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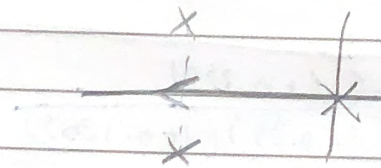
Sheet 5

① a) $G(s) = \frac{K}{s(s^2+8s+20)}$

poles : $0, -4 \pm 2j$
Zero : ——
# of branches = 3
- real axis locus



- Asymptotes
# of asymptotes = 3 - 0 = 3

$\sigma_A = \frac{\sum \text{Poles} - \sum \text{Zero}}{3} = \frac{-8}{3} \simeq 2.667$

$\gamma_A = 60, -60, -180$

$K = -s(s^2+8s+20) = -s^3 - 8s^2 - 20s$

$\frac{dK}{ds} = -3s^2 - 16s - 20 = 0$

$s_1 = -2, \quad s_2 = \frac{-10}{3} \simeq 3.333$

$\frac{d^2K}{ds^2} = -6s - 16$

$\left.\frac{d^2K}{ds^2}\right|_{s=2} = -4 < 0$ breakaway

$\left.\frac{d^2K}{ds^2}\right|_{-\frac{10}{3}} = 4 > 0$ break in

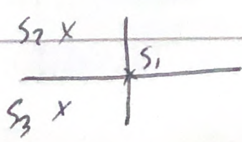
$s^3 + 8s^2 + 20s + K = 0$

s^3	1	20
s^2	8	K
s^1	$\frac{160-K}{8}$	
s^0	K	

$K = 160$

$8s^2 = -160$

$s = \pm 4.47\, i$ (intersection with img axis)

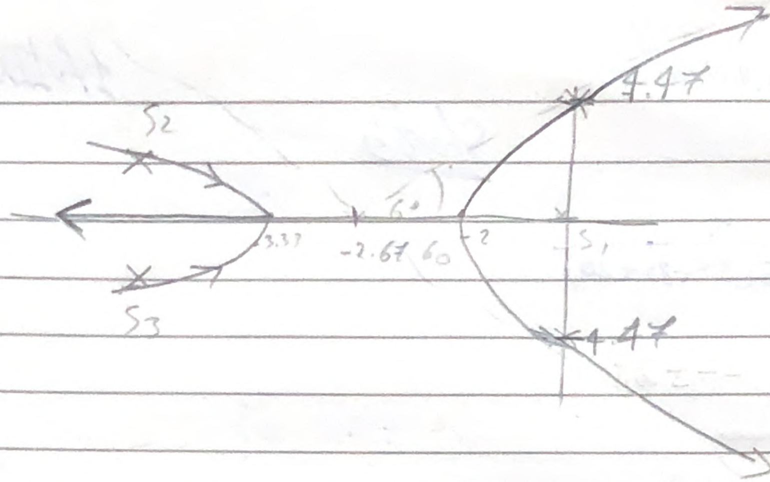


$-90 - \tan^{-1}\frac{7}{4} + \Phi_d = 0$

$\Phi_d = 63.4$ at (S_2)

$\Phi_d = -63.4$ at (S_3)

1



② a) $G(S) = \dfrac{k(1+0.33S)}{S(1+0.2S)(1+0.125S)}$

$G'(S) = \dfrac{K(S+3)}{S(S+5)(S+8)} \Rightarrow K' = \dfrac{40k}{3}$

Poles: 0, -5, -8

Zeros: -3

# of branches = 3

# of asymptotes = 2

$\sigma_A = \dfrac{-5-8+3}{2} = -5$

$\gamma_A = 90, -90$

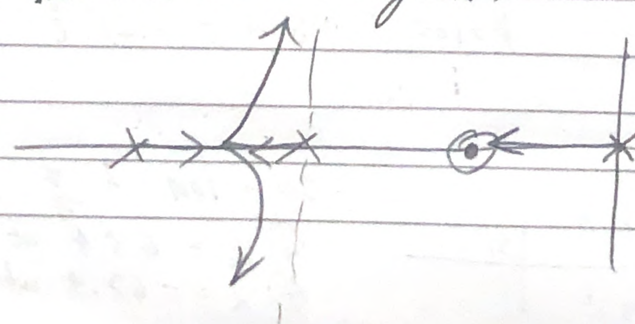
$K' = \dfrac{-S(S+5)(S+8)}{(S+3)}$

$\dfrac{dK'}{ds} = \dfrac{(3S^2+26S+40)(S+3) - (S^3+12S^2+40S)}{(S+3)^2} = 0$

$-(2S^3+22S^2+78S+120) = 0$

$S_1 = -6.34$, $S_{2,3} = -2.33 \pm 2j$ (rejected)

