Superconducting Triangular Islands as a Platform for Manipulating Majorana Zero Modes

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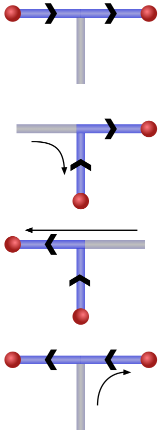
March 8, 2024

Motivation

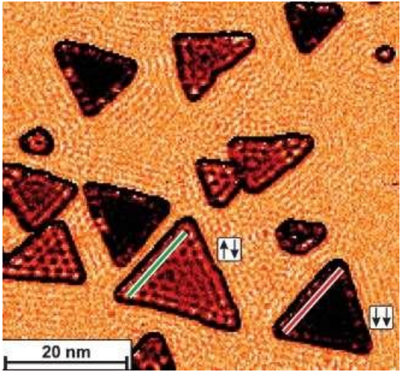
- P-wave superconductors contain half-quantum vortices.
  - Majorana fermions located at core of a vortex.
  - Braiding vortices exhibits Non-Abelian statistics.
- 1D p-wave superconductors host Majorana fermions on end points.
  - Measurements in real systems:
    V. Mourik, *Science* **336**, 1003 (2012)
    S. Nadj-Perge, *Science* **346**, 602 (2014)
    L. Schneider, *Nat. Nanotechnol.* **17**, 384 (2022)
- Quasi-1D T-junction
  - Braiding of Majorana fermions is defined for 2D.
  - In practice challenging to make, but still feasible and seriously pursued.



Alicea, *Nature Phys.* **7**, 412 (2011)

Motivation

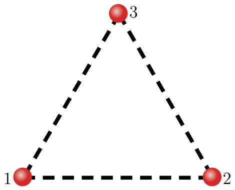
- Consider triangular islands, topologically similar to T-junctions.
- Islands of three-fold rotational symmetry occur naturally in epitaxial growth on close-packed metal surfaces.
- Good platform for transition from 2D to 1D topological superconductor.



Triangular Co islands on Cu(111).
Pietzsch et al., *PRL* **96**, 237203 (2006)

Kitaev Triangle

$$\mathcal{H} = \sum_{\langle j,l \rangle} \left[-t e^{i\phi_{jl}} c_j^\dagger c_l + \Delta e^{i\theta_{jl}} c_j c_l + h.c. \right] - \sum_j \mu c_j^\dagger c_j$$



$$\begin{aligned}(\phi_{12}, \phi_{23}, \phi_{31}) &= \left(0, -\frac{\pi}{3}, -\frac{\pi}{3}\right) = \boldsymbol{\phi}_1 \\ &\rightarrow \left(-\frac{\pi}{3}, -\frac{\pi}{3}, 0\right) = \boldsymbol{\phi}_2 \\ &\rightarrow \left(-\frac{\pi}{3}, 0, -\frac{\pi}{3}\right) = \boldsymbol{\phi}_3 \\ &\rightarrow \boldsymbol{\phi}_1\end{aligned}$$

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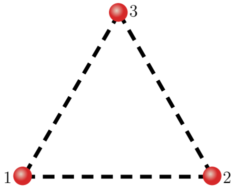

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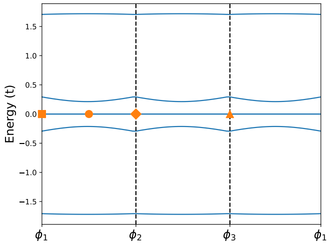
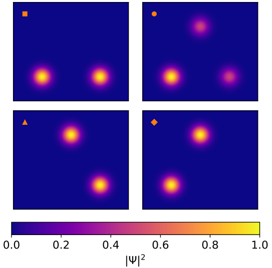
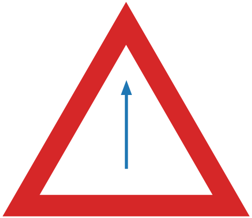
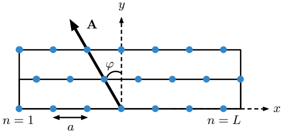




Triangular Ribbon and Topological Phases

$$\phi_{jl} = \frac{e}{\hbar} \int_{\mathbf{r}_j}^{\mathbf{r}_l} \mathbf{A} \cdot d\mathbf{l} = -\phi_{lj}$$





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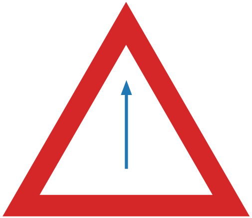

