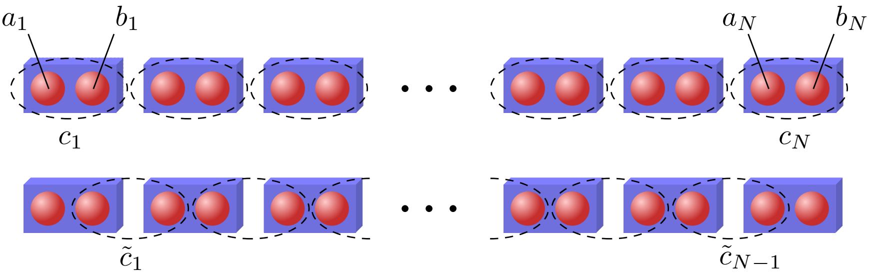
$t = |\Delta| = 0$ and $\mu < 0$, trivial phase

$$\mathcal{H} = -\frac{i\mu}{2} \sum_{j} a_j b_j. \tag{4}$$

$t = |\Delta| > 0$ and $\mu = 0$, non-trivial (topological) phase

$$\mathcal{H} = it \sum_{j} b_j a_{j+1}. \tag{5}$$

Background: MFs in Condensed Matter



Intersite fermion representation

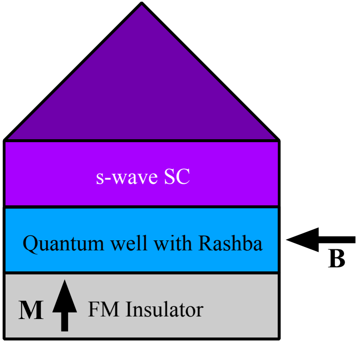
$$\tilde{c}_j = \frac{1}{2}(a_{j+1} + ib_j). \tag{6}$$

The highly non-local fermion state

$$f = \frac{1}{2}(a_1 + ib_N), \tag{7}$$

corresponds to zero energy. This is still true for $|\mu| < 2t$.

Effective *p*-wave superconductor NEEDED?



Alicea, *PRB* **81**, 125318 (2010).

$$c_j = (c_{j\uparrow}, c_{j\downarrow})^T \tag{8}$$

s-wave SC paring term:

$$\mathcal{H}_{SC} = \sum_j \Delta c^\dagger_{j\uparrow} c^\dagger_{j\downarrow} + h.c. \tag{9}$$

Quantum well:

$$\mathcal{H}_0 = \sum_j (6t - \mu) c^\dagger_j c_j - \sum_{\langle j,l \rangle} (t c^\dagger_l c_j + h.c.) \tag{10}$$

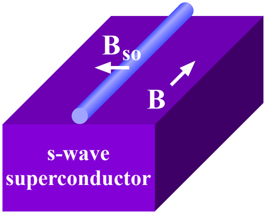
Rashba spin-orbit coupling:

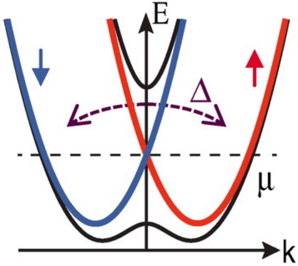
$$\mathcal{H}_R = -it_R \sum_{\langle j,l \rangle \alpha\beta} c^\dagger_{l\alpha} (\boldsymbol{\sigma}_{\alpha\beta} \times \hat{\mathbf{r}}_{lj}) \cdot \hat{\mathbf{z}} c_{j\beta} \tag{11}$$

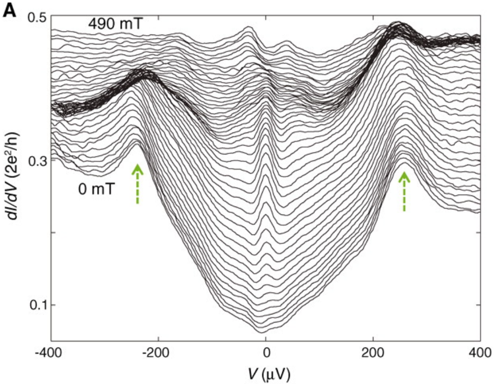
Zeeman field:

$$\mathcal{H}_Z = \sum_j c^\dagger_j \mathbf{V} \cdot \boldsymbol{\sigma} c_j \tag{12}$$

Background: MFs in Condensed Matter

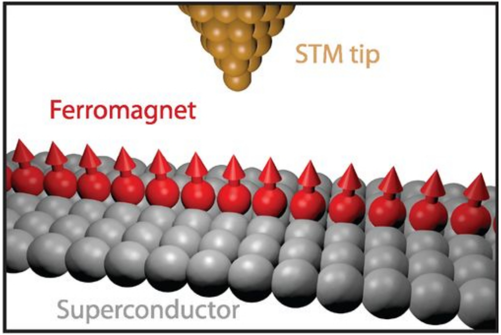


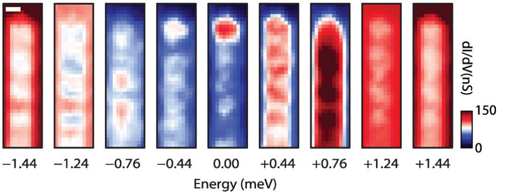




Mourik et al., *Science* **336**, 1003 (2012).

