

AD1 revision, the graph library, and graph search

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Outline

1. Python
2. Data Structures revision
3. Graphs
4. Breadth- and Depth-First Search

Why Python

The coding part of the assignments is done in Python. Why have we decided to use python, when

- other languages are faster (Java, C, C++, etc.),
- you haven't used Python before,
- Python is not your language of choice, etc.

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The ideas and mindset gained from this course are independent of the programming language they are applied to.

Python is just “a means to an end”.

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- accessing an element at index i ($0 \leq i < n$): $\Theta(?)$
- adding an element: $\Theta(?)$
- removing an element: $\Theta(?)$
- checking if A contains the element e : $\Theta(?)$
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- checking if A contains the element e : $\Theta(n)$
- adding an element at index n : $\Theta(1)$

Arrays

Arrays tend to be the default data structure of choice.

Here are some (non-exhaustive) tips on when to use an array:

- checking if an element exists in the array does not worsen the time complexity;
- the order of the elements matter; or
- the elements can be accessed by their (distinct) index fast (does not worsen the time complexity).

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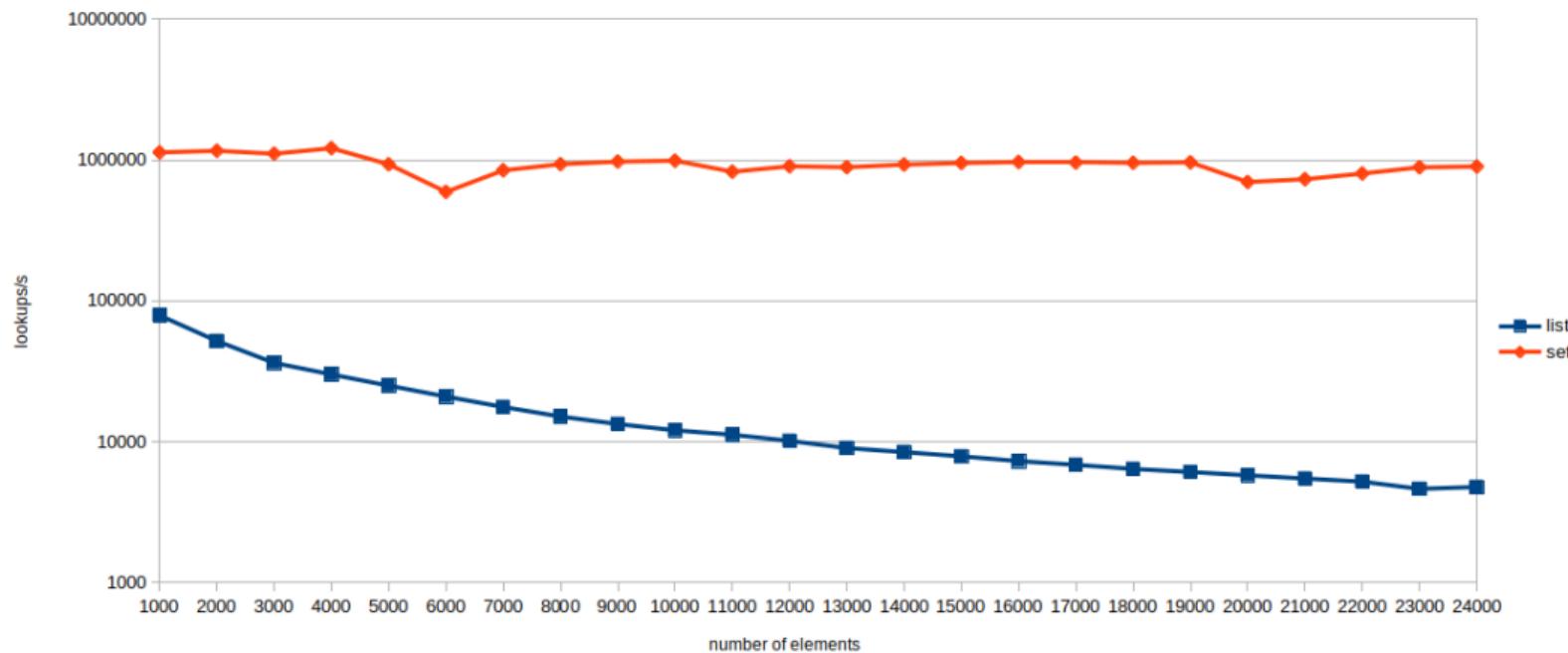
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Sets

Here are some (non-exhaustive) tips on when to use a set:

- checking if an element exists in the set is integral to the task at hand;
- duplicate elements are not allowed; or
- each element can *not* be accessed by a (distinct) index.

