Majorana Corner Modes in Triangular Superconductor Islands

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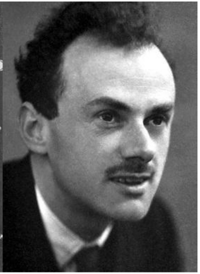
March 18, 2022

Outline

- Background:
  - Majorana fermions in particle physics
  - Majorana fermions in condensed matter
- Motivation:
  - P-wave superconductors and braiding
  - 1D wires and T-junctions
  - Triangular structures for braiding
- Formulation:
  - Peierls substitution in Kitaev's model
- Results:
  - Topological phase
  - Triangular chain
  - Hollow triangles
- Summary
  - Future work
  - Additional projects

Background

Enrico Fermi



Paul Dirac

Ettore Majorana

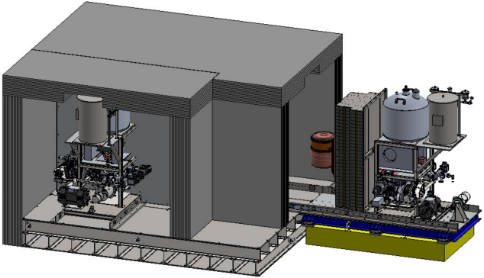
(Dirac) Fermion

- Fermi statistics
- Particle $\neq$ Antiparticle : $c \neq c^\dagger$
- Charged
- Electron

Majorana Fermion

- Fermi statistics
- Particle $=$ Antiparticle : $c = c^\dagger$
- Neutral
- Neutrino? Dark Matter?

Background

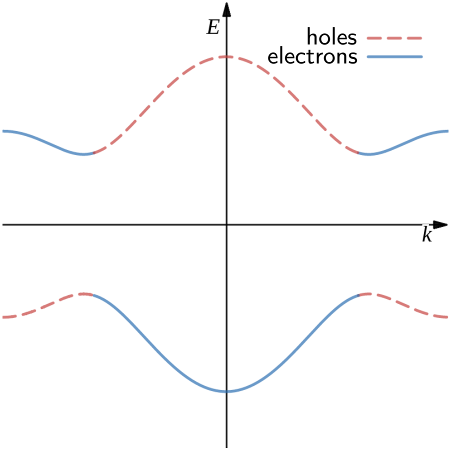


MAJORANA project:
neutrinoless double beta ($0\nu\beta\beta$) decay

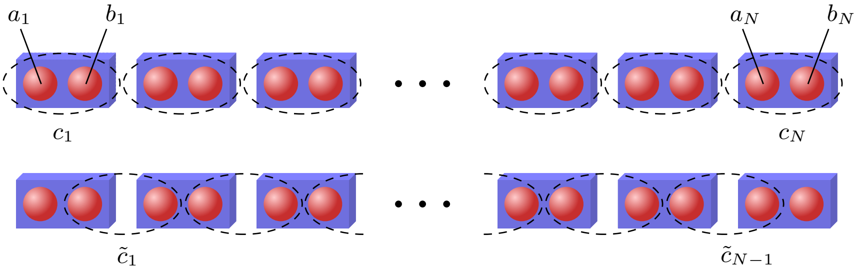
- Are neutrinos Majorana fermions?
- If yes, the standard needs revision
- Negative results for Majorana particles

Background

- Superconductors
  - Cooper pairs
    - Electron-phonon interaction pairs two electrons with opposite spin and momenta.
  - Bogoliubov quasiparticles
    - Excitation from ground state, pairs an electron to a hole.
    - Zero-energy excitations may be Majorana fermions.
    - If so, they come in pairs.



