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① $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{1/s(s^2+5s+6)}{1/s + \frac{1}{s(s^2+5s+6)} * 1}$

$= \frac{1}{s^3+5s^2+6s+1}$

$R(t) = u(t) \Rightarrow R(s) = \frac{1}{s}$

$\therefore C(s) = \frac{1}{s(s^3+5s^2+6s+1)} = \frac{1}{s(s+3.25)(s+0.2)(s+1.55)}$

$= \frac{1}{s} + \frac{-0.06}{s+3.25} + \frac{-1.21}{s+0.2} + \frac{0.28}{s+1.55}$

$C(t) = 1 - 0.06e^{-3.25t} - 1.21e^{-0.2t} + 0.28e^{-1.55t}$

a) final value of $c(t)$ @ $t=\infty = 1$

$t = ?$ @ $c(t) = 0.99$

$1 - 0.06e^{-3.25t} - 1.21e^{-0.2t} + 0.28e^{-1.55t} = 0.99$

$t \simeq 24$ sec

b) $C(0) = \lim_{s\to\infty} s(C(s)) = \lim_{s\to\infty} \frac{1/s^3}{(1+\frac{1.55}{s})(1+\frac{0.2}{s})(1+\frac{3.25}{s})}$

$= 0$

$C'(0) = \lim_{s\to\infty} s^2 C(s) = 0$

$C''(0) = \lim_{s\to\infty} s^3 C(s) = 0$

$C'''(0) = \lim_{s\to\infty} s^4 C(s) = 1$

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Nile STATIONERIES

② a) $1 + GH = 0$

$$\cancel{S + TS^2 + A = 0} \quad S + TS^2 + A = 0$$

$$\cancel{S^2 + \frac{1}{T}S + A} \quad S^2 + \frac{1}{T}S + \frac{A}{T} = 0$$

$$\omega_n = \sqrt{\frac{A}{T}}, \quad 2\zeta\omega_n = \frac{1}{T}; \quad \zeta = \frac{1}{2\sqrt{AT}}$$

$$\Rightarrow A = \frac{1}{4T\zeta^2}$$

$$\frac{A'}{A} = \frac{\cancel{\tfrac{1}{4T}}\zeta^2}{\cancel{\tfrac{1}{4T}}\zeta'^2} = \frac{\zeta^2}{\zeta'^2} = \left(\frac{0.2}{0.6}\right)^2 = \frac{1}{9}$$

$$A' = \frac{1}{9}A$$

b) Max overshoot $= e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} = a$

$$\frac{-\pi\zeta}{\sqrt{1-\zeta^2}} = \ln a$$

$$\frac{1-\zeta^2}{\pi^2\zeta^2} = \frac{1}{\ln^2 a}$$

$$\frac{1}{\zeta^2} - 1 = \frac{\pi^2}{\ln^2 a}$$

$$\zeta^2 = \frac{\ln^2 a}{\ln^2 a + \pi^2}$$

$$\frac{A'}{A} = \frac{\zeta^2}{\zeta'^2} = \frac{\ln^2(0.8)}{\ln^2(0.8) + \pi^2} \times \frac{\ln^2(0.2) + \pi^2}{\ln^2(0.2)} = 0.024$$

$$\underline{A' = 0.024\,A} \quad \#$$

NOTES

③ $1+GH = \dfrac{S(1+0.5S)(1+0.2S)+1}{S(1+0.5S)(1+0.2S)} = 0$

$S(1+0.5S)(1+0.2S)+1=0$

$S(2+S)(5+S)+10=0$

$S^3+7S^2+10S+10=0$

$S_1=-5.5$, $S_{2,3}=-0.74\pm 1.12j$

$(S+5.5)(S^2+1.48S+1.8)=0$

C/Cs $S^2+1.48S+1.8=0$

$\omega_n^2=1.8 \Rightarrow \omega_n=1.34$

$2\zeta\omega_n=1.48 \Rightarrow \zeta=0.55$

$\because 0<\zeta=0.55<1 \Rightarrow$ under damped response

$\because$ unit step input

$\therefore y_{ss}=1$

$\%$ over shoot $= \dfrac{\text{maximum overshoot}}{y_{ss}} \times 100 = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100$

$= 12.6\%$

④ $G(s)=\dfrac{10}{s^2}$ $H(s)=as+b$

a) C/Cs eqn : $1+GH=0$

$1+\dfrac{10(as+b)}{s^2}=0$

$s^2+10as+10b=0$

$\omega_n=\sqrt{10b}$ $\zeta=\dfrac{10a}{2\sqrt{10b}}=\dfrac{5a}{\sqrt{10b}}$ $\zeta=\dfrac{25a^2}{10b}=\dfrac{5a^2}{2b}$ ①

