

Department of Physics and Astronomy

University of Heidelberg

Master thesis

in Physics

submitted by

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2018

Retrieval advances of BrO/SO₂ molar ratios from NOVAC

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(Titel der Masterarbeit - deutsch):

(Abstract in Deutsch)

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Retrieval advances of BrO/SO₂ molar ratios from NOVAC:

Measurements of magnitude and composition of volcanic gas emissions allow insights in magmatic processes. Within the Network for Observation of Volcanic and Atmospheric Change (NOVAC) automatically scanning UV-spectrometers are monitoring gas emission at volcanoes. The emissions of BrO and SO₂ can be retrieved from the recorded spectra by applying Differential Optical Absorption Spectroscopy (DOAS) and comparing the optical absorption of the volcanic plume to the background. Therefore, the background spectrum must not be affected by volcanic influence. Classically, the background spectrum is taken from the same scan but from a elevation angle which has been identified to be outside of the volcanic plume. However, experience shows those background spectra can still be contaminated by volcanic gases. Alternatively reference spectra can be derived from 1) a theoretical solar atlas spectrum or 2) a volcanic-gas-free reference spectrum recorded by the same instrument. 1) comes with a drawback of reduced precision, as the instrumental effects have to be modeled and added to the retrieval. For 2), the alternative reference spectrum should be recorded at similar conditions with respect to meteorology and radiation. We use the first option to check for contamination and the second to evaluate the spectra to maintain a good fit quality. We present our approach and its results when applied on NOVAC data from Tungurahua and Nevado Del Ruiz.

Contents

| | | |
|-----------|---|-----------|
| 1 | Introduction | 6 |
| I | Theoretical Background | 9 |
| 1.1 | Volcanism and volcanic chemistry | 10 |
| 1.1.1 | Volcanism | 10 |
| 1.1.2 | Volcanic degassing | 10 |
| 1.1.3 | Volcanic gases and their impact on the climate | 10 |
| 1.1.4 | Volcanic plume chemistry | 10 |
| 1.1.5 | Sulphur species | 10 |
| 1.1.6 | Bromine oxide | 10 |
| 1.1.7 | Using volcanic gases to study volcanic activity | 10 |
| 1.2 | Remote sensing of volcanic gases | 10 |
| 1.2.1 | Absorption spectroscopy | 10 |
| 1.2.2 | Scattering processes in the atmosphere | 10 |
| 1.2.3 | Beer-Lambert Law | 10 |
| 1.2.4 | Differential Optical Absorption Spectroscopy(DOAS) | 10 |
| II | Evaluation of the Data of Tungurahua and Nevado Del Ruiz | 13 |
| 2 | Network for Observation of Volcanic and Atmospheric Change | 14 |
| 2.1 | Measurement Routine | 15 |
| 3 | Evaluation Routine | 17 |
| 3.1 | NOVAC-Evaluation | 17 |
| 3.2 | Contamination Problem | 19 |
| 4 | Limitations for the evaluation of BrO | 24 |
| 4.1 | BrO Error dependence on external parameters | 24 |
| 4.1.1 | Time | 24 |
| 4.1.2 | Temperature | 26 |
| 4.1.3 | Daytime | 28 |
| 4.1.4 | Colorindex | 30 |
| 4.1.5 | Elevation Angle | 31 |
| 4.1.6 | Exposure Time | 32 |

| | | |
|------------|---|-----------|
| 5 | Method | 33 |
| 5.1 | Fit data | 34 |
| 5.2 | Other approaches | 37 |
| 5.2.1 | Nearest neighbours | 37 |
| 5.2.2 | Iterative | 38 |
| 6 | Comparison with NOVAC Evaluation | 39 |
| 7 | Results | 42 |
| 7.1 | Tungurahua | 42 |
| 7.2 | Nevado Del Ruiz | 42 |
| 8 | Issues of our method | 43 |
| 8.1 | Contamination of the plume | 43 |
| 9 | Conclusion | 44 |
| III | Appendix | 45 |
| A | Lists | 46 |
| A.1 | List of Figures | 46 |
| A.2 | List of Tables | 47 |
| B | Bibliography | 48 |

1 Introduction

Volcanic activities on Earth have always shaped the earth surface and influenced atmospheric processes. Volcanoes are often particularly recognized by their dramatic consequences of a major volcanic eruption. But volcanoes influence our lives in more than this way. Volcanic gases can effect the weather (timescales of days to weeks) or the climate (timescales of months to years) [Schmidt and Robock \[2015\]](#). Examples are the lake eruption in Iceland (1783-1784) followed by a very hot summer and a cold winter in central Europa [Thordarson and Self \[2003\]](#) and the Tambora eruption, indonesia in 1815 which caused the "year without summer" in 1816.

Considering the plate tectonics of earth most volcanoes are caused by diverging or converging of the continental plates and therefore located at the margins of the continental plates. Another possibility for occurrence of volcanoes is the the interior of continental or oceanic shelves. [Schmincke \[2000\]](#)

The most abundant volatile species released during a volcanic eruption are water vapour (H_2O ; relative amount of the plume: 50%-90%) and carbon dioxide (CO_2 ; relative amount of the plume: 1%-40%) [Platt and Bobrowski \[2015\]](#). But the short effects of those two gases are rather low since there effect on atmospheric composition is negligibly due to the high abundance of atmospheric H_2O and CO_2 . But on timescales of the age of the earth the volcanic emission of H_2O and CO_2 are the source of our current atmosphere. [Schmidt et al. \[2015\]](#)

A typically volcanic plume consists of many different gases alongside H_2O and CO_2 sulfur dioxide (SO_2) contributes with 1%-25% to the plume, hydrogen sulfide (H_2S) with 1%-10% and hydrogen chloride with (HCl) 1%-10%. Furthermore there are trace gases for example carbon disulfide (CS_2), carbon sulfide (COS) carbon monoxide (CO) hydrogen fluoride (HF) and hydrogen bromide (HBr) [Platt and Bobrowski \[2015\]](#)

A decrease of stratospheric ozone (O_3) has been observed after the eruption of El Chickon in 1982 and the eruption of mount Pinatubo 1991. A depletion stratospheric O_3 results in ozone holes. The depletion comes from volcanic aerosols which serve anthropogenic chlorine/bromine into more reactive forms [Solomon et al. \[1998\]](#). Volcanic gases can alter the radiative balance of the earth in timescales relevant for climate change due to scatter and absorption of solar radiation [Schmidt et al. \[2015\]](#). The gas composition of the volcano plume change with activity and could be a indication for the processes inside the earth.

In this work we are particularly interested in the ratio of BrO and SO_2 . The halogen sulfur ratio is a proxy for volcanic processes. Therefore we make the assumption that the ratio of BrO and SO_2 contains informations about its degassing source

depth. A change in BrO/SO₂ prior to eruption was observed at Etna and Nevado del Ruiz.

To gain further knowledge about the volcanoes the Network for Observation of Volcanic and Atmospheric Change (NOVAC) was installed. NOVAC is a Network of DOAS Instruments located next to about 30 volcanoes in America, Africa and Europe. At every Volcano there are two to four DOAS Instruments installed, recording record back-scattered solar radiation spectra at different viewing angles.

NOVAC is a network which produces a large amount of data and we have the chance to evaluate long time periods which is a unique opportunity to study correlations of the trace gases.

Since the conditions at volcanoes are rough, the instruments need to be rather simple to keep the maintenance cheap and to assure a longer lifetime of the instruments. So we need to waive on temperature stabilization even at the expense of the quality of the data.

One possibility to measure the volcanic trace gases is to use Differential Optical Absorption Spectroscopy [Platt and Stutz \[2008\]](#). DOAS exploit the wavelength dependency of the absorption of light. Here the gas emissions can be retrieved from the quotient of the absorption signal of the volcanic plume and a reference region. This will be explained in a further chapter.

The reference region, is usually treated as free of volcanic trace gases. If the reference region is for any reason contaminated by volcanic trace gases, the reference spectrum has to be replaced by a volcanic-gas-free reference. Alternative spectra could be for example a theoretical solar atlas spectrum or a volcanic-gas-free reference spectrum recorded in the temporal proximity(eg. a day before) by the same instrument. The first option comes with the drawback of reduced precision, as the instrumental effects have to be modeled and added to the retrieval. The reduction in precision is acceptable for the SO₂ retrieval, but not suitable for a BrO retrieval because then most data would be below the detection limit. For the second option, the alternative reference spectrum should have been recorded at similar conditions with respect to meteorology and radiation as well as in the temporal proximity due to instrumental changes with time and ambient conditions. We combined both options in order to achieve both, enhanced accuracy but still maximum possible precision of the SO₂ and BrO retrievals. We present an algorithm which finds the optimal reference spectrum automatically. As first step, a possible SO₂ contamination of the standard reference is checked by a comparison with the theoretical solar atlas. If a contamination is detected, as second step, the algorithm picks a volcanic-gas-free reference (beforehand automatically checked for contamination) from another scan.

In this work we are mainly dealing with data from Tungurahua in Ecuador in the timespan of 01.08.2008 to 30.07.2009. Later on, we will also show the results of Nevado del Ruiz a volcano located in Colombia.

Part I

Theoretical Background

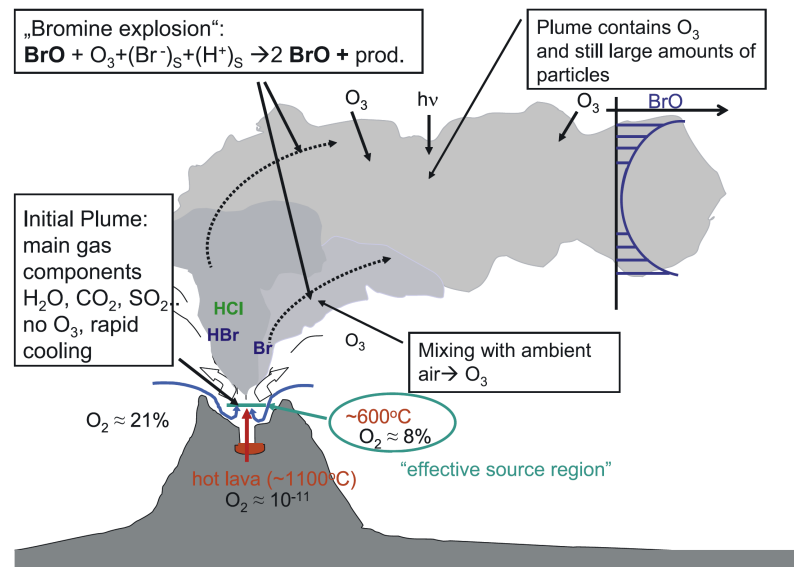


Figure 1.1:

1.1 Volcanism and volcanic chemistry

1.1.1 Volcanism

1.1.2 Volcanic degassing

1.1.3 Volcanic gases and their impact on the climate

1.1.4 Volcanic plume chemistry

1.1.5 Sulphur species

1.1.6 Bromine oxide

1.1.7 Using volcanic gases to study volcanic activity

1.2 Remote sensing of volcanic gases

1.2.1 Absorption spectroscopy

1.2.2 Scattering processes in the atmosphere

1.2.3 Beer-Lambert Law

1.2.4 Differential Optical Absorption Spectroscopy(DOAS)

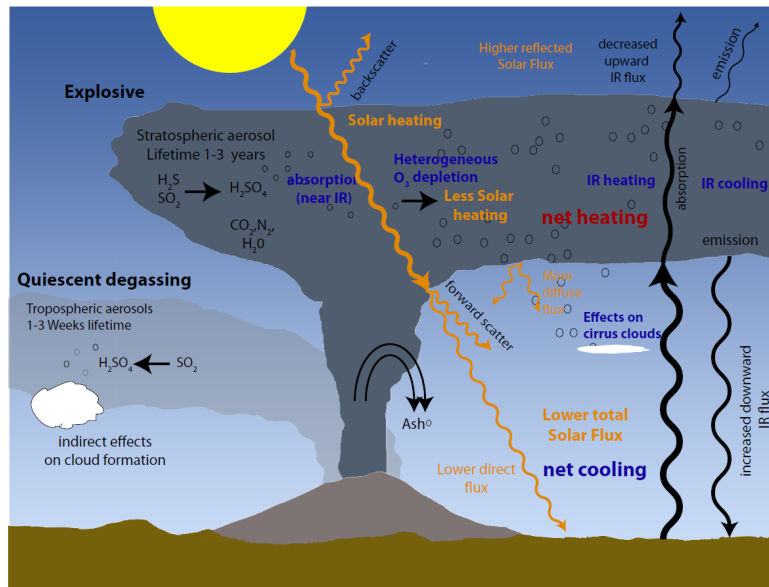


Figure 1.2:

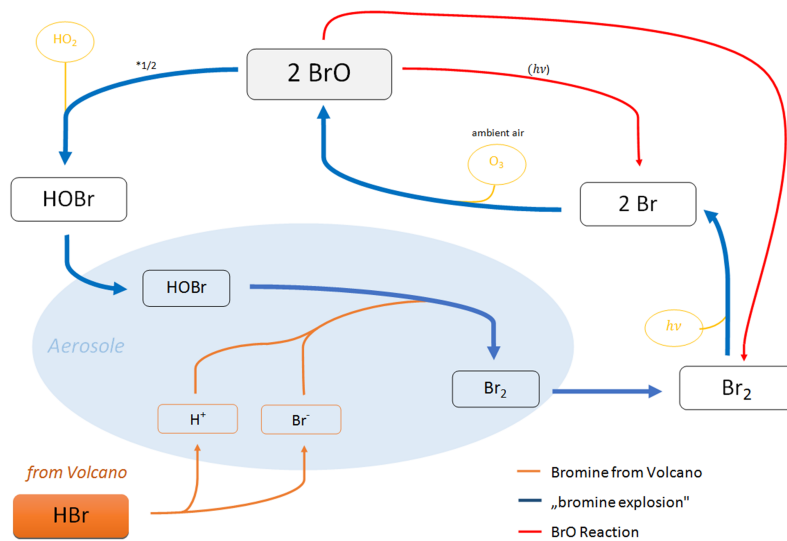


Figure 1.3:

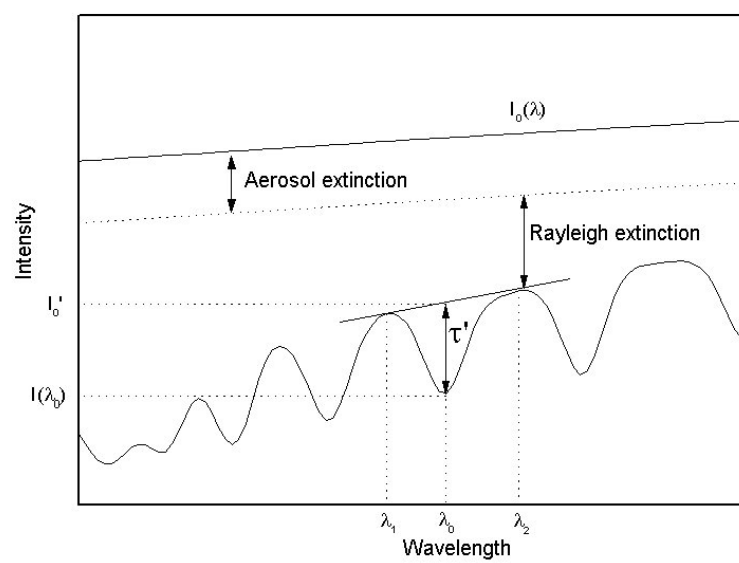


Figure 1.4:

Part II

Evaluation of the Data of Tungurahua and Nevado Del Ruiz

2 Network for Observation of Volcanic and Atmospheric Change

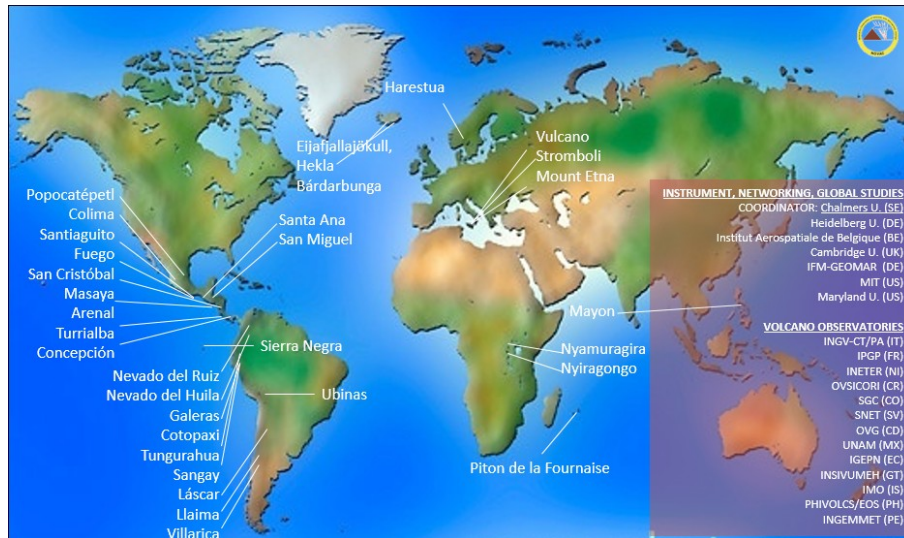


Figure 2.1: Global map of the volcanoes monitored by NOVAC. Used with friendly permission of Santiago Arellano.

Network for Observation of Volcanic and Atmospheric Change (NOVAC) is a network of instruments monitoring volcanoes over the whole world. The aim of NOVAC is to gain another tool for risk assessment, for gas emissions and geophysical researches. Also many other scientific purposes are built on the data from NOVAC. Figure 2.1 shows a map, with all volcanoes of the Network for Observation of Volcanic and Atmospheric Change.

NOVAC was originally funded by the European Union on the first October in 2005. The aim of NOVAC is to establish a global network of stations for the quantitative measurement of volcanic gas emissions. At the beginning NOVAC encompassed observatories of 15 volcanoes in Africa America and Europe, including some of the most active and strongest degassing volcanoes in the world. Although the EU-funding has stopped, the network has been constantly growing since it was founded. In 2017 more than 80 Instruments are installed at over 30 volcanoes in more than 13 countries.

The great advantage of the data monitored in NOVAC is the fact that NOVAC provides continuous gas emission data over many years. Therefore one is able to get more statistically stable results.

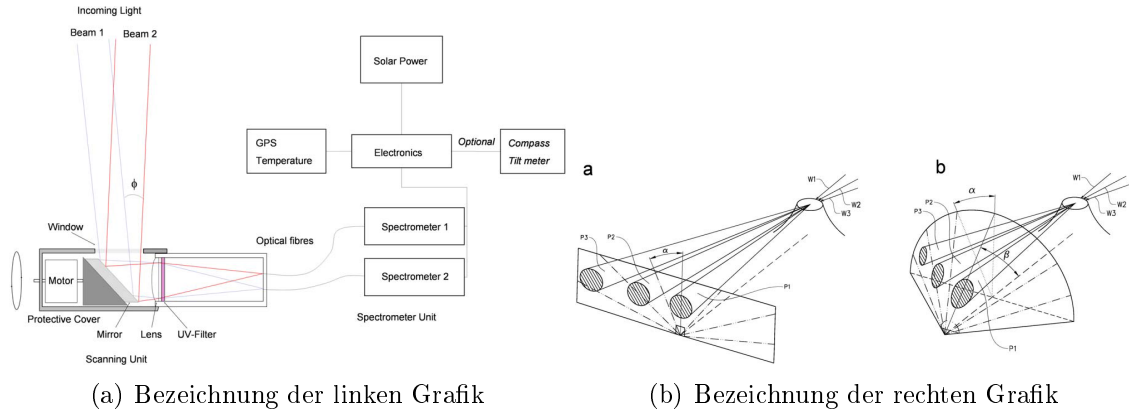


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The instruments used in NOVAC are scanning UV-spectrometer : Mini Doas instruments.

The Mini-DOAS instrument represents a major breakthrough in volcanic gas monitoring as it is capable of real-time semi-continuous unattended measurement of the total emission fluxes of SO₂ and BrO from a volcano. Semi-continuous means in this case that the measurement is only possible during day time when enough Sun light is there.

The basic mini-DOAS system consists of a pointing telescope fiber-coupled to a spectrograph. Ultraviolet light from the sun, scattered from aerosols and molecules in the atmosphere, is collected by means of a telescope with a quartz lens defining a field-of-view of 12 mrad. ??

The spectrometers measure in the UV region in a wavelength range of 280 to 420 nm. In this range are the differential structures of SO₂ and BrO dominant.

The Novac-instruments need to be very robust to stand the conditions around volcanoes. Therefore the design of the instruments is rather simple, this means the instruments do not have internal stabilisation features like temperature stabilization to keep the measurement independent of external parameters (for example Temperature). This comes along with a reduced precision of the data, but the huge amount of data produced by NOVAC compensates this disadvantage.

2.1 Measurement Routine

The Instruments are set up five to ten km downwind of the volcano of the volcano. To cover most of the occurring wind directions two to five instruments are installed at each volcano. Ideally the measurement plane is orthogonal to the plume, to get the best measurement results. In reality the measurement plane could be twisted. The Instruments record spectra in different viewing angles covering a the hole sky from horizon to horizon from -90° to 90°. The zenith is at 0°. The measurement routine starts with a spectrum in zenith direction: The pre-reference. Afterwards

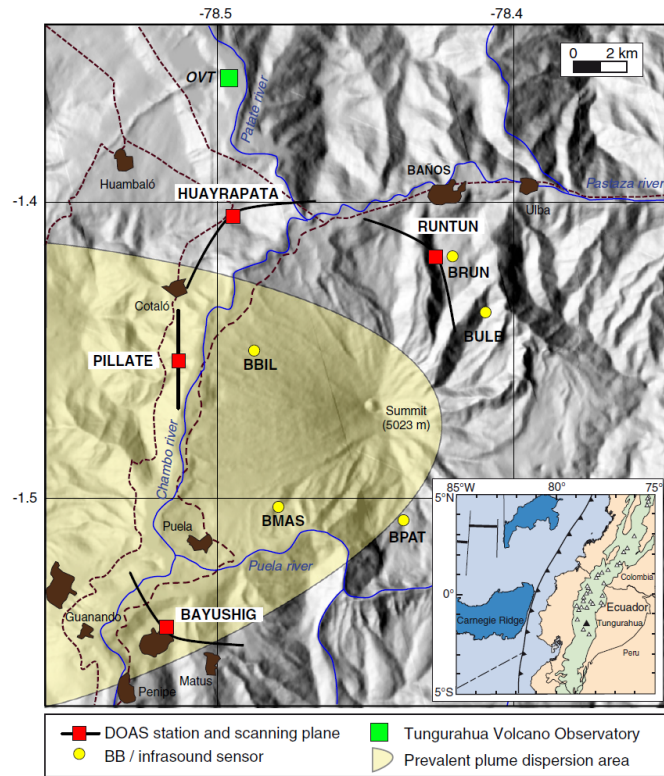


Figure 2.3:

the dark current spectrum is recorded.

Then the Instrument turns automatically to the side, recording spectra at the Elevation Angle from -90° to 90° with steps of 3.6° .

One hole measurement takes from 6 to 15 minutes.