Background



Kitaev, Phys, Uspekhi **44**, 131 (2001)

Majorana fermion notation

$$c_j = \frac{1}{2}(a_j + ib_j). \tag{1}$$

Background

Hamiltonian for a 1D tight-binding chain with spinless *p*-wave superconductivity

$$\mathcal{H}_{chain} = -\mu \sum_{j}^{N} c_j^\dagger c_j - \sum_{j}^{N-1} t c_j^\dagger c_{j+1} + |\Delta| c_j c_{j+1} + h.c. \tag{2}$$

Hamiltonian in Majorana basis

$$\mathcal{H}_{chain} = \frac{i}{2} \sum_{j} -\mu a_j b_j + (t + |\Delta|) b_j a_{j+1} + (-t + |\Delta|) a_j b_{j+1}. \tag{3}$$

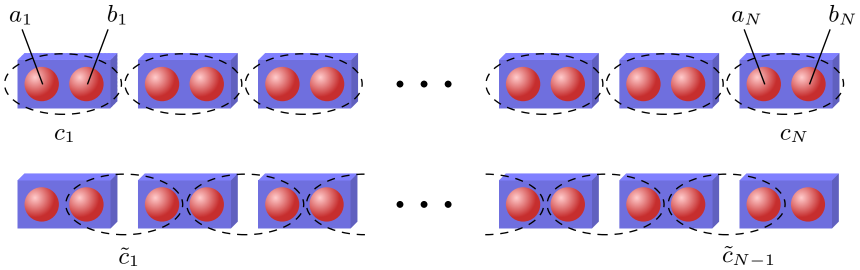
$t = |\Delta| = 0$ and $\mu < 0$, trivial phase

$$\mathcal{H} = -\frac{i\mu}{2} \sum_{j} a_j b_j. \tag{4}$$

$t = |\Delta| > 0$ and $\mu = 0$, non-trivial (topological) phase

$$\mathcal{H} = it \sum_{j} b_j a_{j+1}. \tag{5}$$

Background



Kitaev, *Phys. Uspekhi* **44**, 131 (2001)

Intersite fermion representation

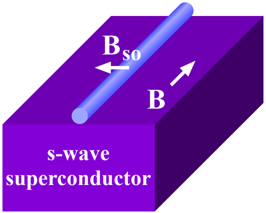
$$\tilde{c}_j = \frac{1}{2}(a_{j+1} + ib_j). \tag{6}$$

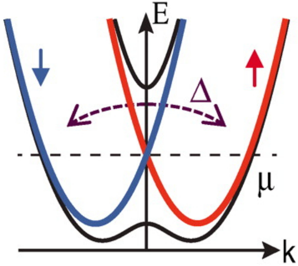
The highly non-local fermion state

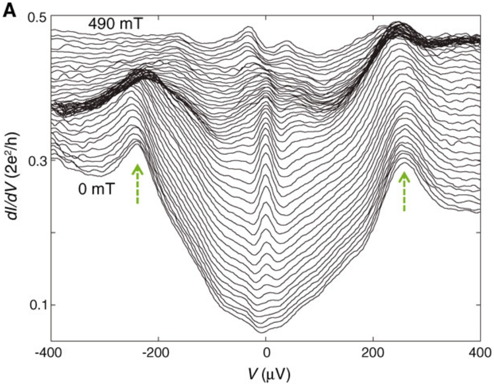
$$f = \frac{1}{2}(a_1 + ib_N), \tag{7}$$

corresponds to zero energy. This is still true for $|\mu| < 2t$

Background

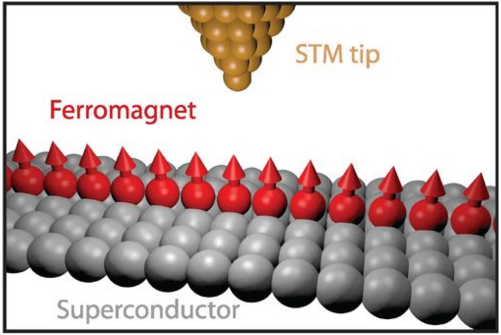


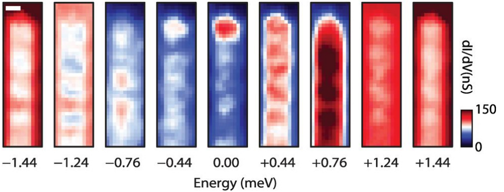




Mourik, *Science* **336**, 1003 (2012).

