May 16th, 2024

Re: BJR1430
Superconducting triangular islands as a platform for manipulating Majorana zero modes

We thank all three referees for their careful evaluation of our revised manuscript. We accept the recommendations of the referees for publication of our revised manuscript as a Regular Article in Physical Review B.

Summary of major changes in the revision:

1. Discussion on possible ways to implement general braiding-based logic operations based on triangles formed by quantum dots is added to the last section.
2. The main text is formatted in compliance with the Regular Article format.

Second Report of the First Referee -- BJR1430/Winblad

Comment: *The authors made several changes to the paper and these did improve the article. However, I don't think the article is best presented as a Letter so I would recommend the paper to be published in PRB as a Regular Article. The reasons are listed as below.*

Reply: We thank the referee for their recommendation of publishing our work as a Regular Article in PRB. Below we briefly reply to their specific comments separately.

(1) The toy model, which the majority of the discussion is focusing on, is rather fine-tuned. The authors mentioned, "...the model can be realized as effective model of carefully engineered devices..." The need for "careful engineering" potentially lowers the interest for the experimental community (and the theoretical community when compared to previous studies), I believe.

Reply: Fine-tuning is the key to our Kitaev triangle model and is exactly why it is applicable to the quantum-dot-based effective Kitaev chains. More elaboration on this point has already been given in our previous response to the second referee's first comment.

(2) The article in a Letter format is still not balanced (after the revision) in my opinion; moreover, the authors made several changes to the supplemental material, as suggested by "summary of major changes points 3 and 4." I believe these contain interesting and important information for the readers, so the content is best presented as a long article. The authors provide explanatory answers to the referees' comments. A lot of these can be added to the paper. For example, the reply to my comment #2, the authors mentioned that, "The gauge field A can be gauged to a phase in the pairing potential." These comments help the non-specialized readers to understand the background.

Reply: Honestly, we do feel that the current format of our work is not balanced after the first revision. Moving the technical details in the Supplementary Material to the main text will directly obscure the otherwise clear messages that the present manuscript tries to convey.

(3) Looking at the second referee's report (as suggested by the editors), I don't think the response given by the authors about previous studies (comment #0) is strong enough for elevating the article to a Letter in PRB, although it is strong enough for granting publication as a Regular Article.

Reply: We believe the positive comments from the second referee on our previous response and on our revised manuscript serve as a strong argument countering this comment. In any event, we appreciate the referee’s overall positive evaluation of work and recommending its publication in Physical Review B.

Second Report of the Second Referee -- BJR1430/Winblad

Comment: *I thank the authors for carefully considering and addressing the questions and concerns raised in my original report.*

Given current interest in the field for the experimental realization of Kitaev chains in quantum dots, I believe this work to be timely and of interest to the community. The focus on geometry is interesting and *could rapidly spark new ideas and experiments in the field. Based on the above, on the modifications made to the manuscript and their replies to my comments, it is my assessment that the revised manuscript now meets the criteria for publication as a Letter in PRB. As such, I recommend publication.*

Reply: We genuinely appreciate the referee's recommendation and recognition of the novelty of our work.

Report of the Third Referee -- BJR1430/Winblad

Comment: *Winblad and Chen presented in their manuscript an idea of manipulating Majorana bound states in triangular platforms by means of changing vector potential and local chemical potential. This is an interesting description, and the calculations seem technically correct. However, I agree with the second referee that this work is based on an idea presented previously in S.-B. Zhang et al. Phys. Rev. B 102, 100503 (2020) [Ref.52 in the manuscript]. Secondly, the authors only showed how to move Majorana bound states similar to the movement in T junction, and it cannot demonstrate non-Abelian character of these Majorana particles which is an outstanding problem in this field. Because of these two reasons I do not see any new idea which can make substantial progress in this field. Hence, I do not recommend this manuscript for publication in Letter section. It can be published in PRB as a Regular Article.*

Below is my another concern for this geometry. I would request the authors to consider and incorporate in their manuscript, which I believe can be useful for some readers.

Realizing the zero-energy Majorana states in quantum dot systems requires very fine tuning of parameters, although in Ref.47 the authors demonstrated in their experiments in a two-quantum dot system. When it will require a triangle or a network of triangles, it will be a challenge to move around the Majorana bound states because these states are weakly protected (as opposed to a Kitaeve chain which gives Majorana bound states due to bulk-boundary correspondence). It will be useful if the authors can discuss how to overcome this challenge in this platform for the braiding scheme shown in Fig. 5.

Reply: We thank the referee for carefully reading our manuscript and previous correspondence with the other two referees. We also appreciate the referee's positive evaluation of our work which leads to their recommendation of publication as a Regular Article. Regarding the timeline of conceiving the triangular structure for manipulating MZM, we gently point out that as far as we know it was the second author who first mentioned this idea in his 2015 APS March Meeting invited talk (slides available at https://meetings.aps.org/Meeting/MAR15/Session/M51.3). In addition, demonstrating non-Abelian braiding based on our triangular network designs is not a fundamental challenge, but just needs more technical preparations that are beyond the scope of the present short paper.

Regarding the referee's concern about quantum-dot systems, our vision of MZM-based topological quantum computation is that demonstrating braiding in precisely tuned few-fermion systems is more important than topological protection by bulk-boundary correspondence in mesoscopic systems, since the latter is always plagued by artifacts resulting in similar in-gap features as the MZM. In the revised manuscript we have added a short discussion on possible designs for implementing braiding more MZM using quantum-dot triangles:

Although we only discussed braiding two MZM in the 3-site Kitaev triangle, generalization of the idea to braiding more MZM in a minimal network formed by quantum dots sitting at triangle vertices similar to that in Fig. 5 is also possible. Moreover, a distinct feature of the minimal quantum-dot

network is that it can be exactly solved due to the small Hilbert space. As such, nontrivial parameter paths defined by bond-dependent vector potentials for braiding MZM, not constrained by bulk-edge correspondence, can be obtained by solving an optimization problem, which can make such systems more advantageous than mesoscopic wire-based networks.