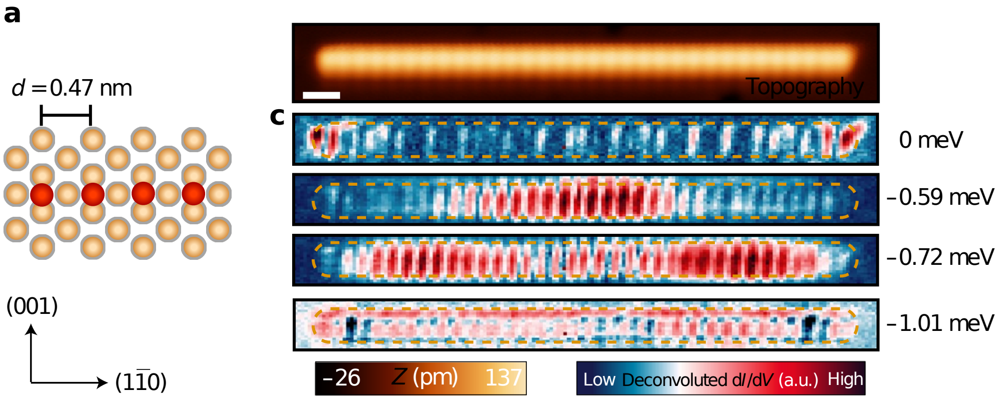
Background: MFs in Condensed Matter





Nadj-Perge et al., *Science* **346**, 602 (2014).

Background: MFs in Condensed Matter



Mn atoms (red spheres) on top of superconducting Nb (brown spheres).
Schneider et al., *Nature Nanotechnology* **17**, 384 (2022).

Motivation: Braiding in a 2D *p*-wave SC

- *p*-wave superconductors can exhibit half-quantum vortices.

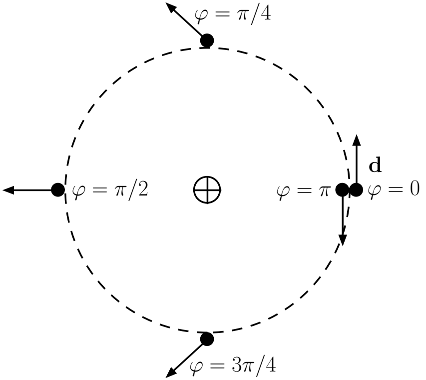
- Triplet pairing

$$\mathbf{d}(\mathbf{k}) = \Delta e^{i\phi}\langle \cos\alpha, \sin\alpha, 0\rangle(k_x + ik_y)$$

- The order phase ϕ and angle α of $\mathbf{d}$ rotate by $\pi : (\phi, \mathbf{d}) \mapsto (\phi + \pi, -\mathbf{d})$.

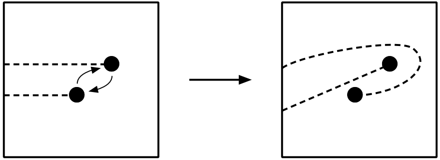
- The order parameter θ maps to itself, $(0, 2\pi)$, under the simultaneous change of both d and $\phi : \theta = \phi + \alpha$.

$$\mathcal{H}_\Delta = \int d^2\mathbf{r}\Delta\left[\Psi^\dagger\left[e^{i\theta} * (\partial_x + i\partial_y)\right]\Psi + h.c.\right]$$



- if overall phase shifts by $\theta : \Psi_\alpha \mapsto e^{i\theta/2}\Psi_\alpha$.
- $(u, v) \mapsto (ue^{i\theta/2}, ve^{-i\theta/2})$

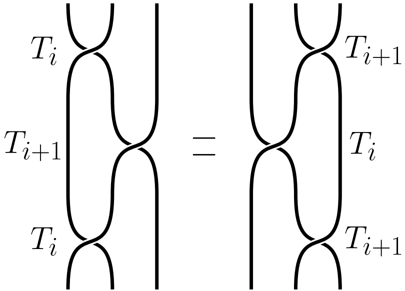
Motivation: Braiding in a 2D p-wave SC



- Interchanging two MFs:

$$\gamma_1 \to \gamma_2$$
$$\gamma_2 \to -\gamma_1$$

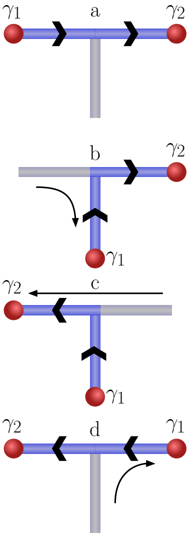
- Exhibit Non-Abelian Statistics
- $a * b \neq b * a$



$$T_i T_j = T_j T_i$$
$$T_i T_{i+1} T_i = T_{i+1} T_i T_{i+1}$$

Ivanov, *PRL* **86**, 268 (2001).