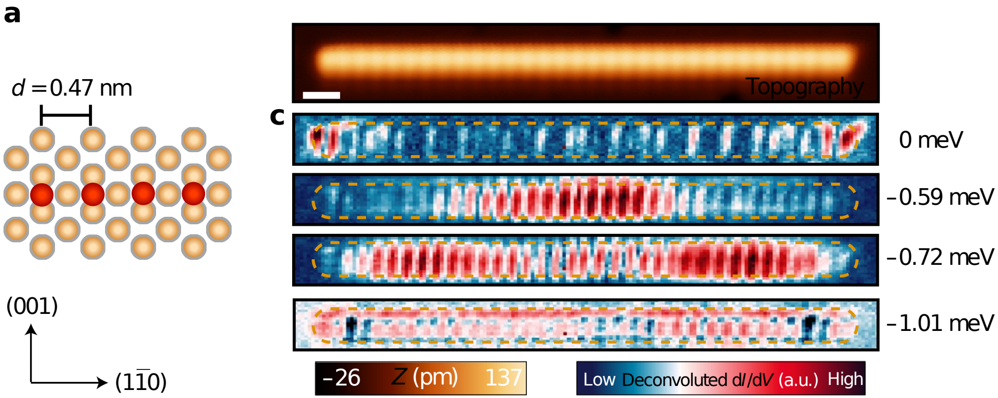
Background





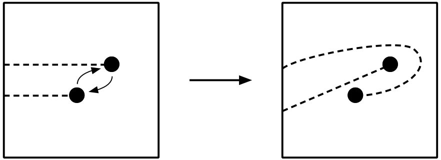
Nadj-Perge, *Science* **346**, 602 (2014).

Background



Mn atoms (red spheres) on top of superconducting Nb (brown spheres).
Schneider et al., *Nature Nanotechnology* **17**, 384 (2022).

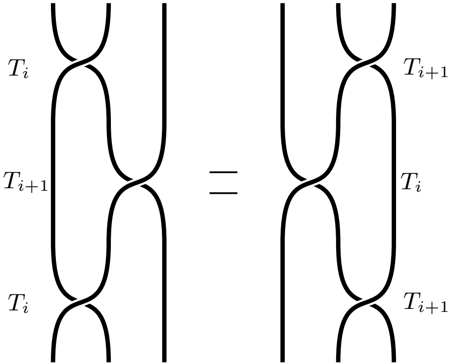
Motivation



$$T_i : \begin{cases} \gamma_i \to \gamma_{i+1} \\ \gamma_{i+1} \to -\gamma_i \\ \gamma_j \to \gamma_j \qquad \text{for } j \text{ and } j+1 \end{cases}$$

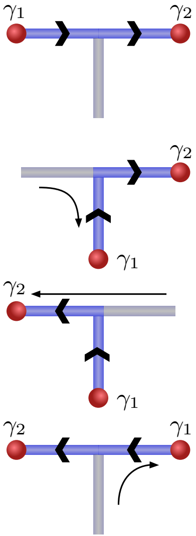
$$\tau(T_i)\gamma_j[\tau(T_i)]^{-1} = T_i(\gamma_j)$$

$$\tau(T_i) = \exp\left(\frac{\pi}{4}\gamma_{i+j}\gamma_i\right) = \frac{1}{\sqrt{2}}(1 + \gamma_{i+j}\gamma_i)$$



Ivanov, *PRL* **86**, 268 (2000).

