Sliding-Mode Control for Systems With Mismatched Uncertainties via a Disturbance Observer

Jun Yang, *Member, IEEE*, Shihua Li, *Senior Member, IEEE*, and Xinghuo Yu, *Fellow, IEEE*

***Abstract*—This paper develops a sliding-mode control (SMC) approach for systems with mismatched uncertainties via a nonlinear disturbance observer (DOB). By designing a novel sliding surface based on the disturbance estimation, a DOB-based SMC method is developed in this paper to counteract the mismatched disturbance. The newly proposed method exhibits the following two attractive features. First, the switching gain is only required to be designed greater than the bound of the disturbance estimation error rather than that of the disturbance; thus, the chattering problem is substantially alleviated. Second, the proposed method retains its nominal performance, which means the proposed method acts the same as the baseline sliding-mode controller in the absence of uncertainties. Simulation results of both the numerical and application examples show that the proposed method exhibits much better control performance than the baseline SMC and the integral SMC (I-SMC) methods, such as reduced chattering and nominal performance recovery.**

***Index Terms*—Chattering reduction, disturbance observer (DOB), mismatched disturbance, nominal performance recovery, sliding-mode control (SMC).**

I. Introduction

Sliding-mode control (SMC) has been widely studied for over 50 years and extensively employed in industrial applications due to its conceptual simplicity, and in particular powerful ability to reject disturbances and plant uncertainties [1]–[4]. It is noticed that most of the existing results on sliding surface design are concentrated on the matched uncertainties attenuation since the sliding motion of the traditional SMC is only insensitive to matched uncertainties [5], which means the uncertainties exist in the same channel as that of the control input [6].

However, the uncertainties existing in many practical systems may not satisfy the so-called matching condition. For example, in the MAGLEV suspension system, the track input disturbance acts on different channel from the control input [7]–[9]. Another example is the permanent magnet synchronous motor system in which the uncertainties consisting of parameter perturbations and the load torque enter system via different channels from the control inputs [10]–[13]. The problem also appears in the flight control systems, in which the lumped disturbance torques caused by unmodeled dynamics, external winds, and parameter variations, always affect the states directly rather than through the input channels [14]. As for these kinds of systems, the sliding motion of the traditional SMC is severely affected by the mismatched uncertainties, and the well-known robustness of SMC does not hold any more [3].

Due to the significance of attenuating mismatched uncertainties in practical applications, many authors devote themselves to the sliding surface design for uncertain systems with mismatched disturbances (see, e.g., [5], [7], [15]–[29] and references therein). In general, the aforementioned SMC methods can be classified into the following two categories.

The first category mainly focuses on the stability (or robust stability) of various systems with mismatched structure uncertainties via some classical control design tools, such as Riccati approach [15], [16], LMI-based approach [5], [17], [18], and adaptive approach [26]. The mismatched uncertainties considered by those methods must be H_2 norm-bounded, or in other words, the mismatched uncertainties must belong to vanishing uncertainties, which is, however, not a reasonable assumption for practical systems since many engineering systems may suffer from mismatched disturbances that do not necessarily satisfy the condition of with a bounded H_2 norm. Taking the aforementioned MAGLEV suspension system and permanent magnet synchronous motor as examples, the lumped uncertainties therein may have nonzero steady-state values and thus do not have bounded H_2 norms [7]–[11].

The second category is referred to as integral sliding-model control (I-SMC) [30], [31]. The idea behind the I-SMC is that a high-frequency switching gain is designed to force the states to achieve the integral sliding surface, and then the integral action in the sliding surface drives the states to the desired equilibrium in the presence of mismatched uncertainties. Compared with the first category SMC for mismatched uncertainties, the I-SMC method is more practical due to its simplicity and robustness and has been reported to be applied to various systems [7], [27]–[29]. However, it is well known that integral action always brings some adverse effects to the control systems, such as large overshoot and long settling time.

It should be pointed out that all the above two categories of SMC methods handle the mismatched uncertainties in a robust way, which implies that the uncertainty attenuation ability is achieved at the price of sacrificing its nominal control performance. In addition, the chattering problem in these methods is still a severe problem to be solved. Recently, several authors introduced a disturbance observer (DOB) for SMC to alleviate the chattering problem and retain its nominal control

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J. Yang and S. Li are with the School of Automation, Southeast University, Nanjing 210096, China (e-mail: yangjun2882700@163.com; lsh@seu.edu.cn).

X. Yu is with the Platform Technologies Research Institute, RMIT University, Melbourne, Vic. 3001, Australia (e-mail: x.yu@rmit.edu.au).

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performance [32]–[38]. The idea is to construct the control law by combining the SMC feedback with the disturbance estimation based-feedforward compensation straightforwardly. However, the methods given in [32]–[38] are only available for the matched uncertain systems.

In this paper, a novel SMC method is proposed to counteract the mismatched uncertainties in the system via a nonlinear DOB. The mismatched uncertainties under consideration are possibly nonvanishing and do not necessarily satisfy the H_2 norm-bounded condition. By designing a new sliding surface based on the disturbance estimation, the system states can be driven to the desired equilibrium asymptotically by sliding motion along the sliding surface even in the presence of mismatched disturbance. And then, a discontinuous control law with high-frequency switching gain is designed to force the initial states to reach the designed sliding surface. There are mainly two remarkable features of the proposed method. First, similar to the methods given in [32]–[38], the high-frequency switching gain in the proposed control law is only required to be designed greater than the bound of the disturbance estimation error rather than that of the disturbance, which substantially alleviates the chattering problem. Second, the proposed method retains its nominal performance since the DOB serves like a patch to the baseline controller and does not cause any adverse effects on the system in the absence of uncertainties, which will be shown by the simulation examples in this paper.

II. Problems of the Existing SMC Methods

Consider the following second-order system with mismatched disturbance, depicted by

$$\begin{cases} \dot{x}_1 = x_2 + d(t), \\ \dot{x_2} = a(x) + b(x)u, \\ y = x_1, \end{cases} \tag{1}$$

where x_1 and x_2 are states, u is the control input, $d(t)$ is the disturbance, and y is the output.

Assumption 1: The disturbance in system (1) is bounded and defined by $d^* = \sup_{t>0} |d(t)|$.

