KTU STUDY MATERIALS

COMPREHENSIVE COURSE WORK

Chapter – 1

Finite Automata and Regular Sets

Level – 1 Questions

01. The class of regular sets are

(a) closed under finite union

(b) sometimes closed under infinite union

(c) closed under finite intersection

(d) all of the above

02. When and NFA is converted to an equivalent DFA accepting the same language, the number of states

(a) necessarily decreases

(b) necessarily increases

(c) always remains the same

(d) sometimes remains the same

03. Which of the following is most approximate in the case of Finite language

(a) It can be regular language

(b) It can be CFL

(c) It can be CSL

(d) It can be REL

04. Choose the correct answer

(a) The class of finite automata can be put into a one to one correspondence with the integers.

(b) The class of finite automata can be put into a one to one correspondence with the formal languages.

(c) The class of push down automata is larger than the class of finite automata and can be put into a one to one correspondence.

(d) None of the above

05. The minimal finite automata accepting the set of all strings in $(0+1)^*$ that has the last two symbols same has

(a) 40 states (b) 5 states

(c) 1000 states (d) 1000000 states

06. The set of all strings over {0, 1} where the 10^{th} symbol from the right end is a 1 and the 10^{th} symbol from the left end is a 0 and the 20^{th} symbol from the right end is a 1 is best described as

(a) regular set (b) CFL

(c) CSL (d) R.E. set

07. The minimal DFA that acts as a mod 3 counter and outputs a 1 whenever the input in binary treated as binary integer is divisible by 3 has

(a) 3 states (b) 4 states

(c) 5 states (d) none of the above

08. The minimal finite automata accepting the set of all strings over {0, 1} where the fourth symbol from the right end is a 1 has

(a) 4 states (b) 8 states

(c) 10 states (d) 16 states

09. The regular expression denoting the set of all strings not containing two consecutive 1's given by

(a) $(0+10)^* (\varepsilon +0)$ (b) $(0+10)^* (\varepsilon +1)$

(c) $(1+01)^*$ (d) $(\varepsilon + 0)(101)^* (\varepsilon+0)$

10. Let L = {Set of all binary strings whose integer equivalent is divisible by 4}. Then the number of states in Minimal FA accepting 'L' is

(a) 2 (b) 3

(c) 4 (d) 5

11. The regular expression with will strings 0' s and 1's with at lest two consecutive 0's is

(a) $(1+10)^*$ (b) $(0+1)^*00\ (0+1)$

(c) $(0+1)^*\ 011$ (d) $0^*1^*2^*$

12. Give the regular expression that derives all strings of a's and b's where each string contain even occurrences of substring ab.

(a) $(abab)^*$ (b) $(b + abab)^*$

(c) $(b + aa^*\ bb^*\ aa^*\ b)^*$ (d) $(b.aa^*bb^*aa^*b)^*$

13. Which of the following strings will the transition diagram given below recognize?

(a) Ending with 10 b) Not ending with 10

(c) Not ending with 01 d) None

14. The regular expression denoting the set of all binary strings not containing 000 as a substring is

(a) $(0+01+001)^* (\varepsilon + 0 + 00)$

(b) $(\varepsilon + 0 + 00)$ $(1+10+100)^*$

(c) $(0+1)^* 000(0+1)^*$

(d) none of the above

15. The minimum state automaton to recognize the set denoted by $(a+b)^*$ abb has

(a) 3 states b) 4 states

(c) 6 states (d) none of the above

16. What is the language expressed by the regular expression $(0^*011^*100^{*})^* \cup (1^*100^*011^{*})^*$

(a) The set of all strings, which contain equal number of 01's and 10's

(b) The set of all strings, which contain 01 or 10 as substring.

