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Sheet #5

① $P_1 \Rightarrow$ Nearest 1st $= q_1 = 0.1$
$\hookrightarrow$ Nearest 2nd $= q_4 = 0.2$

Ratio of $P_1 = \frac{0.1}{0.2} = 0.5 < 0.6 \Rightarrow$ Valid point

$P_2 \Rightarrow$ Nearest 1st $= q_1 \& q_3 = 0.2$
$\hookrightarrow$ Nearest 2nd $= q_4 = 0.3$

Ratio of $P_2 = \frac{0.2}{0.3} \cong 0.667 > 0.6 \Rightarrow$ Invalid point

② 256 x 256 Image 30 θ about pixel [128,128], scaled 1.5, [128, 128] → [40, 15

a) Similarity

b) Step 1 ⇒ Translate [128, 128] to origin

$$M_1 = \begin{bmatrix} 1 & 0 & -128 \\ 0 & 1 & -128 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 2 ⇒ Rotate $30° = \theta$

$$M_2 = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 \\ 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 3 ⇒ Scale by 1.5

$$M_3 = \begin{bmatrix} 1.5 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 4 ⇒ Translate [128, 128] to [110, 150]

$$M_4 = \begin{bmatrix} 1 & 0 & 140 \\ 0 & 1 & 150 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = M_4(M_3(M_2 M_1)) = \begin{bmatrix} 1.299 & -0.75 & -52 \\ 0.75 & 1.299 & -42 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M^{-1} = \begin{bmatrix} 0.577 & 0.333 & 44.022 \\ -0.333 & 0.577 & 6.9153 \\ 0 & 0 & 1 \end{bmatrix}$$

c) $x' = Mx$, $M\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -51.45 \\ -39.95 \\ 1 \end{bmatrix} \Rightarrow$ Not in out put

$$M\begin{bmatrix} 250 \\ 240 \\ 1 \end{bmatrix} = \begin{bmatrix} 92.75 \\ 457.26 \\ 1 \end{bmatrix} \Rightarrow \text{Not in out put}$$

$$M\begin{bmatrix} 120 \\ 190 \\ 1 \end{bmatrix} = \begin{bmatrix} -38.62 \\ 294.81 \\ 1 \end{bmatrix} \Rightarrow \text{Not in out put}$$

d) $M'P' = P$

$$\begin{bmatrix} 50 & 90 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 102.89 & 42.209 \\ 44.933 & 7.158 \end{bmatrix}$$

$P_1 \approx [103 \quad 42]$

$P_2 \approx [45 \quad 7]$

e) $P_1 = (197.2967 \quad 72.3845)$

P_{1x} between 197 198 (197,72) — (198,72)

P_{1y} between 72 73 (197,73) — (198,73)

(197,72) $\Rightarrow w_1 = (1-0.2967) \times (1-0.3845) = 0.4329$

(197,73) $\Rightarrow w_2 = (1-0.2967) \times (0.3845) = 0.27042$

(198,72) $\Rightarrow w_3 = (0.2967) \times (1-0.3845) = 0.18262$

(198,73) $\Rightarrow w_4 = (0.2967) \times (0.3845) = 0.1141$

$w = 1$

③ $(0,0) \to (0,0)$
$(0,1) \to (0.5,1)$
$(1,0) \to (2,0)$
$(1,1) \to (1.5,1)$

$$A = \begin{bmatrix} -x & -y & -1 & 0 & 0 & 0 & xx' & x'y & x' \\ 0 & 0 & 0 & -x & -y & -1 & xy' & y'y & y' \end{bmatrix}$$

$Ah = 0$

$$Ah = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & -1 & 0 & 0 & 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 & -1 & 0 & 0 & 0 & 1.5 & 1.5 & 1.5 \\ 0 & 0 & 0 & -1 & -1 & -1 & 1 & 1 & 1 \end{bmatrix}$$

→ α Constant

$-h_3 = 0$, $-h_6 = 0$ $\Rightarrow$ $\boxed{h_3 = h_6 = 0}$

$-h_2 - h_3 + 0.5h_8 + 0.5h_9 = 0 \Rightarrow -h_2 + 0.5h_8 + 0.5h_9 = 0$
$-h_5 - h_6 + h_8 + h_9 = 0 \Rightarrow -h_5 + h_8 + h_9 = 0 \Rightarrow ①$

$-h_1 - h_3 + 2h_7 + 2h_9 = 0 \Rightarrow -h_1 + 2h_7 + 2h_9 = 0 \Rightarrow -h_1 + 2h_9 = 0$
$-h_4 - h_6 = 0 \Rightarrow \boxed{h_4 = 0}$