A. Traditional SMC

The sliding-mode surface and control law of the traditional SMC are designed as follows:

$$\sigma = x_2 + cx_1, \quad u = -b^{-1}(x)\left[a(x) + cx_2 + ksgn(\sigma)\right]. \tag{2}$$

Combining (1) and (2) gives

$$\dot{\sigma} = -ksgn(\sigma) + cd(t). \tag{3}$$

Equation (3) shows that the states of system (1) initially outside the sliding surface will reach the sliding surface $\sigma = 0$ in finite time as long as the switching gain in the control law (2) is designed such that $k > cd^*$. Taking the condition $\sigma = 0$ in (2), the sliding motion is obtained and given by

$$\dot{x}_1 = -cx_1 + d(t). \tag{4}$$

Remark 1: Equation (4) implies that the states cannot be driven to the desired equilibrium point even the control law (2) can force the system states to reach the sliding surface in finite time. This is the essential reason why the traditional SMC is only insensitive to matched disturbance but sensitive to mismatched disturbance. ■

B. Integral SMC

An efficient solution for counteracting the mismatched disturbances is known as I-SMC [30], [31], which defines the sliding-mode surface as follows:

$$\sigma = x_2 + c_1 x_1 + c_2 \int x_1. \tag{5}$$

The integral SMC law is designed as

$$u = -b^{-1}(x)\left[a(x) + c_1 x_2 + c_2 x_1 + ksgn(\sigma)\right]. \tag{6}$$

Combining (1), (5) and (6), gives

$$\dot{\sigma} = -ksgn(\sigma) + c_1 d(t). \tag{7}$$

The states of system (1) initially outside the sliding surface will reach the sliding surface $\sigma = 0$ in (5) in finite time as long as the switching gain in the control law (6) is designed such that $k > c_1 d^*$. Taking the condition $\sigma = 0$ yields

$$\ddot{x}_1 + c_1\dot{x}_1 + c_2 x_1 = \dot{d}(t) \tag{8}$$

which implies that the state can slide to the desired equilibrium point asymptotically if the system has reached the sliding surface in finite time and the disturbance has a constant steady-state value, i.e., $\lim_{t\to\infty} \dot{d}(t) = 0$.

Remark 2: The I-SMC method is effective to remove the offset asymptotically in the presence of the mismatched disturbance but in a robust way. To this end, the integral action lies in the I-SMC method always brings some adverse effects to the control performance, such as introducing overshoot and destroying its nominal control performance. This will be shown in the latter simulation comparison studies. ■

III. New SMC Method Based on a Disturbance Observer

A. Control Design

Let $x = [x_1, x_2]^T$, then the system (1) can be expressed as

$$\begin{cases} \dot{x} = f(x) + g_1(x)u + g_2 d, \\ y = x_1, \end{cases} \tag{9}$$

where $f(x) = [x_2, a(x)]^T$, $g_1(x) = [0, b(x)]^T$, and $g_2 = [1, 0]^T$.

A nonlinear DOB (NDOB), which can estimate the disturbance in (9), is introduced and depicted by [14], [39]–[42]

$$\begin{cases} \dot{p} = -lg_2 p - l\left[g_2 lx + f(x) + g_1(x)u\right], \\ \hat{d} = p + lx, \end{cases} \tag{10}$$

where $\hat{d}$, p, and l are the estimation of the disturbance, the internal state of the nonlinear observer, the observer gain to be designed, respectively.

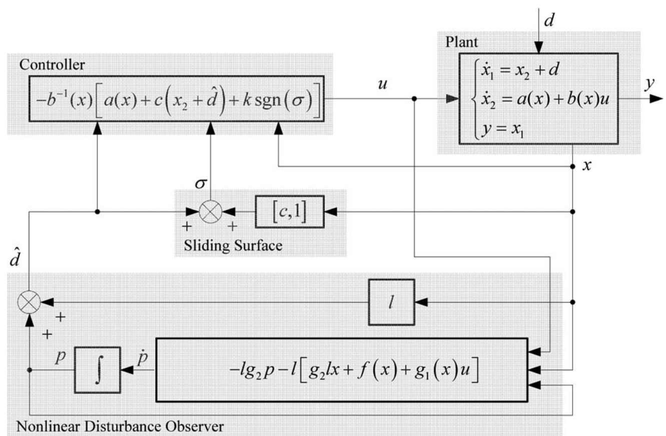

Fig. 1. Block diagram of the proposed disturbance observer based sliding-mode control method.

A novel sliding-mode surface for system (1) under mismatched disturbance is defined based on the disturbance estimation of (10), given by

$$\sigma = x_2 + cx_1 + \hat{d} \tag{11}$$

where $\hat{d}$ is the disturbance estimation given by NDO (10), and $c > 0$ is a control parameter to be designed.

The proposed DOB based SMC law is designed as

$$u = -b^{-1}(x)\left[a(x) + c(x_2 + \hat{d}) + ksgn(\sigma)\right] \tag{12}$$

where k is the switching gain to be designed.

The block diagram on the implementation of the proposed DOB-based SMC is given by Fig. 1.

B. Stability Analysis

Assumption 2: The derivative of the disturbance in system (1) is bounded and satisfies $\lim_{t\to\infty} \dot{d}(t) = 0$.

Lemma 1 [14]: Suppose that Assumptions 1 and 2 are satisfied for system (1). The disturbance estimation $\hat{d}$ of NDO (10) can track the disturbance d of system (1) asymptotically if the observer gain l is chosen such that $lg_2 > 0$ holds, which implies that

$$\dot{e}_d(t) + lg_2 e_d(t) = 0 \tag{13}$$

is globally asymptotically stable, where $e_d(t) = d(t) - \hat{d}(t)$ is the disturbance estimation error.

Assumption 3: The disturbance estimation error in (13) is bounded, defined by $e_d^* = \sup_{t>0} |e_d(t)|$.

Lemma 2 [43]: Consider a nonlinear system $\dot{x} = F(x, w)$ which is input-to-state stable (ISS). If the input satisfies $\lim_{t\to\infty} w(t) = 0$, then the state $\lim_{t\to\infty} x(t) = 0$. ■

Theorem 1: Suppose that Assumptions 1–3 are satisfied for system (1). Considering system (1) under the proposed control law (12), the closed-loop system is asymptotically stable if the switching gain in the control law (12) is designed such that $k > (c + lg_2)e_d^*$ and the observer gain l is chosen such that $lg_2 > 0$ holds.