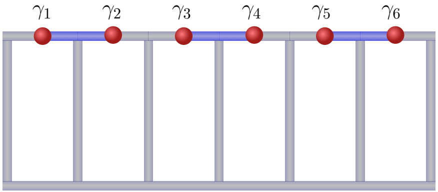
Motivation: T-junction as a Quantum Logic Gate



$$\mathcal{H}_T = -\mu \sum_j c_j^\dagger c_j - \sum_j t c_j^\dagger c_{j+1} + |\Delta| e^{i\phi} c_j c_{j+1} + h.c. \quad (13)$$

$$c_j = e^{-i(\phi/2)} (\gamma_{j+1,1} + i\gamma_{j,2})/2 \quad (14)$$

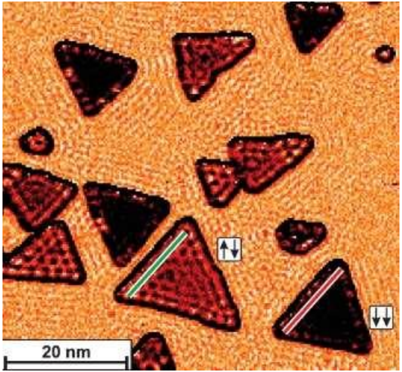
- Take pairing term $|\Delta| e^{i\phi} c_j c_{j+1}$ such that the site indices:
- Increase moving $\rightarrow$ / $\uparrow$ in the horizontal/vertical wires: $\phi = 0$,
- Decrease moving $\leftarrow$ / $\downarrow$ in the horizontal/vertical wires: $\phi = \pi$.



Alicea et al., *Nature Phys.* **7**, 412 (2011)

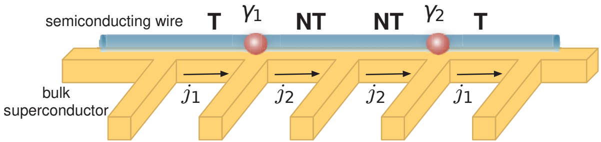
Motivation: Triangular Structures for Braiding

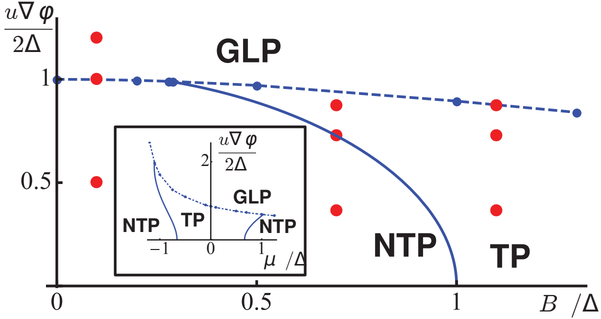
- Consider triangular islands, topologically similar to T-junctions.
- Islands of three-fold rotational symmetry occur naturally in epitaxial growth on close-packed metal surfaces.
- Make a smooth connection from 1D to 2D superconductors.



Triangular Co islands on Cu(111).
Pietzsch et al., *PRL* **96**, 237203 (2006)

Topological phase transition induced by a supercurrent

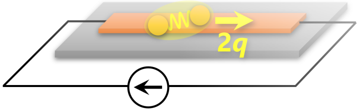




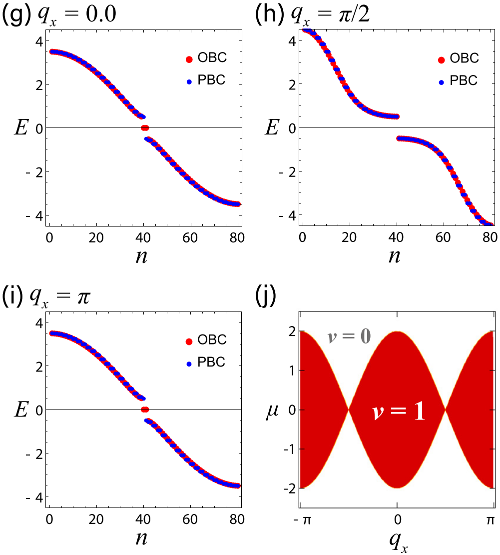
Romito et al., *PRB* **85**, 020502(R) (2012).

Topological phase transition induced by a supercurrent

(a)



Takasan et al., *PRB* **106**, 014508 (2022).



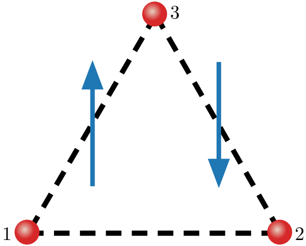
Two Proposals

- Exactly solvable "Kitaev Triangle"
  - Three fermion sites
  - Three edges controlled by Peierls phases

Two Proposals

- Exactly solvable "Kitaev Triangle"
  - Three fermion sites
  - Three edges controlled by Peierls phases
- Finite-size triangle with hollow interior
  - Under uniform vector potential
  - Bulk-edge correspondence

