Majorana Corner Modes in Triangular Superconductor Islands

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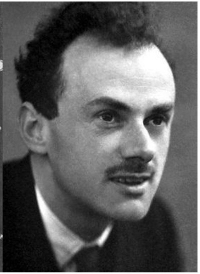
Outline

- Background:
  - Majorana fermions in particle physics
  - Majorana fermions in condensed matter
- Motivation:
  - P-wave superconductors and braiding
  - 1D wires and T-junctions
  - Triangular structures for braiding
- Formulation:
  - Peierls substitution in Kitaev's model
- Results:
  - Topological phase
  - Triangular chain
  - Hollow triangles
- Summary
  - Future work
  - Additional projects

Background

Enrico Fermi　　Paul Dirac

Ettore Majorana

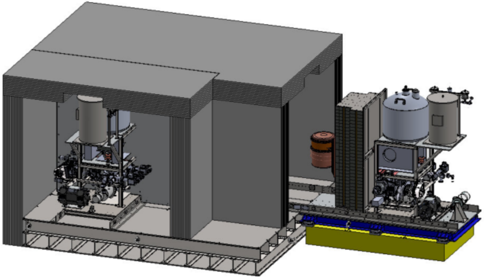
(Dirac) Fermion

- Fermi statistics
- Complex solution
- Particle $\neq$ Antiparticle : $c \neq c^\dagger$
- Charged
- Electron

Majorana Fermion

- Fermi statistics
- Real solution
- Particle $=$ Antiparticle : $c = c^\dagger$
- Neutral
- Neutrino? Dark Matter?

Background



MAJORANA project:
neutrinoless double beta ($0\nu\beta\beta$) decay

- Are neutrinos Majorana fermions?
- If yes, the standard needs revision
- Negative results for Majorana particles

Motivation

- P-wave superconductors contain half-quantum vortices.
  - Majorana fermions located at core of a vortex.
  - Braiding vortices exhibits Non-Abelian statistics.
- 1D p-wave superconductors host Majorana fermions on end points.
  - Possibly measured in real systems:
    Mourik, *Science* **336**, 1003 (2012)
    Nadj-Perge, *Science* **346**, 602 (2014)
- Quasi-1D T-junction
  - Braiding of Majorana fermions is defined for 2D.
  - In practice challenging to make, but still feasible and seriously pursued.

../../images/t-junction.pdf

Alicea, *Nature Phys.* **7**, 412 (2011)

Motivation

- Consider triangular islands, topologically similar to T-junctions.
- Islands of three-fold rotational symmetry occur naturally in epitaxial growth on close-packed metal surfaces.
- Good platform for transition from 2D to 1D topological superconductor.

../../images/triangular-islands.pd

Triangular Co islands on Cu(111).
Pietzsch et al., *PRL* **96**, 237203 (2006)

Previous Work

../../images/tunable-semiconductor.pdf

../../images/dispersion-in-plane.pdf

../../images/energy-spectra-in-plane.pdf

../../images/wavefunction-2-in-plane

Kitaev Limit with Vector Potential on a Triangular Island

../../images/left-triangle-corner.pdf

$$\mathcal{H} = \sum_{<j,l>} \left[-t e^{i\phi_{l,j}} c_l^\dagger c_j + \Delta e^{i\theta_{l,j}} c_l^\dagger c_j^\dagger + h.c. \right] - \sum_j \mu c_j^\dagger c_j$$

$$\phi_{l,j} = -\frac{e}{\hbar} \int_{\mathbf{r}_j}^{\mathbf{r}_l} \mathbf{A} \cdot d\mathbf{l}$$

$$\mathbf{A} = -\frac{2\pi}{3\sqrt{3}a} \hat{\mathbf{y}}$$

Majorana Number of 1D Chain with Vector Potential

μ ../../images/Majorana-number.pdf

ϕ

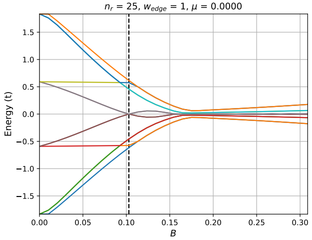
$\phi = \pi$ ../images/kitaev-chain-mu_pi.pdf