Comparison (logarithmic)



Hash Tables

A hash table H (dictionary in Python) of *cardinality* n holds n elements, each identified by a distinct key:

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Hash tables

Hash tables tend to be the data structure of choice for simpler nested data. Here are some (non-exhaustive) tips on when to use a hash table:

- a set with some extra data is required or
- the data is simple, and creating classes (or other custom data structures) for the data is “overkill”.

Graphs

The graph.py Library (1)

A graph is represented similarly to the Adjacency list: the edges are represented as a dictionary with nodes as keys and sets of nodes as elements.

For a graph $G = (V, E)$:

- Space complexity: $\Theta(n + m)$
- checking if G contains an edge (u, v) : $\Theta(1)$
- Identifying all edges: $\Theta(m)$

The graph.py Library (2)

Given a graph $G = (V, E)$ the properties you need for the first assignment are:

- `G.edges` is a (duplicate free) unsorted list containing all edges E and
- `G.nodes` is a (duplicate free) unsorted list containing all vertices V

The graph.py Library (3)

Some graphs, $G = (V, E)$, have additional properties for the edges. In the graph library, an edge (u, v) can have:

- a *flow* that is integer or `None`, (typically) denoting the amount of some commodity that travels over (u, v) ;
- a *capacity* that is integer or `None`, (typically) denoting the maximum amount of some commodity that can travel over (u, v) ; and
- a *weight* that is integer or `None`, (typically) denoting the cost of including or traversing (u, v) .

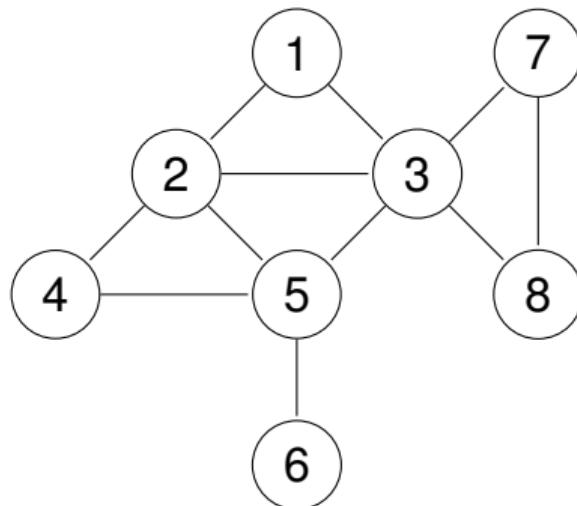
The graph.py Library (4)

Given a graph G and an edge (u, v) , the flow, capacity, and weight of (u, v) be accessed or modified by the methods:

- $G.\text{flow}(u, v)$ and $G.\text{set_flow}(u, v, f)$;
 - $G.\text{capacity}(u, v)$ and $G.\text{set_capacity}(u, v, c)$; and
 - $G.\text{weight}(u, v)$ and $G.\text{set_weight}(u, v, w)$,
- respectively for some $f, c, w \in \{\text{None}\} \cup \mathbb{Z}$.

Breadth- and Depth-First Search

Given an unordered graph $G = (V, E)$, starting at node s , is there a path in G from s to t ?