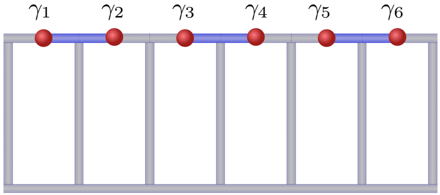
Motivation



$$\mathcal{H}_T = -\mu \sum_j c_j^\dagger c_j - \sum_j t c_j^\dagger c_{j+1} + |\Delta| e^{i\phi} c_j c_{j+1} + h.c. \tag{8}$$

$$c_j = e^{-i(\phi/2)}(\gamma_{j+1,1} + i\gamma_{j,2})/2 \tag{9}$$

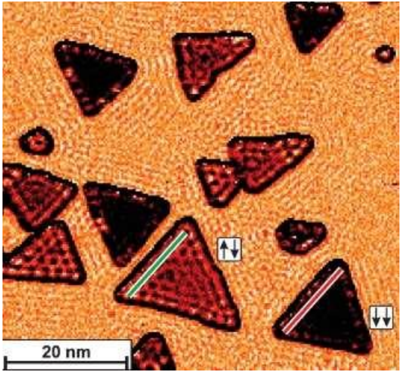
- Take pairing term $|\Delta|e^{i\phi}c_j c_{j+1}$ such that the site indices:
- Increase moving $\rightarrow$ / $\uparrow$ in the horizontal/vertical wires: $\phi = 0$,
- Decrease moving $\leftarrow$ / $\downarrow$ in the horizontal/vertical wires: $\phi = \pi$.



Alicea, *Nature Phys.* **7**, 412 (2011)

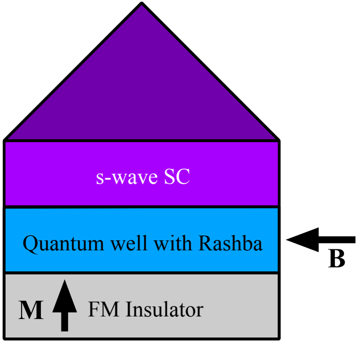
Motivation

- Consider triangular islands, topologically similar to T-junctions.
- Islands of three-fold rotational symmetry occur naturally in epitaxial growth on close-packed metal surfaces.
- Good platform to transition from 2D to 1D topological superconductor.



Triangular Co islands on Cu(111).
Pietzsch et al., *PRL* **96**, 237203 (2006)

Previous Work



Alicea, *PRB* **81**, 125318 (2010).

$$c_j = (c_{j\uparrow}, c_{j\downarrow})^T \tag{10}$$

s-wave SC paring term:

$$\mathcal{H}_{SC} = \sum_j \Delta c^\dagger_{j\uparrow} c^\dagger_{j\downarrow} + h.c. \tag{11}$$

Quantum well:

$$\mathcal{H}_0 = \sum_j (6t - \mu) c^\dagger_j c_j - \sum_{\langle j,l \rangle} (t c^\dagger_l c_j + h.c.) \tag{12}$$

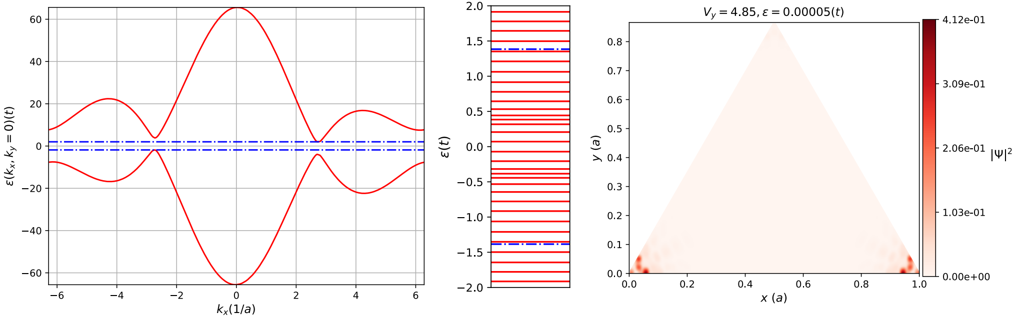
Rashba spin-orbit coupling:

$$\mathcal{H}_R = -it_R \sum_{\langle j,l \rangle \alpha\beta} c^\dagger_{l\alpha} (\boldsymbol{\sigma}_{\alpha\beta} \times \hat{\mathbf{r}}_{lj}) \cdot \hat{\mathbf{z}} c_{j\beta} \tag{13}$$

