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Sheet 1

1) a) $f(t) = 10\,u(t) - 10\,u(t-2)$

$$F(S) = 10/S - 10\,\frac{e^{-2S}}{S}$$

b) $f(t) = 3t^2\ u(t-5)$

$$= 3[(t-5)^2 + 10t - 25]\ u(t-5)$$

$$= [3(t-5)^2 + 30(t-5+5) - 75]\ u(t-5)$$

$$= [3(t-5)^2 + 30\,(t-5) + 75]\,u(t-5)$$

$$F(S) = \frac{e^{-5S}}{S}\left[\frac{3\times 2}{S^2} + \frac{30}{S} + 75\right]$$

c) $f(t) = 5\,u(t)\,u(3-t)$

$$= 5\,[u(t) - u(t-3)]$$

$$F(S) = 5\left[\frac{1}{S} - \frac{e^{-3S}}{S}\right]$$

d) $f(t) = t^n$

let $st = x$

$t = x/s$

$dt = \frac{1}{s}\,dx$

$$F(S) = \int_0^\infty e^{-st}\,t^n\,dt$$

$$= \int_0^\infty e^{-x}\,\frac{x^n}{S^n}\,\frac{1}{S}\,dx$$

$$= \frac{1}{S^{n+1}}\int_0^\infty e^{-x}\,x^n\,dx = \frac{1}{S^{n+1}}\,\Gamma(n+1) = \frac{n!}{S^{n+1}}$$

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2) a) $\frac{d^2 i}{dt^2} - i = 25 + e^{2t}$

$$S^2 I(S) - S i(0) - i'(0) - I(S) = \frac{25}{S} + \frac{1}{S-2}$$

$$I(S)[S^2 - 1] = \frac{25}{S} + \frac{1}{S-2}$$

$$I(S) = \frac{25}{S(S-1)(S+1)} + \frac{1}{(S-2)(S-1)(S+1)}$$

$$= \frac{A}{S} + \frac{B}{S-1} + \frac{C}{S+1} + \frac{D}{S-2} + \frac{E}{S-1} + \frac{F}{S+1}$$

$$I(S) = \frac{-25}{S} + \frac{12}{S-1} + \frac{38/3}{S+1} + \frac{1/3}{S-2}$$

$$i(t) = -25 + 12e^{t} + \frac{38}{3}e^{-t} + \frac{1}{3}e^{2t}$$

b) $\frac{d^2 i}{dt^2} + \frac{di}{dt} = t^2 + 2t$

$$S^2 I(S) - S i(0) - i'(0) + S I(S) - i(0) = \frac{2}{S^3} + \frac{2}{S^2}$$

$$I(S)[S^2 + S] = \frac{2}{S^3} + \frac{2}{S^2} + 4S - 2 + 4$$

$$I(S) = \frac{4S^4 + 2S^3 + 2S + 2}{S^4(S+1)} = \frac{A}{S^4} + \frac{B}{S^3} + \frac{C}{S^2} + \frac{D}{S} + \frac{E}{S+1}$$

$$i(t) = \frac{1}{3}t^3 + 2 + 2e^{-t}$$

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3) a) $F(s) = \frac{2s}{(s^2+1)(s^2+4)} = \frac{2s}{3(s^2+1)} - \frac{2s}{3(s^2+4)}$

