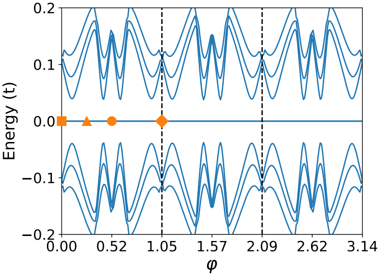
Emergent Topological Phenomena in Low-D Systems Induced by Gauge Potentials



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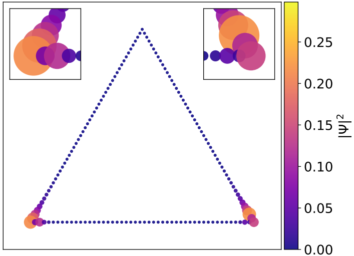
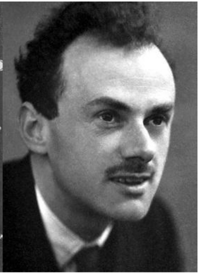


Outline

- Background:
  - Majorana fermions in particle physics and condensed matter
  - Quantum information storage
- Motivation:
  - Braiding in a 2D p-wave SC
  - T-junctions
  - Triangular structures for braiding
- Formulation: Two Approaches
  - Topological phase diagram for linear vector potential on Kitaev chain
    Bulk-edge correspondence for a double chain model
  - Vector potential on a triangular island in the Kitaev limit
- Results:
  - MCMs on 3 triangular structures
- Summary
  - Conclusions
  - Additional projects

Background: MFs in Particle Physics

Enrico Fermi

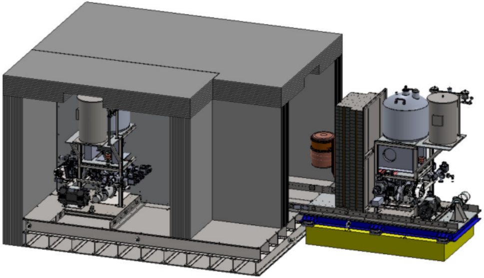


Paul Dirac

Ettore Majorana

- Fermions
  - Half-odd-integer spin
  - Fermi-Dirac statistics
  - Weyl fermions are massless
- Dirac Fermions
  - Particle $\neq$ Antiparticle : $c \neq c^\dagger$
  - Charged
- Majorana Fermions
  - Particle $=$ Antiparticle : $c = c^\dagger$
  - Neutral
  - Neutrino? Dark Matter?

Background: MFs in Particle Physics



MAJORANA project:
neutrinoless double beta ($0\nu\beta\beta$) decay

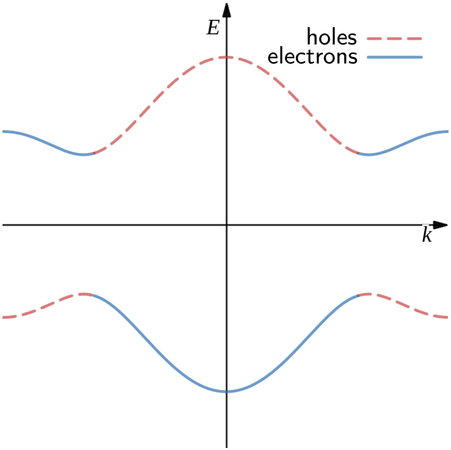
- Are neutrinos Majorana fermions?
- If yes, standard model needs revision
- Negative results for Majorana particles

Background: MFs in Condensed Matter

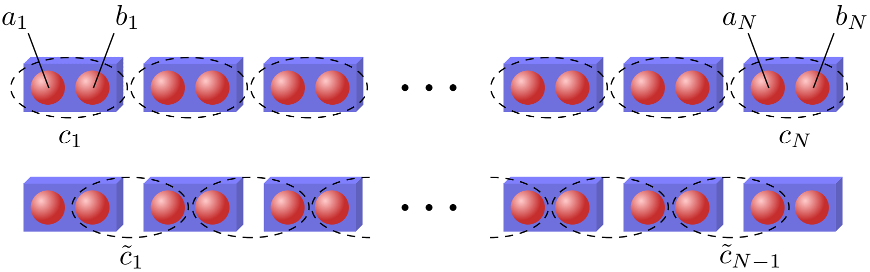
- Superconductors
  - Cooper pairs
    - Electron-phonon interaction pairs two electrons with opposite spin and momenta.
  - Bogoliubov quasiparticles
    - Excitation from ground state, pairs an electron to a hole.

$$H_{BdG} = \begin{bmatrix} \epsilon(k) & \Delta(k) \\ \Delta^*(k) & -\epsilon(-k) \end{bmatrix}$$

    - Zero-energy excitations may be Majorana fermions.
    - If so, they come in pairs.