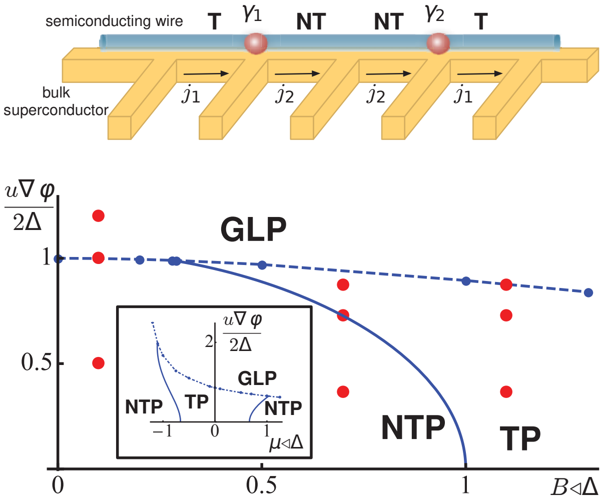
Zeeman field:

$$\mathcal{H}_Z = \sum_j c^\dagger_j \mathbf{V} \cdot \boldsymbol{\sigma} c_j \tag{14}$$

Previous Work

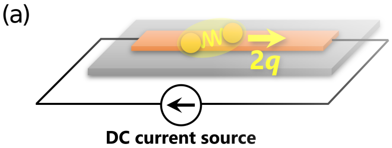


Topological phase transition induced by a supercurrent

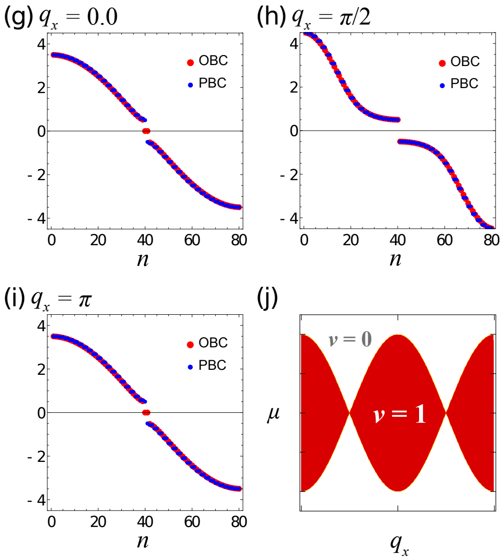


Romito, *PRB* **85**, 020502(R) (2012).

Topological phase transition induced by a supercurrent



Takasan, *PRB* **106**, 014508 (2022).



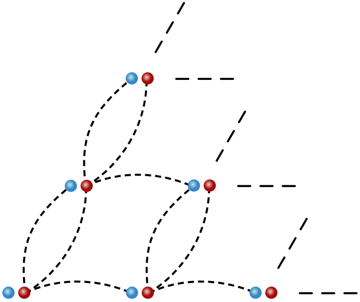
Kitaev chain with vector potential

Peierls substitution

$$
\begin{aligned}
c^{\dagger}_{j+1}c_j &\rightarrow c^{\dagger}_{j+1}c_j \exp\left(-\frac{ie}{\hbar}\int_{r_j}^{r_{j+1}} \mathbf{A}\cdot d\mathbf{l}\right) \\
&\rightarrow c^{\dagger}_{j+1}c_j e^{i\phi_{j+1,j}}.
\end{aligned} \tag{15}
$$

$$
\mathcal{H}_{chain} = \sum_j (-te^{i\phi_{j+1,j}}c^{\dagger}_{j+1}c_j + \Delta c^{\dagger}_{j+1}c^{\dagger}_j + h.c.) - \mu c^{\dagger}_j c_j. \tag{16}
$$