(c) The set of all strings that have same number of disjoint appearances of 10 and 01

(d) None of the above

17. Let Σ = {a, b}. Which of the following regular expression define the language :

L = { $w \in \Sigma^*$/w has atleast one substring ab}

(a) b*a*abb*a*

(b) $(a \cup b)^* (ab)^* (b \cup a)^*$

(c) (a*b*)*ab(a*b*)*

(d) $b^*a^*(ab \cup \phi^*)b^*a^*$

18. The minimal finite automata accepting the set denoted by (0+1)* (00+11) has

(a) 3 states b) 4 states

(c) 5 states d) 6 states

19. The regular expression

(aa)* + a(aa)* + aaaaa*a*

is the same as

(a) (a+aa+aaa)*

(b) aaa* + aaaaa* + aaaaaa*

(c) (aaa)*a****(a*+aa*)a+

(d) none of the above

20. The complement of the language

L = {$a^n b^{n/} n \neq 100$} is _____

(a) Regular b) context free

(b) Context sensitive d) None

21. If P is a prime Number and a is positive integer such that a^{p-1} = r (mod p), then r is

(a) a (b) 1/a

(c) 1 (d) a^2

22. The set of all strings over {0, 1} starting with 00 and ending in 11 is

(a) 0011 (b) 00(0+1)*+11

(c) (00)*(11)* (d) 0*1*

23. The language {ww^R| w is in (a+b)*} is accepted by

a) DFA b) NFA

c) 2DFA d) multi tape, multithreaded turing machine

Level – 2 Questions

01. Choose the true statement :

(a) The minimal finite automata accepting the set of all strings over {0, 1} containing an even number of 0's and an even number of 1' s has 4 states.

(b) The minimal finite automata accepting the set of all strings not containing three consecutive 0's has 5 states.

(c) The minimal finite automata accepting the set of all strings in (0+1)* that start with a 0 or start with a 1 has 3 states.

(d) The minimal finite automata is not always unique.

02. Choose the true statement :

(a) The minimal finite automata accepting the set of all strings over {0, 1} where the 1000 symbol from the right end is a 0 has over a 10^{100} no. of states.

(b) The minimal finite automata accepting the set of all even length strings in (0+1+2+3)* has 4 states.

(c) The minimal finite automata accepting the set of all odd numbered strings over a finite alphabet which has between 100 to 200 symbols has 5 states.

(d) The minimal finite automata accepting the empty set is not unique.

03. The Mealy machine M which recognizes all strings over {a, b} where the last two symbols are the same

(a) no equivalent moore machine

(b) has a moore machine equivalent of 1000 states (minimal)

(c) such a Mealy machine does not exist

(d) has an equivalent Moore machine

04. Which of the following statements is false?

(a) the finite automaton in which the output is associated with state only is called a Moore machine.

(b) the finite automaton in which the output is associated with state and input is called a Mealy machine.

(c) the finite automaton in which the movement of read head is in either direction (left or right) is called a two way finite automaton

(d) None of the above

06. When an NFA is converted to an equivalent DFA, the number of states

(a) is the same

(b) is always more

(c) is always exponential to the number of states of the NFA

(d) is bounded by 2^n, where n is the number of states of the DFA

06. Choose the true answer.

(a) Every formal language can be described using a one symbol alphabet.

(b) Every formal language can be described using a two symbol alphabet.

(c) Every formal language can be described without using ε.

(d) Every formal language does not have a finite description.

07. Consider the set of all strings over {0, 1} not containing two consecutive 0's. The regular expression for the same is

(a) $(1+01)^*(\varepsilon+0)$

(b) $(1+01)^*(\varepsilon+0)(1+01)$

(c) $(1+01)^*(\varepsilon+0)\ (1+01)^*(\varepsilon+0)$

(d) $(1+01)^*(\varepsilon+0)\ (1+01)^*(\varepsilon+0)(1+01)^*(\varepsilon+0)$

08. The set of all strings over {0,1} where every block of 7 consecutive symbols contains at least 2 0's and 3 1's is

(a) a finite set.

(b) a regular set that is not finite

(c) a context free language that is not finite

(d) a.r.e. set that is not finite.