Proof: Taking the derivative of the sliding surface σ defined in (11) along system (1), yields

$$\dot{\sigma} = a(x) + b(x)u + c\left[x_2 + d(t)\right] + \dot{\hat{d}}. \tag{14}$$

Substituting the control law (12) into (14), yields

$$\dot{\sigma} = -ksgn(\sigma) + c\left[d(t) - \hat{d}(t)\right] + \dot{\hat{d}}. \tag{15}$$

It can be derived from (10) that

$$\dot{\hat{d}}(t) = -lg_2\left[\hat{d}(t) - d(t)\right]. \tag{16}$$

Substituting (16) into (15) gives

$$\dot{\sigma} = -ksgn(\sigma) + (c + lg_2)e_d(t). \tag{17}$$

Consider a candidate Lyapunov function as

$$V(\sigma) = \sigma^2/2. \tag{18}$$

Taking the derivative of $V(\sigma)$ in (18), gives

$$\begin{aligned} \dot{V} &= -k|\sigma| + (c + lg_2)e_d(t)\sigma \\ &\leq -\left[k - (c + lg_2)e_d^*\right]|\sigma| \\ &= -\sqrt{2}\left[k - (c + lg_2)e_d^*\right]V^{\frac{1}{2}}. \end{aligned} \tag{19}$$

With the given condition $k > (c + lg_2)e_d^*$, it can be derived from (19) that the system states will reach the defined sliding surface $\sigma = 0$(11) in finite time. The condition $\sigma = 0$ implies that

$$\dot{x}_1 = -cx_1 + d(t) - \hat{d}(t). \tag{20}$$

Combining (20) with the observer dynamics yields

$$\begin{cases} \dot{x}_1 = -cx_1 + e_d, \\ \dot{e}_d = -lg_2 e_d + \dot{d}, \\ x_2 = -cx_1 - \hat{d}. \end{cases} \tag{21}$$

Under the given conditions that $c > 0$ and $lg_2 > 0$, it can be verified that the following system:

$$\begin{cases} \dot{x}_1 = -cx_1 + e_d \\ \dot{e}_d = -lg_2 e_d \end{cases} \tag{22}$$

is exponentially stable. With this result, it can be derived from Lemma 5.5 in [43] that system

$$\begin{cases} \dot{x}_1 = -cx_1 + e_d, \\ \dot{e}_d = -lg_2 e_d + \dot{d} \end{cases} \tag{23}$$

is ISS. Based on the condition given in Assumption 2, it can be derived from Lemma 2 that the states of system (23) satisfy $\lim_{t\to\infty} x_1(t) = 0$ and $\lim_{t\to\infty} e_d(t) = 0$. This implies that the system states will slide to the desired equilibrium point asymptotically under the proposed control law. ■

Remark 3: To ensure stability, the switching gains of the traditional SMC, I-SMC, and DOB-SMC have to be designed such that $k > c|d|$, $k > c_1|d|$, and $k > (c + lg_2)|d - \hat{d}|$, respectively. Since the disturbance has been precisely estimated by the DOB, the magnitude of the estimation error $|d - \hat{d}|$, which is

TABLE I
CONTROL PARAMETERS FOR THE NUMERICAL EXAMPLE IN CASE 1

Controllers	Parameters
SMC	$c = 5,\ k = 3$
I-SMC	$c_1 = 5,\ c_2 = 6,\ k = 3$
DOB-SMC	$c = 5,\ k = 3,\ l$ = [6,0]

expected to converge to 0, can be kept much smaller than the magnitude of the disturbance d. The readers can also refer to [36]–[38] for the same argument. To this end, the switching gain of the proposed method can be designed much smaller than those of the traditional SMC and I-SMC methods, and the chattering problem can be alleviated to some extent, which will be shown by the later simulation studies. ■

Remark 4: In the absence of disturbance, it can be derived from (16) that

$$\dot{\hat{d}}(t) = -lg_2\hat{d}(t) \tag{24}$$

which implies that $\hat{d}(t) \equiv 0$ if its initial value is selected as $\hat{d}(0) = 0$. As for the proposed method, the sliding surface (11) and the control law (12) reduces to those of the traditional SMC in (2). This means that the nominal performance of the proposed method is retained. Such excellent feature will be shown by the numerical example in the next section. ■

IV. NUMERICAL EXAMPLE

Considering the following system for simulation studies

$$\begin{cases} \dot{x}_1 = x_2 + d(t), \\ \dot{x}_2 = -2x_1 - x_2 + e^{x_1} + u, \\ y = x_1. \end{cases} \tag{25}$$

A. Nominal Performance Recovery

Consider the initial states of system (25) as $x(0) = [1, -1]^T$. A step external disturbance $d = 0.5$ is imposed on the system at $t = 6$ sec. For comparison studies, both the traditional SMC and integral SMC methods are employed in the control design for system (25). The control parameters of all the three control methods are listed in Table I.

It can be observed from Figs. 2 and 3 that the states and control input responses under the proposed method are the same as those under the baseline SMC method during the first 6 s, which is an evidence as stated in Remark 3 that the nominal performance of the proposed method is retained. It is also noticed that the integral action of the I-SMC method causes adverse control effects such as undesired overshooting and unsatisfactory settling time.

In the presence of mismatched disturbance, it can be observed from Fig. 2 that the traditional SMC method fails to drive the states to the desired equilibrium, which shows the results in Remark 1 that the traditional SMC is sensitive to mismatched disturbance. Both the proposed method and the I-SMC method can finally suppress the mismatched disturbance, but the proposed method exhibits a much quicker convergence rate than that of the I-SMC method.