$-h_1 - h_2 - h_3 + 1.5h_7 + 1.5h_8 + 1.5h_9 = 0 \Rightarrow -h_1 - h_2 + 1.5h_7 + 1.5h_8 + 1.5h_9 = 0$
$-h_4 - h_5 - h_6 + h_7 + h_8 + h_9 = 0 \Rightarrow -h_5 + h_7 + h_8 + h_9 = 0 \Rightarrow ②$

①②$\Rightarrow$ $\boxed{h_7 = 0}$

$-h_2 + 0.5h_8 + 0.5 = 0$
$-h_5 + h_8 + 1 = 0$
$-h_1 + 2 = 0$
$-h_1 - h_2 + 1.5h_8 + 1.5 = 0$
$-h_5 + h_8 + 1 = 0$

$\boxed{h_1 = 2}$

$-h_2 + 0.5h_8 = -0.5$
$-h_5 + h_8 = -1$
$-h_2 + 1.5h_8 = -1.5$

$h_2 = 0$
$h_8 = -1$
$h_5 = 0$

/3

$$h = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 1 \end{bmatrix} \#$$

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(4) 100 matching pairs
25% outliers
$P = 0.95$

$$N \geq \frac{\log(1-P)}{\log(1-(1-e)^3)}$$ affine $\Rightarrow$ 3 Correspondance

$$N \geq \frac{\log(1-0.95)}{\log(1-(1-0.25)^3)} \geq 5.46 \#$$

N at least $\boxed{6}$ #

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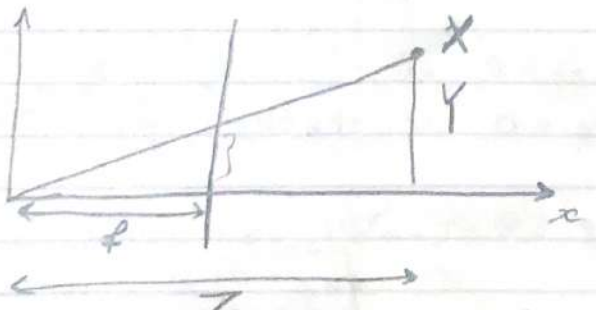
(5)



$$[X\ Y\ Z]^T \Rightarrow \left[\frac{f}{z}X, \frac{f}{z}Y\right]$$

Point 1 $[x_1\ y_1\ d]^T \rightarrow \left[\frac{f}{d}x_1, \frac{f}{d}y_1\right]^T$

Point 2 $[x_2\ y_2\ d]^T \rightarrow \left[\frac{f}{d}x_2, \frac{f}{d}y_2\right]$

height $= \frac{f}{d}(y_1 - y_2) = \frac{f}{d}h$ #

(6) $f = 0.5$

$P_1 = [50, 50, 100]^T$

$P_2 = [75\ 100, 120]^T$

[1] Perspective Model

$[X\ Y\ Z]^T \to [\frac{f}{Z}X, \frac{f}{Z}Y]$

$50 \to 0.25$

$50 \to 0.25$

$P_1' = [0.25\ \ 0.25]^T$

$75 \to 5/16$

$100 \to 5/12$

$P_2' = [5/16\ \ 5/12]$

length $= \sqrt{\|P_1 - P_2\|^2}$

$= 0.18$ cm

[2] Weak perspective

$Z_0 = \frac{100+120}{2} = 110$

$[X\ Y\ Z_0]^T \to [\frac{f}{Z_0}X, \frac{f}{Z_0}Y]$

$P_1' = [\frac{5}{22}\ \ \frac{5}{22}]$

$P_2' = [\frac{15}{44}\ \ \frac{5}{11}]$

length $= \sqrt{\|P_1 - P_2\|^2}$

$= 0.254$ cm

[3] orthographic

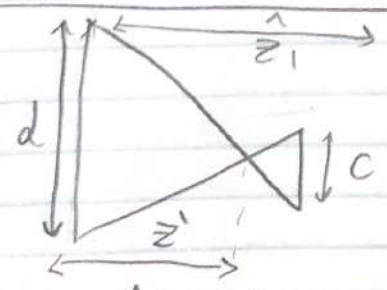
$P_1' = [50, 50]^T$

$P_2' = [75, 100]^T$

length $= \sqrt{\|\ \ \|\ \ \|}$

$= 55.902$ cm

(7)



$\frac{c}{d} = \frac{\hat{z}' - z'}{z'}$

$\rightsquigarrow \hat{z}' < z'$

$\frac{c}{d} = \frac{z' - \hat{z}'}{z'} \Longrightarrow \frac{c}{d} = \frac{|z' - \hat{z}'|}{z'}$

$\rightsquigarrow \boxed{c = d\frac{|z' - \hat{z}'|}{z'}}$ #