Kitaev Limit with Vector Potential on a Triangular Island



$$\mathcal{H} = \sum_{<j,l>} \left[-te^{i\phi_{l,j}} c_l^\dagger c_j + \Delta e^{i\theta_{l,j}} c_l^\dagger c_j^\dagger + h.c. \right] - \sum_j \mu c_j^\dagger c_j$$

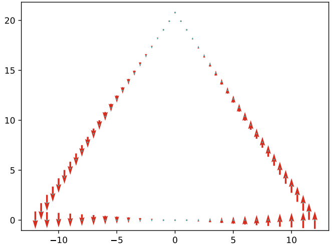
$$\phi_{l,j} = -\frac{e}{\hbar} \int_{\mathbf{r}_j}^{\mathbf{r}_l} \mathbf{A} \cdot d\mathbf{l}$$

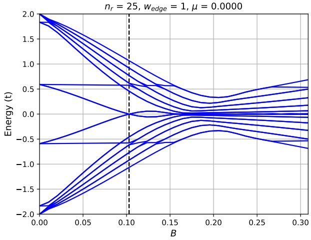
$$\mathbf{A} = -\frac{2\pi}{3\sqrt{3}a} \hat{\mathbf{y}}$$

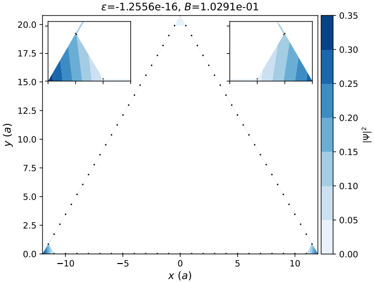
Triangular Chain

$$\mathbf{A}(x) = Bx\hat{\mathbf{y}}$$

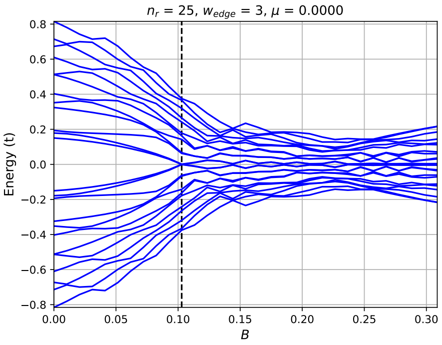
$$B_0 = \frac{8\pi}{3\sqrt{3}a^2(2n_r - 3)}$$