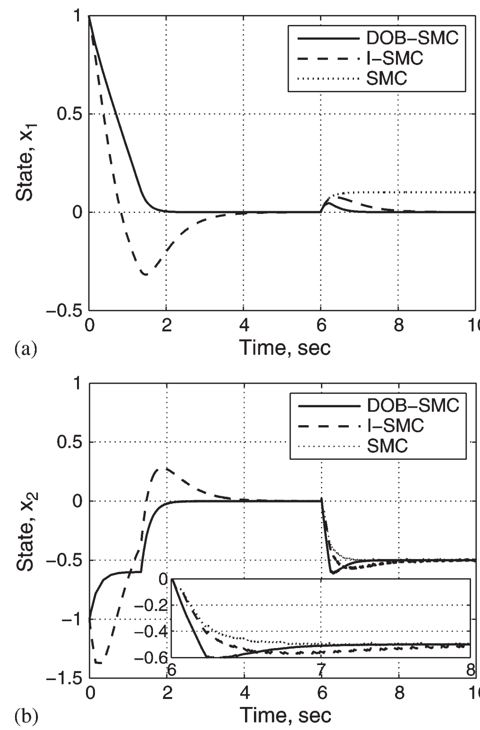

Fig. 2. State variables in simulation scenario 1 (nominal performance recovery).

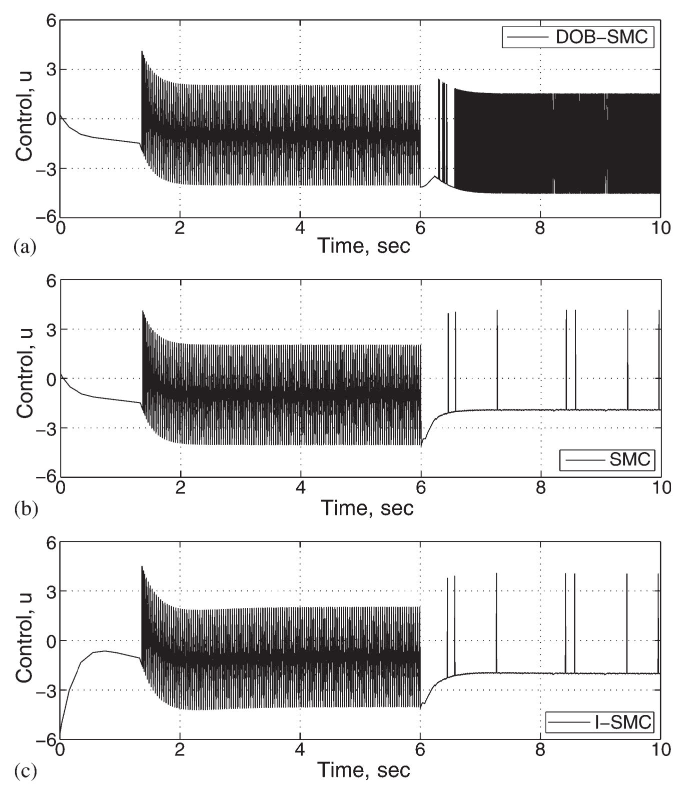

Fig. 3. Control input in simulation scenario 1 (nominal performance recovery).

B. Chattering Reduction

As clearly shown in Fig. 3, all the three controllers have resulted in substantial chattering since a large switching gain has been chosen to attenuate the disturbance. As stated in

TABLE II
CONTROL PARAMETERS FOR THE NUMERICAL EXAMPLE IN CASE 2

Controllers	Parameters
SMC	$c = 5$, $k = 1$
I-SMC	$c_1 = 5$, $c_2 = 6$, $k = 1$
DOB-SMC	$c = 5$, $k = 1$, l = [6,0]

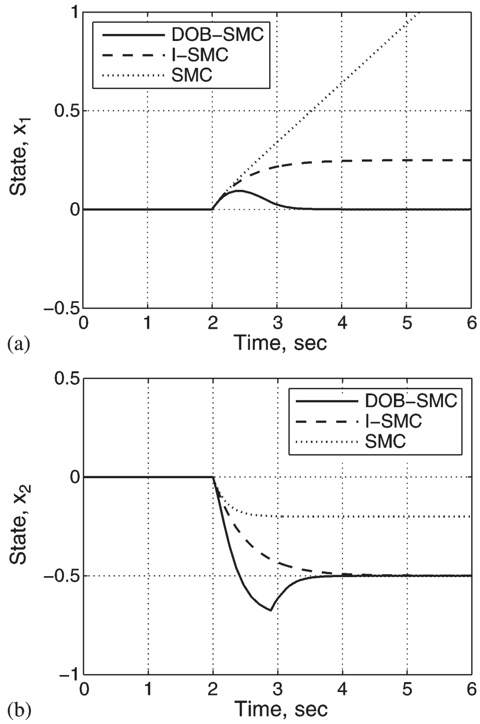

Fig. 4. State variables in simulation scenario 2 (chattering reduction).

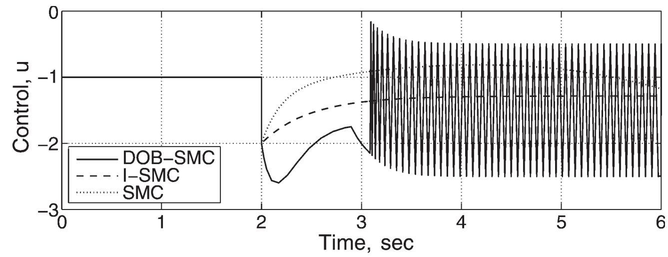

Fig. 5. Control input in simulation scenario 2 (chattering reduction).

Remark 3, the proposed method can alleviate the chattering by setting a relatively smaller switching gain. In this part, we try to verify such feature by simulation studies.

Consider the initial states of system (25) as $x(0) = [0, 0]^T$ now. A step external disturbance $d = 0.5$ is imposed on the system at $t = 2$ sec. The control parameters of all the three control methods are listed in Table II.

It can be observed from Figs. 4 and 5 that the chattering can be reduced for the proposed DOB-SMC method by decreasing the switching gain. In this simulating scenario, the switching gain k is decreased from 3 to 1 for all the three methods. However, as clearly shown in the simulation results, the proposed method can still obtain fine control effects with a much smaller chattering than that in simulation scenario 1. While we try to reduce the chattering under the SMC and I-SMC methods by decreasing the switching gain, it is observed that such two methods failed to reject the disturbances of the system effectively. The simulation results in this simulation scenario verify the conclusions given in Remark 3.

V. APPLICATION TO A MAGLEV SUSPENSION SYSTEM

SMC design of a MAGnetic LEViation (MAGLEV) system, which is subject to mismatched disturbance, is studied in this part.