Background: MFs in Condensed Matter



Complex fermion in Majorana fermion basis

$$c_j = \frac{1}{2}(a_j + ib_j). \tag{1}$$

Background: MFs in Condensed Matter

Hamiltonian for a 1D tight-binding chain with spinless *p*-wave superconductivity

$$\mathcal{H}_{chain} = -\mu \sum_{j}^{N} c_j^\dagger c_j - \sum_{j}^{N-1} t c_j^\dagger c_{j+1} + |\Delta| c_j c_{j+1} + h.c. \tag{2}$$

Hamiltonian in Majorana fermion basis

$$\mathcal{H}_{chain} = \frac{i}{2} \sum_{j} -\mu a_j b_j + (t + |\Delta|) b_j a_{j+1} + (-t + |\Delta|) a_j b_{j+1}. \tag{3}$$

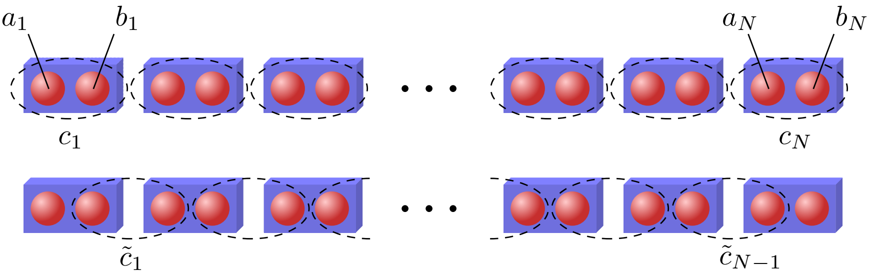
$t = |\Delta| = 0$ and $\mu < 0$, trivial phase

$$\mathcal{H} = -\frac{i\mu}{2} \sum_{j} a_j b_j. \tag{4}$$

$t = |\Delta| > 0$ and $\mu = 0$, non-trivial (topological) phase

$$\mathcal{H} = it \sum_{j} b_j a_{j+1}. \tag{5}$$

Background: MFs in Condensed Matter



Intersite fermion representation

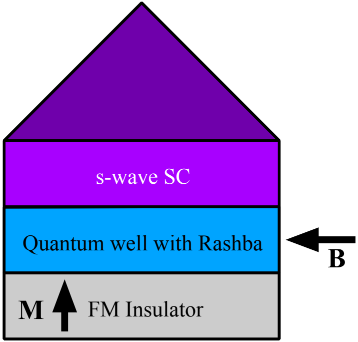
$$\tilde{c}_j = \frac{1}{2}(a_{j+1} + ib_j). \tag{6}$$

The highly non-local fermion state

$$f = \frac{1}{2}(a_1 + ib_N), \tag{7}$$

corresponds to zero energy. This is still true for $|\mu| < 2t$.

Effective p-wave superconductor



Alicea, *PRB* **81**, 125318 (2010).

$$c_j = (c_{j\uparrow}, c_{j\downarrow})^T \tag{8}$$

s-wave SC paring term:

$$\mathcal{H}_{SC} = \sum_j \Delta c^\dagger_{j\uparrow} c^\dagger_{j\downarrow} + h.c. \tag{9}$$

Quantum well:

$$\mathcal{H}_0 = \sum_j (6t - \mu) c^\dagger_j c_j - \sum_{\langle j,l \rangle} (t c^\dagger_l c_j + h.c.) \tag{10}$$

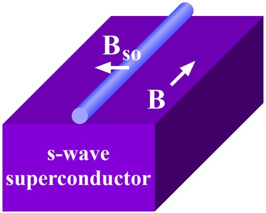
Rashba spin-orbit coupling:

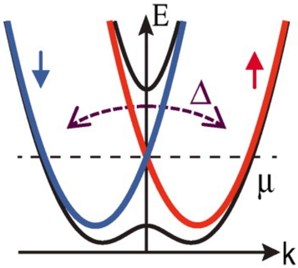
$$\mathcal{H}_R = -it_R \sum_{\langle j,l \rangle \alpha\beta} c^\dagger_{l\alpha} (\boldsymbol{\sigma}_{\alpha\beta} \times \hat{\mathbf{r}}_{lj}) \cdot \hat{\mathbf{z}} c_{j\beta} \tag{11}$$

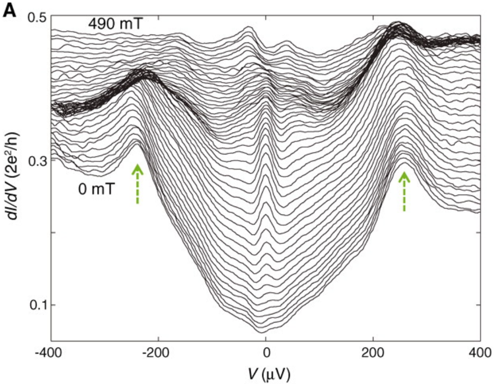
Zeeman field:

$$\mathcal{H}_Z = \sum_j c^\dagger_j \mathbf{V} \cdot \boldsymbol{\sigma} c_j \tag{12}$$

Background: MFs in Condensed Matter

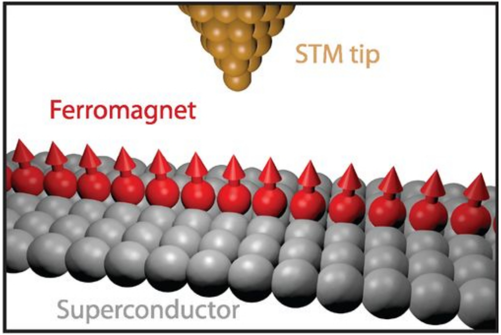


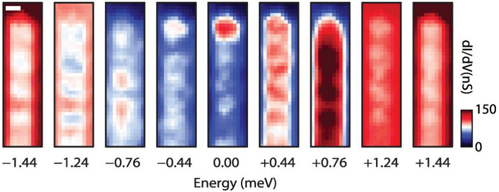




Mourik et al., *Science* **336**, 1003 (2012).

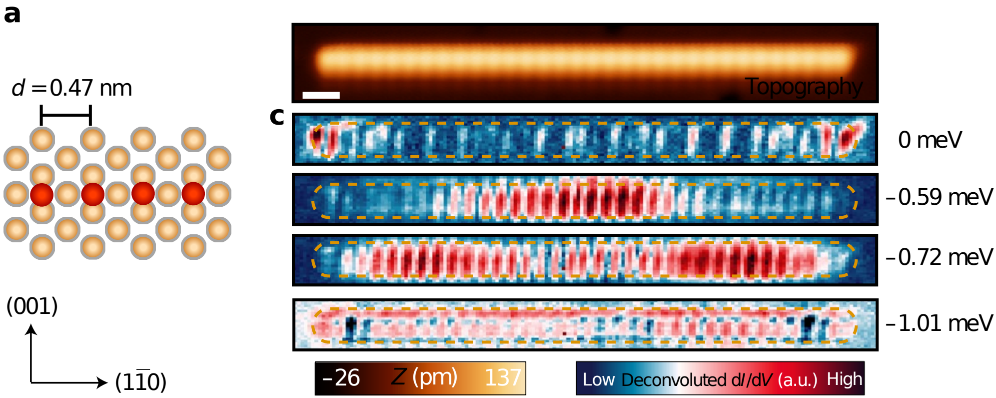
Background: MFs in Condensed Matter





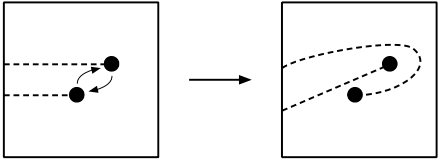
Nadj-Perge et al., *Science* **346**, 602 (2014).