Kitaev Triangle



$$\mathbf{A} = [1 - 2\Theta(x)]\,\frac{2\pi}{3\sqrt{3}}\hat{\mathbf{y}} \tag{15}$$

$$\begin{aligned} c_j^\dagger c_l &\to c_j^\dagger c_l \exp\left(\frac{ie}{\hbar}\int_{r_j}^{r_l} \mathbf{A}\cdot d\mathbf{l}\right) \\ &\to c_j^\dagger c_l e^{i\phi_{jl}} \end{aligned} \tag{16}$$

$$\mathcal{H} = \sum_{\langle j,l\rangle} (-te^{i\phi_{jl}} c_j^\dagger c_l + \Delta c_j^\dagger c_l^\dagger + h.c.) - \mu c_j^\dagger c_j \tag{17}$$

In Kitaev limit, $t = \Delta \neq 0$ and $\mu = 0$,

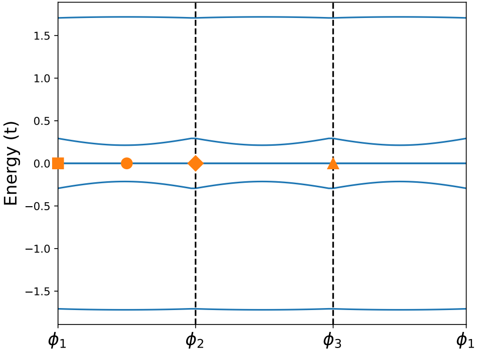
$$(\phi_{12}, \phi_{23}, \phi_{31}) = (0, -\tfrac{\pi}{3}, -\tfrac{\pi}{3}) = \boldsymbol{\phi}_1 \tag{18}$$

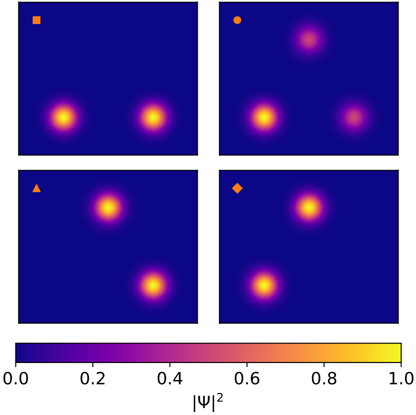
MFs localized at sites 1 and 2

Kitaev Triangle Braiding

A closed parameter path linearly interpolating between the following sets of ϕ_{jk}:

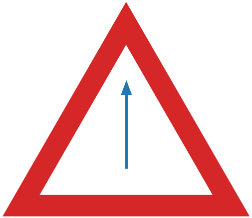
$$(\phi_{12}, \phi_{23}, \phi_{31}) : \boldsymbol{\phi}_1 \to \boldsymbol{\phi}_2 \to \boldsymbol{\phi}_3 \to \boldsymbol{\phi}_1 \tag{19}$$

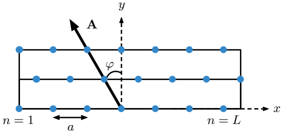




Triangular Ribbon and Topological Phases

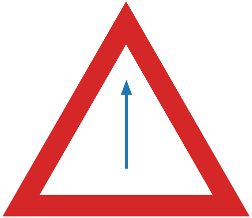
$$\phi_{jl} = \frac{e}{\hbar}\int_{\mathbf{r}_j}^{\mathbf{r}_l} \mathbf{A}\cdot d\mathbf{l} = \mathbf{A}\cdot\mathbf{r}_{jl} = -\phi_{lj}$$

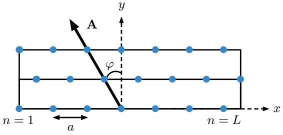


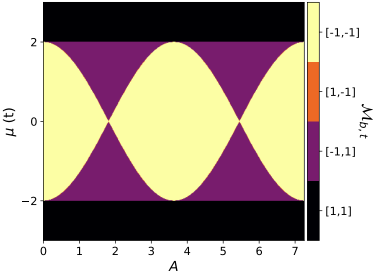


Triangular Ribbon and Topological Phases

$$\phi_{jl} = \frac{e}{\hbar} \int_{\mathbf{r}_j}^{\mathbf{r}_l} \mathbf{A} \cdot d\mathbf{l} = \mathbf{A} \cdot \mathbf{r}_{jl} = -\phi_{lj}$$

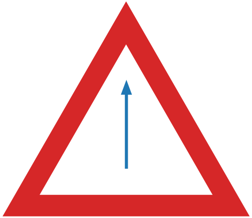


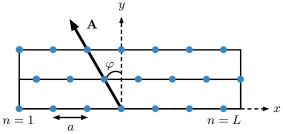


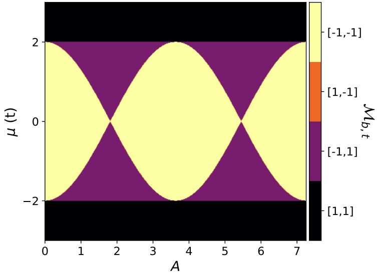


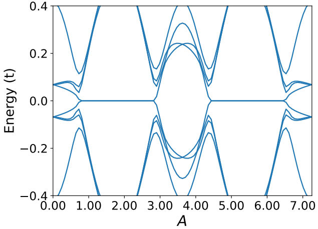
Triangular Ribbon and Topological Phases

$$\phi_{jl} = \frac{e}{\hbar}\int_{\mathbf{r}_j}^{\mathbf{r}_l} \mathbf{A}\cdot d\mathbf{l} = \mathbf{A}\cdot\mathbf{r}_{jl} = -\phi_{lj}$$