TABLE III
PARAMETERS OF MAGLEV SUSPENSION SYSTEM

Parameters	Meaning	Value
M_s	Carriage Mass	1000kg
F_o	Nominal force	9810 N
G_o	Nominal air gap	0.015m
R_c	Coil's Resistance	10Ω
B_o	Nominal flux density	1T
L_c	Coil's Inductance	0.1H
I_o	Nominal current	10A
N_c	Number of turns	2000
V_o	Nominal voltage	100V
A_p	Pole face area	0.01m^2

A. MAGLEV Suspension Dynamic Model

The dynamic model of the MAGLEV suspension system studied here is expressed as [44]

$$\begin{cases} \dot{x} = Ax + B_u u + B_d d, \\ y = Cx, \end{cases} \tag{26}$$

where the states are the current, vertical electromagnet velocity and air gap, i.e., $x = [i, \dot{z}(z_t - z)]^T$, the input $u = u_{coil}$ is the voltage, the track input $d = \dot{z}_t$ is the rail vertical velocity. The controlled variable is the variation of air gap, i.e., $y = z_t - z$. The detailed modeling procedure can be found in [44], here the state matrix A, the input matrix B_u, the disturbance matrix B_d, and the output matrix C are given directly

$$A = \begin{bmatrix} \frac{-R_c}{L_c + K_b N_c \frac{A_p}{G_o}} & \frac{-K_b N_c A_p I_o}{G_o^2\left(L_c + K_b N_c \frac{A_p}{G_o}\right)} & 0 \\ -2K_f \frac{I_o}{M_s G_o^2} & 0 & 2K_f \frac{I_o^2}{M_s G_o^3} \\ 0 & -1 & 0 \end{bmatrix} \tag{27}$$

$$B_u = \begin{bmatrix} \frac{1}{L_c + K_b N_c \frac{A_p}{G_o}} \\ 0 \\ 0 \end{bmatrix} \tag{28}$$

$$B_d = \begin{bmatrix} \frac{K_b N_c A_p I_o}{G_o^2\left(L_c + K_b N_c \frac{A_p}{G_o}\right)} \\ 0 \\ 1 \end{bmatrix} \tag{29}$$

$$C = [0 \ \ 0 \ \ 1]. \tag{30}$$

The physical meanings of the parameters in (27)–(29) are listed in Table III. The diagram of the MAGLEV suspension system is shown by Fig. 6.

The major external disturbance in MAGLEV system comes from the deterministic inputs to the suspension in the vertical direction. Such deterministic inputs are the transitions onto the track gradients. The deterministic input components considered here are referred to [44] and shown in Fig. 7. They represent a gradient of 5% at a vehicle speed of 15 m/s while the jerk level is 1 m/s^3.

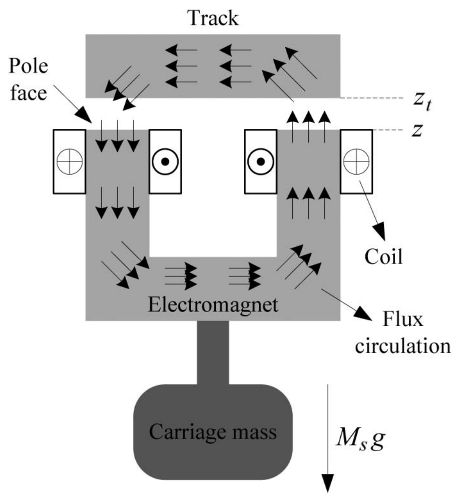

Fig. 6. Diagram of the MAGLEV suspension system.

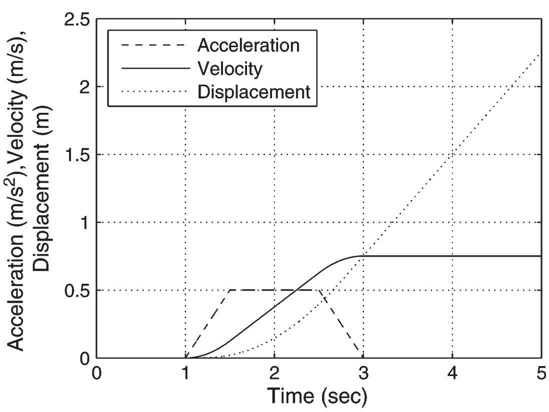

Fig. 7. Track input to the suspension with a vehicle speed of 15 m/s and 5% gradient.

TABLE IV
CONSTRAINTS FOR MAGLEV SUSPENSION SYSTEM

Constraints	Value
Maximum air gap deviation, $((z_t - z)_p)$	$\leq$0.0075m
Maximum input coil voltage, $((u_{coil})_p)$	$\leq$300V($3I_oR_c$)
Settling time, (t_s)	$\leq$3s
Air gap steady state error, $((z_t - z)_{ess})$	=0

It can be observed from (26), (28) and (29) that the disturbances enter the system via different channel from that of the control input. In other words, the disturbances in the MAGLEV system are mismatching ones. The control specifications of the MAGLEV system under consideration of the deterministic track input are given in Table IV [44].

B. Control Design

To implement the proposed DOB-SMC method for the MAGLEV system, the following coordinate transformation is utilized

$$\eta = Tx \tag{31}$$

where

$$T = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}.$$

Under such coordinate transformation, the MAGLEV system is equivalently represented as

$$\dot{\eta} = \bar{A}\eta + \bar{B}_u u + \bar{B}_d d \tag{32}$$

where $\bar{A} = TAT^{-1}$, $\bar{B}_u = TB_u$, and $\bar{B}_d = TB_d$.

Substituting the parameters in Table III into (32), gives

$$\begin{cases} \dot{\eta}_1 = \eta_2 + d, \\ \dot{\eta}_2 = \eta_3, \\ \dot{\eta}_3 = CA^3T^{-1}\eta + CA^2B_u u + CA^2B_d d. \end{cases} \tag{33}$$

The following DOB [8] is employed to estimate the disturbance in the MAGLEV system

$$\begin{cases} \dot{p} = -LB_d(p + L\eta) - L(\bar{A}\eta + \bar{B}_u u) \\ \hat{d} = p + L\eta \end{cases} \tag{34}$$

where $\hat{d}$ is the disturbance estimate, p is an auxiliary vector and L is the observer gain matrix to be designed. It can be derived from the DOB (34) that

$$\dot{\hat{d}} = -LB_d(\hat{d} - d). \tag{35}$$

The sliding surface of the proposed method is designed as follows:

$$\sigma = c_1\eta_1 + c_2(\eta_2 + \hat{d}) + c_3\eta_3 \tag{36}$$

where the parameters c_i $(i = 1, 2, 3)$ have to be designed such that the polynomial

$$p_o(s) = c_3s^2 + c_2s + c_1 = 0 \tag{37}$$

is Hurwitz.