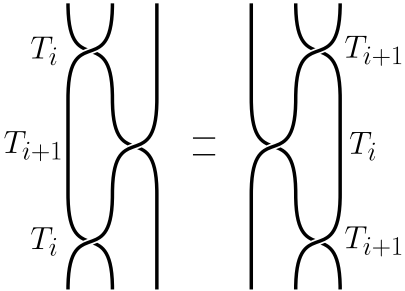
Background: MFs in Condensed Matter



Mn atoms (red spheres) on top of superconducting Nb (brown spheres).
Schneider et al., *Nature Nanotechnology* **17**, 384 (2022).

Motivation: Braiding in a 2D p-wave SC





- Interchanging two MFs:

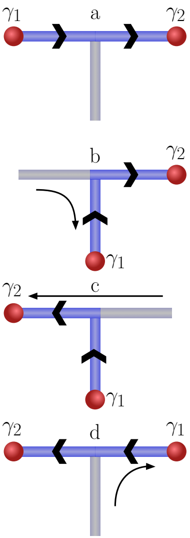
$$\gamma_1 \to \gamma_2$$
$$\gamma_2 \to -\gamma_1$$

- Exhibit Non-Abelian Statistics
- $a * b \neq b * a$

$$T_i T_j = T_j T_i$$
$$T_i T_{i+1} T_i = T_{i+1} T_i T_{i+1}$$

Ivanov, *PRL* **86**, 268 (2001).

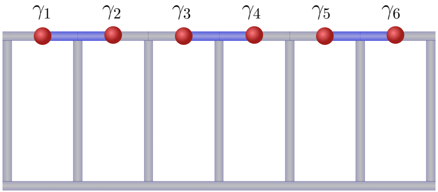
Motivation: T-junction as a Quantum Logic Gate



$$\mathcal{H}_T = -\mu \sum_j c_j^\dagger c_j - \sum_j t c_j^\dagger c_{j+1} + |\Delta| e^{i\phi} c_j c_{j+1} + h.c. \quad (13)$$

$$c_j = e^{-i(\phi/2)} (\gamma_{j+1,1} + i\gamma_{j,2})/2 \quad (14)$$

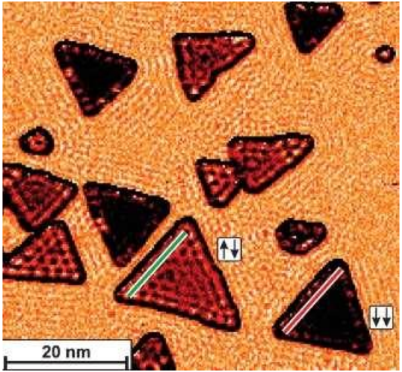
- Take pairing term $|\Delta| e^{i\phi} c_j c_{j+1}$ such that the site indices:
- Increase moving $\rightarrow$ / $\uparrow$ in the horizontal/vertical wires: $\phi = 0$,
- Decrease moving $\leftarrow$ / $\downarrow$ in the horizontal/vertical wires: $\phi = \pi$.



Alicea et al., *Nature Phys.* **7**, 412 (2011)

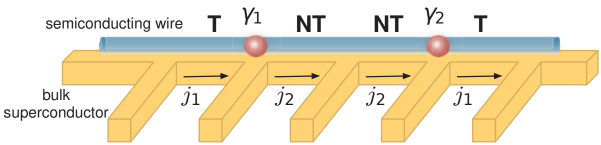
Motivation: Triangular Structures for Braiding

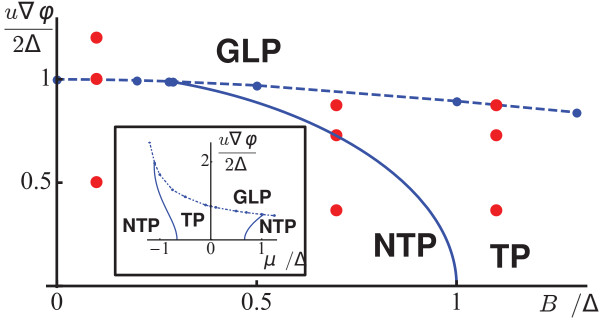
- Consider triangular islands, topologically similar to T-junctions.
- Islands of three-fold rotational symmetry occur naturally in epitaxial growth on close-packed metal surfaces.
- Make a smooth connection from 1D to 2D superconductors.



