AbdelRahman Adel
1701296

Sheet 1

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Question 1:-

$$x(t) = \frac{1}{a}(1 - e^{-at})$$

$$X(z) = Z\left[\frac{1}{a}(1 - e^{-at})\right]$$

$$= Z\left(\frac{1}{a}\right) Z(1 - e^{-at})$$

$$= \frac{1}{a}\left[Z(1) - Z(e^{-at})\right]$$

$$= \frac{1}{a}\left(\frac{1}{1 - z^{-1}} - \frac{1}{1 - e^{-aT} z^{-1}}\right)$$

$$= \frac{1}{a} \frac{(1 - e^{-aT} z^{-1}) - (1 - z^{-1})}{(1 - z^{-1})(1 - e^{-aT} z^{-1})}$$

$$= \frac{1}{a} \frac{z^{-1}(1 - e^{-aT})}{(1 - z^{-1})(1 - e^{-aT} z^{-1})}$$

Question 2:-

$x(0) = 0$, $x(1) = 0$, $x(2) = 0$, $x(3) = \frac{1}{3}$, $x(4) = \frac{2}{3}$

$x(5) = 1$, $x(k) = 1$ $k = 5, \ldots \infty$

$$X(z) = \sum_{k=0}^{\infty} x(k) z^{-k}$$

$$= \frac{1}{3} z^{-3} + \frac{2}{3} z^{-4} + z^{-5} + z^{-6} \ldots$$

$$= \frac{1}{3}(z^{-3} + 2z^{-4}) + \frac{z^{-5}}{1 - z^{-1}}$$

$$= \frac{1}{3} \frac{(z^{-3} + 2z^{-4})(1 - z^{-1}) + 3z^{-5}}{1 - z^{-1}}$$

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Question 3:-

$$X(z) = \frac{z^{-1}(0.5 - z^{-1})}{(1-0.5z^{-1})(1-0.8z^{-1})^2} \times z^3$$

$$= \frac{z(0.5z - 1)}{(z-0.5)(z-0.8)^2}$$

$$\frac{X(z)}{z} = \frac{A}{z-0.5} + \frac{B}{z-0.8} + \frac{C}{(z-0.8)^2} = \frac{(0.5z-1)}{(z-0.5)(z-0.8)^2}$$

$$A(z-0.8)^2 + B(z-0.5)(z-0.8) + C(z-0.5) = (0.5z-1)$$

$$A(z^2 - 1.6z + 0.64) + B(z^2 - 1.3z + 0.4) + C(z - 0.5) = (0.5z - 1)$$

$$A + B = 0 \qquad A = -\frac{25}{3}$$

$$-1.6A - 1.3B + C = 0.5 \qquad B = \frac{25}{3}$$

$$0.64A + 0.4B - 0.5C = -1 \qquad C = -2$$

$$\frac{X(z)}{z} = \frac{-25/3}{z-0.5} + \frac{25/3}{z-0.8} + \frac{-2}{(z-0.8)^2}$$

$$X(z) = \frac{-25/3}{1-0.5z^{-1}} + \frac{25/3}{1-0.8z^{-1}} - \frac{2z^{-1}}{(1-0.8z^{-1})^2}$$

$$x(k) = \frac{-25}{3}(0.5)^k + \frac{25}{3}(0.8)^k - 2k(0.8)^{k-1}, \quad k = 0, 1, 2$$

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Question 4:-

$$X(z) = \frac{1 + z^{-1} - z^{-2}}{1 - z^{-1}} = \frac{z^2 + z - 1}{(z-1)z}$$

$$X(z)\, z^{k-1} = \frac{(z^2 + z - 1) z^{k-1}}{(z-1)z}$$

~~for k=0~~

$$X(z)\, z^{k-1} = \frac{z^2 + z - 1}{(z-1) z^2}$$

$$x(0) = \lim_{z \to 1} \left[(z-1) \frac{z^2 + z - 1}{(z-1) z^2} \right] + \frac{1}{(z-1)!} \lim_{z \to 0} \frac{d}{dz} \left[z^2 \frac{z^2 + z - 1}{(z-1) z^2} \right] = 1 + 0 = 1$$

For k=1 :-

$$X(z)\, z^{k-1} = \frac{z^2 + z - 1}{(z-1) z}$$

$$x(k) = \lim_{z \to 1} \left[z-1 \frac{z^2 + z - 1}{(z-1) z} \right] + \lim_{z \to 0} \left[z \frac{z^2 + z - 1}{(z-1) z} \right]$$

~~for k=2~~ $= 1 + 1 = 2$

For k=2 :- $X(z)\, z^{k-1} = \dfrac{(z^2 + z - 1) z^{k-2}}{z - 1}$

$$X(k) = \lim_{z \to 1} \left[(z-1) \frac{(z^2 + z - 1) z^{k-2}}{z - 1} \right] = 1$$

$x(0) = 1$

$x(1) = 2$

$x(k) = 1$ for $k = 2, 3, 4 \ldots$

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Question 5:

$$x(k+2) - x(k+1) + 0.25x(k) = u(k+2)$$

$x(0)=1 \quad x(1)=2 \quad u(k)=1 \quad k=0,1,2.$

$$[z^2X(z) - z^2x(0) - zx(1)] - [zX(z) - zx(0)] + 0.25X(z) = z^2U(z) - z^2U(0) - zU(1)$$

$$\Rightarrow [z^2X(z) - z^2 - 2z] - [zX(z) - z] + 0.25X(z) = z^2U(z) - z^2 - z$$

$$z^2X(z) - z^2 - 2z - zX(z) + z + 0.25X(z) = z^2U(z) - z^2 - z$$

$$X(z)[z^2 - z + 0.25] = z^2U(z) = z^2 \frac{z}{z-1}$$

$$X(z) = \frac{z^3}{(z-1)(z^2 - z + 0.25)}$$

$$\frac{X(z)}{z} = \frac{z^2}{(z-1)(z-0.5)^2}$$

$$\frac{A}{(z-1)} + \frac{B}{(z-0.5)} + \frac{C}{(z-0.5)^2} = \frac{X(z)}{z}$$

$A+B=1$

$-A-1.5B+C=0$

$0.25A+0.5B-C=0$

$$A(z-0.5)^2 + B(z-1)(z-0.5) + C(z-1) = z^2$$

$$A(z^2 - z + 0.25) + B(z^2 - 1.5z + 0.5) + C(z-1) = z^2$$

$$\frac{4}{z-1} + \frac{-3}{(z-0.5)} + \frac{-0.5}{(z-0.5)^2} \Rightarrow x(k) = 4 - (3+k)(0.5)^k$$

$k=0,1,2,3$

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