3 Evaluation Routine

3.1 NOVAC-Evaluation

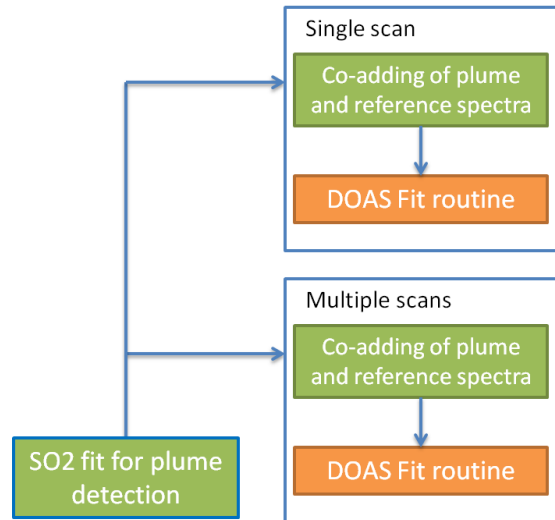


Figure 3.1:

In the following we describe the technical implementation of the DOAS approach using the data of NOVAC instruments:

The first important task is to locate volcano plume and the reference region using the data from the measurement routine described above.

To do so we use the pre-reference (the spectra recorded at an elevation angle of 0°) to evaluate spectra for SO_2 at every elevation angle as described in chapter 1.2.4, that means we divide each recorded spectra by the pre-reference and take the logarithm to get rid of the Fraunhofer structures and to be able to just look at the important structures of the plume. To get the gas amounts of the evaluated spectra, one fits the absorption spectrum of all important gases on the spectrum. In our case we take all gases written in tab. ?? into account. The result will be an SO_2 curve as it is shown in fig. ?. Figure ?? shows the relative SO_2 column density to the pre-reference as a function of the elevation angle. We can clearly observe a maximum of SO_2 and a minimum. Inside the plume the SO_2 amount is much higher than in the outside the plume. Therefore we assume that the location of the SO_2 maximum match with the location of the plume. We assume that the minimum of the SO_2 curve refers to a region outside of the plume which is in most

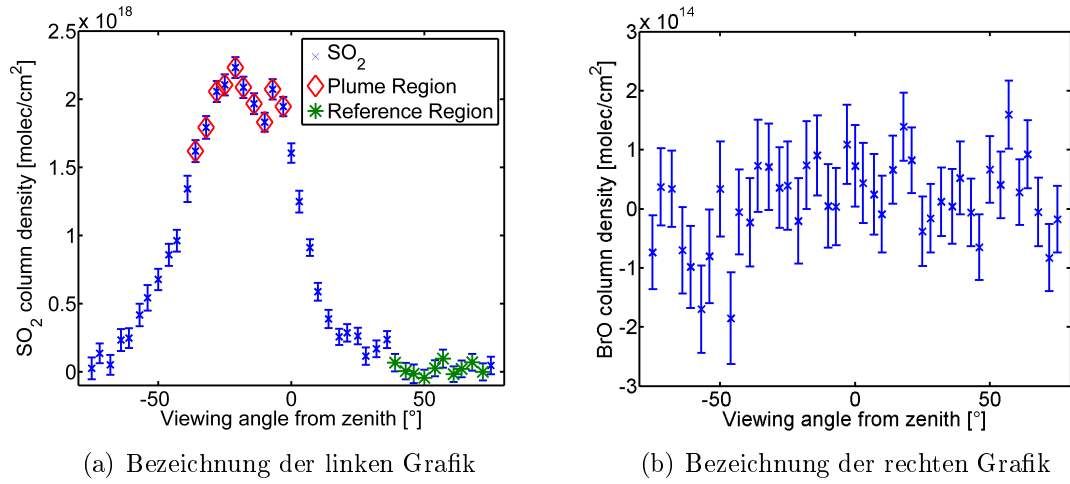


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times the case. The SO₂ amount in the earth's atmosphere is negligible so we take it as a region of zero SO₂. Now it is possible to locate the plume region as the SO₂ maximum, whereas the minimum of the SO₂ curve the reference region is.

To technically detect the plume region we use a gauss fit of the SO₂ curve. To increase the quality and to get a more robust result the sum over several plume spectra is taken. If the gauss curve is too wide we use only the 10 spectra with the highest SO₂ amount. For the reference we use the sum of 10 spectra with the lowest SO₂ amount.

The so found reference spectrum is used to fit it on the SO₂ absorption lines of Gases to get the absolute column densities of SO₂ and BrO in the plume spectrum

Since the BrO column density is much lower than the SO₂ column density and lies just slightly above the detection limit the plume is hard to detect using the BrO column density as it is shown in fig. ???. Therefore we use plume location we found by using SO₂ to evaluate the BrO column density.

For the evaluation we use the data of more than one measurement, to increase the fit quality.

We are mainly interested in the BrO/SO₂ ratio, with the calculations described above it is now possible to get this ratio. In ??? is the NOVAC Evaluation visualized.

Taking the BrO/SO₂ ratio if the column densities are close to zero yields unpredictable and unrealistic results. Thus spectra measured outside of the volcano plume need to be excluded. This could be achieved by setting a BrO or/and a SO₂ threshold. A reasonable BrO threshold need to be at least in the order of the DOAS fit error. But this could lead to elevated BrO/SO₂ ratios, since the BrO error is often close to the detection limit, and thus exclude all low BrO column densities from the evaluation.

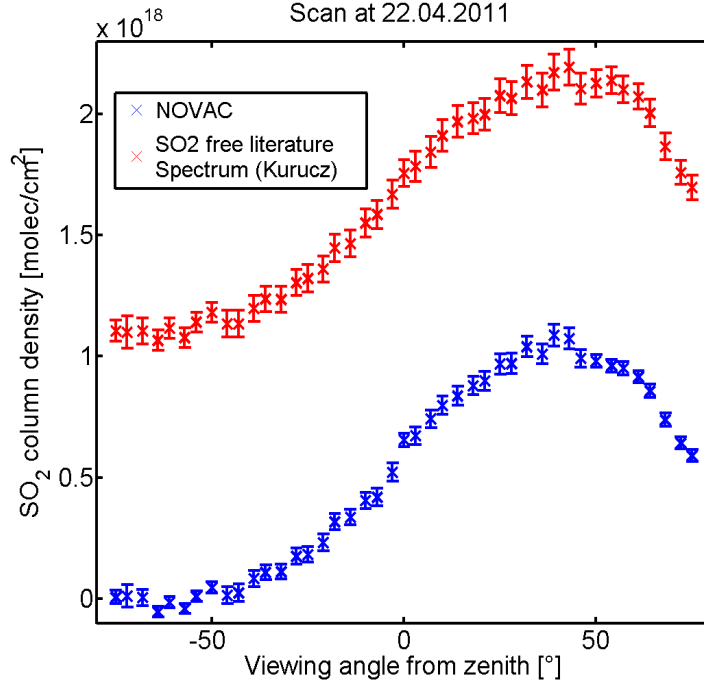


Figure 3.3:

To avoid this problem, an SO₂ threshold of $7 \cdot 10^{17} \frac{\text{molec}}{\text{cm}^2}$ was used to select spectra for the evaluation of the BrO/SO₂ ratio. This threshold is a relatively high SO₂ column density. However, for the lower values of the BrO/SO₂ ratio in this would result in a BrO column density as low as $4 \cdot 10^{13} \frac{\text{molec}}{\text{cm}^2}$, a value only slightly higher than the average DOAS retrieval error for BrO. This approach assures that scans not seeing significant amounts of volcanic gas are filtered out and thus will not significantly influence the BrO/SO₂ ratio. [Lübcke et al., 2014](#)

3.2 Contamination Problem

———— genaue angebane tungurahua u NEVADO

It might occur that in rare (ca. 10% of the data) scenarios, the volcanic plume covers the whole scan region. This could happen if for example the volcanic plume of the day before still extend over the hole scan area as a consequence of windless conditions. In consequence, the reference is contaminated with volcanic trace gases. Thus the gas amount is underestimated by the NOVAC-Evaluation: In fig. 3.3 we see an example from April 2011 (Tungurahua) where the reference region is contaminated by volcanic trace gases. The blue SO₂ curve shows our calculations with the NOVAC-Evaluation, but since there is still SO₂ in the reference region, therefore the assumption, that the SO₂ amount could be set to zero in the reference region

is wrong. The red curve shows the real SO₂ curve, and we will underestimate the total SO₂ amount of the plume. Contamination occur in approximately 10% of the data.

If the reference region is for any reason contaminated by volcanic trace gases, the reference spectrum has to be replaced by a volcanic-gas-free reference. Alternative spectra are a theoretical solar atlas spectrum (the use of a solar atlas spectrum will be described in section 3.2) or a volcanic-gas-free reference spectrum recorded by the same instrument.

In the following we will discuss both of these options:

Evaluation using a Solar Atlas Spectrum

An alternative to choose the region with the lowest column density as reference region is to use a theoretical high resolution solar atlas spectrum as reference [Chance and Kurucz \[2010\]](#). The use of a theoretical solar atlas spectrum as a reference which is completely volcanic-trace-gases-free was first proposed by [Lübcke et al. \[2014\]](#). The advantage of using a solar atlas spectrum as reference is, that we know that there are no volcanic trace gases, we do not need to assume, that the minimum SO₂ amount is zero. The disadvantage is, that using a solar atlas spectrum comes along with a drawback of precision: A theoretical solar atlas spectrum is far more precise than the spectra of the NOVAC instruments therefore the instrument functions need to be modeled and added to the retrieval.

The reduction of precision is acceptable for the SO₂ retrieval but not suitable for a BrO retrieval because then most data would be below the detection limit.

Possible contaminations can be checked by a theoretical solar atlas spectrum to evaluate the SO₂ amount in the reference.

Evaluation using a Spectrum of the same Instrument

An alternative reference spectrum could be a volcanic-gas-free reference spectrum recorded by the same instrument. When using such a reference several problems occur:

As described in chapter 2 the instruments used in NOVAC do not include features like temperature stabilisation due to that the measurements are not independent from external parameters. So we need to choose a reference recorded at similar conditions with respect to meteorology and radiation as well as in the temporal proximity due to instrumental changes with time and ambient conditions. Ideally the external conditions should be equal to the conditions when the plume was recorded.

In this work we will combine both options in order to achieve both, enhanced ac-

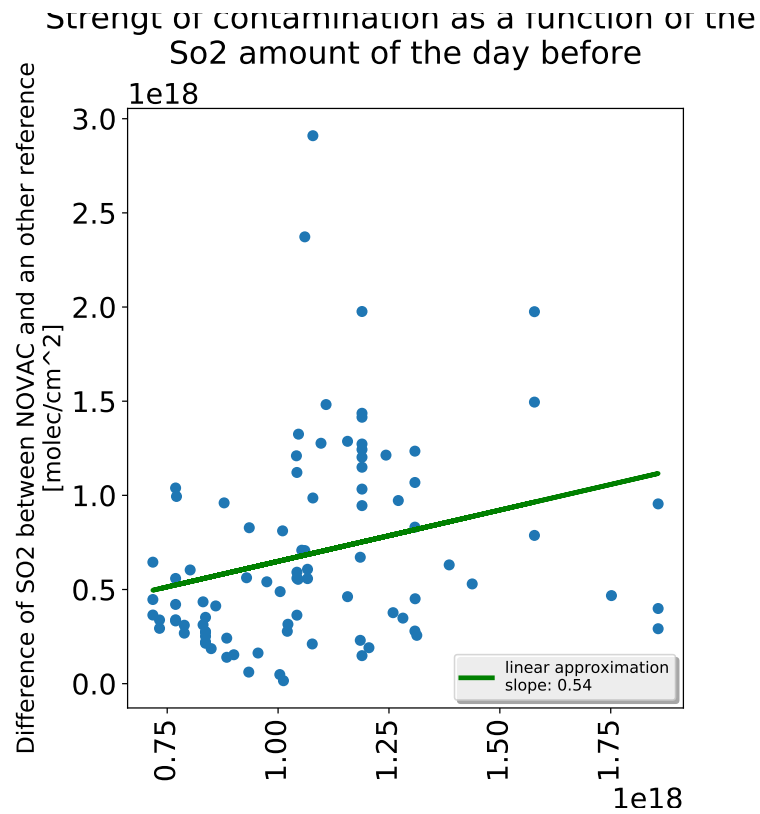


Figure 3.4:

curacy but still maximum possible precision of the SO₂ and BrO retrievals. So we use the solar atlas spectrum to check for contamination and a reference spectrum recorded in temporal proximity by the same instrument as reference.