09. The minimal finite automata accepting the set of all strings over {0,1,2} that interpreted as a base 3 number is congruent to 0 modulo 7 and also congruent to 0 modulo 5 has

(a) 35 states

(b) 12 states

(c) 36 states

(d) none of the above

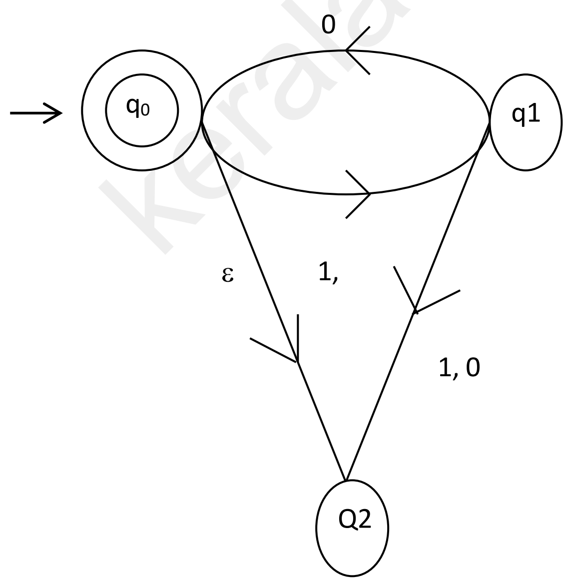
10. The minimal finite automata accepting the set of all strings over $(0 + 1)^*$ that do not have the sub string 0001 has

(a) 5 states b) 6 states

c) 7 states d) none of these

Common Data for Q11 & Q12 is given below.

Consider the transition Diagram.



11. Language accepted by FA is

(a) $L = \{(101)^n : n \geq 0\}$

(b) $L = \{(10)^n : n \geq 0\}$

(c) L = {10^n : n $\geq$ 0}

(d) L = {(1^n0) : n $\geq$ 0}

12. By referring to the above result, what happens when this automaton is presented with the string 111?

(a) It will be accepted

(b) It will be rejected by moving towards q1

(c) It will be rejected by moving towards dead state

(d) Both (b) & (c)

13. The minimal finite automata accepting the set of all strings over {0,1} starting with a 1 that interpreted as the binary representation of am integer are congruent to zero modulo 29 has

(a) 25 states (b) 26 states

(c) 29 states (d) 31 states

14. One of the following regular expressions is not the same as others. Which one?

(a) (a* + b*a*)*

(b) (a*b* + b*a*)* (a*b*)*

(c) ((ab)* + a *)*

(d) (a+b)*a*b*a*b*

15. The language {ww|w in (0+1)* and $|w|$<100000} is

A. a finite set B. regular set

C. CFL D. recursive set

(a) D>C>B>A (b) A>B>C>D

(c) B>D>C>A (d) C>B>A>D

16. Choose the non regular set

(a) $L = \{a^i b^j c^k \mid i < j < k < 10^{100}\}$

(b) $L = \{a^i b^j c^k \mid i < j < k < 10^{100}\}$

(c) $L = \{a^i b^j | j = I * I \text{ and } j < 10^{100}\}$

(d) $L = \{0^m 1^n 0^{m+n'} | m,n >= 1 \text{ and } m, n < 10^{100}\}$

17. Consider the languages $L_1 L_2$ and L_3 as given below.

$L_1 =$ $\{0^p 1^q \mid p, q \in N\}$

$L_2 =$ $\{0^p 1^q \mid p, q \in N \text{ and } p = q\}$ and

$L_3 =$ $\{0^p 1^q 0^r \mid p,q,r \in N \text{ and } p=q=r\}$.

Which of the following statements is not

TRUE?

(a) Pushdown automata (PDA) can be used to recognize L_1 and L_2

(b) L_1 is a regular language

(c) All the three languages are context free

(d) Turing machines can be used to recognize all the languages

18. Consider the languages

L_1 : $\{www| \ |w| < 3000, w \in (a+b)^*\}$

L_2 : $\{a^n b^m \ |m >= 2000*n\}$

Choose the correct statement

(a) L_1 and L_2 are both regular

(b) L_1 is regular and L_2 is context free but nor regular

(c) L_2 is regular and L_1 is context free

(d) None of the above

19. Consider the regular expression identities

i. $r^* + s^* + t^* = (r + s + t)^*$

ii. $(rrss)^*rr = rr\,(ssrr)^*$

iii. $(r + s + t)^* = (r^*s^*t^* + \varepsilon)^*$

(a) all are valid

(b) (i) and (ii) are valid but (iii) is not

(c) (ii) and (iii) are valid but (i) is not

(d) (i) and (iii) are valid but (ii) is not

20. The minimal DFA accepting the set $(a+\varepsilon^*)\,(a+\varepsilon)$ has ____ states

(a) 1 (b) 2

(c) 3 (d) 4

21. The minimal finite automata accepting the set of all strings over {0, 1} where the number of 0's is divisible by three hundred and the number of 1's is divisible by two hundred has, choose the true statement