The control law of the proposed method for the MAGLEV suspension system is designed as

$$u = (CA^2B_u)^{-1}\Big\{-CA^3T^{-1}\eta - CA^2B_d\hat{d} + c_3^{-1}\Big[-ksgn(\sigma) - c_1(\eta_2 + \hat{d}) - c_2\eta_3)\Big]\Big\}. \tag{38}$$

Taking the derivative of the sliding surface (36) along system (33) and substituting the designed control law (38) into it gives

$$\dot{\sigma} = -ksgn(\sigma) + (c_1 + c_2LB_d + c_3CA^2B_d)e_d(t). \tag{39}$$

This implies that the proposed control law can force the system state to reach the sliding surface $\sigma = 0$ in finite time as long as the switching gain is selected such that $k > (c_1 + c_2LB_d + c_3CA^2B_d)e_d^*$.

Combining (33) with (36), the condition $\sigma = 0$ implies

$$c_3\ddot{\eta}_1 + c_2\dot{\eta}_1 + c_1\eta_1 = c_2e_d + c_3\dot{d}. \tag{40}$$

Let $\xi = [\xi_1, \xi_2, \xi_3]^T = [\eta_1, \dot{\eta}_1, e_d]^T$. Combining (35) with (40), gives

$$\dot{\xi} = A_\xi\xi + B_\xi\dot{d} \tag{41}$$

TABLE V
CONTROL PARAMETERS FOR THE MAGLEV SUSPENSION IN CASE 1

Controllers	Parameters
SMC	$c_1 = 100$, $c_2 = 20$, $c_3 = 1$, $k = 60$
I-SMC	$c_0 = 200$, $c_1 = 100$, $c_2 = 20$, $c_3 = 1$, $k = 60$
DOB-SMC	$c_1 = 100$, $c_2 = 20$, $c_3 = 1$, $k = 60$, L =[100,0,0]

where

$$A_\xi = \left[\begin{array}{cc|c} 0 & 1 & 0 \\ -c_1 c_3^{-1} & -c_2 c_3^{-1} & c_2 c_3^{-1} \\ \hline 0 & 0 & -LB_d \end{array}\right], \; B_\xi = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

It can be verified that the matrix A_ξ is Hurwitz under the conditions that both the polynomial (37) and matrix $-LB_d$ are Hurwitz, which also proves that system $\dot{\xi} = A_\xi \xi$ is exponentially stable. Similar with the proof of Theorem 1, it can be derived from [43, Lemma 5.5] that system (41) is ISS. Based on the condition in Assumption 2, it can be derived from Lemma 2 that the states of system (41) satisfy $\lim_{t\to\infty} \xi(t) = 0$. To this end, the proposed control law can guarantee the sliding motion, i.e., the system states in the sliding surface can move to its equilibrium point asymptotically.

C. Simulation Results

To evaluate the efficiency of the proposed method, the SMC and the I-SMC methods are employed in the simulations for the purpose of comparisons. The sliding surface of the SMC method for the MAGLEV suspension system is designed as

$$\sigma = c_1\eta_1 + c_2\eta_2 + c_3\eta_3 \tag{42}$$

the corresponding SMC law is designed as

$$u = \left(CA^2B_u\right)^{-1}\left\{c_3^{-1}\left[-ksgn(\sigma) - c_1\eta_2 - c_2\eta_3\right] - CA^3T^{-1}\eta\right\}. \tag{43}$$

The sliding surface of the I-SMC method for the MAGLEV suspension system is designed as

$$\sigma = c_1\eta_1 + c_2\eta_2 + c_3\eta_3 + c_0\int \eta_1 dt \tag{44}$$

the corresponding I-SMC law is designed as

$$u = \left(CA^2B_u\right)^{-1}\left\{-CA^3T^{-1}\eta + c_3^{-1}\left[-ksgn(\sigma) - c_1\eta_2 - c_2\eta_3 - c_0\eta_1\right]\right\}. \tag{45}$$

1) Nominal Performance Recovery: Consider the initial states of MAGLEV suspension system (26) as $[i(0), \dot{z}(0), (z_t - z)(0)]^T = [0, 0, 0.003]^T$. The external disturbance shown by Fig. 7 is imposed on the system. For comparison studies, both the SMC and I-SMC are employed for the control design here again. The control parameters of all the three control methods are listed in Table V.

Response curves of the output and input of the MAGLEV system under the three controllers are shown in Figs. 8 and 9, respectively. Response curves of the corresponding states are shown in Fig. 10.

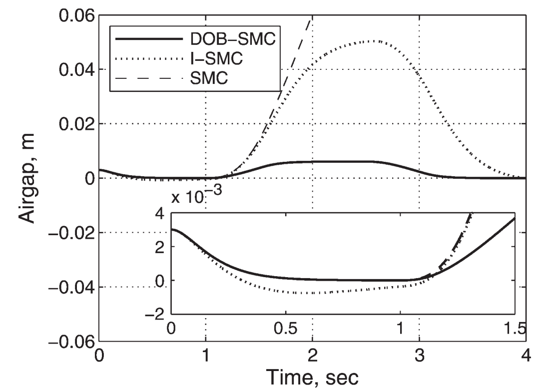

Fig. 8. Response curves of the air gap $z_t - z$ of the MAGLEV suspension system in simulation scenario 1 (nominal performance recovery).