No intersection with img axis



b) $1 + GH = 0$

$S(S+5)(S+8) + K'(S+3) = 0$

$S^3 + 13S^2 + 40S + \frac{40}{3}K(S+3) = 0$

$3S^3 + 39S^2 + 120S + 40KS + 120K = 0$

S^3	3	$120+40K$
S^2	39	$120K$
S^1	A	
S^0	$120K$	

$A = \dfrac{39(120+40K) - 360K}{39}$

$A > 0$ & $120K > 0 \Rightarrow K > 0$

$39(120+40K) - 360K > 0$

$K > -3.9$

$K > 0 \Rightarrow$ for stable system

3) a) $G(s) = \dfrac{K(1+S/5)}{S^2(1+S/12)}$ $\quad H(S) = (1 + S/10)$

$GH = \dfrac{K(1+S/5)(1+S/10)}{S^2(1+S/12)} = \dfrac{K'(S+5)(S+10)}{S^2(S+12)}, \; K' = \frac{50}{12}$

poles: $0, 0, -12$

Zeros: $-5, -10$

# of branches $= 3$

# of asymptotes $= 3-2 = 1$

$\sigma = \dfrac{-12+5+10}{1} = 3$

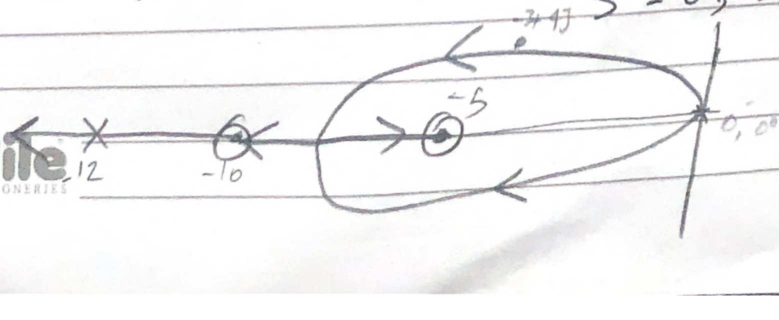
$\gamma_A = 180°$

$K' = \dfrac{-(S^3+12S^2)}{S^2+15S+50}$

$\dfrac{dK'}{ds} = \dfrac{-(3S^2+24S)(S^2+15S+50) - (2S+15)(S^3+12S^2)}{(S^2+15S+50)^2} =$

$S^4 + 30S^3 + 330S^2 + 1200S = 0$

$S = 0, -7.3, -11.3 \pm 4.9J$ X



b) $C = 1/3 \quad \alpha = 3 \quad \zeta\omega_n = 3$

$S = -3 + yJ$

$+\tan^{-1}\frac{y}{3} + \tan^{-1}\frac{y}{3} + \tan^{-1}\frac{y}{2} + \tan^{-1}\frac{y}{7} - \tan^{-1}\frac{y}{9} = 180$