(a) 3000 states is the best case

(b) 40000 states or more

(c) 60000 states in the minimal machine

(d) 8000 states only in the minimal machine

22. Choose the correct statement

(a) The set of all formal languages is countable

(b) The set of all languages is not countable

(c) The regular sets are not countably infinite.

(d) The family of all finite automata is not countably infinite.

23. The transition function for the transition graph is

(a) $\varsigma : Q \times \Sigma^* \longrightarrow Q$

(b) $\varsigma : Q \times \Sigma^* \longrightarrow Q$

(c) $\varsigma : Q \times \Sigma^* \longrightarrow 2^{Q \times \Sigma}$

(d) All of the above

Solutions

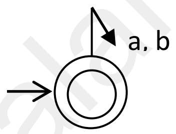
Level – 1

01. Ans : (d)

Sol : The operation of finite union & finite intersection preserve the regular sets. The set $\{\varepsilon\} \cup \{aa\} \cup \{aaa\} \ldots\ldots = a^*$ is the example of the intersection and union of regular sets being regular.

02. Ans : (d)

Sol : consider the NFA



03. Ans : (a)

Sol : All finite languages are regular.

04. Ans : (a)

Sol : The finite automata can be recursively enumerated.

05. Ans : (b)

Sol : It has 5 states

06. Ans : (d)

Sol : The set can be described by a regular expression.

07. **Ans : (a)**

Sol : A mod 3 counter needs 3 states in the minimal DFA.

08. **Ans : (d)**

Sol : This is the example of a DFA where the NFA to DFA requires an exponential number of states.

09. **Ans : (b)**

Sol : (a), (c) & (d) contain ε. Which is a negative string that does not contain two consecutive 1's.

10. **Ans : (b)**

Sol : The decimal 4 of L needs three bits to represent it. So a 3 state modulo machine is needed.

11. **Ans : (b)**

Sol : ………….00………….. :2 0's are needed

So (0+1)* 00 (0+1)*

12. **Ans : (c)**

Sol : we need

[(a+b)*a (a+b)* b(a+b)* a(a+b)* b (a+b)*

13. **Ans : (b)**

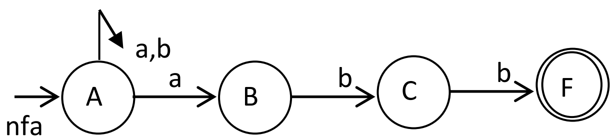
Sol : 10 is not recognized & 01 is recognized

14. **Ans (a) & (b)**

Sol : After ε, 0, or 00 after 1 appears. (c) contains 000.

15. **Ans (b)**

Sol :



16. **Ans (c)**

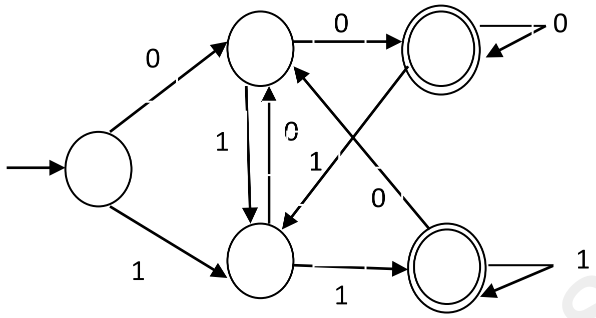
Sol : Use the method of elimination with the strings 0 & 1

17. **Ans : All of the choices**

Sol : The sub string ab is in all the choices (a) – (d)

18. **Ans (c)**

Sol :



19. **Ans : (a)**

Sol : (a) is a* which is the given R.E. (b) does not allow a & (c) does not allow a.

Sol : Use the method of elimination with the strings 0 & 1

20. **Ans : (c)**

Sol : The complement is a DCFL that is not regular. DCFLs are closed under complement.

