

$$\textcircled{1} \quad P(A, B | C) = P(A | C) P(B | C)$$

$$\frac{P(A, B, C)}{P(C)} = \frac{P(A, C) P(B, C)}{P(C) P(C)}$$

$$\boxed{P(A, B, C) = \frac{P(A, C) P(B, C)}{P(C)}} \Rightarrow \textcircled{1}$$

From \textcircled{1}

$$P(A | B, C) P(B, C) = \frac{P(A, C) P(B, C)}{\cancel{P(C)}}$$

$$\boxed{P(A | B, C) = P(A | C)} \Rightarrow \text{first one}$$

$$\text{From } \textcircled{1} \quad P(B | A, C) \cancel{P(A, C)} = \frac{P(A, C) P(B, C)}{P(C)}$$

$$\boxed{P(B | A, C) = P(B | C)} \Rightarrow \text{Second One}$$

$$\textcircled{2} \quad P(\text{toothache}) = \sum_{x,y} P(\text{toothache}, x, y)$$

$$\bullet P(\text{toothache}) = \sum_{x,y} P(\text{toothache}, x, y)$$

$$= 0.108 + 0.012 + 0.016 + 0.064$$

$$\boxed{P(\text{toothache}) = 0.2}$$

$$\bullet P(\text{cavity}) = \sum_{x,y} P(\text{cavity}, x, y)$$

$$= 0.108 + 0.012 + 0.012 + 0.008$$

$$\boxed{P(\text{cavity}) = 0.2}$$

$$\cdot P(\text{Toothache} | \text{Cavity}) = \frac{P(\text{Toothache}, \text{Cavity})}{P(\text{Cavity})}$$

$$P(\text{Toothache} | \text{Cavity}) = \frac{0.108 + 0.012}{0.2} = \frac{3}{6} = 0.6$$

Normalization

$$P(x, y, \text{Cav. ty}) \quad \text{Normalized}$$

	toothache	Not toothache	→		Toothache	Not toothache
Cavity	0.108	0.012	Z = 0.2	Cavity	0.54	0.06
Not	0.08	0.98		Not	0.04	0.96

$$P(T|C) = 0.6$$

$$P(\text{Toothache} | \text{Cavity}) = \frac{P(\text{Cavity}, (\text{Toothache} \vee \text{catch}))}{P(\text{Toothache} \vee \text{catch})}$$

$$P(\text{Cavity} | \text{Toothache} \vee \text{catch}) = \frac{P(\text{Cavity}, (\text{Toothache} \vee \text{catch}))}{P(\text{Toothache} \vee \text{catch})}$$

$$= \frac{P(\text{Cavity}, (\text{Toothache} \vee \text{catch}))}{P(T \vee C)}$$

$$P(\text{Catch}, \text{toothache}, x)$$

Normalize

	toothache v catch	→		toothache v catch
Cavity	0.108	Z = 0.124	Cavity	0.871
not	0.016		not	0.129

$$P(\text{Cavity} | \text{toothache} \vee \text{catch}) = 0.871$$

$$\textcircled{3} \quad P(\text{Seems B} | \text{Actual B}) = 0.75$$

$$P(SB | AG) = 0.25$$

$$P(SG | AG) = 0.75$$

$$P(SG | AB) = 0.25$$

$$P(AB | SB) = \frac{P(AB, SB)}{P(SB)} = \frac{P(SB | AB) P(AB)}{P(SB)}$$

$$= \frac{0.75 P(AB)}{P(SB)}$$

$$P(SB) = \sum_x P(SB, x) = P(SB, AG) + P(SB, AB)$$

$$P(AB | SB) = \frac{0.75 P(AB)}{P(SB, AG) + P(SB, AB)} = \frac{0.75 P(AB)}{P(SB | AG) P(AG) + P(SB | AB) P(AB)}$$

$$= \frac{0.75 P(AB)}{0.25 P(AG) + 0.75 P(AB)}$$

Not Possible without $P(AG)$ & $P(AB)$

then if

$$P(AG) = 0.9$$

$$P(AB) = 0.1$$

$$P(AB | SB) = \frac{0.75 \times 0.1}{0.25 \times 0.9 + 0.75 \times 0.1} = \frac{1}{4} = 0.25$$

The Car is more likely to be green $\#$