

AbdelRahman Adel
17012296

Sheet 2

No.

Date. / /

① Image Filtering

① 128×128 using 8 bits \Rightarrow Size of image = 131072 bits
= 16384 bytes
= 16 KBytes

Gaussian Pyramid

$$128 \times 128 \xrightarrow{\times 8} 16 \text{ KB}$$

$$64 \times 64 \xrightarrow{\times 8} 4 \text{ KB}$$

$$32 \times 32 \xrightarrow{\times 8} 1 \text{ KB}$$

$$16 \times 16 \xrightarrow{\times 8} 0.25 \text{ KB}$$

$$8 \times 8 \xrightarrow{\times 8} \frac{1}{16} \text{ KB}$$

$$\rightarrow \boxed{21 \frac{5}{16} \text{ KBytes}}$$

$$\approx \frac{4}{3} \times 16 = 21 \frac{1}{3} \text{ KBytes}$$

Laplacian

$$128 \times 128 \xrightarrow{\times 4} 8 \text{ KB}$$

$$64 \times 64 \xrightarrow{\times 4} 2 \text{ KB}$$

$$32 \times 32 \xrightarrow{\times 4} \frac{1}{2} \text{ KB}$$

$$16 \times 16 \xrightarrow{\times 4} \frac{1}{8} \text{ KB}$$

$$8 \times 8 \xrightarrow{\times 4} \frac{1}{16} \text{ KB}$$

$$\rightarrow \boxed{10 \frac{11}{16} \text{ KB}}$$

$$\frac{\text{Laplacian}}{\text{Gaussian}} = \frac{171}{341} \approx 50.15\% \text{ of saving}$$

$$\textcircled{2} BF[I]_p = \frac{1}{W_p} \sum_{q \in S} \underbrace{G_{\omega_s}(\|p - q\|)}_{BF_s} \underbrace{G_{\omega_r}(\|I_p - I_q\|)}_{BF_r} \cdot I_q$$

$$G_{\omega}(x) = \frac{1}{\omega \sqrt{2\pi}} e^{-\frac{x^2}{2\omega^2}}$$

$$G_{\omega_s}(x) = \frac{1}{1\sqrt{2\pi}} e^{-\frac{x^2}{2 \times 1}} = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$G_{\omega_r}(x) = \frac{1}{20\sqrt{2\pi}} e^{-\frac{x^2}{800}}$$

$$BF_s = \frac{1}{\sqrt{2\pi}} e^{-\frac{-(\|p - q\|)^2}{2}}, \quad BF_r = \frac{1}{20\sqrt{2\pi}} e^{-\frac{-(\|I_p - I_q\|)^2}{800}}$$

$$\|p - q\| = \begin{bmatrix} \sqrt{2} & 1 & \sqrt{2} \\ 1 & 0 & 1 \\ \sqrt{2} & 1 & \sqrt{2} \end{bmatrix}$$

$$\|I_p - I_q\| = \begin{bmatrix} 40 & 30 & 40 \\ 40 & 0 & 0 \\ 30 & 10 & 10 \end{bmatrix}$$

$$BF_s = \begin{bmatrix} 0.147 & 0.242 & 0.147 \\ 0.242 & 0.399 & 0.242 \\ 0.147 & 0.242 & 0.147 \end{bmatrix}$$

$$BF_r = \begin{bmatrix} 2.7 \times 10^{-3} & 6.5 \times 10^{-3} & 2.7 \times 10^{-3} \\ 2.7 \times 10^{-3} & 0.02 & 0.02 \\ 6.5 \times 10^{-3} & 0.018 & 0.018 \end{bmatrix}$$

$$BF_s \times BF_r = \begin{bmatrix} 2.01 \times 10^{-3} & 8.44 \times 10^{-3} & 7.88 \times 10^{-3} \\ 3.30 \times 10^{-3} & 0.014 & 0.013 \\ 2.01 \times 10^{-3} & 8.44 \times 10^{-3} & 7.88 \times 10^{-3} \end{bmatrix}$$

$$W_p = 0.06696$$

$$BF_p = \begin{bmatrix} 0.03 & 0.126 & 0.12 \\ 0.05 & 0.21 & 0.19 \\ 0.03 & 0.126 & 0.12 \end{bmatrix}$$

③ Sharpening the image

Hough Transform

$$\textcircled{1} \rho = x \cos \theta + y \sin \theta \quad \begin{array}{l} \theta \in [0, 360] \\ x \in [1, W] \\ y \in [1, H] \end{array}$$

Range of $\theta = 0 \rightarrow 360$

$$\frac{\partial \rho}{\partial x} = \cos \theta \quad U = \frac{\partial \rho}{\partial x} = \frac{\partial \rho}{\partial y} = \cos \theta + \sin \theta$$

$$\frac{\partial \rho}{\partial y} = \sin \theta$$

$$U = \cos \theta + \sin \theta$$

$$\frac{\partial U}{\partial \theta} = -\sin \theta + \cos \theta$$

$$\boxed{\sin \theta = \cos \theta}$$

$$\theta = 45$$

$$\rho = x \cos 45 + y \sin 45$$

$$\boxed{\rho = \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}}} \Rightarrow \text{Max} = \frac{1}{\sqrt{2}} [W + H]$$

$$\textcircled{2} \rho = (7, 10) \quad \text{gradient of } \rho = [0.707, 0.707]^T$$

$$r = 20$$

$$a = x + r \cos \theta \rightarrow 0.707$$

$$b = y + r \sin \theta \rightarrow 0.707$$

$$a = 7 + 20 \times 0.707 = 21.14$$

$$b = 10 + 20 \times 0.707 = 24.14$$

$$\text{Center} = (21.14, 24.14)$$

③ $y = ax^2 + bx + c$

a) 3D space

b) Cubic Curve

c) a straight line

d) Cubic Curve with an offset

e) $\frac{dy}{dx} = 2ax + b$

we would use the tangents

for figuring out the polar coordinates