Sol : Use the method of elimination with the strings 0 & 1

21. **Ans : (c)**

Sol : This is a statement of Fermat's Theorem

22. **Ans : (b)**

Sol : 00 _ _ start with 00

_ _ 11 end with 11

00 (0+1)* 11 start with 00 & end with 11

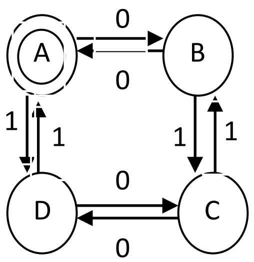
23. Ans : (d)

Sol : The language of palindrome is not regular or a DCFL. It is a CFL & hence R.E. So (d) is the answer.

Level – 2

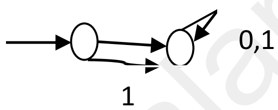
01. Ans : (a)

Sol : a)



(b) is false as a minimal FA is unique up to a homomophism.

(c) 0 (0+1)* + 1 (0+1)*



Is false

02. Ans : (a)

Sol : The DFA has 2^{1000} states & hence has more than a 10^{1000} no. of states.

03. Ans : (d)

Sol : Every moore machine has an equivalent mealy m/c & vice versa.

04. Ans : (d)

Sol : These are the standard definition of the Moore & Mealy machines & the two way finite automata

05. Ans : (d)

Sol : If NFA contains n states, then DFA contains atmost 2^n states.

06. Ans : (c)

Sol : ε is just symbol so some other symbol like λ can be used for the empty string.

07. Ans : (a)

Sol : After one or no 0's, a '1' compulsorily occurs.

08. Ans : (b)

Sol : Set = $(00)^+ + (111)^+$ + other

So it is infinite. It can be expressed as a regular expression so it is regular.

09. Ans : (d)

Sol : The answer is a modulo 7 m/c multiplied by a modulo 5 m/c.

10. Ans : (a)

Sol : FA has 5 states

11. Ans : (b)

Sol : Evidently $(1\ 0)^*$ is accepted by the FA.

12. Ans : (c)

Sol : 111 will go to dead state

13. Ans : (d)

Sol : A modulo 29 machine has 31 states as it start with 1.

14. Ans : (c)

Sol : (a), (b) & (d) are $(a+b)^*$

Where (c) does not contain 'b'

15. **Ans : (a)**

Sol : The given set is finite. Every finite set is regular. Every regular set is DCFL. Every DCFL is a CFL. Every CFL is DSL. Every CSL is recursive. Every recursive set is R.E.

16. **Ans : (b)**

Sol : By the Pumping Lemma for regular sets, (b) is not regular. The remaining choice are all finite sets.

17. **Ans : (d)**

Sol : L_1 requires no stack

L_2 requires one stack

L_3 requires 2 stack

So L_1 is Regular language, L_2 is CFL, and

L_3 is CSL and every RL, CFL, CSL are RES, accepted by TM

18. **Ans : (b)**

Sol : L_1 is a finite set & hence is regular. L_2 is a CFL that is not regular.

19. **Ans : (c)**

Sol : $(ra)^* r = r(ar)^*$

So (ii) is true

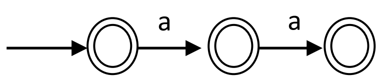
$(r + s + t)^* \quad = (r^*s^*t^* + \varepsilon)^*$

$= (r^*s^*t^*)^*$

$= (r + s + t)^*$

20. **Ans : 3**

Sol : The minimal DFA is



21. **Ans : (c)**

Sol : M_1 0' need 300 states

M_2 1' need 200 states

$M_3 = M_1 \times M_2 = 60000$ states

22. **Ans : (b)**

Sol : Set of all formal languages is uncountable, since there is no one – to – one relation between set of formal languages and N.

23. **Ans : (c)**

Sol : The Question deals with the conversion of a transition diagram having ε-moves. The non-determinism requires all possibilities to be considered.