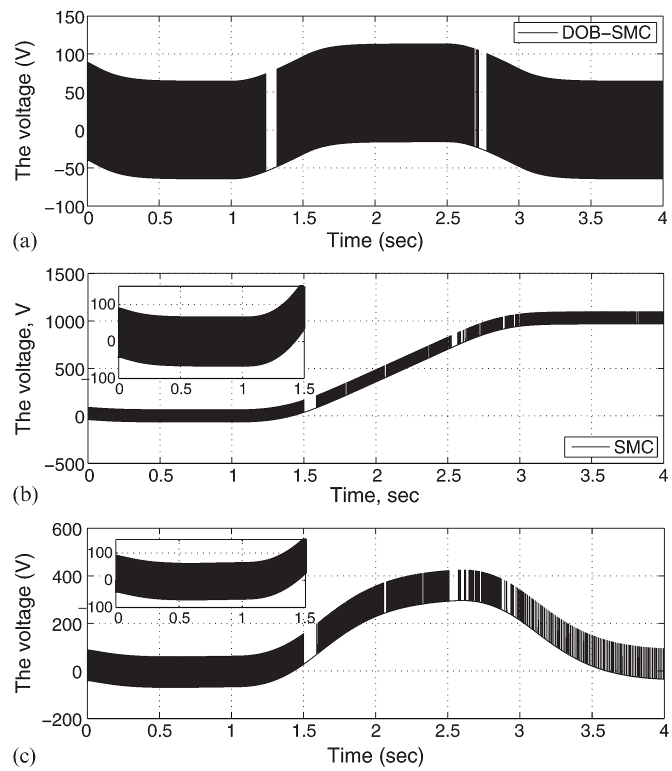

Fig. 9. Response curves of the voltage of the coil u_{coil} of the MAGLEV suspension system in simulation scenario 1 (nominal performance recovery).

A brief observation from Figs. 8 and 10 in the first sec (there is no disturbance during such interval) shows that the proposed method results in the same responses as those of the baseline SMC method, which verifies the nominal performance recovery of the proposed methods. In addition, it can be observed from Figs. 8 and 10 that the proposed DOB-SMC obtains fine disturbance rejection property, while the other two methods exhibit poor performance when the system suffers such mismatched disturbance.

2) Chattering Reduction: Consider the initial states of MAGLEV suspension system (26) as $[i(0), \dot{z}(0), z_t(0) - z(0)]^T = [0, 0, 0]^T$. To alleviate the chattering problem, the switching gains of all the three methods are substantially reduced here. The control parameters of all the three control methods are listed in Table VI.

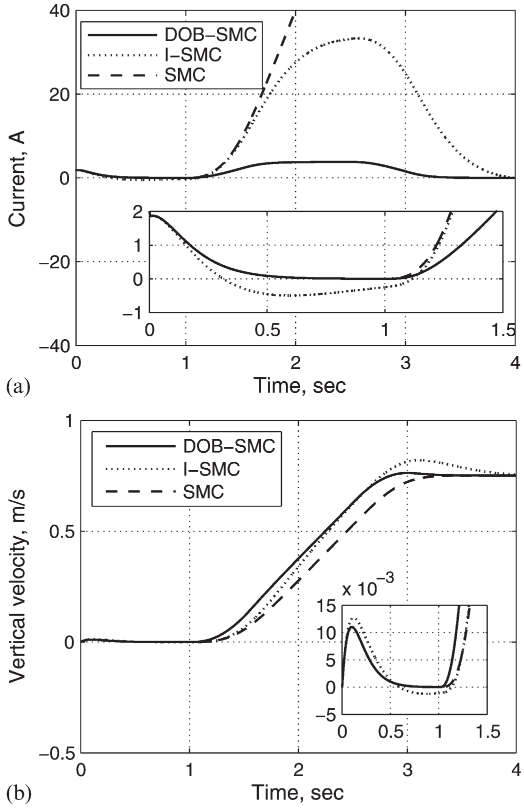

Fig. 10. Response curves of the states of the MAGLEV suspension system in simulation scenario 1 (nominal performance recovery): (a) the current, i, (b) the vertical electromagnet velocity, $\dot{z}$.

TABLE VI
CONTROL PARAMETERS FOR THE MAGLEV SUSPENSION IN CASE 2

Controllers	Parameters
SMC	$c_1 = 100$, $c_2 = 20$, $c_3 = 1$, $k = 15$
I-SMC	$c_0 = 200$, $c_1 = 100$, $c_2 = 20$, $c_3 = 1$, $k = 15$
DOB-SMC	$c_1 = 100$, $c_2 = 20$, $c_3 = 1$, $k = 15$, L =[100,0,0]

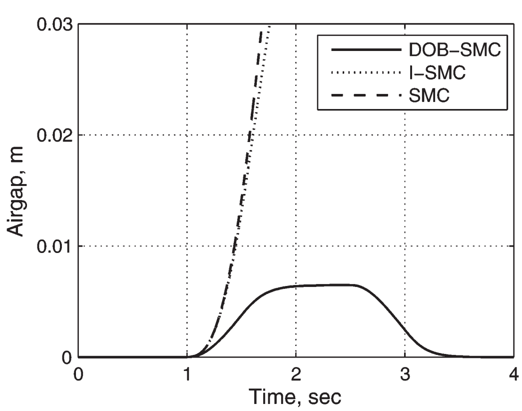

Fig. 11. Response curves of the air gap $z_t - z$ of the MAGLEV suspension system in simulation scenario 2 (chattering reduction).

The response curves of the output, input, and states in this case are shown by Figs. 11–13, respectively. As compared with the first simulation scenario, it can be observed from Fig. 12 that the chattering has been significantly reduced for the proposed method. Both the dynamic and static performances of the proposed method are fine and satisfy the control specification listed in Table IV. However, it is observed that both the SMC and I-SMC methods fail to reject the disturbances in the MAGLEV suspension system effectively when we try to reduce the chattering by decreasing the switching gain. This result verifies the conclusions given in Remark 3.