Triangular Co islands on Cu(111).
Pietzsch et al., *PRL* **96**, 237203 (2006)

Topological phase transition induced by a supercurrent

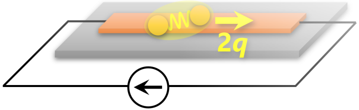




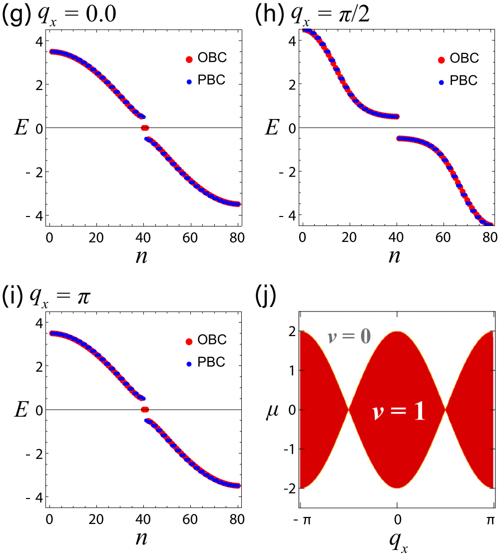
Romito et al., *PRB* **85**, 020502(R) (2012).

Topological phase transition induced by a supercurrent

(a)



Takasan et al., *PRB* **106**, 014508 (2022).



Formulation: Two Approaches

- Topological phase diagram for linear vector potential on Kitaev chain
  Bulk-edge correspondence for a double chain model
- Vector potential on a triangular island in the Kitaev limit

Linear vector potential and Majorana number for Kitaev chain

Peierls substitution

$$\mathbf{A} = B\mathbf{x} \tag{15}$$

$$c_{j+1}^\dagger c_j \to c_{j+1}^\dagger c_j \exp\left(-\frac{ie}{\hbar}\int_{r_j}^{r_{j+1}} \mathbf{A}(x)\cdot d\mathbf{l}\right) = c_{j+1}^\dagger c_j e^{i\phi_{j+1,j}}. \tag{16}$$

$$\mathcal{H}_{ch} = \sum_j (-te^{i\phi_{j+1,j}} c_{j+1}^\dagger c_j + \Delta c_{j+1}^\dagger c_j^\dagger + h.c.) - \mu c_j^\dagger c_j. \tag{17}$$

Majorana number

$$U = u \otimes I_N, \quad u = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix} \tag{18}$$

$$A_{ch} = -iU\mathcal{H}_{ch}U^\dagger \tag{19}$$

$$\mathcal{M} = \mathsf{sgn}[\mathsf{Pf}(A_{ch})] \tag{20}$$

Topological phase transition due to a linear vector potential

./figures/linear-chain-majorana-number-full-range.pdf

Topological phase transition due to a linear vector potential

./figures/linear-chain-majorana-number-full-range.pdf

to force Majorana fermions at the interface between differing topologies with large enough gaps.

- Double chain toy model:

$$\mathbf{A} = B\mathbf{x}$$

./figures/double-chain.pdf

$$\mathbf{A} = 0$$

Bulk-edge correspondence on a double chain

$\mathbf{A} = 0.16\pi\mathbf{x}$

./figures/double-chain.pdf

./figures/linear-chain-majorana-number-B-1_6.pdf

$\mathbf{A} = 0$

$\mu = 1.4t$

./figures/double-chain-mu-p1_4-B-0_16pi.pdf

Bulk-edge correspondence on a double chain

$\mathbf{A} = 0.16\pi\mathbf{x}$

./figures/linear-chain-majorana-number-B-1_6.pdf

./figures/double-chain.pdf

$\mathbf{A} = 0$

$\mu = 1.4t$

$\mu = 1.1t$

./figures/double-chain-mu-p1_4-B-0_16pi.pdf

./figures/double-chain-mu-p1_1-B-0_16pi.pdf

Bulk-edge correspondence on a double chain

$\mathbf{A} = 0.16\pi\mathbf{x}$

./figures/linear-chain-majorana-number-B-1_6.pdf

./figures/double-chain.pdf

$\mathbf{A} = 0$

$\mu = 1.4t$ | $\mu = 1.1t$ | $\mu = 0.7t$

./figures/double-chain-mu-1_4-B-0_16.pdf

./figures/double-chain-mu-1_1-B-0_16.pdf

./figures/double-chain-mu

Bulk-edge correspondence on a double chain

$\mathbf{A} = 0.15\pi\mathbf{x}$

./figures/double-chain.pdf

$\mathbf{A} = 0$

./figures/linear-chain-majorana-number-B-1_5.pdf

$\mu = 1.4t$

./figures/double-chain-mu-p1_4-B-0_15pi.pdf

Bulk-edge correspondence on a double chain

$\mathbf{A} = 0.15\pi\mathbf{x}$

./figures/linear-chain-majorana-number-B-1_5.pdf

./figures/double-chain.pdf

$\mathbf{A} = 0$

$\mu = 1.4t$

$\mu = 1.1t$

./figures/double-chain-mu-p1_4-B-0_15pi.pdf

./figures/double-chain-mu-p1_1-B-0_15pi.pdf

Bulk-edge correspondence on a double chain

$\mathbf{A} = 0.15\pi\mathbf{x}$

./figures/linear-chain-majorana-number-B-1_5.pdf

./figures/double-chain.pdf

$\mathbf{A} = 0$

$\mu = 1.4t$ $\mu = 1.1t$ $\mu = 0.7t$

./figures/double-chain-mu-1_4-B-0_15.pdf ./figures/double-chain-mu-1_1-B-0_15.pdf ./figures/double-chain-mu

Triangular *p*-wave superconductor with vector potential

p-wave superconductor Hamiltonian with vector potential

$$\mathcal{H} = \sum_{\langle j,l \rangle} \left[-te^{i\phi_{l,j}} c_l^\dagger c_j + \Delta e^{i\theta_{l,j}} c_l^\dagger c_j^\dagger + h.c. \right] - \sum_j \mu c_j^\dagger c_j, \tag{21}$$

where

$$\phi_{l,j} = -\frac{ie}{\hbar} \int_{\mathbf{r}_j}^{\mathbf{r}_l} \mathbf{A}(x) \cdot d\mathbf{l}. \tag{22}$$

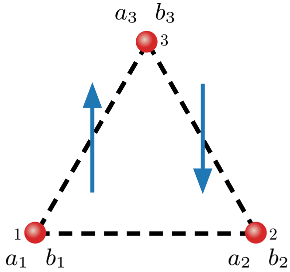
p-wave superconductor Hamiltonian in Majorana fermion basis

$$\begin{aligned} \mathcal{H} = & -\frac{i\mu}{4} \sum_j (a_j b_j - b_j a_j) \\ & -\frac{i}{2} \sum_{\langle j,l \rangle} [(t \sin \phi_{l.j} - \Delta \sin \theta_{l,j}) a_l a_j + (t \sin \phi_{l,j} + \Delta \sin \theta_{l,j}) b_l b_j \\ & +(t \cos \phi_{l,j} + \Delta \cos \theta_{l,j}) a_l b_j - (t \cos \phi_{l,j} - \Delta \cos \theta_{l,j}) b_l a_j]. \end{aligned} \tag{23}$$

Conditions for MZMs on a triangular island

Start with Kitaev limit $t = \Delta \neq 0$ and $\mu = 0$.

$$t(\sin\phi_{l,j} - \sin\theta_{l,j})a_l a_j, \quad (24)$$
$$t(\sin\phi_{l,j} + \sin\theta_{l,j})b_l b_j, \quad (25)$$
$$t(\cos\phi_{l,j} + \cos\theta_{l,j})a_l b_j, \quad (26)$$
$$t(\cos\phi_{l,j} - \cos\theta_{l,j})b_l a_j \quad (27)$$



- Need a_3a_1, b_3a_1, b_2a_3 and b_2b_3 to have zero weight.
- We find $\phi_{3,1} = \pi/3$ and $\phi_{2,3} = \pi/3$.
- This critical condition is met for odd function vector potentials. For a linear vector potential:

$$\mathbf{A}_0(x) = \frac{8\pi}{3\sqrt{3}a^2} x\hat{\mathbf{y}} = B_0 x\hat{\mathbf{y}}. \quad (28)$$

Triangular island

Extrapolate the critical vector strength from a 3-point triangle to a triangle with n_r rows.