Chapter – 2

Context – free Languages & Push down Automata

Level – 1 Questions

01. The intersection of two CFL's

(a) is always a CFL (b) may be a CFL

(c) is never a CFL (d) is always a CSL

02. Which of the following statements is true?

(a) if a language is context free it can always be accepted by a deterministic push down automaton

(b) The union of two CFLs is a CFL

(c) The context free languages are closed under intersection

(d) The complement of a CFL is always a CSL

03. Consider the following two languages

$L_1 = \{1^n 0^n 1^n 0 / n \geq 0\}$

$L_2 = \{a^n b^k / n \leq k \leq 2n\}$

Which of the following statement is true.

(a) both L_1 and L_2 are context free

(b) L_1 is context free but not L_2

(c) L_2 is context free but not L_1

(d) neither L_1 nor L_2 is context free

04. The language generated by the following context free grammar is,

$S \rightarrow AB$, $A \rightarrow a A/a$, $B \rightarrow bBc/\varepsilon$

(a) $\{a^m b^n c^n / m \geq 0, n \geq 0\}$

(b) $\{a^n b^m c^n / m \geq 0, n \geq 0\}$

(c) $\{a^n b^n c^m / m \geq 0, n \geq 0\}$

(d) None of the above

05. The recursive sets, CFL_s and R.E. sets are closed under

(a) Union and complement

(b) Intersection and complement

(c) Union and intersection

(d) Union

06. CFL is not closed under

(a) Intersection

(b) complement

(c) difference

(d) all the above

07. Which of the following language cannot be accepted by any deterministic push down automaton

(a) The set of all strings over {a, b} consisting of equal number of a's and b's.

(b) A language of palindromes

$\{x \in \{a, b\}^* / x = rev(x)\}$

(c) $\{wcw^R / w \in \{a, b\}^*\}$

(d) all of the above

08. The language which is accepted by LBA is called as

(a) Regular

(b) Context free

(c) Context dependant

(d) None of the above

09. Choose the correct statement

(a) The nondeterministic & deterministic finite automata are not equivalent.

(b) The nondeterministic & deterministic PDA are not equivalent.

(c) The nondeterministic & deterministic TMs are not equivalent.

(d) The nondeterministic & deterministic halting TMs are not equivalent.

10. In the case of non – determinism choose the incorrect statement

(a) Deterministic and nondeterministic finite automata are equivalent in power.

(b) Deterministic and nondeterministic push down automata are equivalent in power.

(c) Deterministic and non deterministic two push down tape machines are equivalent in power.

(d) Deterministic and nondeterministic three counter machines are equivalent in power.

11. The language $0^* \cup \{0^n1^n/n\geq1\} \cup \{0^n1^{2n}/n\geq1\}$ is accepted by

(a) 2 DFA

(b) DPDA

(c) PDA

(d) LBA but not by a PDA

12. Consider a PDA M accepting a CFL L. Choose the correct statement.

(a) A PDA can never loop on its input and makes as many moves as the size of the input.

(b) In a PDA the maximum size of the stack for an input of size n, is a polynomial p (n).

(c) In a PDA the maximum size of the stack for an input of size n, is an exponential of $O(2^n)$

(d) A PDA can be defined in two equivalent models having acceptance by final state or acceptance by empty store.

13. Consider the CFG G generating the CFL $L = \{ww^R \mid w \in \{a, b\}^*\}$. Choose the correct statement.

(a) As L is a CFL, g must be ambiguous.

(b) L is CFL that is also a DCFL.

(c) If L is defined over a single alphabet {a} then L is accepted by non deterministic finite automata.

(d) the exists an LR (k) that generates L.

14. Let L be a language over the alphabet {a} & be generated by some CFG G. Choose the correct statement.

(a) L is a CSL that is not a CFL

(b) L is a CFL that is not a DCFL

(c) L is a DCFL that is not regular

(d) L can be denoted by some regular expression

15. Consider the language $L = \{ww \mid w \in \Sigma^*\}$.

(a) L is a CSL that is not a CFL as down by the pumping Lemma for CFLs.

(b) L is a CSL as shown by the pumping Lemma for CFLs.

(c) L does not model the definition before use requirements in some programming languages.

(d) L can be described by extended regular expression which arranges the standard definition of regular expression with the intersection & complement operators.

