Comutational Physics Assignment 2

Name: Alapan Das (DTP, I-PhD)

Link to github folder

Problem-1:

To solve the differential equation $-\frac{d^2u}{dx^2} = (3x + x^2)e^x = f(x)$ we disctretize it in N steps and turn this equation into a set of linear equations which coupled with the information of boundary conditions $u(0) = u(1) = 0$ we get a $(N + 1) \times (N + 1)$ linear equations.

The matrix equation is following:

$$\frac{-1}{h^2}\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & -2 & 1 & 0 & \cdots & 0 \\ 0 & 1 & -2 & 1 & \cdots & 0 \\ \vdots & \vdots & \cdots & \cdots & \vdots & \vdots \\ 0 & \cdots & 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ u_1 \\ u_2 \\ \vdots \\ u_N \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ r_1 \\ r_2 \\ \vdots \\ r_N \\ 0 \end{pmatrix}$$

$r_k = f(x_k), x_k = kh, h = \frac{1}{N+1}$

Now we solve this matrix equation to get the u_k column vector.To solve this system of linear equation we use Forward and Backward substitution of Gauss elemination method.

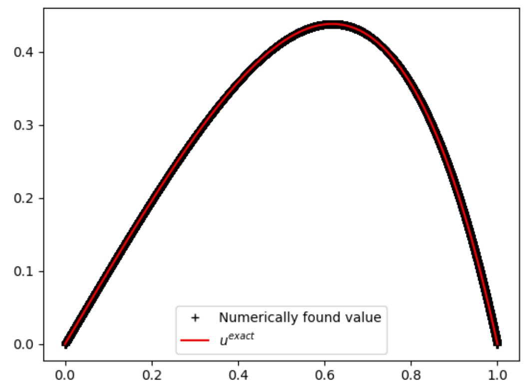

Fig-1:Plot for 0.1 million data-points, minimum error

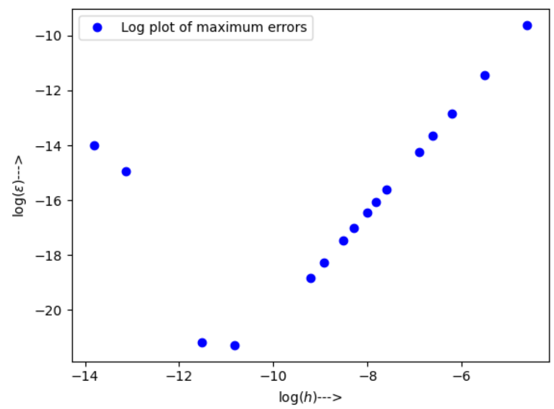

Fig-2: Logarithmic maximum error plot for different h

Problem-2:

Among the given seven equations, two are linearly dependent on the other five equations.
So, we exclude those two equations:
1) As, $I_1 + I_4 = I_0$ and $I_2 + I_3 = I_0$ and from 4th and 4rd equation we get $I_1 + I_4 - I_2 - I_3 = 0$ which is nothing but the subtraction of 2) from 1).
Hence, we exclude 4th equation.
2) Similarly Adding 5th and 6th equations we obtain the 7th equayion, hence we exclude 7.
So, the matrix formulation of our linear equations is

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & R_2 & -R_3 & 0 & -R_5 \\ R_1 & 0 & 0 & -R_4 & R_5 \\ 1 & -1 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{pmatrix} = \begin{pmatrix} I_0 \\ I_0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$(R1, R2, R3, R4, R5 = 2, 4, 6, 8, 10 \text{ and } I_0 = 1)$$

We use LU decomposition and obtain the soultion as following:

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 \\ 0 & 4 & -10 & 1 & 1 \end{pmatrix}, U = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & -10 & 10 \\ 0 & 0 & 0 & 0 & -30 \end{pmatrix}$$

Hence, the solution is $I_1 = 0.7333333333333334 I2 = 0.6666666666666666 I3 = 0.3333333333333337 I4 = 0.2666666666666666 I5 = 0.0666666666666667$

We write this numerically found values in irreducible fractional forms
$I_1 = \frac{11}{15}, I_2 = \frac{2}{3}, I_3 = \frac{1}{3}, I_4 = \frac{4}{15}, I_5 = \frac{1}{15}$