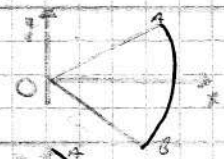
Laborator 3 - Centre de greutate

~~20~~ 27.10.2021

Centrul de greutate este sistemul de forțe paralele format din greutățile $\vec{G}_i$, $i = \overline{1,n}$ ale punctelor din care se compune corpul.

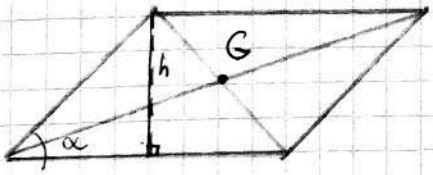
- Pasul 1. Se împarte corpul complex în corpuri simple, ale căror centre de greutate sunt mai ușor de aflat.

I. Bare

dreaptă		$OA = l$	$x_G = l/2$	$y_G = 0$
arc circular				$y_c = 0$
semicirculară		$AB = \pi \cdot R$	$x_C = R$	$y_c = \frac{2R}{\pi}$
sfert de cerc				$y_c = 0$

II. Plăci

- paralelogram (dreptunghi)



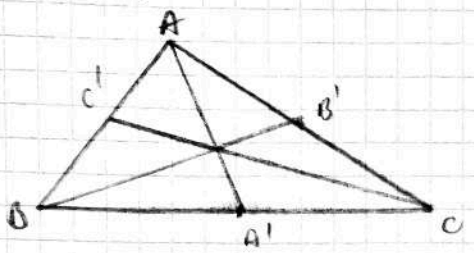
$$A = b \cdot h$$

$$\begin{cases} x_G = \frac{b + h \cdot \cos\alpha}{2} \\ y_G = \frac{h}{2} \end{cases}$$

$$A = l \cdot L$$

$$\begin{cases} x_G = \frac{L}{2} \\ y_G = \frac{l}{2} \end{cases}$$

- triunghi



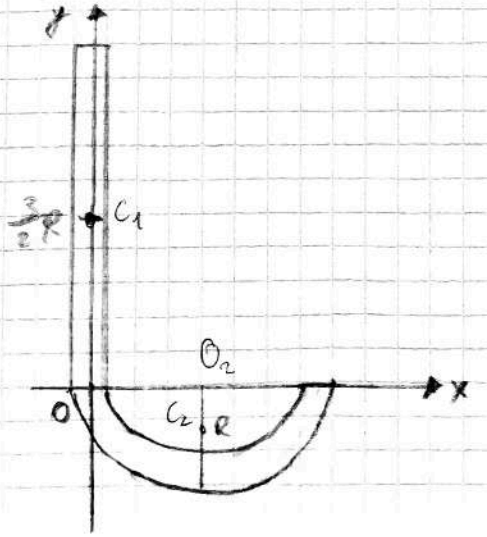
$$A = \frac{b \cdot h}{2}$$

$$\begin{cases} x_G = \frac{x_A + x_B + x_C}{3} \\ y_G = \frac{y_A + y_B + y_C}{3} \end{cases}$$

- Pasul 2. Se alege un sist. de coordonate
- Pasul 3. Se determină poz. c.d.g. pentru corpurile simple și se calcul. elem. geometrice ale acestora
- Pasul 4. Se completează tabelul

Nr. d.	G_i	x_i	y_i	z_i	$G_i \cdot x_i$	$G_i \cdot y_i$	$G_i \cdot z_i$
$\sum$	$\sum$	$\sum$	$\sum$	$\sum$	$\sum$	$\sum$	$\sum$

Problema 1



Pasul 1: obținem o bară verticală cu $l = 3R$ și un semicerc de rază R

Pasul 2: Sistem xOy

Pasul 3:

$$C_1 = \begin{cases} x_{C_1} = 0 \\ y_{C_1} = \frac{3}{2}R \end{cases}$$

$C_1 \in$ [illegible] $3R$

$C_2 = \begin{cases} x_{C_2} = R \\ y_{C_2} = -\frac{2R}{\pi} \end{cases} \Rightarrow O_2C_2 = R\frac{\sin\alpha}{\alpha} = \frac{2R}{\pi}$

$l_2 = \pi R$

Nr. crt.	l_i	x_i	y_i	$l_i \cdot x_i$	$l_i \cdot y_i$
1	$3R$	0	$\frac{3R}{2}$	0	$\frac{9R^2}{2}$
$\cup$	πR	R	$-\frac{2R}{\pi}$	πR^2	$-2R^2$
	$\sum l_i$	-	-	$\sum l_i x_i$	$\sum l_i y_i$

$\sum l_i = 3R + \pi R$

$\sum l_i \cdot x_i = 0 + \pi R^2 = \pi R^2$

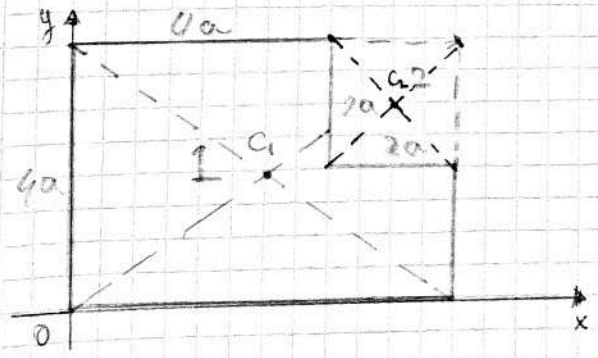
$\sum l_i \cdot y_i = \frac{9R^2}{2} - 2R^2 = \frac{5R^2}{2}$

Coordonatele lui C_g sunt:

$x_C = \frac{\sum l_i x_i}{\sum l_i} = \frac{\pi R}{3+\pi} = 0{,}57R$

$y_C = \frac{\sum l_i y_i}{\sum l_i} = \frac{5R^2}{6R+2\pi R} = \frac{5R}{3+2\pi} = 0{,}753R$

Problema 2



$A_1 = 24a^2$

$C_1 = \begin{cases} x_C = 3a \\ y_C = 2a \end{cases}$

$A_2 = -4a^2$

$C_2 = \begin{cases} x_2 = 5a \\ y_2 = 3a \end{cases}$

	A_i	x_i	y_i	$A_i x_i$	$A_i y_i$
6 × 4	$24a^2$	$3a$	$2a$	$72a^3$	$48a^3$
2 × 2	$-4a^2$	$5a$	$3a$	$-20a^3$	$-12a^3$
	$\sum A_i$	–	–	$\sum A_i x_i$	$\sum A_i y_i$

$\sum A_i = 20a^2$

$\sum A_i x_i = 52a^3$

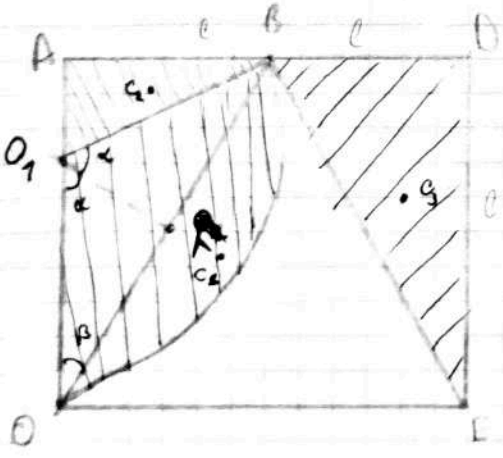
$\sum A_i y_i = 36a^3$

Coordonatele lui C:

$x_C = \frac{52a^3}{20a^2} = \frac{13}{5}a = 2{,}6a$

$y_C = \frac{36a^3}{20a^2} = \frac{9}{5}a = 1{,}8a$

Problema 3



$OB = \sqrt{3\ell^2 + \ell^2} = 2\ell$

$OF = \frac{OB}{2} = \ell$

$tg\,\beta = \frac{AB}{OA} = \frac{\ell}{\ell\sqrt{3}} = \frac{\sqrt{3}}{3} \Rightarrow \beta = \frac{\pi}{6}$

$\alpha = \pi - \frac{\pi}{2} - \beta = \frac{\pi}{3}$

$\overset{\frown}{OO_1B} = 2\alpha = \frac{2\pi}{3}$

$OO_1 = O_1B = \frac{OF}{\cos\beta} = \frac{\ell}{\frac{\sqrt{3}}{2}} = \frac{2\ell\sqrt{3}}{3} = R$

$OB = BE = OE = 2\ell \Rightarrow \triangle OBE$ – echilateral $\Rightarrow \measuredangle OBE = \frac{\pi}{3}$

$\overset{\frown}{OO_1B} = \overset{\frown}{O_1OB} = \frac{\pi}{6} \Rightarrow \overset{\frown}{O_1BE} = \frac{\pi}{2}$

$$A_1 = \frac{\pi}{3} R^2 = \frac{4\pi \ell^2}{9}$$

$$C_1 = \begin{cases} x_C = O_1C_1 \cdot \sin\alpha = \frac{\ell\sqrt{3}}{\pi} \\ y_C = R - O_1C_1\cos\alpha = \frac{2\pi\sqrt{3}\cdot\ell - 3\ell}{3\pi} = \ell\left(\frac{2\sqrt{3}}{3} - \frac{1}{\pi}\right) \end{cases}$$

$$A_2 = \frac{\ell^2\sqrt{3}}{6}$$

$$C_2 = \begin{cases} x_2 = \frac{\ell}{3} \\ y_2 = \frac{8\ell\sqrt{3}}{9} \end{cases}$$

$$A_3 = \frac{\ell^2\sqrt{3}}{2}$$

$$C_3 = \begin{cases} x_3 = \frac{5\ell}{3} \\ y_3 = \frac{2\ell\sqrt{3}}{3} \end{cases}$$

$$\begin{cases} x_C = \frac{12\sqrt{3}}{6\sqrt{3}+4\pi}\ell = 0{,}9\ell \\ y_C = \frac{27+8\pi\sqrt{3}}{9(6\sqrt{3}+4\pi)}\ell = 1{,}02\ell \end{cases}$$

Comp	A_i	x_i	y_i	$A_i x_i$	$A_i y_i$
1	$\frac{4\pi\ell^2}{9}$	$\frac{\ell\sqrt{3}}{\pi}$	$\ell\left(\frac{2\sqrt{3}}{3} - \frac{1}{\pi}\right)$	$\frac{4\sqrt{3}\ell^3}{9}$	$\ell^3\left(\frac{8\pi\sqrt{3}}{27} - \frac{4}{9}\right)$
2	$\frac{\ell^2\sqrt{3}}{6}$	$\frac{\ell}{3}$	$\frac{8\ell\sqrt{3}}{9}$	$\frac{\ell^3\sqrt{3}}{18}$	$\frac{4\ell^3}{9}$
3	$\frac{\ell^2\sqrt{3}}{2}$	$\frac{5\ell}{3}$	$\frac{2\ell\sqrt{3}}{3}$	$\frac{5\ell^3\sqrt{3}}{6}$	ℓ^3
$\sum$	$\ell^2\frac{6\sqrt{3}+4\pi}{9}$	—	—	$\frac{4\ell^3\sqrt{3}}{3}$	$\ell^3\left(1 + \frac{8\pi\sqrt{3}}{27}\right)$