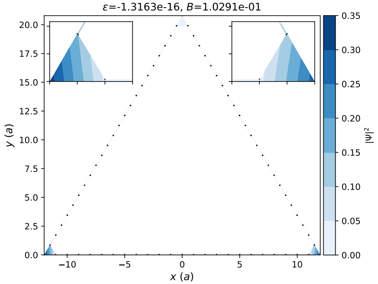
Triangular Chain

$$\mathbf{A}(x) = Bx\hat{\mathbf{y}}$$

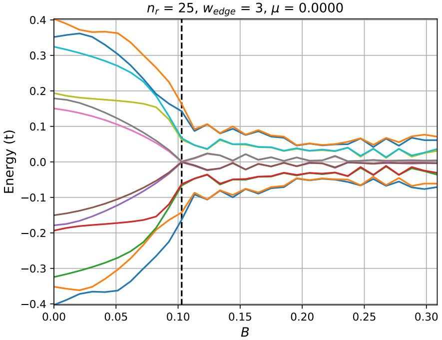
$$B_0 = \frac{8\pi}{3\sqrt{3}a^2(2n_r - 3)}$$

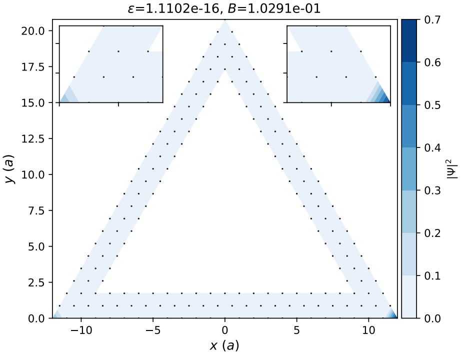
../../images/vector-potential-field.pdf





Hollow Triangle





Summary

- Introduction of vector potential allows for additional tunability of topology.
- Triangular islands with a gapped interior can be a promising platform for hosting and manipulating MZMs.
- Next steps
  - Search for safe MZMs in hollow triangles outside the Kitaev limit.
  - Develop a robust braiding scheme.

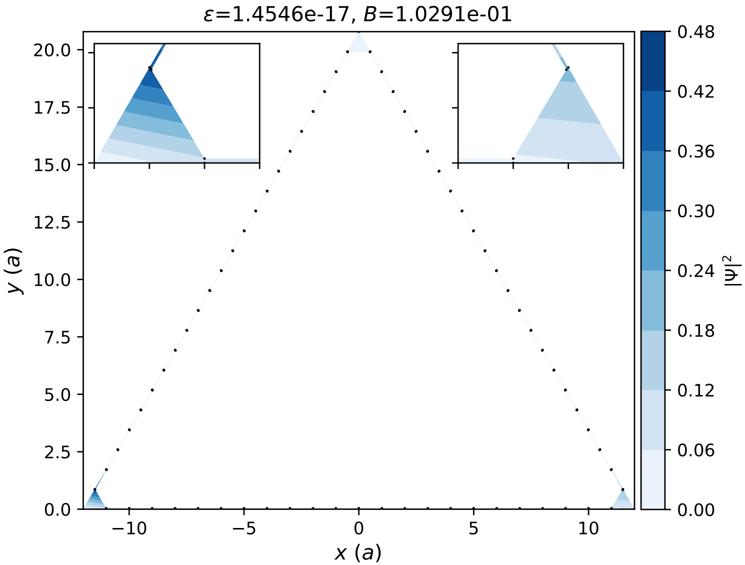
Majorana fermion notation and coupling isolations

The complex fermion operator can be written as a superposition of two Majorana fermions $c_j = \frac{1}{2}(a_j + ib_j)$. Due to the nature of Majorana fermions, $a_j^\dagger = a_j$, the creation operator is $c_j^\dagger = \frac{1}{2}(a_j - ib_j)$.

$$H = -\frac{i\mu}{4}\sum_j (a_j b_j - b_j a_j) - \frac{i}{4}\sum_{<j,l>}[(t\sin\phi - \Delta\sin\theta)a_l a_j + (t\sin\phi + \Delta\sin\theta)b_l b_j \\ +(t\cos\phi + \Delta\cos\theta)a_l b_j - (t\cos\phi - \Delta\cos\theta)b_l a_j].$$

$$(t\sin\phi_{j,l} - \Delta\sin\theta_{j,l})a_l a_j, \tag{1}$$
$$(t\sin\phi_{j,l} + \Delta\sin\theta_{j,l})b_l b_j, \tag{2}$$
$$(t\cos\phi_{j,l} + \Delta\cos\theta_{j,l})a_l b_j, \tag{3}$$
$$(t\cos\phi_{j,l} - \Delta\cos\theta_{j,l})b_l a_j \tag{4}$$

Triangular chain degeneracy



Hollow triangle degeneracy?