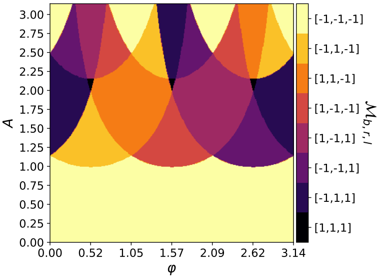




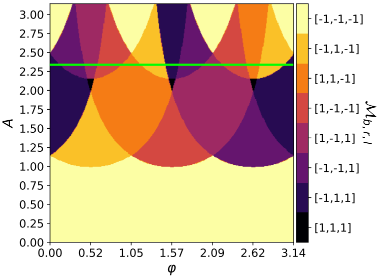




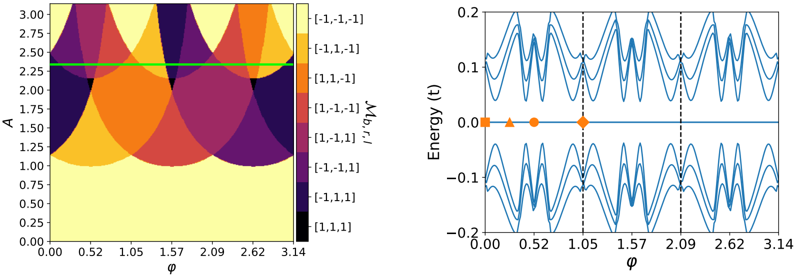
Rotating MFs on a Triangular Chain (W=1)



Rotating MFs on a Triangular Chain (W=1)

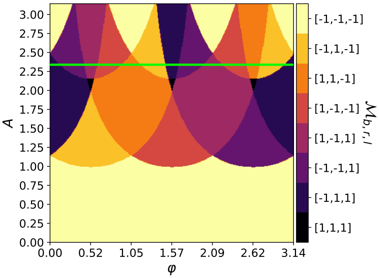


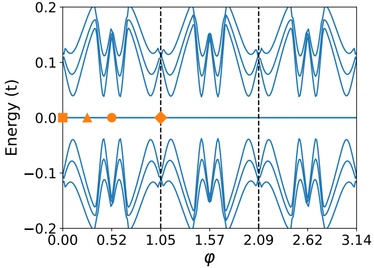
Rotating MFs on a Triangular Chain (W=1)



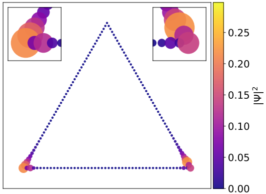
$$L = 50,\ W = 1,\ \mu = 1.1,\ A = 2.35$$

Rotating MFs on a Triangular Chain (W=1)

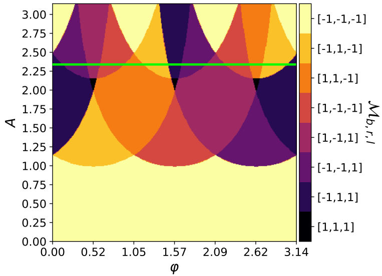


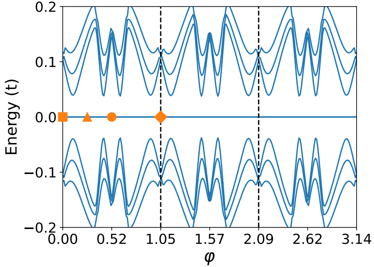


$$L = 50,\ W = 1,\ \mu = 1.1,\ A = 2.35$$

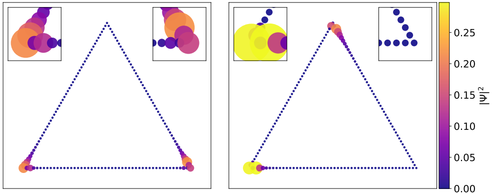


Rotating MFs on a Triangular Chain (W=1)

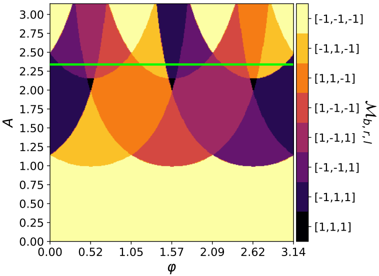


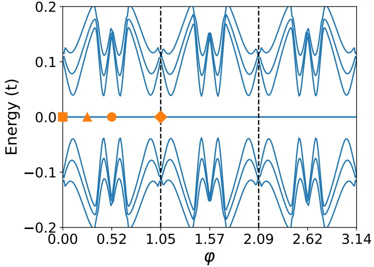


$$L = 50,\ W = 1,\ \mu = 1.1,\ A = 2.35$$

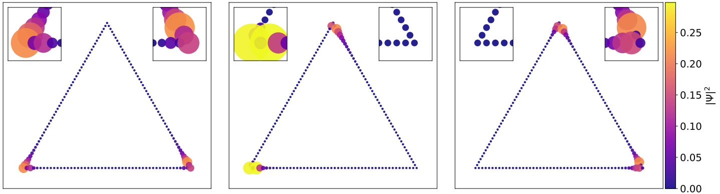


Rotating MFs on a Triangular Chain (W=1)