$e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}=0.16$

$\dfrac{-\pi\zeta}{\sqrt{1-\zeta^2}}=\ln 0.16$

Nile STATIONERIES

NOTES.

$$\frac{1-\zeta^2}{\zeta^2} = \frac{\pi^2}{\ln^2 0.16}$$

$$\zeta^2 = \frac{\ln^2 0.16}{\ln^2 0.16 + \pi^2} \quad (2)$$

$$\tau = 0.1 = \frac{1}{\alpha} \Rightarrow \alpha = 10$$

$$\zeta \omega_n = 10$$

$$\frac{5a}{\sqrt{10b}} \times \sqrt{10b} = 10$$

$$a = 2$$

from 1 & (2)

$$\frac{5(2^2)}{2b} = \frac{\ln^2 0.16}{\ln^2 0.16 + \pi^2}$$

$$b = 39.39 \simeq 40$$

b) $\because \zeta = \frac{10a}{2\sqrt{10b}} = \frac{ka}{2\sqrt{kb}} = \sqrt{k}\,\frac{a}{2\sqrt{b}}$

$\therefore \zeta \propto \sqrt{k}$ as k decreases $\Rightarrow \zeta$ decreases

c) $G(s) = \frac{k}{s^2}$ $\quad H(s) = 2s + 40$

C/cs eqn $= 1 + GH = 0$

$$1 + \frac{k}{s^2}(2s+40) = 0$$

$$s^2 + 2ks + 40k = 0$$

$$s_{1,2} = \frac{-2k \pm \sqrt{4k^2 - 4 \times 40k}}{2} = -k \pm \sqrt{k(k-40)}$$

@ $k = 40$ $\quad s_1 = s_2 = -k = -40$

@ $k \geq 40$ $\quad s_{1,2}$ are real are negative

@ $k < 40$ $\quad s_{1,2}$ are complex

$x = -k \quad y = \sqrt{k(40-k)}$

$$y^2 = k(40-k)$$

$$y^2 = -x(40+x)$$

$$y^2 + (x+20)^2 = (20)^2$$

Nile STATIONERIES

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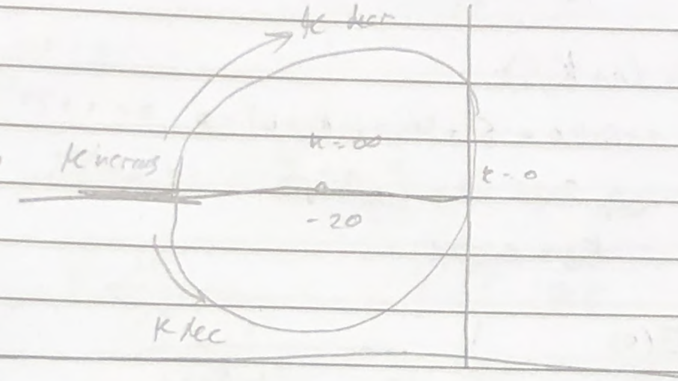
@ $K = \infty$ $\quad S_{1,2} = \lim_{k \to \infty} \dfrac{k^2 - k(k-40)}{-k \mp \sqrt{k(k-40)}} = \lim_{k \to \infty} \dfrac{40k}{-k \mp \sqrt{k(k+40)}}$ NOTES

$$= \frac{40}{-1 \mp 1} = \begin{cases} -20 \\ \infty \end{cases}$$

@ $K = 0$ $\quad S_{1,2} = 0$



⑤ % overshoot = 5%

Same as eqn in Problem (4)

$$\zeta^2 = \frac{\ln^2 0.05}{\ln^2 0.05 + \pi^2} \Rightarrow \zeta = 0.69011 > 0.69$$

$$t_s = 2 = \frac{4.5\zeta}{\omega_n} = \frac{4.5 \times 0.69011}{\omega_n}$$

$$\omega_n = 1.553$$

⑥ $1 + GH = 0$

$$1 + \frac{10}{s(s+1)} \times 1 = 0$$

$$s^2 + s + 10 = 0$$

$\omega_n^2 = 10$ $\qquad \omega_n = \sqrt{10}$

$$\zeta = \frac{1}{2\sqrt{10}}$$

$$E(s) = R(s) - C(s)H(s)$$

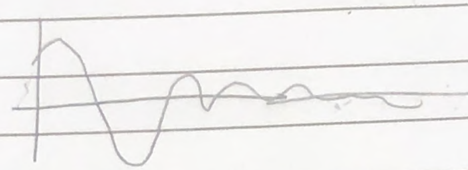
$$\frac{E(s)}{R(s)} = \frac{s(s+1)}{s^2+s+10}$$

@ $R(s) = \dfrac{1}{s}$ $\qquad E(s) = \dfrac{s+1}{s^2+s+10}$

$$e_{ss} = \lim_{s \to 0} sE(s) = 0$$

NOTES.

