

Sheet 2

$$\textcircled{1} u_1 = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix}, u_2 = \begin{bmatrix} -4 \\ 3 \\ 0 \end{bmatrix}$$

a) if $x \cdot y = 0$ then $x \perp y$

$$\begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} -4 \\ 3 \\ 0 \end{bmatrix} = 3 \times -4 + 4 \times 3 + 0 \times 0 \\ = -12 + 12 + 0 = 0$$

$$u_1 \perp u_2$$

b) • Projection of y on $u_1 = \frac{u_1 \cdot y}{|y|}$

$$u_1 \cdot y = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix} = 3 \times 6 + 4 \times 3 + 0 \times 2 \\ = 30$$

$$|y| = 7$$

$$\frac{u_1 \cdot y}{|y|} = \frac{30}{7} = 4 \frac{2}{7} \neq$$

• Projection of y on $u_2 = \frac{u_2 \cdot y}{|y|} = \frac{-15}{7} = -2 \frac{1}{7} \neq$

② LDA

NOTES.

	x_1	x_2	Class
n_1	2	3	▲
n_2	3	3	▲
n_3	3	4	▲
n_4	5	4	○
n_5	5	8	▲
n_6	6	5	○
n_7	7	4	○
n_8	7	5	○
n_9	7	7	▲
n_{10}	8	2	○
n_{11}	9	4	○

$$S^{-1} = \begin{bmatrix} 0.056 & -0.029 \\ -0.029 & 0.052 \end{bmatrix}$$

a) $\mu_{\Delta x_1} = 4$

$$\mu_{\Delta} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$\mu_{\Delta x_2} = 5$

$\mu_{0 x_1} = 7$

$$\mu_0 = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$$

$\mu_{0 x_2} = 4$

$$\mu_{\Delta} - \mu_0 = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

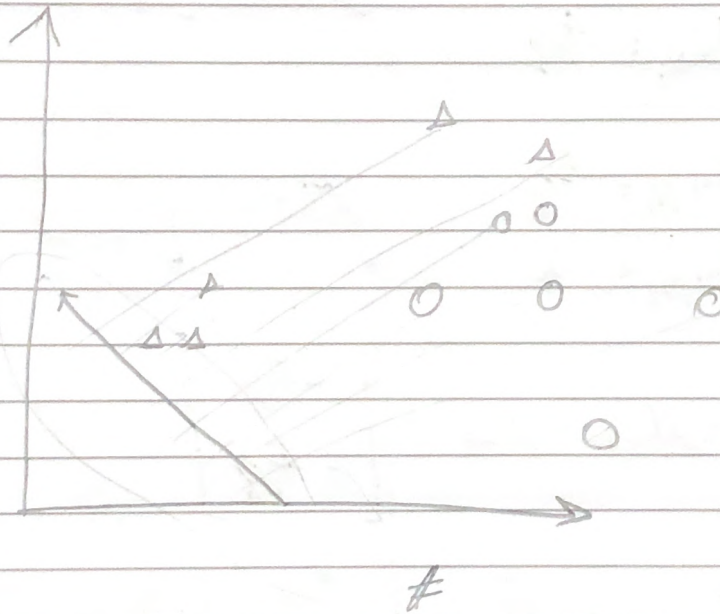
$$B_{\Delta} = (\mu_{\Delta} - \mu_0)(\mu_{\Delta} - \mu_0)^T = \begin{bmatrix} 9 & -3 \\ -3 & 1 \end{bmatrix}$$

$$b) S^{-1}B = \begin{pmatrix} 0.591 & -0.197 \\ -0.417 & 0.139 \end{pmatrix}$$

eigenvalue ($S^{-1}B$) ~~is~~

$$\lambda = \frac{73}{100}, \quad \lambda = 0$$

$$\begin{pmatrix} -1.41726 \\ 1 \end{pmatrix}, \begin{pmatrix} 0.3333 \\ 1 \end{pmatrix}$$



(3)

NOTES

i	N_1	N_2	y_i
x_1	1	1	1
x_2	2	1	1
x_3	1	2	1
x_4	2	2	-1
x_5	3	2	-1

$$a) \mu_{+1N_1} = 4/3 \quad \mu_{+1} = \begin{bmatrix} 4/3 \\ 4/3 \end{bmatrix}$$

$$\mu_{+1N_2} = 4/3$$

$$\mu_{-1N_1} = 5/2 \quad \mu_{-1} = \begin{bmatrix} 5/2 \\ 2 \end{bmatrix}$$

$$\mu_{-1N_2} = 2$$

$$A = \mu_{+1} - \mu_{-1} = \begin{bmatrix} -1/6 \\ -4/6 \end{bmatrix}$$

$$B = A A^T = \begin{bmatrix} -1/6 \\ -4/6 \end{bmatrix} \begin{bmatrix} -1/6 & -4/6 \end{bmatrix}$$

$$B = \begin{bmatrix} 1/36 & 2/9 \\ 2/9 & 16/9 \end{bmatrix}$$

$$b) Z_{+1} = \begin{bmatrix} -\frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

$$Z_{-1} = \begin{bmatrix} -\frac{1}{2} & 0 \\ \frac{1}{2} & 0 \end{bmatrix}$$

$$S_H = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

$$S_{-1} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0 \end{bmatrix}$$

$$S = \begin{bmatrix} \frac{7}{6} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