In the following we will discuss how to find the an optimal reference from another scan automatically.

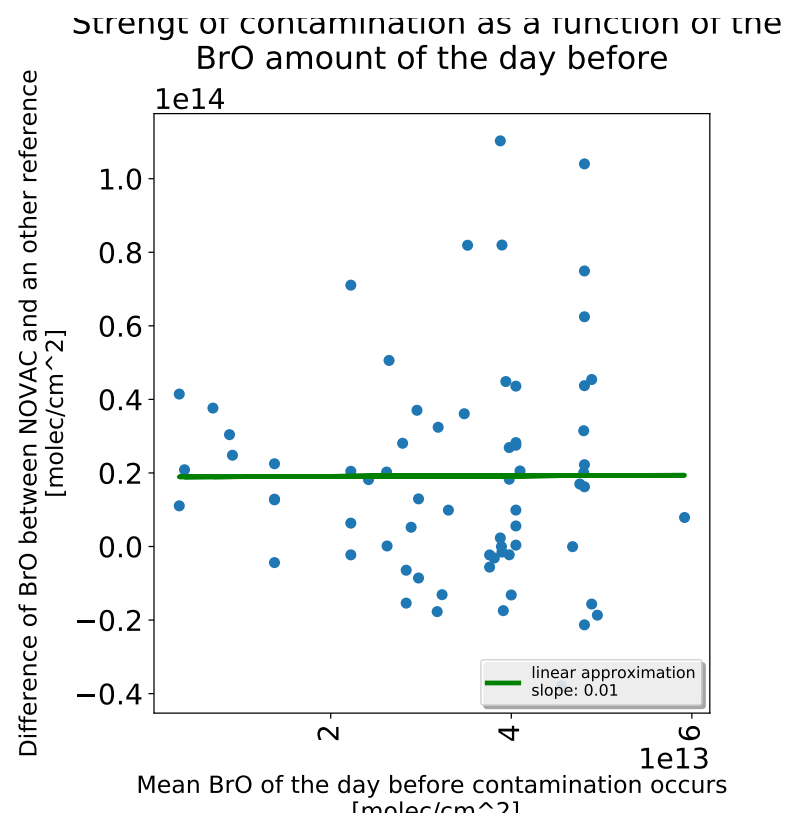


Figure 3.5:

Strengt of contamination as a function of the
BrO/SO2 amount of the day before

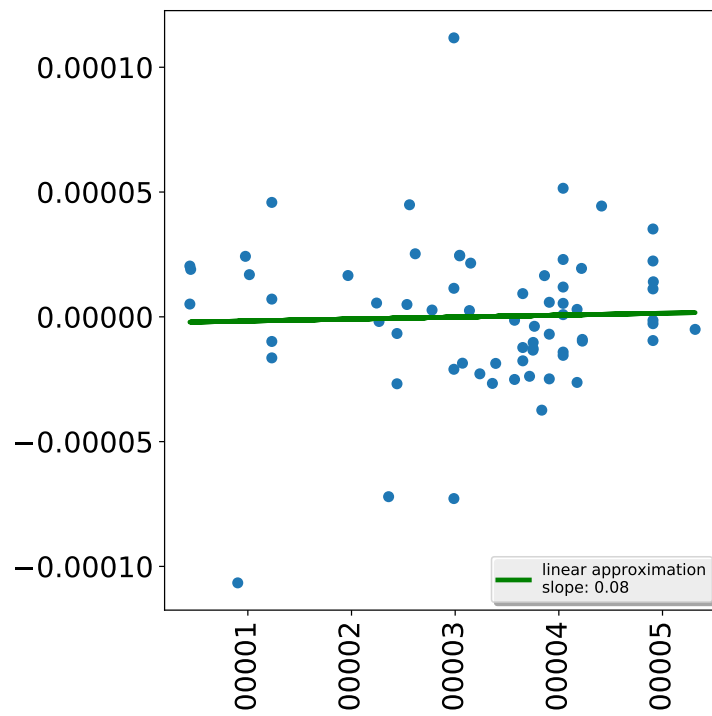


Figure 3.6:

4 Limitations for the evaluation of BrO

Since the SO₂ amount in a volcano plume is rather high (magnitude of SO₂ at Tungurahua $\approx 1e^{18}$, [Warnach \[2015\]](#)), the evaluation of SO₂ is unproblematic compared to BrO.

Evaluating BrO is more difficult since the amount is much smaller and the measurement error relative to the column density much larger. Since we want to get the BrO/SO₂ we need to maximize the accuracy of BrO. Therefore the aim is to choose the reference with respect to the BrO error, to minimize the BrO Error and to increase the amount of reliable BrO/SO₂ ratio data.

We figured out, that the BrO Error depends strongly on the surrounding conditions when recording the plume and the reference. In the following, we will take a closer look at the dependence of the BrO error on external parameters.

4.1 BrO Error dependence on external parameters

The measurement and evaluation depends on the surrounding conditions like temperature or cloudiness [Lübcke \[2014\]](#)

If choosing a new reference we need to take the surrounding conditions into account. The better the surrounding conditions of the time where the reference is measured coincide with the conditions of the time when the plume is measured, the lower is the BrO error.

The surrounding conditions we take into account are temperature, colorindex, exposure time, elevation-angle, daytime and the temporal difference.

In almost all cases (99%) the absolute BrO Error is minimal when using the reference recorded at the same time as the plume spectrum. So we won't be able to get an BrO Error which is smaller than the "Same Time Error".

4.1.1 Time

Due to instrument drifts the fit quality decreases with the time difference between recording the plume and the reference. Therefore it is better to use an reference in temporal proximity.

Figure 4.1 shows the Instrumental drift as a function of time, to create fig. 4.1 we used Tungurahua data, 2008 from June to November. We can observe that the drift changes with time. If we use the reference and plume spectra of the same time, we do not need to care about these effects, since the shift is equal for the plume and reference spectrum, but if the recording time is not the same the quality of the fit changes with the differences in wavelength shift which increases with the time

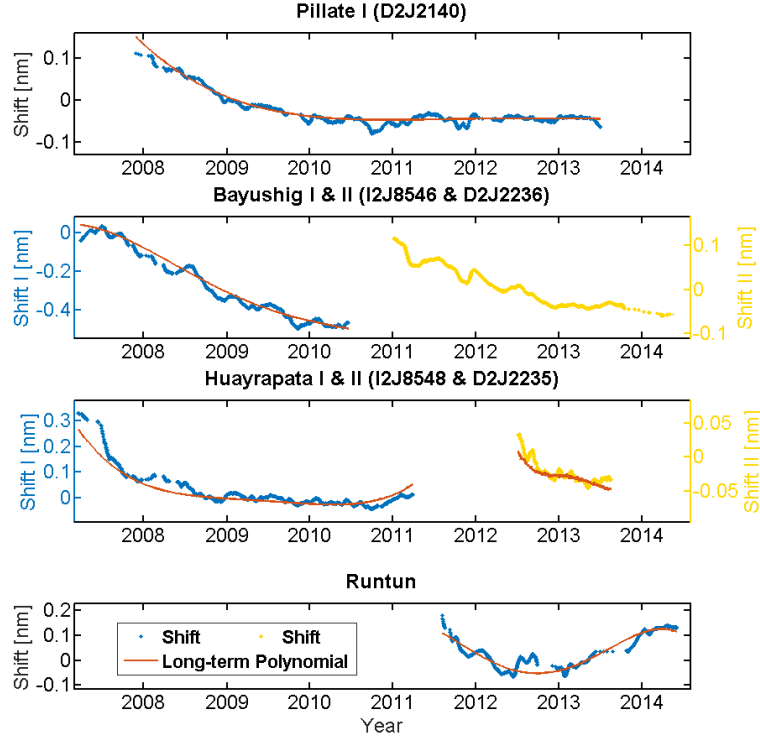


Figure 4.1:

difference.

In fig. 4.3 the BrO Error as a function of the time difference between recording the plume and the reference is shown. The running mean is drawn with a black line. The BrO Error increases with time difference.

To evaluate the maximal time difference, were we still get reliable results we calculated for all possible reference-plume pairs the corresponding BrO Error. With this data we are able to find for all plume spectra the associated reference where the BrO Error is minimal. In ?? a histogram is plotted with the probability of picking the best reference as a function of the time difference. Obviously the best results are if the day of measuring the reference is the same day as measuring the reference that means, if the time difference is smaller than one day. We allow all time difference which are in one sigma area.

We found out that the time interval where it is still reasonable to use references is about 14 days. Therefore we only use references where the recording time difference between plume and reference is smaller than two weeks. When using a references with a temporal difference to the plume of more than 14 days the probability, that the fit quality and thus the BrO error increases to much for our purposes.

4.1.2 Temperature

The Instrument design of the NOVAC instruments compromise between accuracy and longevity as explained in chapter 2. In particular there are no internal thermal stabilizations installed as an attempt to reduce the need for power. This can influence the recorded spectra.

Each pixel of the spectrometer, which is used for the DOAS experiment, collects photons of a certain wavelength range.

The calibration for the wavelength to pixel mapping (WMP) is commonly done with a Mercury lamp or by the comparison with the high defined Kuruz spectrum. As the WMP depends on the optical alignment of the spectrometer, which itself depends on the temperature, it is not constant. Changes in the spectrometers temperature can cause changes in the instrument line function and shifts in the WMP (Pinardi et al. [2007]). Moreover, Warnach [2015] show that, short term shifts are related to the instrument temperature (see Figure 4.4).

The above discussed temperature dependence of the WMP causes a reduction of the fitquality with increasing instrument temperature between plume and reference. Thus the BrO Error increases as well with the temperature difference. To quantify the BrO error dependency on the temperature all plume spectra of Tungurahua from August 2008 to August 2009 (Nevado del Ruiz from to) were evaluated with all plume spectra of the same time. In this time span 1647 "multi-add" spectra from three different instruments were recorded, so we get approximately 1646^2 plume reference pairs and their corresponding BrO error and temperature. The BrO error as function of the temperature difference can be seen in fig. 4.5. The blue dots show the mean BrO error at the specific temperature difference, the standard deviation is illustrated with gray bars.

When compare the data of Tungurahua and Nevado Del Ruiz it is noteworthy that the BrO error on temperature dependence of the Data of Nevado Del Ruiz is stronger and the deviation is weaker than at Tungurahua, this may occur due to the larger temperature fluctuation at Nevado Del Ruiz. ?????????? quellen bitte!

When looking at all discussed external parameters, temperature has the strongest impact on the BrO error due to the strong impact on the WMP. [h!]

