AbdelRahman Adel
17012296

Sheet 2

Question 1:-

① Method of residue

$$X(Z) = \left[\text{reside of } \frac{X(S)Z}{Z-e^{Ts}} \text{ at pole} = -1\right] + \left[\text{reside of } \frac{X(S)Z}{Z-e^{Ts}} \text{ at pole} = -2\right]$$

$$= \lim_{s\to -1}\left[(s+1)\frac{s+3}{(s+1)(s+2)}\frac{Z}{Z-e^{Ts}}\right] + \lim_{s\to -2}\left[(s+2)\frac{s+3}{(s+1)(s+2)}\frac{Z}{Z-e^{Ts}}\right]$$

$$= \lim_{s\to -1}\left[\frac{s+3}{s+2}\frac{Z}{Z-e^{Ts}}\right] + \lim_{s\to -2}\left[\frac{s+3}{s+1}\frac{Z}{Z-e^{Ts}}\right]$$

$$= \frac{2Z}{Z-e^{-T}} + \frac{Z}{Z-e^{-2T}} = \frac{2}{1-e^{-T}Z^{-1}} - \frac{1}{1-e^{-2T}Z^{-1}}$$

$$= \frac{2(1-e^{-2T}Z^{-1}) - (1-e^{-T}Z^{-1})}{(1-e^{-T}Z^{-1})(1-e^{-2T}Z^{-1})}$$

$$= \frac{2-2e^{-2T}Z^{-1} - 1 + e^{-T}Z^{-1}}{//}$$

$$= \frac{1-2e^{-2T}Z^{-1} + e^{-T}Z^{-1}}{//} = \frac{1+e^{-T}(1-2e^{-T})Z^{-1}}{(1-e^{-T}Z^{-1})(1-e^{-2T}Z^{-1})}$$

② Method of impulse

$$X(S) = \frac{s+3}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2} = \frac{2}{s+1} - \frac{1}{s+2}$$

$$s+3 = AS+2A + BS+B \Rightarrow A+B=1 \Rightarrow A=2$$
$$\Rightarrow 2A+B=3 \quad B=-1$$

Inverse of laplace $\Rightarrow x(t) = 2e^{-t} - 2e^{-2t}$

$$X(Z) = \frac{2}{1-e^{-T}Z^{-1}} - \frac{1}{1-e^{-2T}Z^{-1}} = \frac{1+e^{-T}(1-2e^{-T})Z^{-1}}{(1-e^{-T}Z^{-1})(1-e^{-2T}Z^{-1})}$$

#

PAGE
DATE

Question 2 :-

$$y(k) - y(k-1) + 0.24\,y(k-2) = x(k) + x(k-1)$$

Z-transform :-

$$Y(z) - z^{-1}Y(z) + 0.24\,z^{-2}Y(z) = X(z) + z^{-1}X(z)$$

$$Y(z)\left(1 - z^{-1} + 0.24\,z^{-2}\right) = X(z)\left(1 + z^{-1}\right)$$

$$G(z) = \frac{Y(z)}{X(z)} = \frac{1+z^{-1}}{1-z^{-1}+0.24z^{-2}} = \frac{1+z^{-1}}{(1-0.6z^{-1})(1-0.4z^{-1})}$$

$$\frac{1+z^{-1}}{(1-0.6z^{-1})(1-0.4z^{-1})} = \frac{A}{(1-0.6z^{-1})} + \frac{B}{(1-0.4z^{-1})}$$

$$1+z^{-1} = A - 0.4Az^{-1} + B - 0.6Bz^{-1}$$

$A+B=1$

$-0.4A-0.6B=+1$ $\Rightarrow$ $A=8$, $B=-7$

then $g(k) = 8(0.6)^k - 7(0.4)^k$

for unit step where $X(z) = \frac{1}{1-z^{-1}}$

then $$Y(z) = \frac{1+z^{-1}}{1-z^{-1}+0.24z^{-2}}\;\frac{1}{1-z^{-1}} = \boxed{\frac{14/3}{(1-0.4z^{-1})} - \frac{12}{(1-0.6z^{-1})} + \frac{25/3}{(1-z^{-1})}}$$

$$= \frac{1+z^{-1}}{(1-0.4z^{-1})(1-0.6z^{-1})(1-z^{-1})} = \frac{A}{(1-0.4z^{-1})} + \frac{B}{(1-0.6z^{-1})} + \frac{C}{(1-z^{-1})}$$

$$1+z^{-1} = A(1-0.6z^{-1})(1-z^{-1}) + B(1-0.4z^{-1})(1-z^{-1}) + C(1-0.4z^{-1})(1-0.6z^{-1})$$

$$= A(1-z^{-1}-0.6z^{-1}+0.6z^{-2}) + B(1-z^{-1}-0.4z^{-1}+0.4z^{-2}) + C(1-0.6z^{-1}-0.4z^{-1}+0.24z^{-2})$$

$$= A(1-1.6z^{-1}+0.6z^{-2}) + B(1-1.4z^{-1}+0.4z^{-2}) + C(1-z^{-1}+0.24z^{-2})$$

$A+B+C=1$

$-1.6A-1.4B-C=1$ $\qquad A=\frac{14}{3},\ B=-12\quad C=\frac{25}{3}$

$0.6A+0.4B+0.24C=0$

$$y(k) = \frac{14}{3}(0.4)^k - 12(0.6)^k + \frac{25}{3} \quad \text{for } k=0,1,2\ldots$$