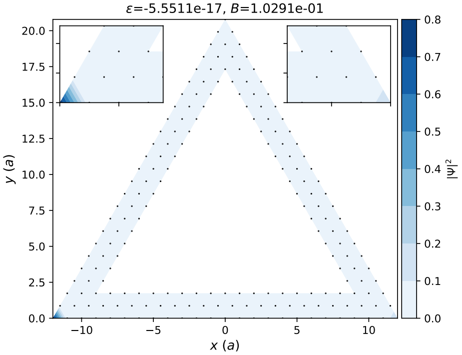






Hollow Triangle

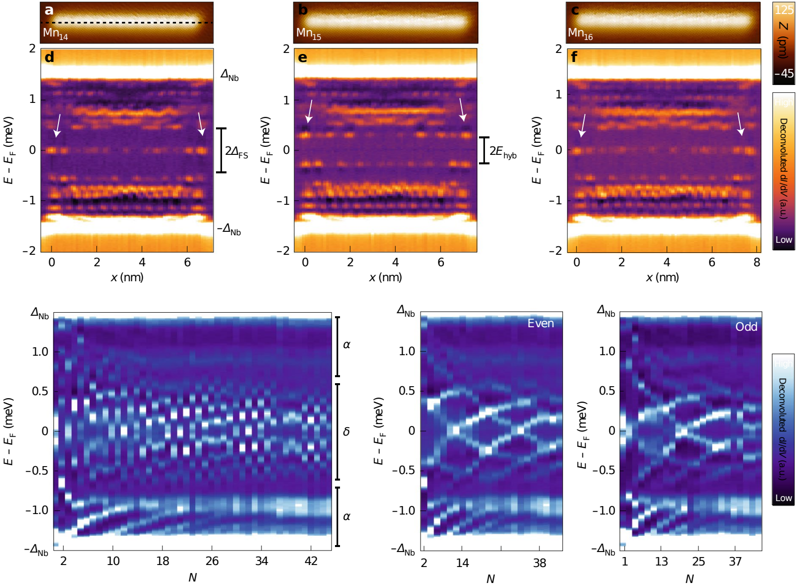




Summary

- Introduction of vector potential allows for additional tunability of topology.
- Triangular islands with a gapped interior can be a promising platform for hosting and manipulating MZMs.
- Next steps
  - Search for safe MZMs in hollow triangles outside the Kitaev limit.
  - Develop a robust braiding scheme.

Additional results from Schneider et al.



Majorana fermion notation and coupling isolations

The complex fermion operator can be written as a superposition of two Majorana fermions $c_j = \frac{1}{2}(a_j + ib_j)$. Due to the nature of Majorana fermions, $a_j^\dagger = a_j$, the creation operator is $c_j^\dagger = \frac{1}{2}(a_j - ib_j)$.

$$H = -\frac{i\mu}{4}\sum_j (a_j b_j - b_j a_j) - \frac{i}{4}\sum_{<j,l>}[(t\sin\phi - \Delta\sin\theta)a_l a_j + (t\sin\phi + \Delta\sin\theta)b_l b_j + (t\cos\phi + \Delta\cos\theta)a_l b_j - (t\cos\phi - \Delta\cos\theta)b_l a_j].$$

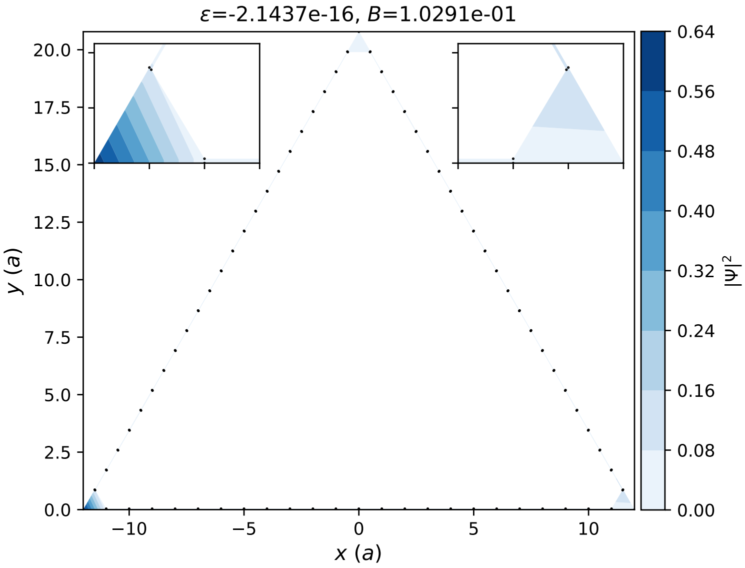
$$(t\sin\phi_{j,l} - \Delta\sin\theta_{j,l})a_l a_j, \tag{17}$$

$$(t\sin\phi_{j,l} + \Delta\sin\theta_{j,l})b_l b_j, \tag{18}$$

$$(t\cos\phi_{j,l} + \Delta\cos\theta_{j,l})a_l b_j, \tag{19}$$

$$(t\cos\phi_{j,l} - \Delta\cos\theta_{j,l})b_l a_j \tag{20}$$

Triangular chain degeneracy



Hollow triangle degeneracy?