- The BrO error has the strongest dependence on the temperature difference. At Tungurahua (Nevado Del Ruiz) the BrO error increases by factor of $3.53 \cdot 10^{12}$ per degree.

$$\begin{aligned} \rightarrow BrO_{Error} &= f(ext.P) + 3.53 \cdot 10^{12} \cdot \frac{\Delta T}{1C^\circ} + \mathcal{O}() & Tungurahua \\ \rightarrow BrO_{Error} &= f(ext.P) + 7.56 \cdot 10^{12} \cdot \frac{\Delta T}{1C^\circ} + \mathcal{O}() & NevadoDelRuiz \end{aligned}$$

between monitoring the plume
and reference region

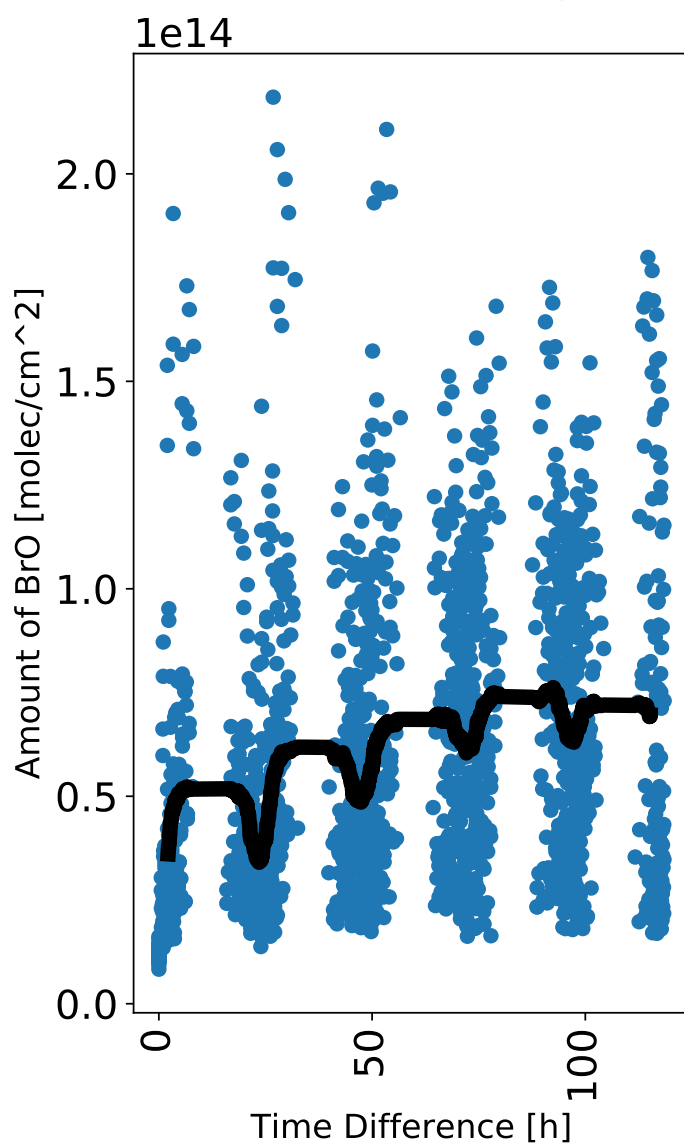


Figure 4.2:

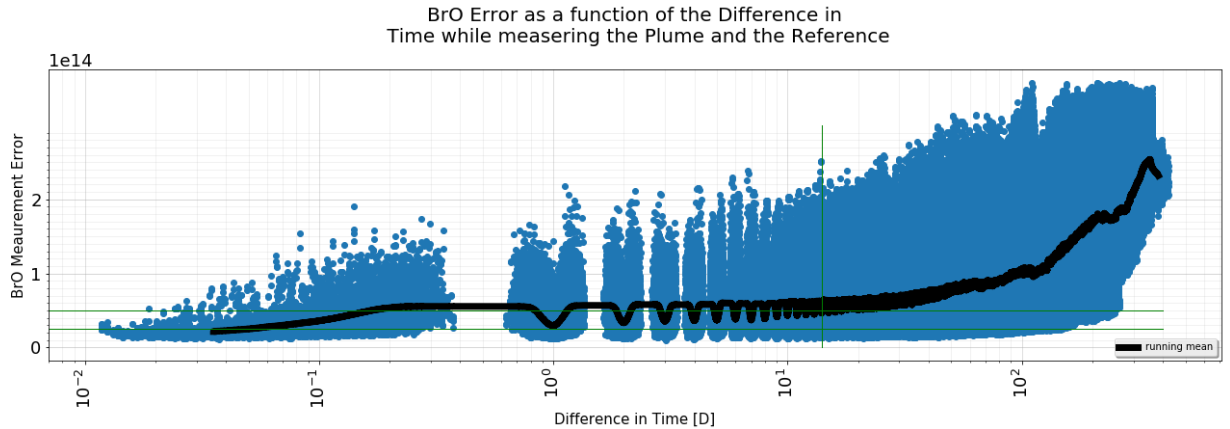


Figure 4.3:

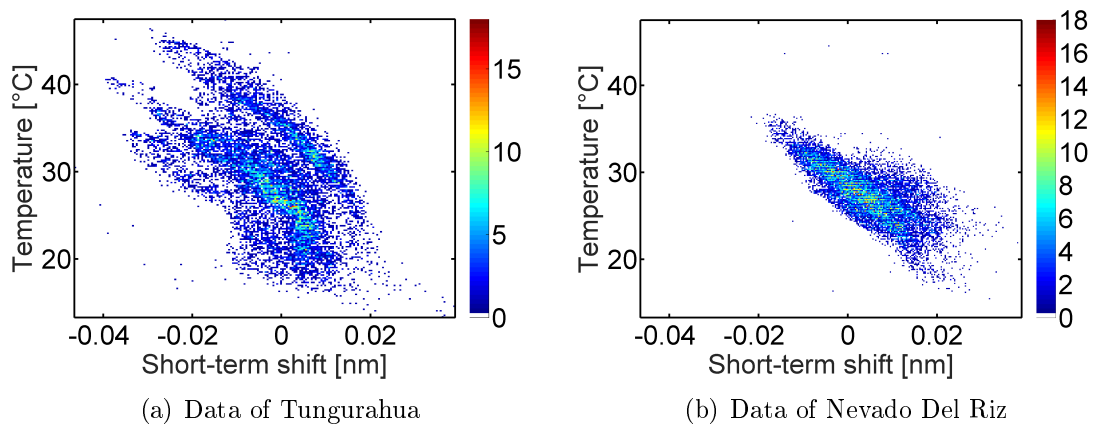
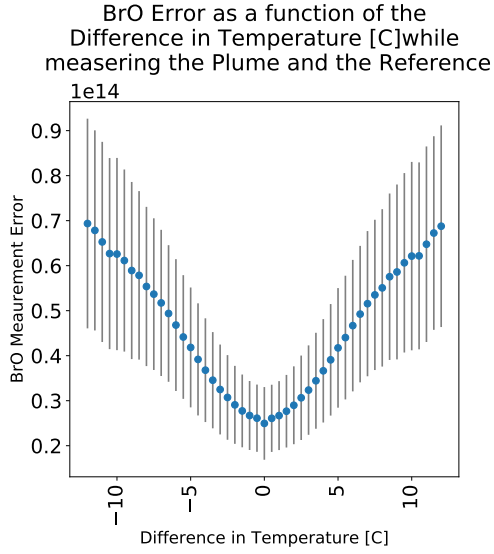


Figure 4.4: Titel unterm gesamten Bild

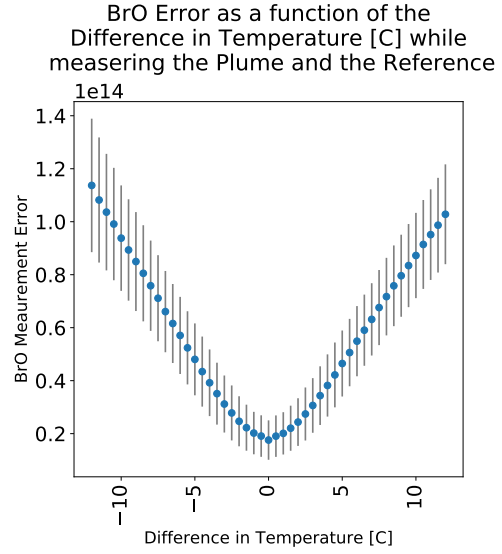
4.1.3 Daytime

During the day a lot of external parameters like temperature, solar altitude etc. change. In particular the solar altitude could have an impact on the fit quality since the light path of the sun is much longer at the evening than at noon. Figure 4.6 shows the dependency of the BrO error on the daytime. The data are calculated as described for the temperature. As for the temperature the dependence of Nevado Del Ruiz is much larger than of Tungurahua, this might occur during the larger distance from the equator of Nevado Del Ruiz -> besser beschreiben

- We found a dependency of the BrO error on the daytime. We assume, that this dependency comes from other external parameters which change during the day.
- The BrO Error increases with the daytime differences like:

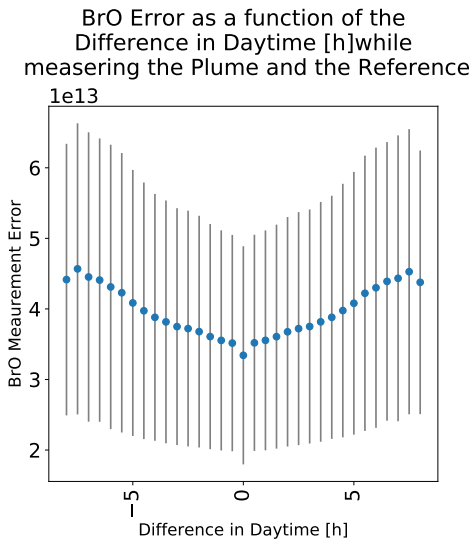


(a) Data of Tungurahua

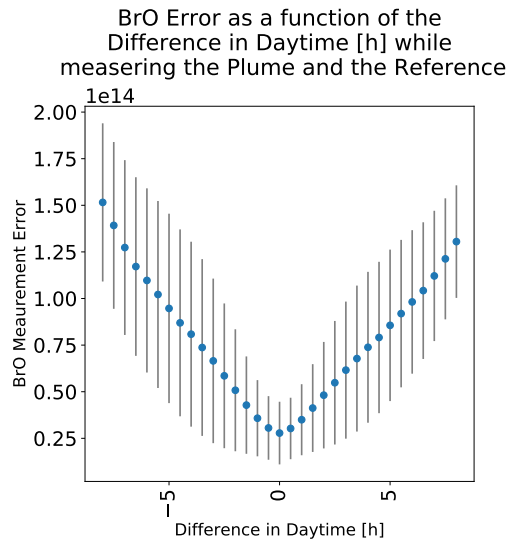


(b) Data of Nevado Del Riz

Figure 4.5: Titel unterm gesamten Bild



(a) Data of Tungurahua



(b) Data of Nevado Del Riz

Figure 4.6: Titel unterm gesamten Bild

$$\rightarrow BrO_{Error} = f(ext.P) + 1.33 \cdot 10^{12} \cdot \frac{\Delta DT}{1h} + \mathcal{O}() \quad Tungurahua$$

$$\rightarrow BrO_{Error} = f(ext.P) + 1.58 \cdot 10^{13} \cdot \frac{\Delta DT}{1h} + \mathcal{O}() \quad NevadoDelRuiz$$

4.1.4 Colorindex

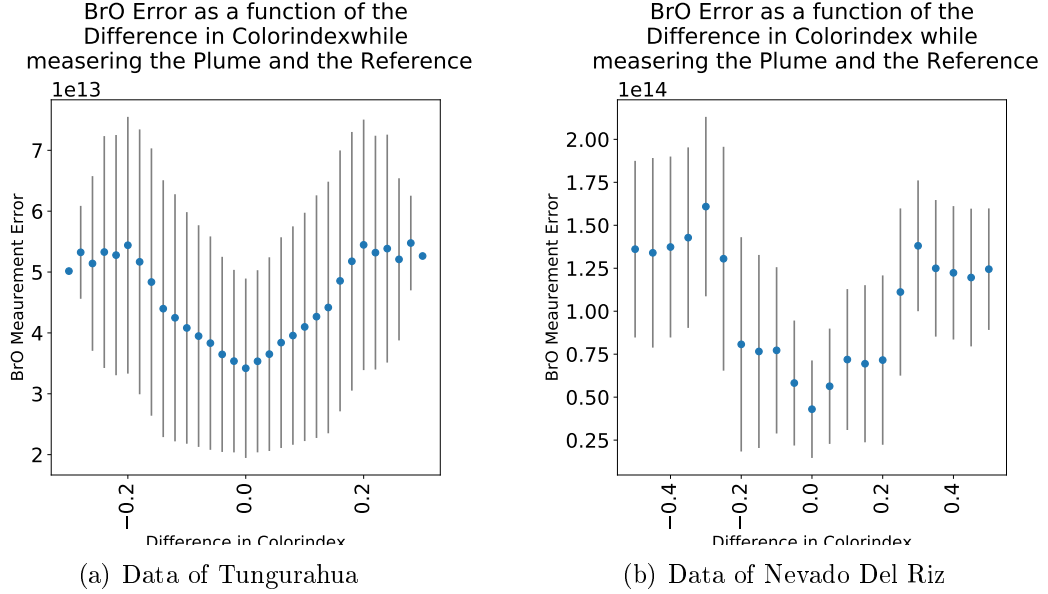


Figure 4.7: Titel unterm gesamten Bild

Clouds have a strong influence on the atmospheric radiative transfer and thus affect the interpretation and analysis of DOAS - observations [Wagner et al. \[2014\]](#).