NOTES

$f(t) = \frac{2}{3}\cos(t) - \frac{2}{3}\cos(2t)$

b) $F(s) = \frac{e^{-2s}}{(s+1)(s+2)}$

$f(t) = u(t-2)\left(e^{-t+2} - e^{-2(t-2)}\right)$

c) $F(s) = \frac{s}{(s^2+A^2)^2}$

$\frac{d}{ds}\left[\frac{A}{A^2+s^2}\right] = \frac{-2As}{(s^2+A^2)^2}$

$F(s) = \frac{1}{2A} * \frac{-d}{ds}\frac{A}{s^2+A^2}$

$f(t) = \frac{1}{2A}\, t \sin At$

d) $F(s) = \frac{1}{(s^2+A^2)^2}$

$= \frac{1}{s}\frac{s}{(s^2+A^2)^2} = \frac{1}{s}G(s)$

$f(t) = \int_0^t g(u)\,du = \int_0^t \left(\frac{1}{2A} u \sin Au\right) du$

$= \frac{1}{2A}\int_0^t u \sin Au\,du = \frac{-1}{2A^2}\int_0^t u\, d\cos Au$

$= \frac{-1}{2A^2}\left(t\cos At - \frac{1}{A}\sin At\right)$

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4) $f(t) = \sin(t)\,[u(t) - u(t - \frac{\pi}{2})$
$+ (\sin(t) + \cos(t - \frac{\pi}{4}))\,[u(t - \frac{\pi}{4})]$

$= \sin(t)\,u(t) - \sin(t)\,u(t - \frac{\pi}{4})$
$+ \sin(t)\,u(t - \frac{\pi}{4}) + \cos(t - \frac{\pi}{4})\,u(t - \frac{\pi}{4})$

$= \sin(t)\,u(t) + \cos(t - \frac{\pi}{4})\,u(t - \frac{\pi}{4})$

$$F(s) = \frac{1}{s^2+1} + \frac{s}{s^2+1} \cdot e^{-\frac{\pi}{4}s}$$

5)	Open Loop System	Closed Loop System
definition	• input is independent of output • Only good for ideal enviroments	• i/p is dependent on o/p • Feed back informs the system • balanced from feedback
example	• Light switch	• Air Conditioning
Adv	• Simple • economical • easy to maintain	• More accurate • Facilates automation • less affected by noise
Disadv	• Need perfect conditions • less accurate • unreliable • Any chang in o/p cant be corrected automaticaly	• More Complex & Costly • Require more maintance • stability is a problem

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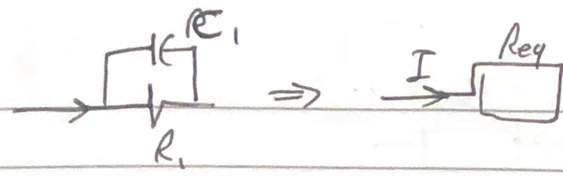
6)



$$R_{eq} = \frac{R_1 \frac{1}{C_1 s}}{R_1 + \frac{1}{C_1 s}} = \frac{R_1}{R_1 C_1 S + 1}$$

$$E_0 = I R_2$$

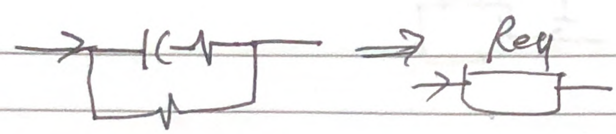
$$E_1 = I(R_{eq} + R_2)$$

$$\frac{E_0}{E_1} = \frac{R_2}{R_2 + R_{eq}} = \frac{R}{R_2 + \frac{R_1}{R_1 C_1 S + 1}}$$

$$= \frac{R_2(1 + R_1 C_1 S)}{R_2 R_1 C_1 S + (R_2 + R_1)}$$

$$= \frac{R_2}{R_1 + R_2} \quad \frac{1 + R_1 C_1 S}{1 + R_1 C_1 S}$$

$$= T_2 \cdot \frac{1 + T_1 s}{T_1 T_2 S} \quad \#$$

7) 

$$Req = \frac{R_1\left(R_3 + \frac{1}{C_1 s}\right)}{R_1 + \left(\frac{1}{C_1 s} + R_3\right)} = \frac{R_1 + R_1 R_3 C_1 S}{R_1 C_1 S + R_3 C_1 S + 1}$$

$$E_i = I\,Req + E_o$$

$$E_o = I\left[R_2 + \frac{1}{C_2 S}\right]$$

$$E_i = \frac{E_o}{R_2 + \frac{1}{C_2 s}} \cdot \frac{R_1 + R_1 R_3 C_1 S}{R_1 C_1 S + R_3 C_1 S + 1} + E_o$$

