

5. b) Given an orthonormal basis for $L_2([0,1])$, describe how to construct an orthonormal basis for $L_2([0,1]^D)$ - the space of functions of D continuous-valued variables $x(t)$ such that

$$\int_0^1 \dots \int_0^1 |x(t)|^2 dt_1 \dots dt_D < \infty.$$

Based on our results from 5a) we have shown that constructing an orthonormal basis for $L_2([0,1]^2)$ can be done as a composition of orthonormal functions on the $L_2([0,1])$ space. Specifically $v_{k,l} = \psi_k(s)\psi_l(t)$ is an orthonormal basis for $L_2([0,1]^2)$. Extending this logic, $v_{k,l,\dots,D}$ can be an orthonormal basis for $L_2([0,1]^D)$ where

$$v_{k,l,\dots,D} = \prod_{i=1}^D \psi_i(t_i).$$

Essentially there is an orthonormal function corresponding to each input dimension.