BFS – pseudocode

```
1 algorithm BFS( $G, s, t$ )
2    $Visited \leftarrow \{s\}$ 
3    $Q \leftarrow$  an empty queue
4   ENQUEUE( $Q, s$ )
5   while  $|Q| > 0$  do                                // Variant: ...
6      $u \leftarrow$  DEQUEUE( $Q$ )
7     if  $u = t$  then
8       return true
9     for each  $(w, v) \in E$  where  $w = u$  do          // Variant: ...
10      if  $v \notin Visited$  then
11        ENQUEUE( $Q, v$ )
12         $Visited \leftarrow Visited \cup \{v\}$ 
13    return false
```

BFS – pseudocode

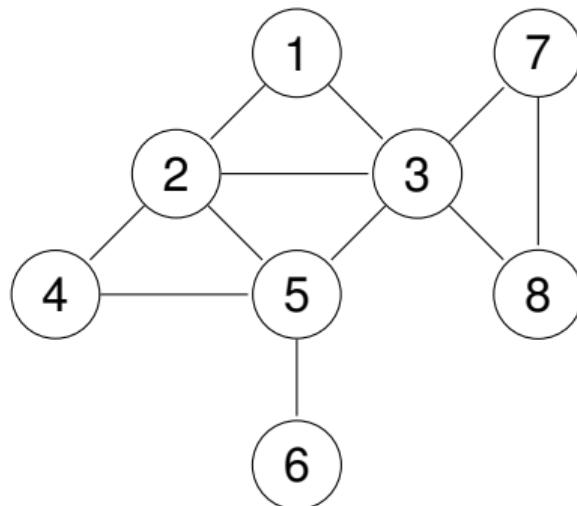
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3    $Q \leftarrow$  an empty queue
4   ENQUEUE( $Q, s$ )
5   while  $|Q| > 0$  do                                // Variant:  $n - |Visited|$ 
6      $u \leftarrow$  DEQUEUE( $Q$ )
7     if  $u = t$  then
8       return true
9     for each  $(w, v) \in E$  where  $w = u$  do      // Variant:  $\text{degree}(u) - \#\text{iterations}$ 
10      if  $v \notin Visited$  then
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BFS – Python code

```
1 def bfs(graph: Graph, s: str, t: str) -> bool:
2     visited = {s}
3     queue = deque()
4     queue.append(s)
5     while len(queue) > 0:
6         u = queue.popleft()
7         if u == t:
8             return True
9         for v in graph.neighbors(u):
10            if v not in visited:
11                queue.append(v)
12                visited.add(v)
13    return False
```

BFS

With $s = 1$ and $t = 6$, what set of nodes will be visited during $\text{BFS}(G, s, t)$?
Can the set differ between runs?



DFS – pseudocode

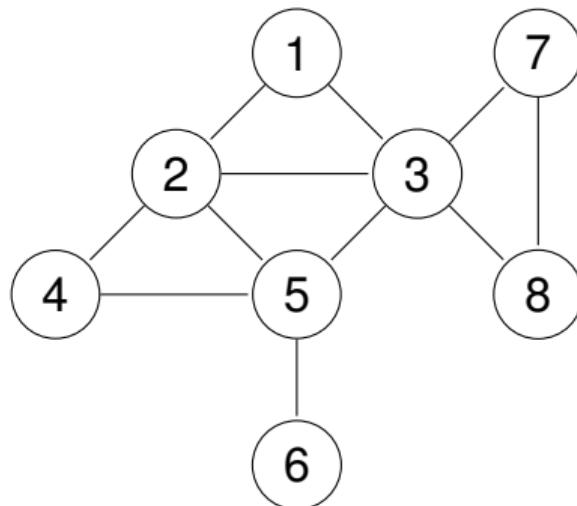
```
1 algorithm DFS( $G, s, t$ )
2    $Visited \leftarrow \{s\}$ 
3    $S \leftarrow$  an empty stack
4   PUSH( $S, s$ )
5   while  $|S| > 0$  do                                // Variant:  $n - |Visited|$ 
6      $u \leftarrow \text{POP}(S)$ 
7     if  $u = t$  then
8       return true
9     for each  $(w, v) \in E$  where  $w = u$  do      // Variant:  $\text{degree}(u) - \#\text{iterations}$ 
10       if  $v \notin Visited$  then
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4   PUSH( $S, s$ )
5   while  $|S| > 0$  do                                // Variant:  $n - |Visited|$ 
6      $u \leftarrow$  POP( $S$ )
7     if  $u = t$  then
8       return true
9     for each  $(w, v) \in E$  where  $w = u$  do      // Variant:  $\text{degree}(u) - \#\text{iterations}$ 
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```

DFS

With $s = 1$ and $t = 6$, what set of nodes will be visited during $\text{DFS}(G, s, t)$?
Can the set differ between runs?



DFS – Python code