$$B_0 = \frac{8\pi}{3\sqrt{3}a^2}\frac{1}{2n_r - 3} \tag{29}$$

$$\mathbf{A} = Bx\hat{\mathbf{y}}$$

../../../research-code/mf-quantum

./figures/vector-potential-field.pdf

../../../research-code/mf-quantum

Triangular Chain

../../../research-code/mf-quantum-logic-gate-scripts/data/figures/linear-

../../../research-code/mf-quantum-logic

Hollow Triangle

../../../research-code/mf-quantum-logic-gate-scripts/data/figures/linear-

../../../research-code/mf-quantum-logi

Summary

- Triangular islands with a gapped interior can be a promising platform for hosting and manipulating MZMs.
- Next steps
  - Search for robust MZMs in hollow triangles outside the Kitaev limit using a Topological phase diagram.
  - Reapply the methodology for a Rashba SC heterostructure.
  - Develop a practical braiding scheme.

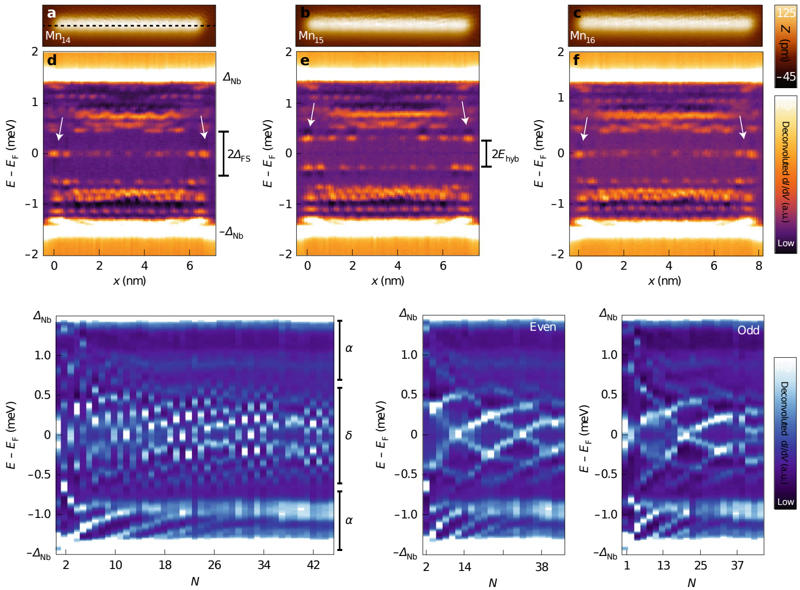
./figures/linear-chain-majorana-number-full-range.pdf

/mf-quantum-logi

Additional projects

- Using a semi-infinite tight binding model to find Floquet Landau levels for Graphene and 2DEGs using two linearly polarized lights.
- Kitaev mapped spins to fermions using the Jordan-Wigner transformation. Can we achieve similar results using one of our triangular structures?

Additional results from Schneider et al.



Majorana fermion notation and coupling isolations

The complex fermion operator can be written as a superposition of two Majorana fermions $c_j = \frac{1}{2}(a_j + ib_j)$. Due to the nature of Majorana fermions, $a_j^\dagger = a_j$, the creation operator is $c_j^\dagger = \frac{1}{2}(a_j - ib_j)$.

$$H = -\frac{i\mu}{4}\sum_j (a_j b_j - b_j a_j) - \frac{i}{4}\sum_{<j,l>}[(t\sin\phi - \Delta\sin\theta)a_l a_j + (t\sin\phi + \Delta\sin\theta)b_l b_j + (t\cos\phi + \Delta\cos\theta)a_l b_j - (t\cos\phi - \Delta\cos\theta)b_l a_j].$$

$$(t\sin\phi_{j,l} - \Delta\sin\theta_{j,l})a_l a_j, \tag{30}$$
$$(t\sin\phi_{j,l} + \Delta\sin\theta_{j,l})b_l b_j, \tag{31}$$
$$(t\cos\phi_{j,l} + \Delta\cos\theta_{j,l})a_l b_j, \tag{32}$$
$$(t\cos\phi_{j,l} - \Delta\cos\theta_{j,l})b_l a_j \tag{33}$$

Triangular chain degeneracy

../../../research-code/mf-quantum-logic-gate-scripts/data/figures/

Hollow triangle degeneracy?

../../../research-code/mf-quantum-logic-gate-scripts/data/figures/