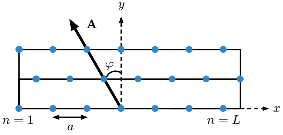

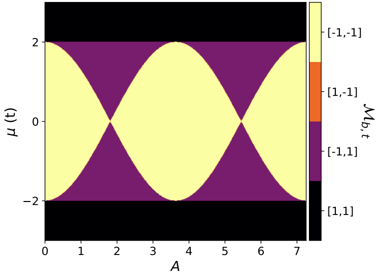

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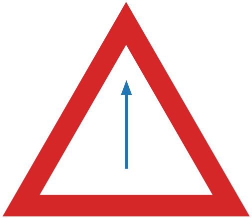
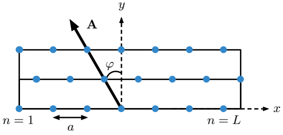
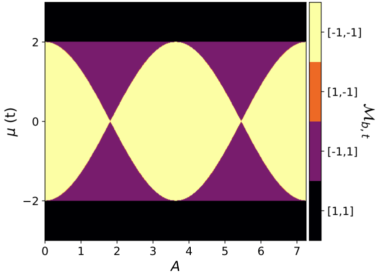
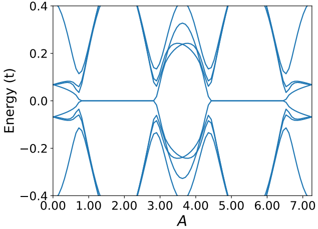
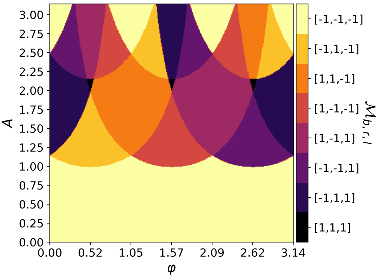








Rotating MZMs on a Triangular Chain (W=1)



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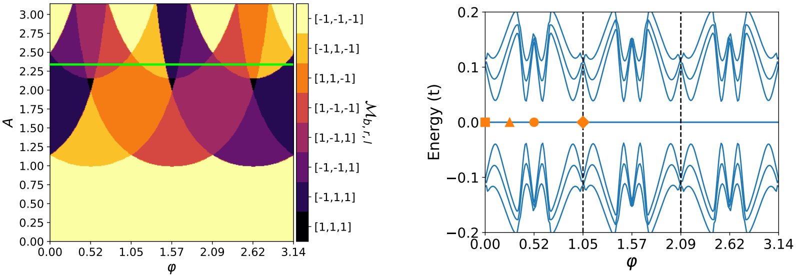

$$L = 50,\ W = 1,\ \mu = 1.1,\ A = 2.35$$

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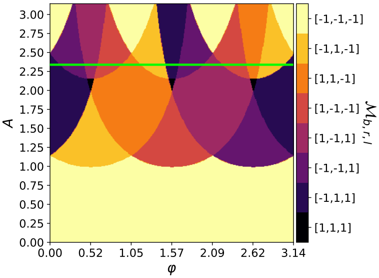

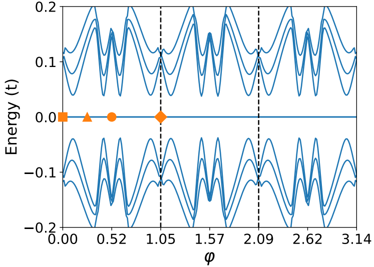

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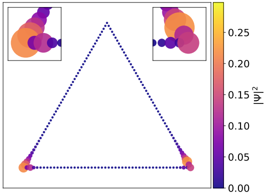

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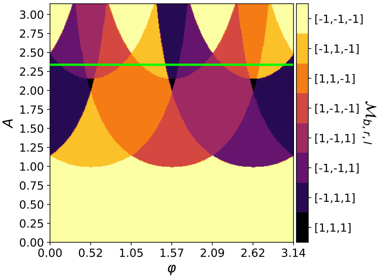

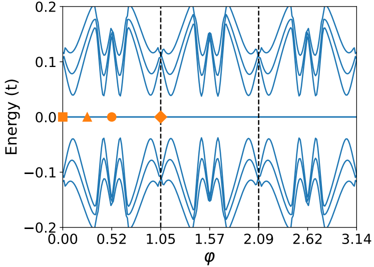

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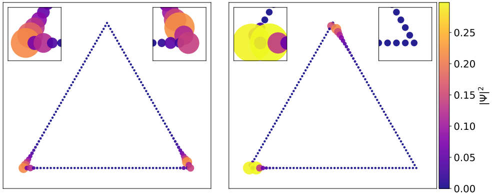