$y = 4.25$

$S = -3 \pm 4.25J$

(4) $G(s) = \frac{K(S+3)}{S(S+2)}$

$1 + GH = 0 \qquad S(S+2) + K(S+3) = 0$

$S^2 + (2+K)S + 3K = 0$

$S = \frac{-2+K}{2} \pm \frac{1}{2}J\sqrt{8K - K^2 - 4}$

let $x = \frac{-2+K}{2} \qquad y = \frac{1}{2}\sqrt{8K - K^2 - 4}$

$K = -2(x+1)$

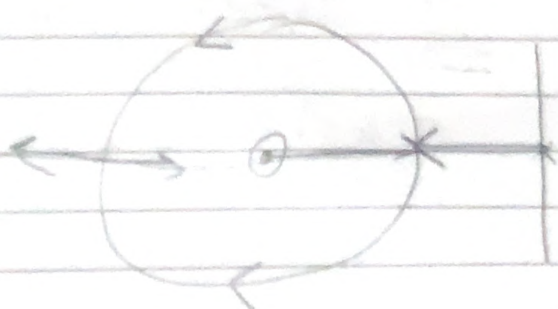
$y^2 = \frac{1}{4}(8K - K^2 4)$

$= \frac{1}{4}(-4x^2 - 24x - 24) = -x^2 - 6x - 6$

$y^2 + x^2 + 6x + 6 = 0$

$(x+3)^2 + y^2 = 3$

center at $(-3, 0)$ and $r = \sqrt{3}$



5 a) $G = \dfrac{K}{s^2(s+1)}$

poles: 0, 0, -1

Zeros: —

branches = 3

Asymptotes = 3

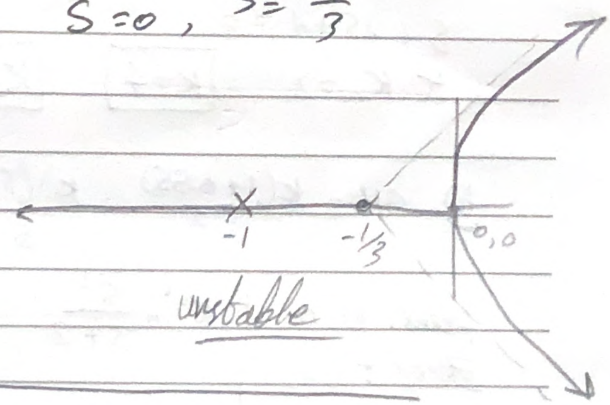
$\sigma_A = \dfrac{-1}{3} = -0.333$

$\gamma_A = 60, -60, -180$

$K = s^2(s+1) = s^3 + s^2$

$\dfrac{dK}{ds} = 3s^2 + 2s = 0$

$s = 0, \quad s = \dfrac{-2}{3}$



b) $G(s) = \dfrac{K(s+a)}{s^2(s+1)}$

poles: 0, 0, -1

Zeros: $-a$

branches = 3

Asymptotes = 2

$\sigma_A = \dfrac{(a-1)}{2} \Rightarrow 0 < a < 1$

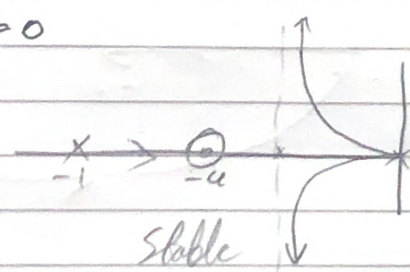
$\gamma_A = 90, -90$

$K = -\dfrac{s^3 + s^2}{s+a}$

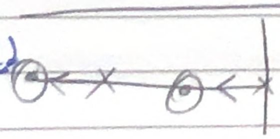
$\dfrac{dK}{ds} = \dfrac{(3s^2+2s)(s+a) - (s^3+s^2)}{(s+a)^2} = 0$

$2s^3 + (3a+1)s^2 + 2as = 0$

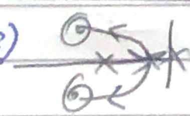
$s = 0$



6 a)



c)



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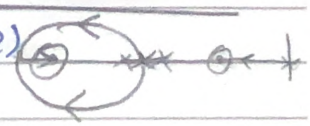
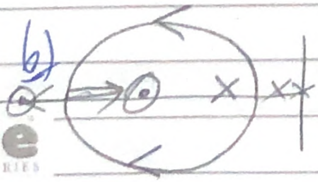
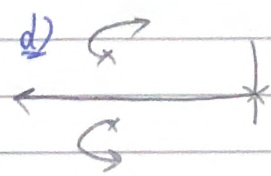


b)



d)



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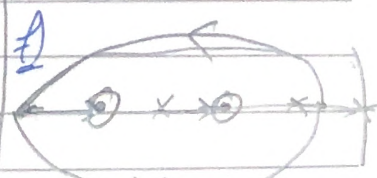


7 a) $G = \frac{K}{S^2}$, $H = 1 + K_n S$

$1 + GH = 1 + \frac{K}{S^2}(1 + K_n S) = 0$

$S^2 + K + KK_n S = 0$

$S = -1 \pm \sqrt{3} J$

$S + 2S + 4 = 0$

$K K_n = 2$ $\boxed{K = 4}$ $\boxed{K_n = 0.5}$

b) $GH = \frac{K(1 + 0.5S)}{S^2} = \frac{K'(S+2)}{S^2}$, $K' = 2K$

poles: 0, 0

zeros: -2

branches: 2

asymptotes: 1

$\sigma_A = 0$

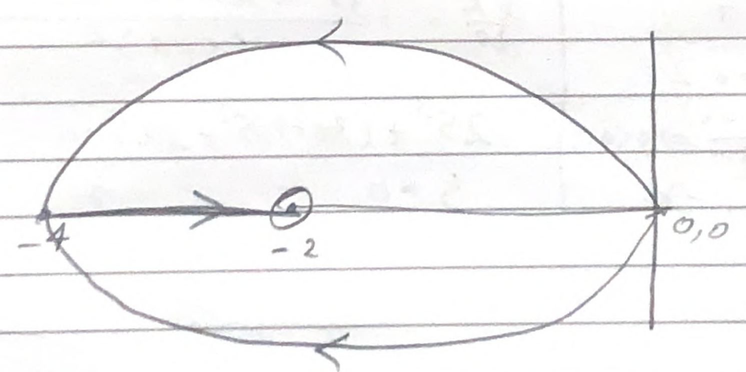
$\gamma_A = 180$

$K' = \frac{S^2}{S+2}$ $\frac{dK'}{dS} = \frac{2S(S+2) - S^2}{(S+2)^2} = 0$

$2S^2 + 4S - S^2 = 0$

$S^2 + 4S = 0$

$S = 0$ $S = -4$



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