$$e(t) = e^{-0.5t}\left[\cos\left(\frac{\sqrt{79}}{2}t\right) + \frac{1}{\sqrt{29}}\sin\left(\frac{\sqrt{39}}{2}\right)t\right]$$



(b) $H = (1 + K_6 S)$

$1 + GH = 0 = S^2 + S(1 + 10K_6 + 6) = 0$

$\omega_n^2 = 10 \Rightarrow \omega_n = \sqrt{10}$

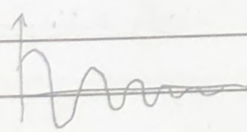
$K_6 = 0.216$

$$\frac{E(S)}{R(S)} = \frac{1}{1 + \frac{10}{S(S+1)}(1 + K_6 S)}$$

$$= \frac{S(S+1)}{S(S+1) + 10 + 10K_6 S} = \frac{S(S+1)}{S^2 + 3S + 10}$$

$$E(S) = \frac{S(S+1)}{S^2+3S+10} \rightarrow e_{ss} = \lim_{s \to 0} S E(S) = 0$$

$$e(t) = e^{-1.5t}\left[\cos\left(\frac{\sqrt{31}}{2}\right)t - \frac{1}{\sqrt{31}}\sin\left(\frac{\sqrt{31}}{2}\right)t\right]$$



(7) $$\frac{C(S)}{R(S)} = \frac{0.4S + 1}{S^2 + S + 1}$$

a) @ $R = \frac{1}{s}$ $$C = \frac{0.4S + 1}{S(S^2 + S + 1)} = \frac{1}{S} - \frac{S + 0.6}{S^2 + S + 1}$$

$$c(t) = 1 - e^{-0.5t}\left[\cos\left(\frac{\sqrt{3}}{2}t\right) + \frac{0.2}{\sqrt{3}}\sin\left(\frac{\sqrt{3}}{2}t\right)\right]$$

$$= 1 - e^{-0.5t}\sin\left(\frac{\sqrt{3}}{2}t + 83.4^\circ\right)$$

b) $t_r \rightarrow$ 10% to 90% of $c(t)$ @ $t = \infty$

@ 10% $$0.1 = 1 - e^{-0.5t}\sin\left(\frac{\sqrt{3}}{2}t + 83.4\right)$$

$t = 0.2$ sec

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NOTES

@ 90% $0.9 = 1 - e^{-0.5t} \sin(\frac{\sqrt{3}}{2} t + 83.4)$

$t = 1.67$ sec

$t_r = 1.67 - 0.2 = 1.47$ sec

c) c/cs eqn $1 + GH = 0$

$s^2 + s + 1 = 0$

$\omega_n = 1$, $2\zeta\omega_n = 1 \Rightarrow \zeta = 0.5$

$t_{max} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{1 \times \sqrt{1-0.5^2}} = 3.63$ sec

d) Max overshoot

$= 100 e^{-\pi\zeta/\sqrt{1-\zeta^2}} = 16.3\%$

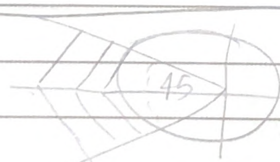
e) $\frac{E(s)}{R(s)} = \frac{s(s+0.6)}{s^2+s+1}$, $E(s) = \frac{s+0.6}{s^2+s+1}$

$e_{ss} = \lim_{s \to 0} sE(s) = 0$

(8) a) $\zeta \geq 0.707$

$0.707 \leq \zeta \leq 1$

$45 \geq \theta \geq 0$



$\omega_n \geq 2$

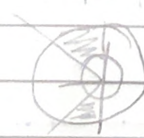
b) $0 \leq \zeta \leq 0.707$

$90 \geq \theta \geq 45$

$\omega_n < 2$

c) $\zeta \leq 0.5$, $\theta \geq 60$

$2 \leq \omega_n \leq 4$



d) $0.5 \leq \zeta \leq 0.707$

$60 \geq \theta \geq 45$

ω_n

Nile STATIONERIES

NOTES

(9) $G(s) = \frac{500}{s(1+0.1s)}$

a) $r(t) = t^2/2 \Rightarrow R(s) = \frac{1}{s^3}$

$E(s) = \frac{C(s)}{G(s)} = \frac{G}{1+G} \times \frac{1}{G} \times R = \frac{R(s)}{1+G(s)}$

$E(s) = \frac{1/s^3}{1+\frac{500}{s(1+0.1s)}} = \frac{1+0.1s}{s^3(1+0.1s)+500s^2}$

$e_{ss} = \lim_{s\to 0} sE(s) = \infty$

b) $r(t) = 1+2t+t^2 \Rightarrow R(s) = \frac{1}{s} + \frac{2}{s^2} + \frac{2}{s^3}$
$= \frac{s^2+2s+2}{s^3}$