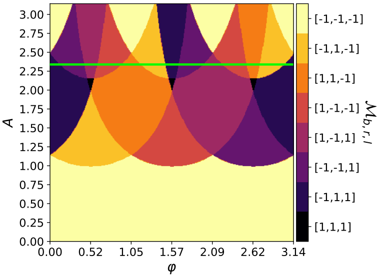


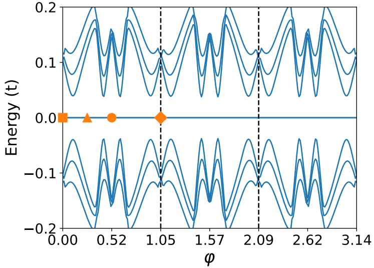


$$L = 50,\ W = 1,\ \mu = 1.1,\ A = 2.35$$

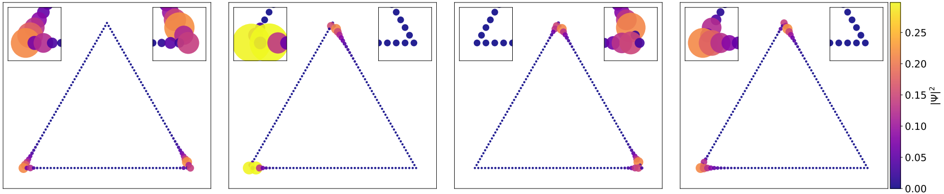


Rotating MFs on a Triangular Chain (W=1)

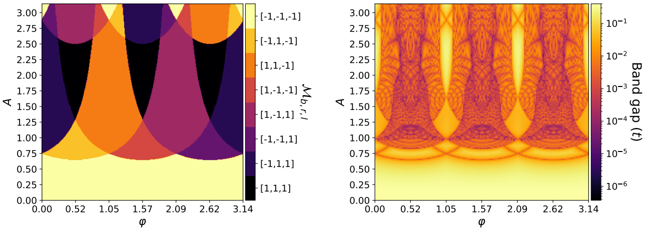




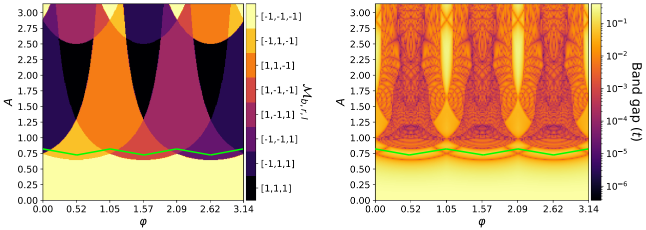
$$L = 50,\ W = 1,\ \mu = 1.1,\ A = 2.35$$



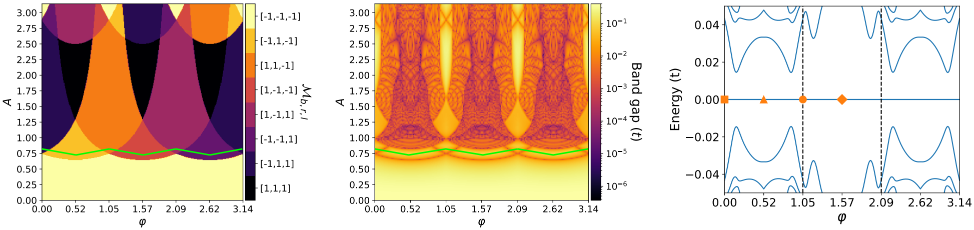
Rotating MFs on a Hollow Triangle (W=3)



Rotating MFs on a Hollow Triangle (W=3)

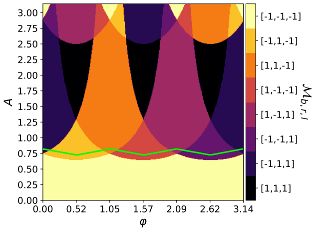


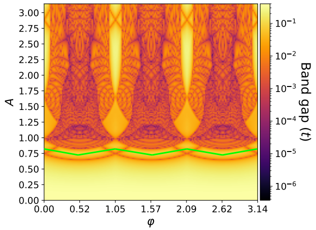
Rotating MFs on a Hollow Triangle (W=3)

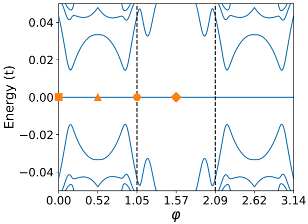


$$L = 80,\ W = 3,\ \mu = 1.6,\ (A, \varphi) = (0.83, 0) \rightarrow (0.77, \tfrac{\pi}{6}) \rightarrow (0.83, \tfrac{\pi}{3}) \rightarrow (0.77, \tfrac{\pi}{2}) \ldots$$

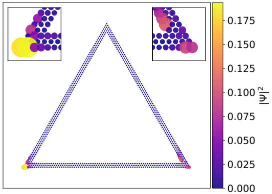
Rotating MFs on a Hollow Triangle (W=3)