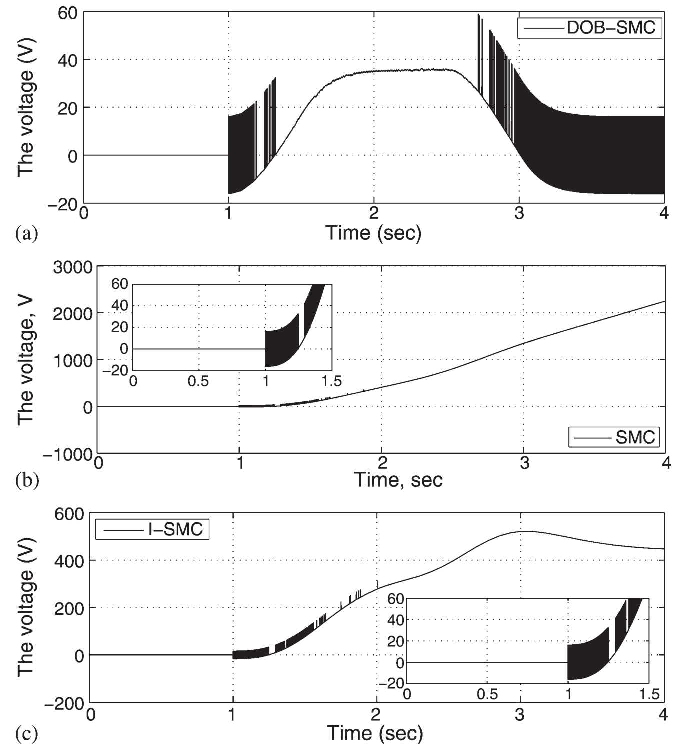

Fig. 12. Response curves of the voltage of the coil u_{coil} of the MAGLEV suspension system in simulation scenario 2 (chattering reduction).

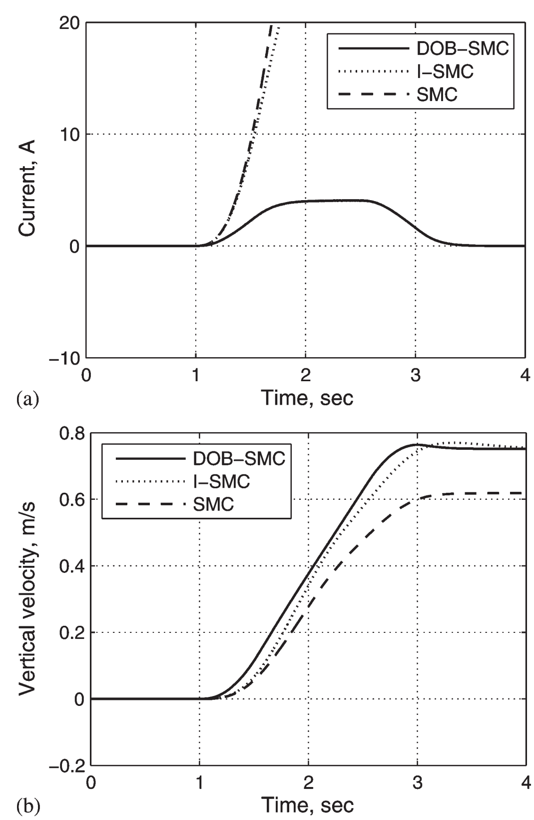

Fig. 13. Response curves of the states of the MAGLEV suspension system in simulation scenario 2 (chattering reduction): (a) the current, i, (b) the vertical electromagnet velocity, $\dot{z}$.

VI. Conclusion

In this paper, a novel DOB-based SMC approach has been proposed to attenuate the mismatched uncertainties. The main contribution here is to design a new sliding-mode surface which includes the disturbance estimation such that the sliding motion along the sliding surface can drive the states to the desired equilibrium point in the presence of mismatched uncertainties. As compared with the traditional SMC and integral SMC methods, the proposed method has exhibited two superiorities including nominal performance recovery and chattering reduction. Both numerical and application examples have been simulated to demonstrate the effectiveness as well as the superiorities of the proposed method. The results have shown that the proposed method exhibits the properties of nominal performance recovery and chattering reduction as well as excellent dynamic and static performance as compared with the SMC and I-SMC methods.

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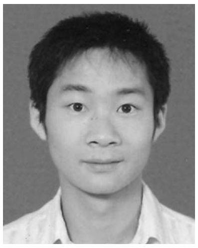

Jun Yang (M'11) was born in Anlu, Hubei Province, China, in 1984. He received the B.Sc. degree in the Department of Automatic Control from Northeastern University, Shenyang, China, in 2006. In 2011, he received the Ph.D. degree in control theory and control engineering from School of Automation, Southeast University, Nanjing, China, where he is currently a lecturer.

His research interests include disturbance estimation and compensation, advanced control theory, and its application to flight control systems and motion control systems.

Shihua Li (M'05–SM'10) was born in Pingxiang, Jiangxi Province, China, in 1975. He received the B.Sc., M.Sc., and Ph.D. degrees all in automatic control from Southeast University, Nanjing, China, in 1995, 1998, and 2001, respectively.

Since 2001, he has been with School of Automation, Southeast University, where he is currently a professor. His main research interests include nonlinear control theory with applications to robots, spacecraft, ac motors, and other mechanical systems.

Xinghuo Yu (M'91–SM'98–F'08) received the B.Sc. and M.Sc. degrees from the University of Science and Technology of China, Hefei, China, in 1982 and 1984, and Ph.D. degree from South-East University, Nanjing, China, in 1988, respectively.

He is currently with Royal Melbourne Institute of Technology (RMIT) University, Melbourne, Australia, where he is the Foundation Director of RMIT's Platform Technologies Research Institute. His research interests include variable structure and nonlinear control, complex and intelligent systems, and industrial applications. He has published over 400 refereed papers in technical journals, books and conference proceedings.

Dr. Yu has served as an Associate Editor of the IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS PART I, the IEEE TRANSACTIONS ON INDUSTRIAL INFORMATICS, the IEEE TRANSACTIONS ON INDUSTRIAL ELECTRONICS, and several other scholarly journals. He received an award under the Thousand Talents Program of the Chinese Government in 2010, a Chang Jiang Scholar (Chair Professor) Award from the Ministry of Education of China in 2009, the 1995 Central Queensland University Vice Chancellor's Award for Research, and was made Emeritus Professor of Central Queensland University in 2002 for his long-term contributions. Dr. Yu is a Fellow of the IEEE, Vice-President for Publications and an IEEE Distinguished Lecturer of the IEEE Industrial Electronics Society. He is also a Fellow of the Institution of Engineers Australia and the Australian Computer Society. Professor Yu is Chair of the Technical Committee on Smart Grids of IEEE Industrial Electronics Society.