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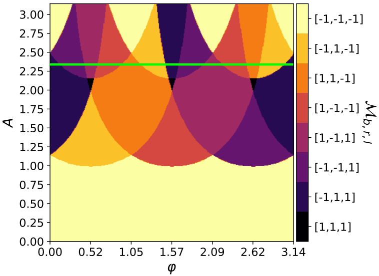

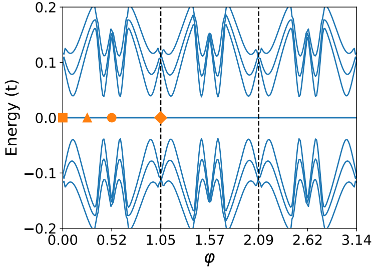

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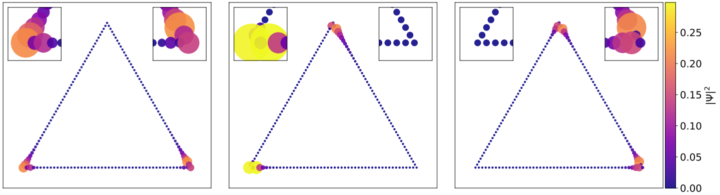

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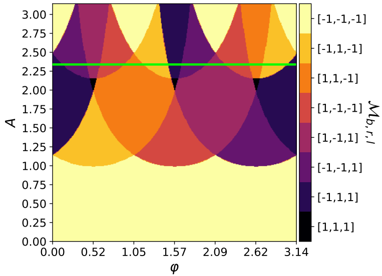

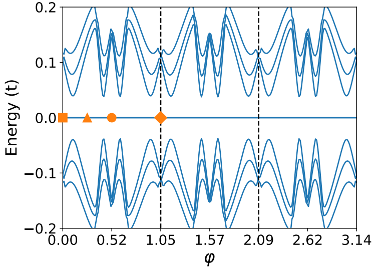

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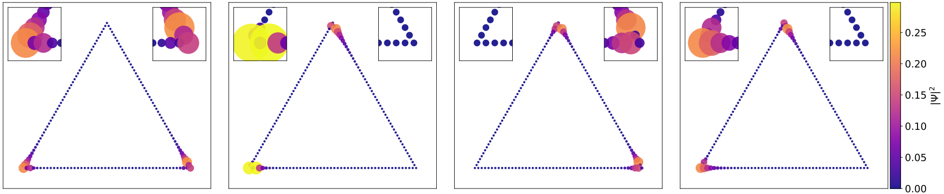

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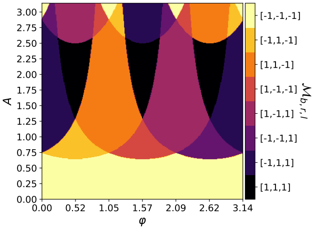

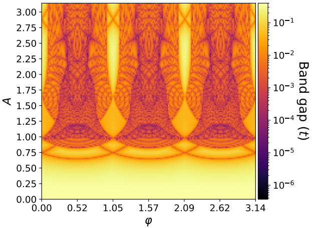

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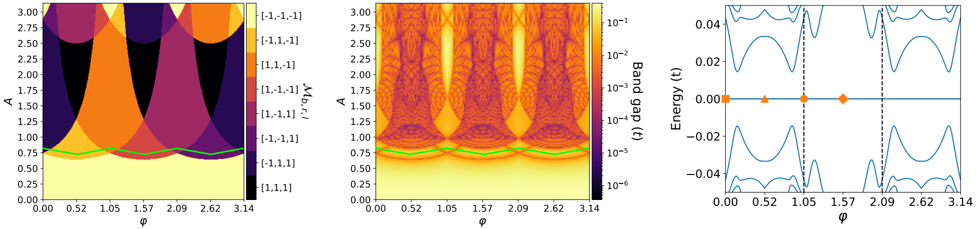

$$L = 80,\ W = 3,\ \mu = 1.6,\ (A, \varphi) = (0.83, 0) \rightarrow (0.77, \pi/6) \rightarrow (0.83, \pi/3) \rightarrow \ldots$$