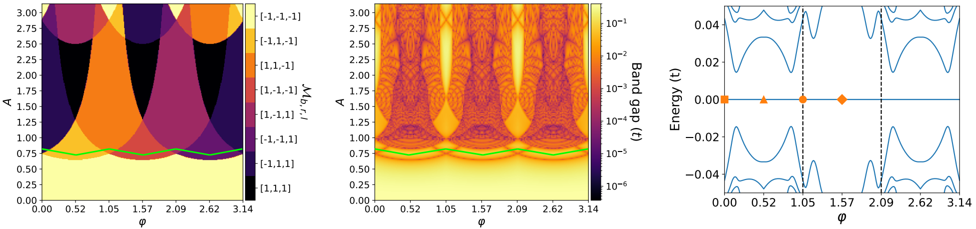




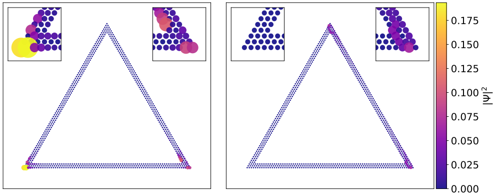
$$L = 80,\ W = 3,\ \mu = 1.6,\ (A, \varphi) = (0.83, 0) \to (0.77, \tfrac{\pi}{6}) \to (0.83, \tfrac{\pi}{3}) \to (0.77, \tfrac{\pi}{2}) \ldots$$



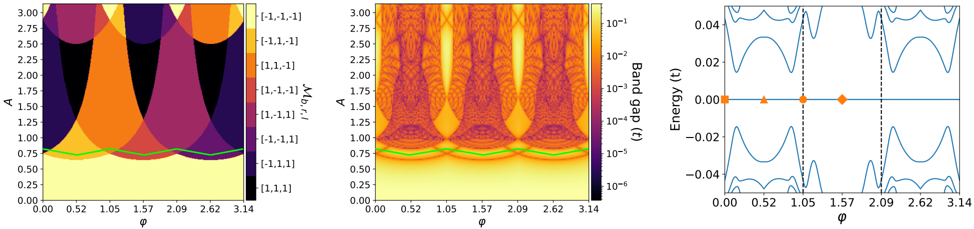
Rotating MFs on a Hollow Triangle (W=3)



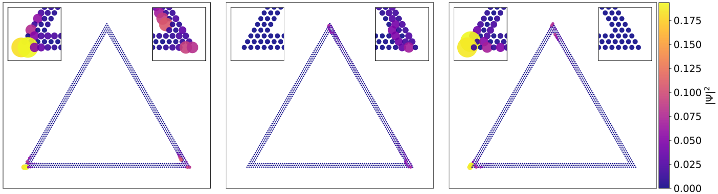
$$L = 80,\ W = 3,\ \mu = 1.6,\ (A, \varphi) = (0.83, 0) \rightarrow (0.77, \tfrac{\pi}{6}) \rightarrow (0.83, \tfrac{\pi}{3}) \rightarrow (0.77, \tfrac{\pi}{2}) \ldots$$



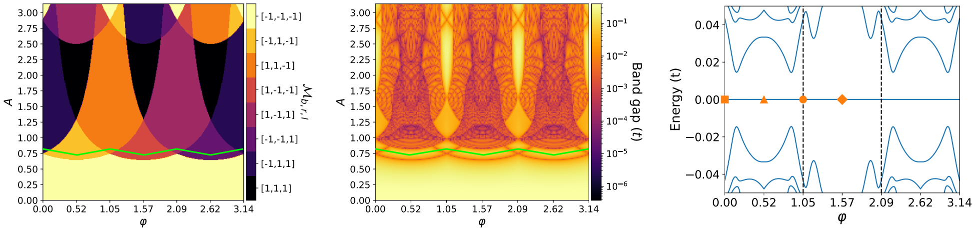
Rotating MFs on a Hollow Triangle (W=3)



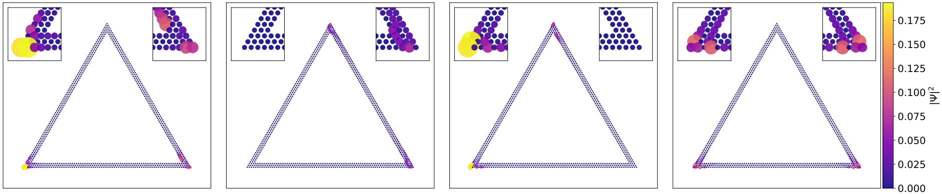
$$L = 80,\ W = 3,\ \mu = 1.6,\ (A, \varphi) = (0.83, 0) \rightarrow (0.77, \tfrac{\pi}{6}) \rightarrow (0.83, \tfrac{\pi}{3}) \rightarrow (0.77, \tfrac{\pi}{2}) \ldots$$



