AbdRahman Adel AbdelFattah 17012296

Sheet 5

① $S_x = S_y$ or $\theta = n\pi$

Case 1:- $S_x = S_y = S$ but $\theta \neq n\pi$

① $\begin{bmatrix} S & 0 & 0 \\ 0 & S & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} S\cos\theta & -S\sin\theta & 0 \\ S\sin\theta & S\cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

② $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} S & 0 & 0 \\ 0 & S & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} S\cos\theta & -S\sin\theta & 0 \\ S\sin\theta & S\cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

① = ② #

Case 2:- $S_x \neq S_y$, $\theta = n\pi$ $\qquad \sin n\pi = 0 \quad \cos n\pi = \pm 1$ ($n = 0$ to ∞)

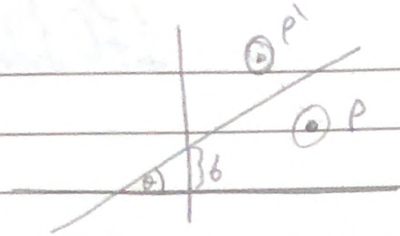
① $\begin{bmatrix} \pm 1 & 0 & 0 \\ 0 & \pm 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \pm 1 & 0 & 0 \\ 0 & \pm 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} \pm S_x & 0 & 0 \\ 0 & \pm S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$ #

odd $\cos\pi = -1$
even $\cos\pi = 1$

② It's similar to reflecting the shape on the y-axis

③



1- Translate line down to intercept the origin
2- Rotate Line $-\theta$ degrees to make it parallel to X
3- Reflect wrt X axis
4- Inverse Rotate (θ)
5- Invers translate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) & 0 \\ \sin(-\theta) & \cos(-\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -b \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

#

④ Scale to twice $S_x = S_y = 2$

step 1: Translate C to (0,0) $\quad \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} = T_1$

step 2: Scale $\quad \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = R_1$

step 3:- Inverse translate C to (5,2) $\quad \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = T_1^{-1}$

$$M = T_1^{-1} R_1 T_1$$

$$= \begin{bmatrix} 2 & 0 & -5 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = M \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$C = (5,2)$
$B = (-3,0)$
$A = (-5,-2)$

NOTES

5) a) 100 2D points = 100 vectors $\begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$

1 translate = $\begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}$ ~~Require 3 multiplications and 3 Additions~~

each row require 3 multiplication and 2 additions

x 3 rows

= 9 multiplication

each of Rotate and Translate require 9 multiplications & 9 additions

for each point 9+9 = 18 Multiplication & additions

6 + 6 = 12 additions

Total = 1800 Multiplications & 1600 additions

b) $T \times R = \begin{bmatrix} \theta & 0 & 0 \\ 0 & \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}$ = each cell = 3 multiplications + 2 additions

9 Cells = 27 multiplication + 18 additions

Combined x point = 9 Multiplications + 6 additions

100x = 900 Multiplications + 600 additions

Total = 927 Multiplications + 618 additions

$\frac{\text{Sequential}}{\text{Concatenated}}$ ~~$\frac{927}{1800}$~~ = $\frac{1800}{927}$ = 1.94 ≃ 2

= $\frac{1600}{618}$ = 2.58 ≃ 2.6 #

A lot less additions & multiplications #

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