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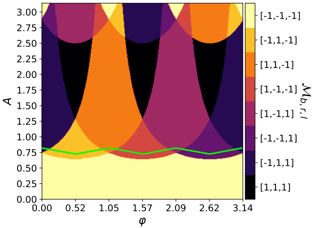

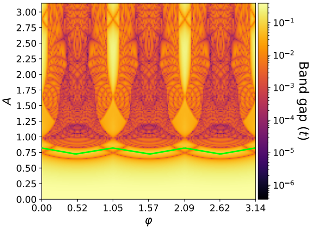

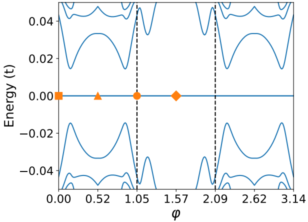

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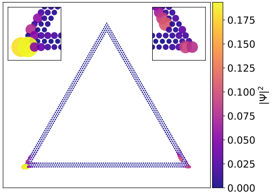

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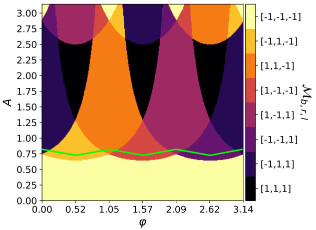

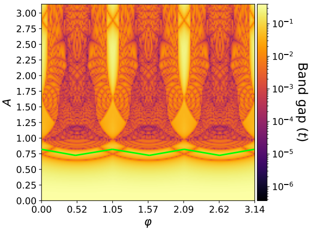

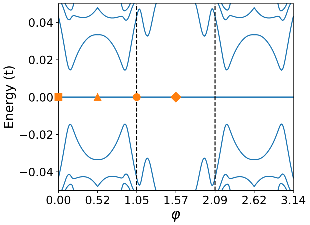

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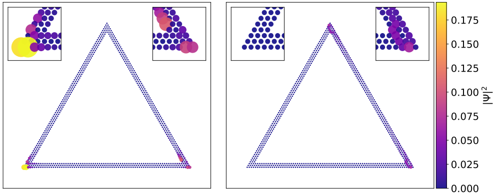

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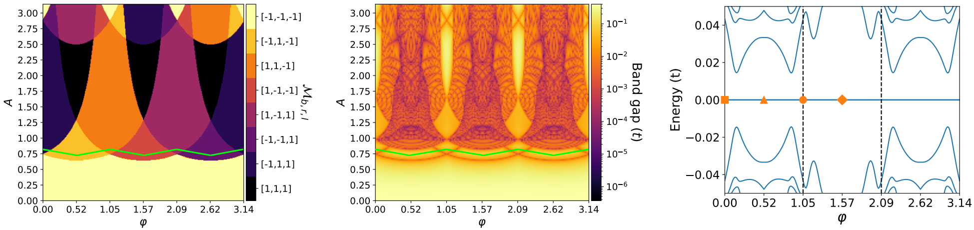

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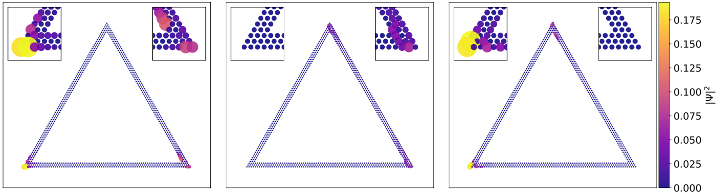

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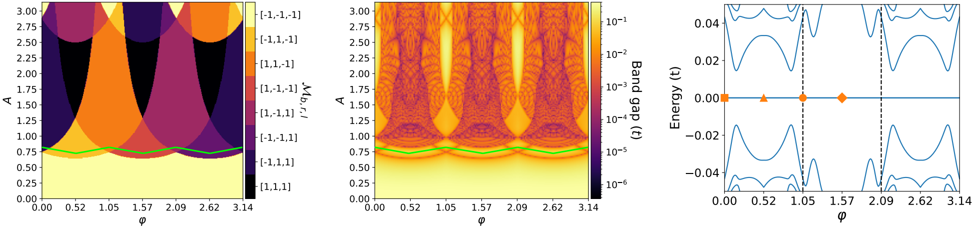

