Algorithms on Strings

Problems Set 2

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Proof. Since we know that $|t| \leq 2|p|$, each pair of the pattern occurrences has non-empty intersection. Assume $q_1, q_2, \ldots, q_n$ - all the occurrences of p in t

1. $|t| < 2|p|$
   Assume $q_1 = 0$. Consider two occurrences of p in t - q_1, q_2. By the above fact there is a subword w having a border $|p|$. By the lemma from the Problem Set 1, we conclude, that q_2 - periods of w, so, especially, q_1, q_2 - periods of p. Considering all possible pairs of q_i, q_j gives us $q_1, \ldots, q_n$ - periods of p. By the another lemma from the Problem Set 1, $gcd(q_i, q_j)$ - period of p. So $gcd(q_2, \ldots, q_n) = d$ is also a period of p and hence
   $$p = u^k v \tag{1}$$
   where $|d| = u$ and
   $$q_i = \alpha d$$
   for all $i \in \{1, \ldots, n\}$. Easy to prove, that $q_i = id$. Consider the first occurence such that $q_i = (i+k)d$, $k > 0$. Then there should be a mismatch at id, what stays in contradiction to matches at q_{i-1}, q_i and the form 1.
   For $q_1 \neq 0$ we need to subtract q_1 everywhere above.

2. $|t| = 2|p|$
   Trivial, there are only at most two occurrences of p in t.

□

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Proof. Induction by n.
We can write the thesis in an equivalent way:

$$f_n f_{n+1} = c(f_{n+1} f_n)$$

Observation:

$$c(ab) = ac(b) \ \textit{if} \ |b| > 1 \tag{2}$$

Induction 1 (Base)**.**

$$\begin{aligned} f_1 f_2 &= c(f_2 f_1) \\ f2_1 f_3 &= c(f_3 f_2) \end{aligned} \tag{3}$$

Proof.

$$\begin{aligned} ba &= c(ab) \\ bab &= c(bba) \end{aligned} \tag{4}$$

$\square$

Induction 2 (Step)**.**

$$f_n f_{n+1} = c(f_{n+1} f_n \implies f_{n+1} f_{n+1} = c(f_{n+2} f_{n+1}) \tag{5}$$

Proof.

$$\begin{aligned} f_{n+1} f_{n+2} &= f_{n+1} f_{n+1} f_n \\ &= f_{n+1} f_n f_{n-1} f_n \\ &= f_{n+2} f_{n-1} f_n \\ &\text{induction} \\ &= f_{n+2} c(f_n f_{n-1}) \\ &\text{observation 1} \\ &= f_{n+2} c(f_{n+1}) \\ &= c(f_{n+2} f_{n+1}) \end{aligned} \tag{6}$$

$\square$

$\square$