Clouds can be identified by several measurement quantities that they influence. As Mie scattering is dominant in clouds the wavelength of the light that is scattered is different than the Rayleigh sky. Thus, clouds can be easily identified by their white color. Therefore, the cloudiness of the sky can be quantified in a scalar measure defined by the ratio of the measured intensity at two wavelengths, the so-called colour index. [Wagner et al. \[2014\]](#) showed that for a zenith-looking instrument the measured radiation intensity is enhanced by clouds. Thus, clouds can cause large errors for the retrieved gas column density and the corresponding uncertainties. Cloud effects are especially severe if the cloudiness for the recorded plume and reference spectra strongly differ. Also for broken clouds the described effect can be observed as measurements at some elevation angles might be influenced by clouds while others are not. In this work the Colour Index (CI) is the ratio between the intensities at 320nm and 360 nm. These two wavelengths are as far apart as the filter used for stray-light prevention in the spectrometers allows. On the other hand, the lower wavelength avoids the deep UV range where SO₂ and O₃ absorption plays a dominant role. The Mie scattering in the clouds is responsible for the higher amount of radiation from larger wavelengths. This results in a decrease of the CI ([Lübcke \[2014\]](#)).

We evaluated the CI at the zenith, to increase the stability of the fit we added in each cases 10 intensities. Using always the zenith to evaluate the colour index makes

the colour index more comparable, but if broken clouds occur, the CI of the reference and the plume could differ from the calculated CI of the zenith. This could be a reason for the large deviations of the mean BrO error as function of the colour index (see fig. 4.7)

- The BrO Error increases with the Colorindex differences as

$$\rightarrow BrO_{Error} = f(ext.P) + 1.01 \cdot 10^{13} \cdot \frac{\Delta Cidx}{0.1} + \mathcal{O}() \quad Tungurahua$$

$$\rightarrow BrO_{Error} = f(ext.P) + 4 \cdot 10^{13} \cdot \frac{\Delta Cidx}{0.1} + \mathcal{O}() \quad NevadoDelRuiz$$

4.1.5 Elevation Angle

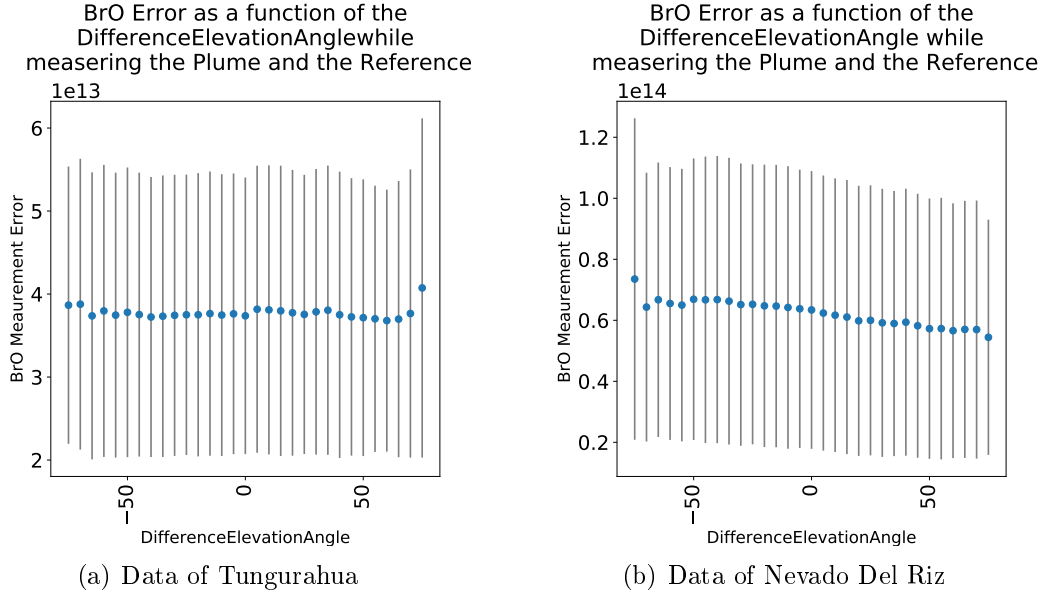


Figure 4.8: Titel unterm gesamten Bild

The elevation angle describes the angle between the horizon and the zenith. When using the plume spectrum and the reference spectrum of the same time, the difference in elevation angle cannot be zero, since the plume is always somewhere else than the reference located.

The BrO error doesn't depend significantly on the difference between the Elevation Angles. This could have several reasons. One problem is, that the Elevation Angle of Plume and Reference spectrum is not the same. This could also be a reason of uncertainty of the evaluations of the plume spectrum.

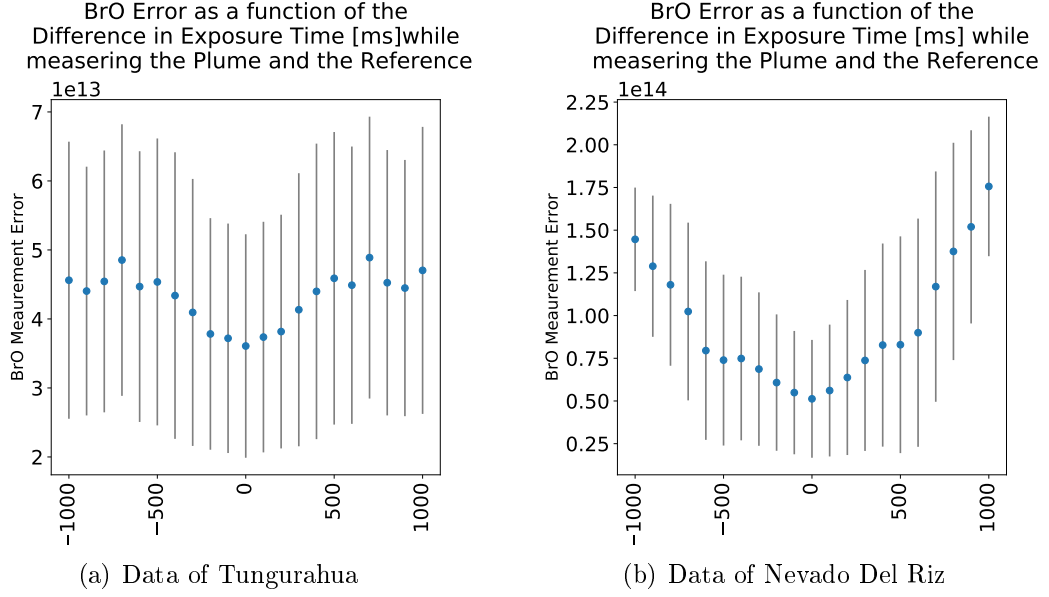


Figure 4.9: Titel unterm gesamten Bild

4.1.6 Exposure Time

The Exposure Time is a degree of sky lightness. The exposure time is the length of time the sensor of the NOVAC instrument is exposed to light. The amount of light that reaches the film or image sensor is proportional to the exposure time. The exposure time is adjusted in the way that the maximum intensity does not overly the capacity of the sensor.

We can observe an small dependency of the BrO error on the Exposure time at Tungurahua and Nevado Del Ruiz as it is shown in fig. 4.9

- The BrO Error increases with the exposure time differences as

$$\begin{aligned} \rightarrow BrO_{Error} &= f(ext.P) + 1.92 \cdot 10^{12} \cdot \frac{\Delta ET}{10^{-2}s} + \mathcal{O}() && Tungurahua \\ \rightarrow BrO_{Error} &= f(ext.P) + 1.0 \cdot 10^{13} \cdot \frac{\Delta T}{10^{-2}s} + \mathcal{O}() && NevadoDelRuiz \end{aligned}$$

5 Method

Based on the findings about the influence of external parameters on the BrO error we developed an algorithm which is able to pick an appropriate volcanic-trace-gas free reference.

The first step is, to evaluate every reference with solar atlas spectrum, to check for contamination. A Spectrum is treated as contaminated if the SO₂ column density of the reference (evaluated with a solar atlas spectrum) is larger as $2 \cdot 10^{17} \frac{\text{molec}}{\text{cm}^2}$.

If the reference is contaminated:

- We have a list of possible references where all references are not contaminated and the temporal distance to the plume date is no longer than 14 days.
- we calculate of all possible references the differences in the external parameters
- We use the analysis of external parameters described above to estimate the BrO error of all references
- We choose the reference with the smallest estimated BrO error as new reference
- We evaluate the plume spectra with the new reference.

The assumption is, that the BrO error ϵ_{BrO} can be described as the sum of ϵ_0 and the deviation of ϵ_{BrO} with respect to all external parameters. ϵ_0 is the BrO error when evaluate the plume spectrum with the "same-time-reference", it is determined due to the accurateness of the NOVAC-instruments.

$$\epsilon_{BrO} = \epsilon_0 + \frac{d\epsilon}{dt} + \frac{d\epsilon}{d^\circ} + \frac{d\epsilon}{dT} + \frac{d\epsilon}{ddt} + \frac{d\epsilon}{dc} + \mathcal{O}(OE) \quad (5.1)$$

$$\rightarrow \Delta\epsilon_{BrO} = \epsilon_{BrO} - \epsilon_0 = \frac{d\epsilon}{dt} + \frac{d\epsilon}{d^\circ} + \frac{d\epsilon}{dT} + \frac{d\epsilon}{ddt} + \frac{d\epsilon}{dc} + \mathcal{O}(OP) \quad (5.2)$$

With ϵ_{BrO} describes the BrO Error, t: time between plumetime and referencetime, T, temperaure; dt: daytime, c: colorindex, OP: other excluded external parameters. The task occurring at this stage is to find the best representation for the deviations. An then find the reference which minimize $\Delta\epsilon_{BrO}$

The easiest way is to just calculate the BrO error of all possible references for every plume. Using this method we would be able to just choose the reference where the BrO error is minimal. But this takes to much time since the evaluation would be proportional to the number of possible references because the evaluation need to be done for every plume-reference pair. Doing the evaluation for every plume-reference

pair would make it impossible to do the evaluation in real, or near real time. But we use this optimal evaluation to rate our model and compare them among each other. The optimal evaluation always choose the reference with the smallest absolute error. We don't use the relative error due to his vulnerability. Using the relative error could lead to a less preciseness.

Hier ein Bild, das eine Plume gegen viele referenzen auswertet und hier die Abweichungen zeigt

The results of the algorithm which chooses the reference automatically are described relative to an optimal evaluation. If the relative error is larger than 5 we don't use the data.

We tried several methods for choosing the best reference based on the analysis of external parameters. Fitting the data with a first order polynomial brought the best results.

5.1 Fit data

When looking at the analysis of the external parameters (see fig. 4.5-4.9) we can observe curves which are symmetric to the zero position. Therefore we conclude that it makes no difference whether the difference of a specific external parameter is positiv or negativ. Thus we take the absolute values for our calculations.

If we assume that all differentiations are linear, than we can write eq. (5.2) as:

$$\Delta\epsilon_{BrO} = a_t \cdot \Delta t + a_o \cdot \Delta^\circ + a_T \cdot \Delta T + a_{dt} \cdot \Delta dt + a_c \cdot \Delta c + \mathcal{O}(OP) \quad (5.3)$$

We used the same data as in fig. 4.5-4.9 to get the coefficients a_x of eq. (5.3). We used on ordinary least square linear regression to get the coefficients a_x . In particular we used the python function LinearRegression from the library sklearn [SKL](#).