//

2

Question 3:-

$$Y(S) = \frac{1}{(S+1)(S+2)} X^*(S) = \left(\frac{1}{S+1} - \frac{1}{S+2}\right) X^*(S)$$

$$\frac{1}{(S+1)(S+2)} = \frac{A}{S+1} + \frac{B}{S+2} \Rightarrow 1 = AS + 2A + SB + B$$

$A + B = 0$ $\quad A = 1$

$2A + B = 1$ $\quad B = -1$

Take stated laplaccian

$$Y^*(S) = \left(\frac{1}{S+1}\right)^* X^*(S) - \left(\frac{1}{S+2}\right)^* X^*(S)$$

$$Y(z) = Z\left[\frac{1}{S+1}\right] X(z) - Z\left[\frac{1}{S+2}\right] X(z)$$

$$= \frac{1}{1-e^{-T}z^{-1}} X(z) - \frac{1}{1-e^{-2T}z^{-1}} X(z)$$

$$X(z) = \frac{1}{1-z^{-1}}$$

then

$$Y(z) = \frac{1}{1-e^{-T}z^{-1}} \frac{1}{1-z^{-1}} - \frac{1}{1-e^{-2T}z^{-1}} \frac{1}{1-z^{-1}}$$

$$= \frac{e^{-T}}{e^{-T}-1} \frac{1}{1-e^{-T}z^{-1}} + \frac{1}{1-e^{-T}} \frac{1}{1-z^{-1}}$$

$$- \frac{e^{-2T}}{e^{-2T}-1} \frac{1}{1-e^{-2T}z^{-1}} - \frac{1}{1-e^{-2T}} \frac{1}{1-z^{-1}}$$

then

$$y(kT) = \frac{e^{-T}}{e^{-T}-1}(e^{-T})^k + \frac{1}{1-e^{-T}} - \frac{e^{-2T}}{e^{-2T}-1}(e^{-2T})^k - \frac{1}{1-e^{-2T}}$$

For $T = 0.1$

$$y(k) = -9.51(0.905)^k + 4.52(0.82)^k + 4.99$$

#

3

PAGE
DATE

Question 4 :-

$$G(z) = Z\left[\frac{1-e^{-Ts}}{s}\frac{K}{s}\right] = (1-z^{-1})Z\left[\frac{K}{s^2}\right]$$

$$= \cancel{(1-z^{-1})}\frac{KTz^{-1}}{\cancel{(1-z^{-1})^2}} \quad \text{with } T=1,\ G(z) = \frac{Kz^{-1}}{(1-z^{-1})}$$

$$\frac{C(z)}{R(z)} = \frac{G(z)}{1+G(z)} = \frac{Kz^{-1}/\cancel{(1-z^{-1})} \times 1-z^{-1}}{1+Kz^{-1}/\cancel{(1-z^{-1})} \times 1-z^{-1}}$$

$$= \frac{Kz^{-1}}{(1-z^{-1})+Kz^{-1}} = \frac{Kz^{-1}}{1+(K-1)z^{-1}}$$

$$R(z) = \frac{1}{1-z^{-1}}$$

$$C(z) = \frac{Kz^{-1}}{1+(K-1)z^{-1}} \times \frac{1}{1-z^{-1}} = \frac{Kz^{-1}}{1+(K-1)z^{-1}}\ \frac{1}{1-z^{-1}}$$

$$= \frac{A}{1+(K-1)z^{-1}} + \frac{B}{1-z^{-1}} = \frac{-1}{1+(K+1)z^{-1}} + \frac{1}{1-z^{-1}}$$

$$Kz^{-1} = A(1-z^{-1}) + B(1+(K-1)z^{-1})$$

$A + B = 0$

$-A + B(K-1) = K \qquad \text{let } K=1 = \quad -A = 1\ \&\ B = 1$

inverse z transform

$$c(k) = -(1-K)^k + 1$$

output will converge if $0 < K < 2$ only

Continuous

$$C(s) = \left(\frac{K}{s}\frac{1-e^{-Ts}}{s}\right) E^*(s)$$

$$E(s) = R(s) - C(s) \Longrightarrow E^*(s) = R^*(s) - C^*(s)$$

$$E^*(s) = R^*(s) - \left(\frac{K}{s}\frac{1-e^{-Ts}}{s}\right)^* E^*(s)$$

$$E^*(s)\left(1 + \left(\frac{K}{s}\frac{1-e^{-Ts}}{s}\right)\right) = R^*(s)$$

$$E^*(s) = R^*(s) \Big/ 1 + \left(\frac{K}{s}\frac{1-e^{-Ts}}{s}\right)^*$$

then

$$C(s) = \frac{\frac{K}{s}\frac{1-e^{-Ts}}{s}}{\left(1 + \frac{K}{s}\frac{1-e^{-Ts}}{s}\right)^*} R^* s$$

then

$$Z\left[\frac{K}{s}\frac{1-e^{-Ts}}{s}\right] = 1 - z^{-1} Z\left[\frac{K}{s^2}\right] = \frac{Kz^{-1}}{1-z^{-1}}$$

substitute $z = e^{Ts} = e^{s}$

$$\left(\frac{K}{s}\frac{1-e^{-Ts}}{s}\right)^* = \frac{Ke^{-s}}{1-e^{-s}}$$

$$C(s) = \frac{K}{s^2}\frac{1-e^{-s}}{1+(K+1)e^{-s}} = \frac{K}{s^2}\left[1 - Ke^{-s} + K(K-1)e^{-2s} - K(K-1)^2e^{-3s} + \dots\right.$$

Reverse laplace

$$C(t) = Kt - K^2(t-1)1(t-1) + K^2(K-1)(t-2)1(t-2) - K^2(K-1)^2(t-3)1(t-3) + \dots$$

5