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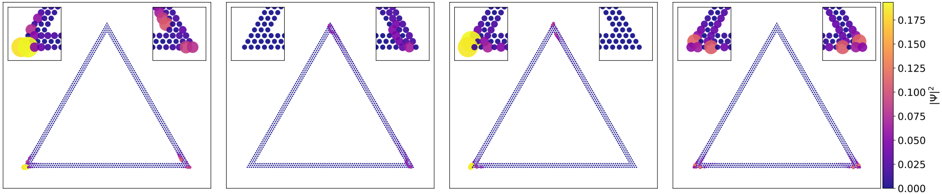
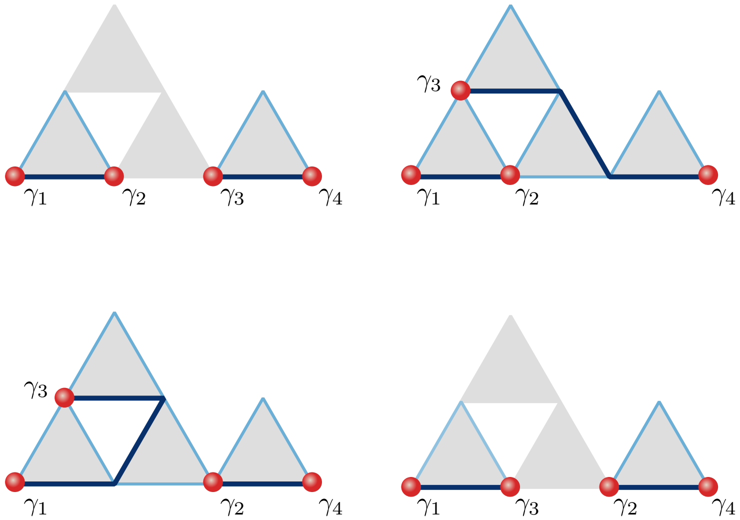


Braiding Two of Four MZMs



Summary

- Introduction of Peierls phase allows for a minimal Kitaev triangle, reducing fermionic sites down to 3.
- Vector potential field and its rotation allows additional tunability of topology.
- MZMs can be hosted and braided on a network of triangular islands.

Majorana fermion notation and coupling isolations

The complex fermion operator can be written as a superposition of two Majorana fermions $c_j = \frac{1}{2}(a_j + ib_j)$. Due to the nature of Majorana fermions, $a_j^\dagger = a_j$, the creation operator is $c_j^\dagger = \frac{1}{2}(a_j - ib_j)$.

$$
\begin{aligned}
H = -\frac{i\mu}{2}\sum_j a_j b_j - \frac{i}{2}\sum_{\langle jl\rangle}[(t\sin\phi_{jl} - \Delta\sin\theta_{jl})a_l a_j + (t\sin\phi_{jl} + \Delta\sin\theta_{jl})b_l b_j \\
+(t\cos\phi_{jl} - \Delta\cos\theta_{jl})a_l b_j - (t\cos\phi_{jl} + \Delta\cos\theta_{jl})b_l a_j].
\end{aligned}
$$

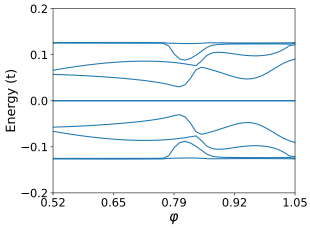
$$(t\sin\phi_{jl} - \Delta\sin\theta_{jl})a_l a_j, \tag{1}$$

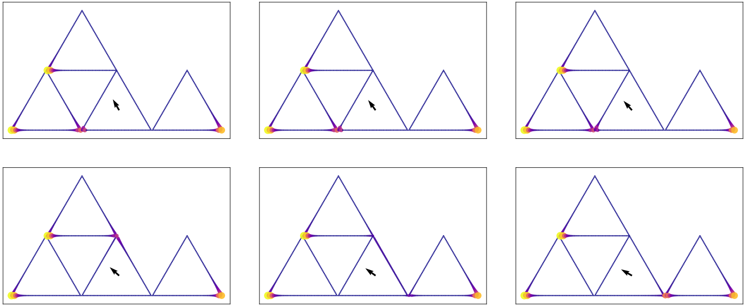
$$(t\sin\phi_{jl} + \Delta\sin\theta_{jl})b_l b_j, \tag{2}$$

$$(t\cos\phi_{jl} + \Delta\cos\theta_{jl})a_l b_j, \tag{3}$$

$$(t\cos\phi_{jl} - \Delta\cos\theta_{jl})b_l a_j \tag{4}$$

Braiding MZM in a Small Network of Triangles





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