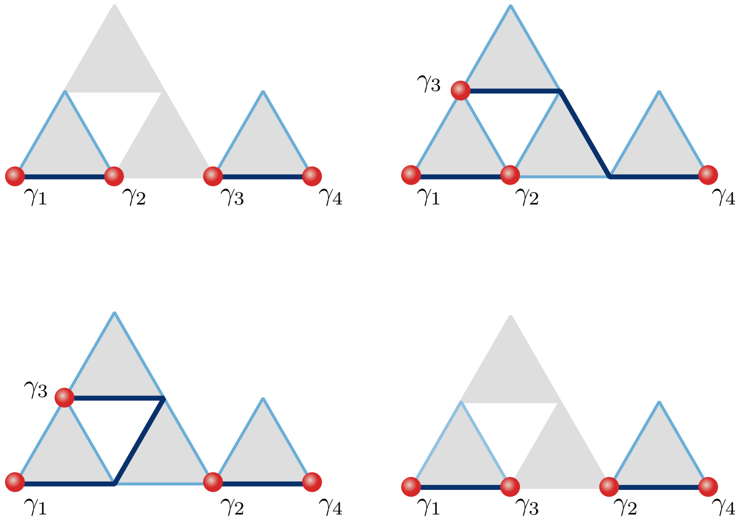
Rotating MFs on a Hollow Triangle (W=3)



$$L = 80,\ W = 3,\ \mu = 1.6,\ (A, \varphi) = (0.83, 0) \to (0.77, \tfrac{\pi}{6}) \to (0.83, \tfrac{\pi}{3}) \to (0.77, \tfrac{\pi}{2}) \ldots$$



Braiding Two of Four MFs



Summary

- Introduction of Peierls phase allows for a minimal Kitaev triangle, reducing fermionic sites down to 3.
- Vector potential field and its rotation allows additional tunability of topology.
- MFs can be hosted and braided on a network of triangular islands.

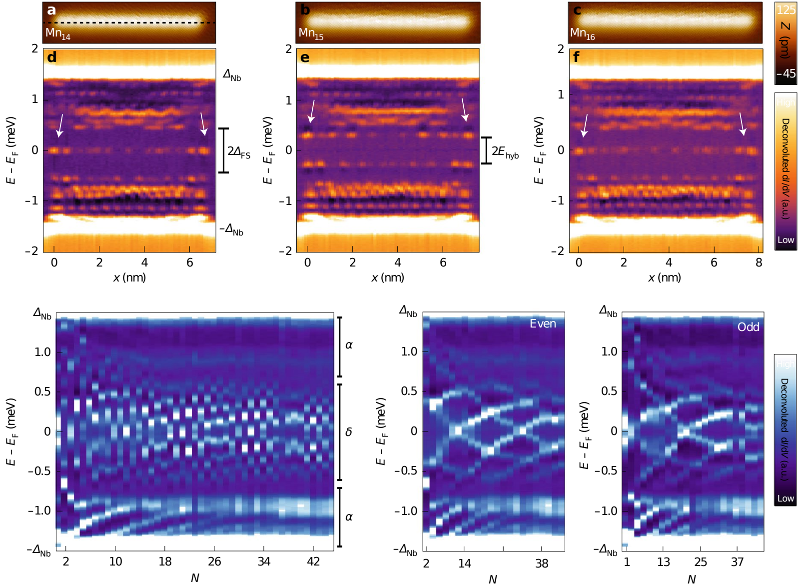
Additional projects

- Using a semi-infinite tight binding model to find Floquet Landau levels for Graphene and 2DEGs using two linearly polarized lights.
- Kitaev mapped spins to fermions using the Jordan-Wigner transformation. Can we achieve similar results using one of our triangular structures?

Emergent topological phenomena in low-D systems snduced by gauge potentials

Aidan Winblad

Additional results from Schneider et al.



Majorana fermion notation and coupling isolations

The complex fermion operator can be written as a superposition of two Majorana fermions $c_j = \frac{1}{2}(a_j + ib_j)$. Due to the nature of Majorana fermions, $a_j^\dagger = a_j$, the creation operator is $c_j^\dagger = \frac{1}{2}(a_j - ib_j)$.

$$H = -\frac{i\mu}{4}\sum_j (a_j b_j - b_j a_j) - \frac{i}{4}\sum_{<j,l>} [(t\sin\phi - \Delta\sin\theta)a_l a_j + (t\sin\phi + \Delta\sin\theta)b_l b_j + (t\cos\phi + \Delta\cos\theta)a_l b_j - (t\cos\phi - \Delta\cos\theta)b_l a_j].$$

$$(t\sin\phi_{j,l} - \Delta\sin\theta_{j,l})a_l a_j, \tag{20}$$

$$(t\sin\phi_{j,l} + \Delta\sin\theta_{j,l})b_l b_j, \tag{21}$$

$$(t\cos\phi_{j,l} + \Delta\cos\theta_{j,l})a_l b_j, \tag{22}$$

$$(t\cos\phi_{j,l} - \Delta\cos\theta_{j,l})b_l a_j \tag{23}$$

Emergent topological phenomena in low-D systems snduced by gauge potentials

Aidan Winblad

Triangular chain degeneracy

../../../research-code/mf-quantum-logic-gate-scripts/data/figures/

Hollow triangle degeneracy?

`../../../research-code/mf-quantum-logic-gate-scripts/data/figures/`