Algorithms on Strings

Problems Set 1

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1

Proof.

$$\begin{aligned}
|w| - p \textit{ is a border } &\equiv w[1, |w| - p] = w[|w| - (|w| - p), |w|] \\
&\equiv w[1, |w| - p] = w[p, |w|] \\
&\equiv w[i] = w[i + p] \textit{ for } i \in \{1, 2, \dots, |w| - p\} \\
&\equiv \ p \textit{ is a period of } w
\end{aligned} \tag{1}$$

□

2

2.1

Proof.

$$\begin{aligned}
p \textit{ is a period of } w &\equiv w \textit{ is a subword of some } x^k \textit{ with } |x| = p \textit{ and } k > 0 \\
&\equiv w = ux^j v \textit{ where } j \leq k, ux \textit{ and } vx
\end{aligned} \tag{2}$$

Since u and v - suffix and prefix of x, respectively, we can proceed as follows:

$$w = ux^j v = u(zu)^j v = (uz)^j uv \tag{3}$$

Hence, the period condition holds for i $\in \{1, 2, \dots, |w| - p - |v|\}$. On the other hand:

$$w = ux^j v = ux^{j-1} vyv \tag{4}$$

So the period condition holds for i $\in \{|w| - p - |v|, |w| - p - |v| + 1, \dots, |w| - p\}$. From 3 and 4 the proof is done. □

2.2

Proof. Assume $|w| = kp + l$. From the period condition we have:

$$w[1,p] = w[p+1,2p] = \cdots = w[(k-1]p+1,kp] \tag{5}$$

and

$$w[kp+1,kp+l] = w[(k-1)p+1,(k-1)p+l] \tag{6}$$

Basing on 5, we can write $w = y^k u$, such that $|y| = p$ and $|u| = l$. On the other hand, by 6, we have $w = xuvu$, where $|u| = l$, $|v| = p - l$, $|x| = |w| - p - l$. Combining these two conclusions, $y = uv$, so $w = (uv)^k u$. ∎

2.3

Proof.

$$\begin{aligned} p \textit{ is a period of } w &\equiv \text{—}w\text{— - } p \textit{ is a border of } w \textit{ (from the 1.)} \\ &\equiv \exists_{x,\ y,\ z}(|y| = |w| - p \land xy = yz = w) \\ &\equiv \exists_{x,\ y,\ z}(|x| = |y| = p \land xy = yz = w) \end{aligned} \tag{7}$$

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