$$= E_o \frac{S^2 (R_1 + R_1 R_3 C_1 S) + (R_2 C_2 S + 1)(R_1 C_1 S + R_3 C_1 S + 1)}{(R_2 C_2 S + 1)(R_1 C_1 S + R_3 C_1 S + 1)}$$

$$\frac{E_o}{E_i} = \frac{a_2 S^2 + a_1 S + a_0}{b_2 S^2 + b_1 S + b_0}$$

$a_0 = 1$

$a_1 = R_2 C_2 + R_1 C_1 + R_3 C_1$

$a_2 = R_1 R_2 C_1 C_2 + R_2 R_3 C_1 C_2$

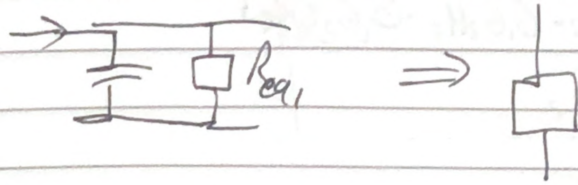
$b_0 = 1$

$b_1 = R_1 C_2 + R_2 C_2 + R_1 C_1 + R_3 C_1 = a_1 + R_1 C_2$

$b_2 = a_2 + R_1 R_3 C_1 C_2$

NOTES

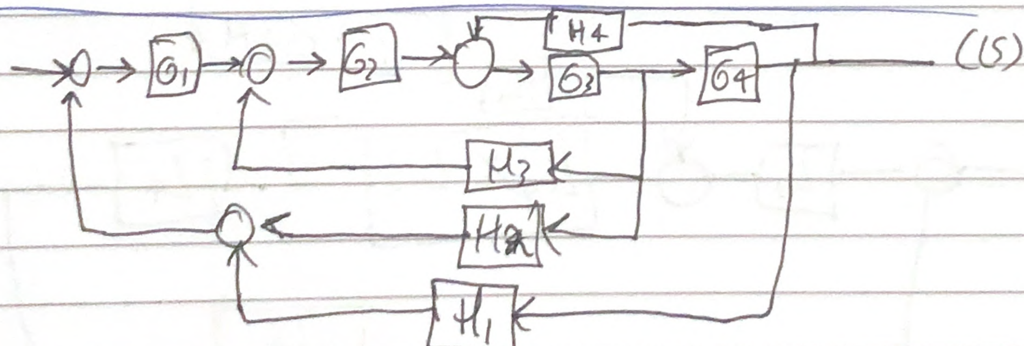
8)  $R_{eq_1} = L_1 S + R_1$

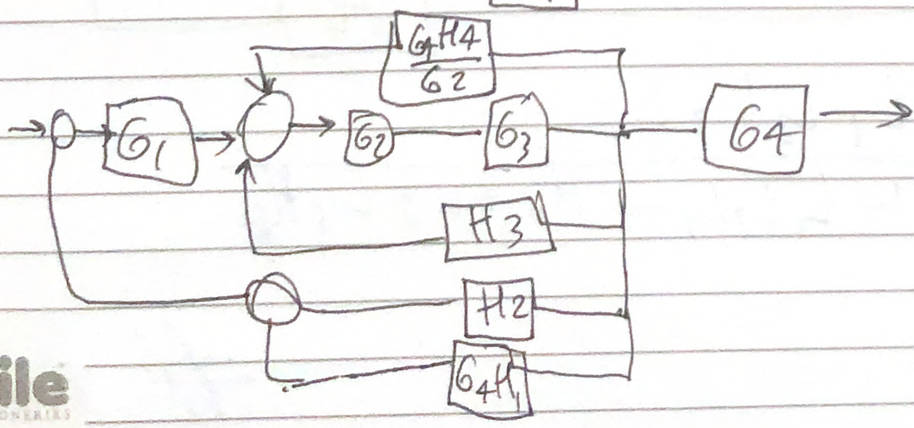
 $R_{eq_2} = \dfrac{R_{eq_1}}{R_{eq_1} C_1 S + 1}$