The constants for Tungurahua and Nevado Del Ruiz are:

- Plume data are reliable if the SO2 column density is larger as $7 \cdot 10^{17} \frac{\text{molec}}{\text{cm}^2}$
- Data are above the detection limit if the column density as two times larger than the fit error.
- If the reference is contaminated:

(a) Data of Nevado Del Riz D2J2201_0

| Constant | value | importance |
|----------|-----------|------------|
| a_T | 7.338e+12 | 0.840 |
| a_{ET} | 1.545e+10 | 0.045 |
| a_t | -2.6e+09 | 0.0 |
| a_{dt} | 1.805e+12 | 0.091 |
| a_c | 2.301e+13 | 0.031 |

(b) Data of Nevado Del Riz D2J2200_0

| Constant | value | importance |
|----------|-----------|------------|
| a_T | 1.162e+13 | 0.908 |
| a_{ET} | 2.811e+10 | 0.046 |
| a_t | -1.7e+09 | 0.0 |
| a_{dt} | 1.076e+12 | 0.034 |
| a_c | 3.587e+13 | 0.016 |

(c) Data of Nevado Del Both Instruments

| Constant | value | importance |
|----------|------------|------------|
| a_T | 1.073e+13 | 0.973 |
| a_{ET} | 3.478e+10 | 0.070 |
| a_t | -9.1e+08 | 0.0 |
| a_{dt} | 1.523e+11 | 0.006 |
| a_c | -6.811e+13 | -0.047 |

Table 5.1: (a)Data from Nevado Del Ruiz from the D2J2201_0 instrument. All external parameter where taken into account. $\epsilon_0 == 5.404e + 12$ (b)Data from Nevado Del Ruiz from the D2J2200_0 instrument. All external parameter where taken into account. $\epsilon_0 == 1.105e + 13$ (c) Data from Nevado Del Ruiz from both instrument. All external parameter where taken into account. $\epsilon_0 = 1.260e + 13$

percentage of contaminated data
function of the minimum SO2 amol

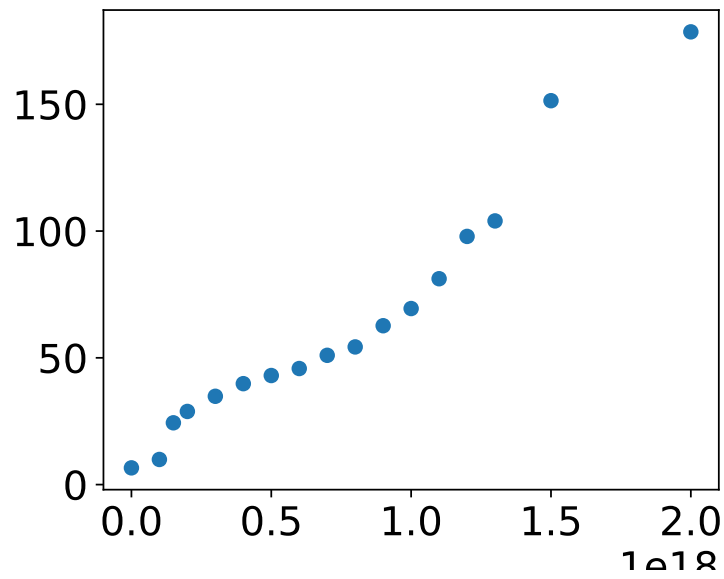


Figure 5.1:

total amount of data as
function of the minimum SO2 amol

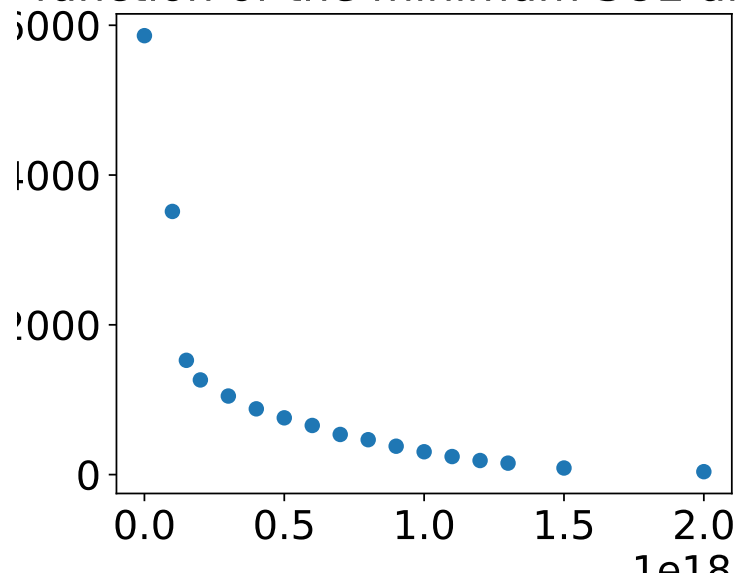


Figure 5.2:

| | | Error | Amount of Data | valid data |
|------------------------|-------------|-------|-------------------|---------------|
| All Variables | independent | 1.51 | 95% | 10,5% |
| | dependent | 1.40 | 98% | 8% |
| Exposure Time | independent | 1.47 | 97% | 10% |
| | All | 1.39 | 98% | 7% |
| Exp.Time u Coloridx | independent | 1.40 | 98% | 11 |
| | All | 1.35 | 98% | 7% |

| Evaluation of all contaminated data from Nevado Del Ruiz | | | |
|--|-------------------------------------|-------------------|--|
| Instrument | Dev from opt eval. (mean/median) | valid data | |
| <i>D2J2201_0</i> | 1.14 / 0.82293 | 128/283 = 45.2% | |
| <i>D2J2200_0</i> | 1.5 / 0.89965 | 954/1109 = 86.0% | |
| Both | 1.26/0.847 | 1073/1392 = 77.1% | |

Table 5.2: Data from Nevado Del Ruiz from the D2J2201_0 instrument. All external parameter where taken into account. $\epsilon_0 == 5.404e + 12$

5.2 Other approaches

- In the optimal results are 15% valid data
 - We also tried other possibilities than fitting to find the reference where the BrO error is minimal. In the following we present two additional possibilities but compared to fitting the results are not as good.

5.2.1 Nearest neighbours

- Description of the Nearest Neighbours Method

Nearest neighbor search (NNS), as a form of proximity search, is the optimization problem of finding the point in a given set that is closest (or most similar) to a given point. Closeness is typically expressed in terms of a dissimilarity function: the less similar the objects, the larger the function values. Formally, the nearest-neighbor (NN) search problem is defined as follows: given a set S of points in a space M and a query point $q \in M$, find the closest point in S to q . Donald Knuth in vol. 3 of *The Art of Computer Programming* (1973) called it the post-office problem, referring to an application of assigning to a residence the nearest post office. A direct generalization of this problem is a k -NN search, where we need to find the k closest points. Most

commonly M is a metric space and dissimilarity is expressed as a distance metric, which is symmetric and satisfies the triangle inequality. Even more common, M is taken to be the d -dimensional vector space where dissimilarity is measured using the Euclidean distance, Manhattan distance or other distance metric. However, the dissimilarity function can be arbitrary. One example are asymmetric Bregman divergences, for which the triangle inequality does not hold.

5.2.2 Iterative

- Description of the iterative Method

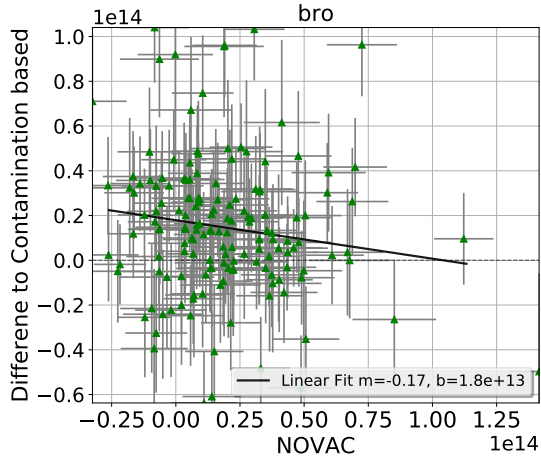
The idea of the iterative method was, that the importance of the individual external parameters are very different, that means if we have the list of possible references, we took all referenes where the temperature difference is minimal, so we get a new, much smaller list of possible referenecs. From this list we choose all references where the next external parameter for example the daytime is minimal and get again a new list. We proceed this way with the following external parameters. We experiment with the sequence of the parameters, to increase the success of the method. The final sequence was:

Temperature •

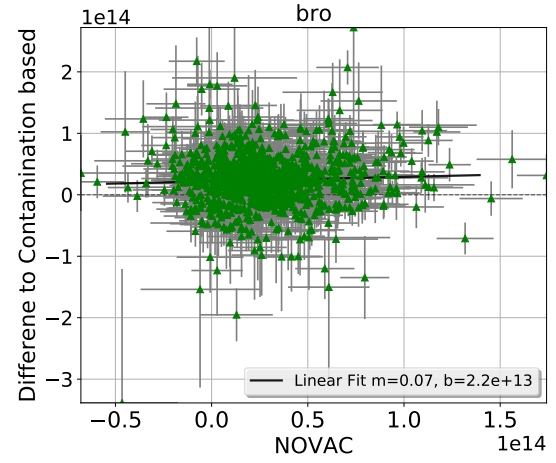
6 Comparison with NOVAC Evaluation

We want to

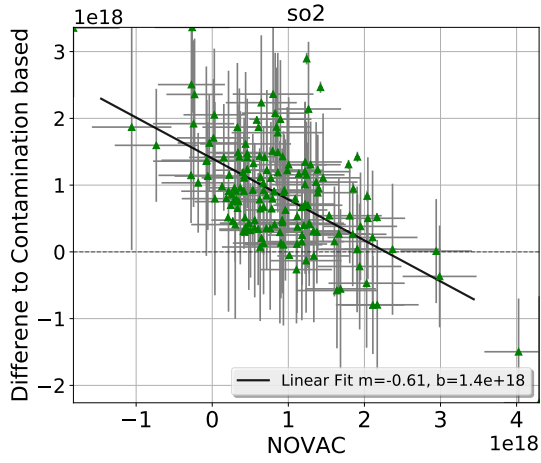
- Results only for contaminated data
 - Difference in SO₂ data evaluated with NOVAC-method and contamination-based evaluation
 - Difference in BrO data evaluated with NOVAC-method and contamination-based evaluation
 - Difference in BrO/SO₂ Ratio data evaluated with NOVAC-method and contamination-based evaluation
- Amount of BrO data more than before (valid and not valid and above detection limit)
- Amount of SO₂ data more than before (valid and not valid and above detection limit)
- Amount of BrO/SO₂ data more than before (valid and not valid and above detection limit)
- More BrO data: 51%
- More valid BrO data: 38%
- Compare the daily means: how many more data? due to higher SO₂ values
-