$E(s) = \frac{R}{1+G} = \frac{s^2+2s+2}{s^3+\frac{500s^3}{s(1+0.1s)}}$

$e_{ss} = \lim_{s\to 0} sE(s) = \infty$

(10) $C/R = \frac{a_{n-1}s+a_n}{s^n+a_1s^{n-1}\dots+a_{n-1}s+a_n}$

$r(t) = t\ u(t) \Rightarrow R(s) = \frac{1}{s^2}$

$C(s) = \frac{a_{n-1}s+a_n}{s^2(s^n+a_1s^{n-1}\dots a_{n-1}s+a_n)}$

$E(s) = R(s) - C(s)$

$= \frac{1}{s^2} - C(s)$

$= \frac{(s^{n-2}+a_1s^{n-3}+\dots a_{n-2})}{(s^n+a_1s^{n-1}+\dots a_{n-1}s+a_n)}$

$e_{ss} = \lim_{s\to 0} sE(s) = 0$

Nile
STATIONERIES

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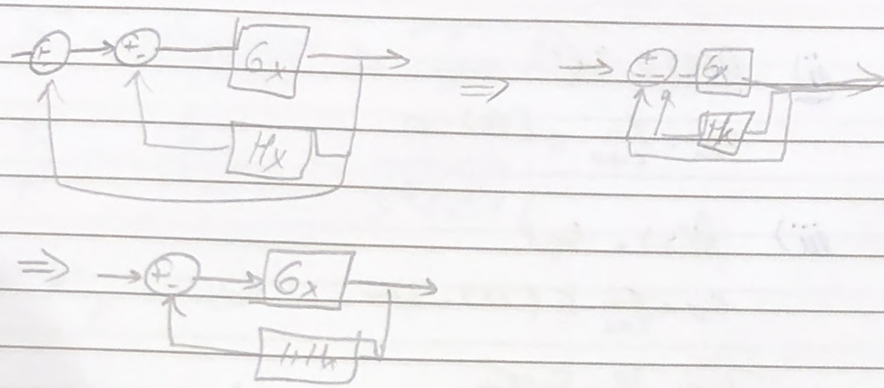
11 a) $\frac{C}{R} = \frac{1500(1+0.5S)}{S(S+5)(0.01S+1)(1+0.5S)+1500}$

b)



$\therefore \quad 1 + H_x = H$

$H_x = H - 1 = \frac{1}{1+0.5S} - 1 = \frac{-0.5S}{1+0.5S}$

c) $G' = \frac{G_x}{1+G_x H_x} = \frac{1500(1+0.5S)}{S(S+5)(0.01S+1)(1+0.5S) - 7500S}$

$\Rightarrow$ type 1 System

d) $K_p = \lim_{s\to 0} G'(S) = \infty$

$K_v = \lim_{s\to 0} sG'(s) = \frac{1500}{5-7500} \simeq -2$

$K_a = \lim_{s\to 0} s^2G'(s) = 0$

e) $R(s) = \frac{1}{s}$

$E(s) = \frac{R(s)}{1+G(s)H(s)} = \frac{\frac{1}{s}}{1 + \frac{1500(1+0.5S)}{S[(S+5)(0.01S+1)(1+0.5S) - 7500]}}$

$e_{ss}(t) = \lim_{s\to 0} sE(S) = 0$

(12) a) i) $R(s) = 5/s$

$e_{ss} = \lim_{s \to 0} s\,E(s) = 0$

ii) $R(s) = 3/s^2$

$e_{ss} = \lim_{s \to 0} s\,E(s) = 0$

iii) $R(s) = 2/s^3$

$e_{ss} = \lim_{s \to 0} s\,E(s) = 24$

Type 2 System

$K_p = \infty$, $K_v = \infty$

$K_a = \lim_{s \to 0} s^2 G(s) = \frac{1}{12}$

b) i) $e_{ss} = \frac{5}{1+15} = \frac{5}{16}$

ii) $e_{ss} = \infty$

iii) $e_{ss} = \infty$

c) i) $e_{ss} = 0$

ii) $e_{ss} = \frac{-1}{25}$

iii) $e_{ss} = \infty$

d) i) $e_{ss} = 0$

ii) $e_{ss} = 0$

iii) $e_{ss} = -1$

NOTES.

Nile STATIONERIES

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13 a) $G' = \frac{C}{E} = \frac{G}{1+GH} = \frac{1}{s(s+6)}$

b) $G' = \frac{1}{s(s+6)} \Rightarrow$ Type 1

c) $\frac{C}{R} = \frac{G'}{1+G'} = \frac{1}{s^2+6s+1}$

d) $K_p = \infty$

$K_v = \lim_{s \to 0} s\,G'(s) = \frac{1}{6}$

$K_a = 0$

NOTES

Nile STATIONERIES