$$\frac{E_0}{E_1} = \frac{\cancel{I}\, R_{eq_2}}{\cancel{I}\,(R_2 + R_{eq_2})} = \frac{R_1 + L_1 S}{(R_1 + L_1 S)C_1 R_2 S + R_2 + R_1 + L_1 S}$$

$$= \frac{L_1 S + R_1}{L_1 C_1 R_2 S^2 + (R_1 R_2 C_1 + L_1) S + (R_1 + R_2)} \quad /R_1 \quad /R_2$$

$$= \frac{a_1 S + a_0}{b_2 S^2 + b_1 S + b_0}$$

9) 



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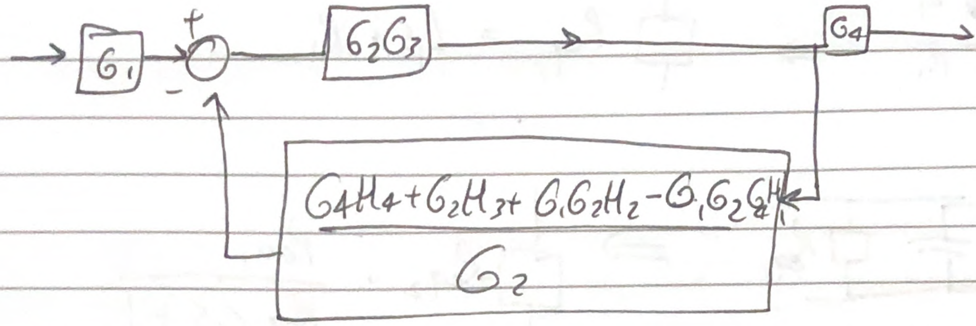
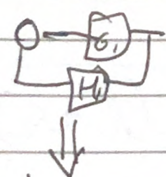


$$\frac{C}{R}=\frac{G_1G_2G_3G_4}{1+G_1G_4H_4+G_2G_3H_3+G_1G_2G_3H_2-G_1G_2G_3G_4H_1}$$

b)

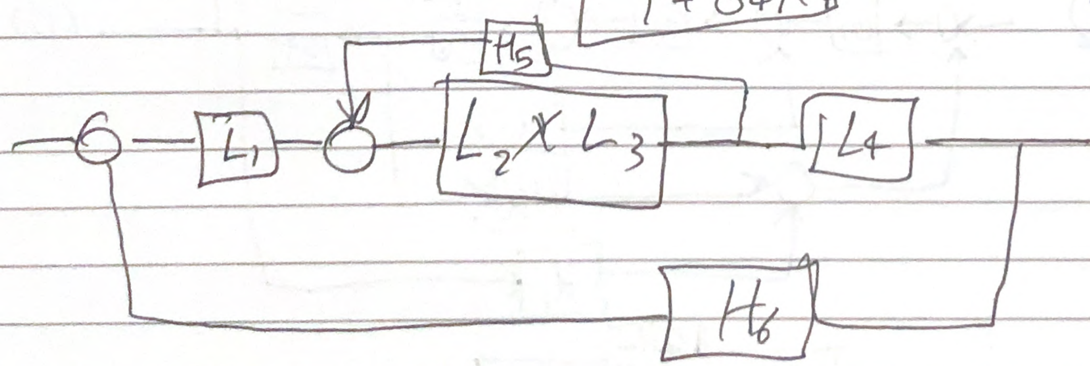


L_1 $\boxed{\frac{G_1}{1+G_1H_1}}$

L_2 $\boxed{\frac{G_2}{1+G_2H_2}}$

L_3 $\boxed{\frac{G_3}{1+G_3H_3}}$

L_4 $\boxed{\frac{G_4}{1+G_4H_4}}$



$$\frac{C}{R}=\frac{L_1L_2L_3L_4}{1+L_1L_2L_3L_4H_6+L_2L_3H_5}$$

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9)