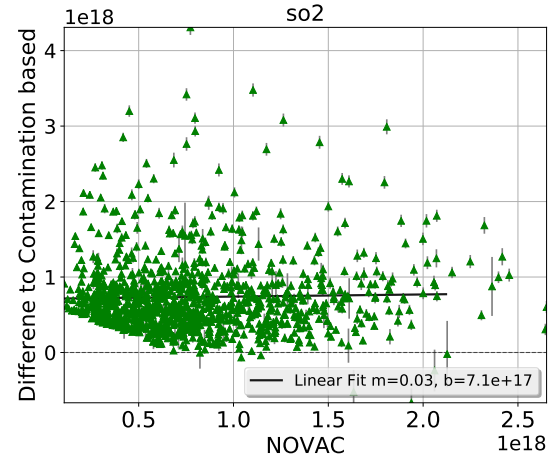
(a) Data of Tungurahua



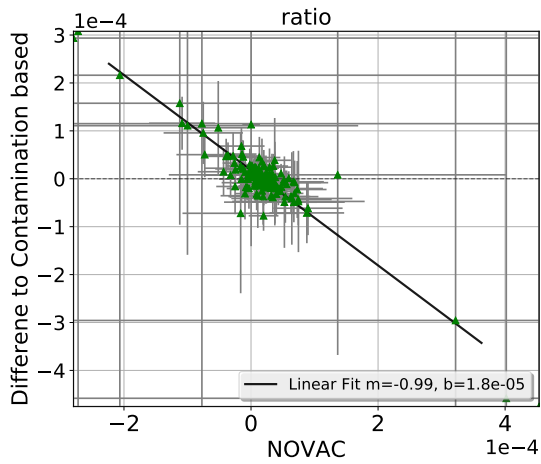
(b) Data of Nevado Del Riz



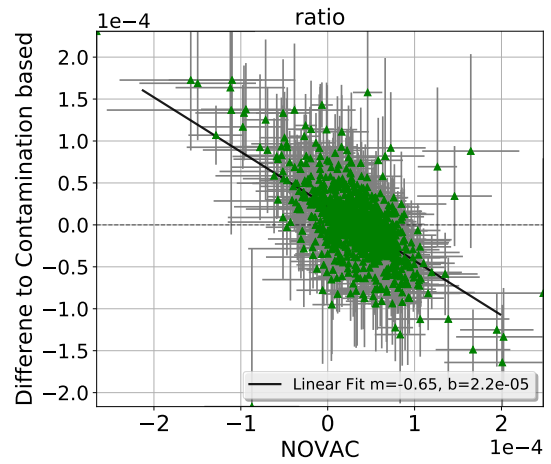
(c) Data of Tungurahua



(d) Data of Nevado Del Riz

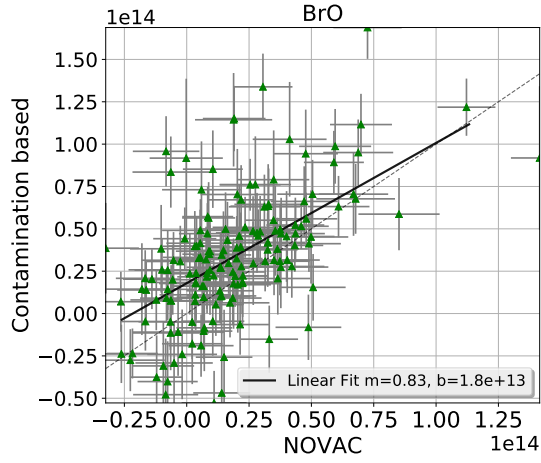


(e) Data of Tungurahua

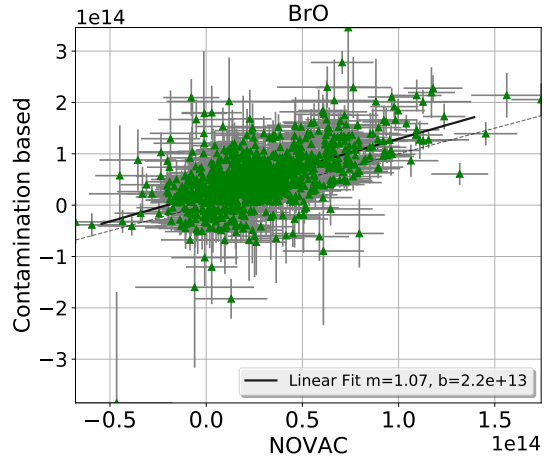


(f) Data of Nevado Del Riz

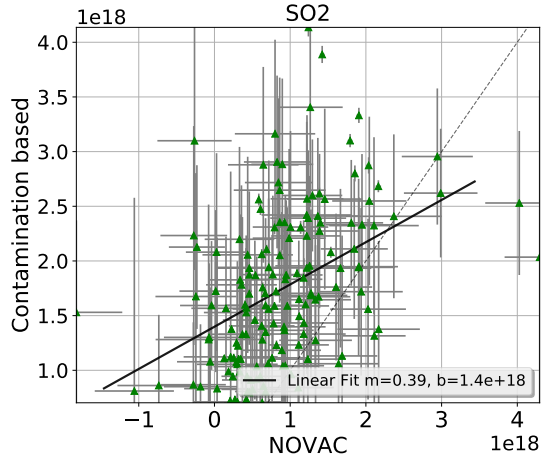
Figure 6.1: The dependency of the Difference between contamination based data and NOVAC to the data evaluated with the NOVAC data



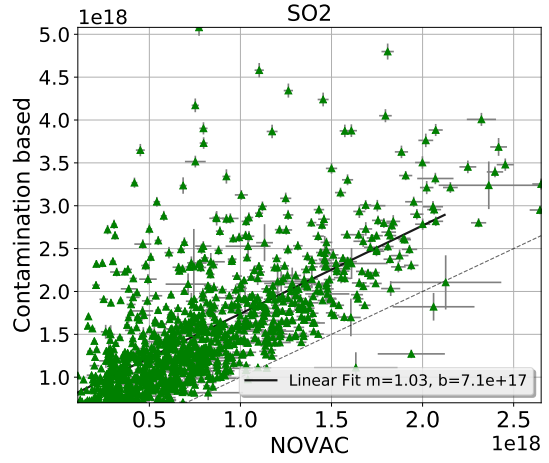
(a) Data of Tungurahua



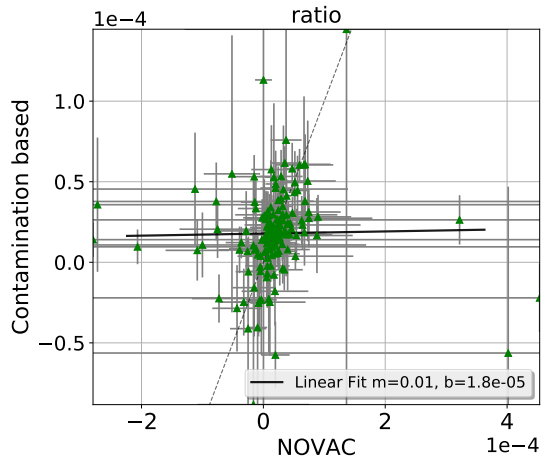
(b) Data of Nevado Del Riz



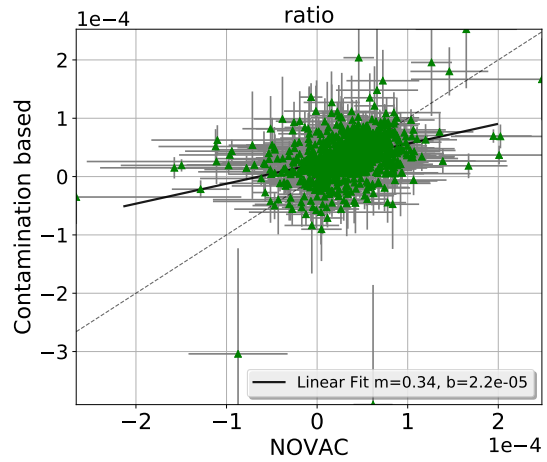
(c) Data of Tungurahua



(d) Data of Nevado Del Riz



(e) Data of Tungurahua



(f) Data of Nevado Del Riz

Figure 6.2: The dependency of the Difference between contamination based data and NOVAC to the data evaluated with the NOVAC data

7 Results

Interpretation of the BrO/SO₂ ratio time-series

7.1 Tungurahua

| | | |
|---|---------------|-------|
| <i>MengeanDateninsgesamt :</i> | 5883 \equiv | 1 |
| <i>Davon : (NOVAC Auswertung)berplumelimit</i> | 712 \equiv | 0.121 |
| <i>Davon : MengeanDaten, dienichtKontaminiertsind :</i> | 5504 \equiv | 0.936 |
| <i>DavonimPlume – limit :</i> | 599 \equiv | 0.102 |
| <i>DavonberdemDetectionLimit :</i> | 36 \equiv | 0.006 |
| <i>Davonsindkontaminiert :</i> | 379 \equiv | 0.064 |
| <i>Davon(mitNOVACAusgewertet)berplumelimit :</i> | 114 \equiv | 0.301 |
| <i>Davon(NeueAuswertung)berplumelimit</i> | 185 \equiv | 0.488 |

Dh in den kontaminierten daten sind mit NOVAC ausgewerteten daten 2.485 häufiger über dem plume limit

7.2 Nevado Del Ruiz

| | | |
|--|---------------|-------|
| <i>MengeanDateninsgesamt :</i> | 8962 \equiv | 1 |
| <i>Davon : (NOVAC Auswertung)berplumelimit</i> | 142 \equiv | 0.016 |
| <i>Davon : nichtkontaminiertedaten :</i> | 8596 \equiv | 0.959 |
| <i>DavonimPlume – limit :</i> | 123 \equiv | 0.014 |
| <i>DavonberdemDetectionLimit :</i> | 53 \equiv | 0.006 |
| <i>Davonsindkontaminiert :</i> | 366 \equiv | 0.041 |
| <i>Davon(mitNOVACAusgewertet)berplumelimit :</i> | 20 \equiv | 0.055 |
| <i>Davon(NeueAuswertung)berplumelimit</i> | 179 \equiv | 0.489 |

Dh in den kontaminierten daten sind mit NOVAC ausgewerteten daten 3.449 häufiger über dem plume limit

8 Issues of our method

8.1 Contamination of the plume

- As discussed above it might occur, that, that the reference is contaminated for example by the plume of the day before. If that happens, we underestimate the gas amount by using a contaminated reference. But another possibility is, that the plume is also contaminated. This might be the case if the volcanic gas of the volcano is not taken away by the wind, but accumulates in the plume. If this is the case, using an other reference would lead to an overestimation of the column density of gases.

9 Conclusion

....

Part III

Appendix

A Lists

A.1 List of Figures

| | | |
|-----|--|----|
| 1.1 | | 10 |
| 1.2 | | 11 |
| 1.3 | | 11 |
| 1.4 | | 12 |
| 2.1 | Global map of the volcanoes monitored by NOVAC. Used with friendly permission of Santiago Arellano. | 14 |
| 2.2 | Titel unterm gesamten Bild | 15 |
| 2.3 | | 16 |
| 3.1 | | 17 |
| 3.2 | Titel unterm gesamten Bild | 18 |
| 3.3 | | 19 |
| 3.4 | | 21 |
| 3.5 | | 22 |
| 3.6 | | 23 |
| 4.1 | | 25 |
| 4.2 | | 27 |
| 4.3 | | 28 |
| 4.4 | Titel unterm gesamten Bild | 28 |
| 4.5 | Titel unterm gesamten Bild | 29 |
| 4.6 | Titel unterm gesamten Bild | 29 |
| 4.7 | Titel unterm gesamten Bild | 30 |
| 4.8 | Titel unterm gesamten Bild | 31 |
| 4.9 | Titel unterm gesamten Bild | 32 |
| 5.1 | | 36 |
| 5.2 | | 36 |
| 6.1 | The dependency of the Difference between contamination based data and NOVAC to the data evaluated with the NOVAC data | 40 |
| 6.2 | The dependency of the Difference between contamination based data and NOVAC to the data evaluated with the NOVAC data | 41 |

A.2 List of Tables

| | | |
|-----|---|----|
| 5.1 | (a)Data from Nevado Del Ruiz from the D2J2201_0 instrument. All external parameter where taken into account. $\epsilon_0 == 5.404e + 12$ | |
| | (b)Data from Nevado Del Ruiz from the D2J2200_0 instrument. All external parameter where taken into account. $\epsilon_0 == 1.105e + 13$ | |
| | (c) Data from Nevado Del Ruiz from both instrument. All external parameter where taken into account. $\epsilon_0 = 1.260e + 13$ | 35 |
| 5.2 | Data from Nevado Del Ruiz from the D2J2201_0 instrument. All external parameter where taken into account. $\epsilon_0 == 5.404e + 12$. . . | 37 |

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Erklärung:

Ich versichere, dass ich diese Arbeit selbstständig verfasst habe und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe.

Heidelberg, den (Datum)