```
1 def dfs(graph: Graph, s: str, t: str) -> bool:
2     visited = {s}
3     stack = []
4     stack.append(s)
5     while len(stack) > 0:
6         u = stack.pop()
7         if u == t:
8             return True
9         for v in graph.neighbors(u):
10            if v not in visited:
11                stack.append(v)
12                visited.add(v)
13    return False
```

DFS – Python code (contd)

Why can we use a list as a stack without worsening the time complexity?

DFS – Python code (contd)

Why can we use a list as a stack without worsening the time complexity?

Adding an element to the end of a list or array has (amortised) time complexity $\Theta(1)$. We discuss amortised time complexity in AD3.

We could also initialize a list with n elements and keep a “stack pointer”:

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We construct a function g_f returning integer v , such that v (monotonically) decreases as n_f increases towards some well-defined lower bound (typically 0).

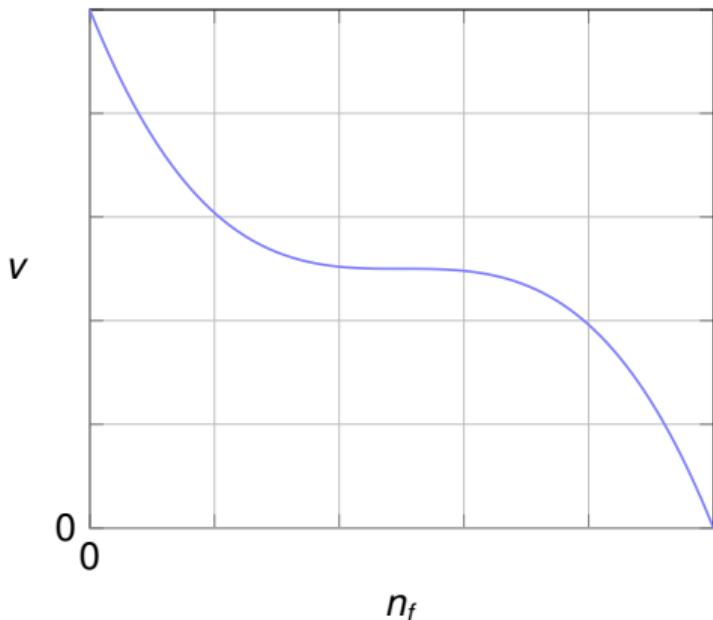
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Function g_f defines the *variant* v of f and is a guarantee that f will terminate.

Recursion Variant (contd)



As the number of completed recursions n_f of recursive function f increases, the variant v of f monotonically decreases.

Recursion Variant – Example

```
1 @recursion_variant(
2     lambda graph, visited: len(graph.nodes) - len(visited) )
3 def dfs_rec(graph: Graph, u: str, t: str,
4             visited: Set[str]) -> bool:
5     if u == t:
6         return True
7     for v in graph.neighbors(u):
8         if v not in visited:
9             visited.add(v)
10            if dfs_rec(graph, v, t, visited):
11                return True
12    return False
```

Recursion Variant – Example

```
1 @recursion_variant (
2     lambda graph, visited: len(graph.nodes) - len(visited) )
```

The size of `graph.nodes` is static. After each completed recursion, the size of `visited` either remains the same or decreases. If all nodes of `graph` have been visited, then there will be no more recursions.

DFS with Stack Pointer – Python code

```
1 def dfs_sp(graph: Graph, s: str, t: str) -> bool:
2     visited = {s}
3     stack = [s] + ([None] * (len(graph.nodes) - 1))
4     top = 0 # stack[top] = top element of stack
5     while top >= 0:
6         u = stack[top]
7         top -= 1
8         if u == t:
9             return True
10        for v in graph.neighbors(u):
11            if v not in visited:
12                top += 1
13                stack[top] = v
14                visited.add(v)
15    return False
```