16. Consider the language

$L = \{a^i b^j a^i b^j \mid i,j \geq 1\}$.

Choose the correct statement.

(a) L can be accepted by some PDA

(b) L can be described by the intersection of regular expression.

(c) L models the requirement that the number of parameters in procedure definition & use be the same.

(d) The pumping Lemma for regular sets shows that L is a CSL.

17. Consider the languages

i. $L_1 = \{a^n b^{2n} | n \geq 1\}$

ii. $L_2 = \{a^n b^{2n} | n \geq 1\}$

Choose the false statement

(a) L_1 is a DCFL

(b) L_2 is a DCFL

(c) $L_1 \cup L_2$ is accepted by some PDA

(d) $L_1 \cup L_2$ is a DCFL

18. Consider the languages

$L_1 = \{a^n b^n c^i | n, I \geq 1\}$ &

$L_2 = \{a^m b^j c^j | m, I \geq 1\}$ &

Choose the false statement.

(a) L_1 & L_2 are CFLs

(b) L_1 may have an ambiguous CSG generating it

(c) L_2 may have and ambiguous CFG generating it.

(d) $L_1 \cup L_2$ is not inherently ambiguous.

19. Define a linear grammar G having rules of the form

$A \rightarrow a, \quad A \rightarrow aB$

$A \rightarrow Ba, \quad A \rightarrow B,$

$A \rightarrow ab.$

Where A, B are non-terminals. & a, b are terminals, Let G generates a CFL L.

Choose the correct statement

(a) Every CFL does not have a linear grammar.

(b) Every regular set does not have a linear grammar.

(c) A linear grammar does not describe all the languages that can be denoted by regular expressions.

(d) A linear grammar can generate the set $\{a^n b c^n \mid n \geq 1\}$

Level – 2 Questions

01. The language $\{ww^R\ w\ w\ w\ w\ w\ w\ w^R\ w\ w\ w\ w\ w^R \mid w \text{ in } (a+b)^*\}$ is accepted by

(a) PDA

(b) NFA

(C) 2DFA

(d) multitape, multiheaded turing machine

02. **Statement (I) :** The language L = set of all strings not containing 101 as a substring is regular set.

Statement (II) : L satisfies the pumping Lemma for regular sets.

(a) I is true, II is false
(b) I is false II is true
(c) I & II are true
(d) I & II are false

03. A is given a CFG, G_1 that generates language F is not a DCFL L_1 & a LR(k) grammar G_2 that generates a languages $L_2\$$. Choose the correct statement

(a) For every G_1 there exists a grammar G_2.

(b) Every G_1 & G_2 are unambiguous.

(c) L_1 L_2 is always a DCFL.

(d) L_2 is always a DCFL if $ is a symbol not in the vocabulary

04. A DPDA's that is accepted by empty store. Choose the correct statement.

(a) For every DCFL, there exists a DPDA that accepted by empty store.

(b) For every DCFL that satisfies the prefix property, there does not exist, a DPDA that is accepted by empty store.

(c) If L is a DCFL then L$ can be generated by some LR(k) grammar.

(d) Every DCFL can be defined to be accepted by an empty store or final sate.

05. Consider the machines

M_1 = a NPDA accepting L_1

M_2 = a 2 way PDA accepting L_2

M_3 = a PDA with two stacks accepting L_3.

Choose the false statement

(a) For any CFL L, there exists some M_1, M_2, & M_3.
(b) For some CSL L, there exits some M_1, M_2, & M_3.
(c) For every recursive set, there exits some M_2, & M_3.
(d) The power of the machines are not $M_1 \leq M_2 < M_3$.

06. Consider the properties of a context – free grammar G in the chomsky normal from (CNF), the Greibach Normal From (GNF) & an operator grammar (OG). Choose the false statement.

(a) In CNF, the r.h.s of a rule is either a terminal or two adjacent non – terminals.

(b) In GNF, the r.h.s of a rule is either a terminal or a terminal followed by a string of non – terminals

(c) In OG, the r.h.s of every rule does not have two adjacent non – terminals.

(d) No grammar cannot be in CNF, GNF & OG at the same time

07. We are given a type 0 grammar G_0, a type 1 grammar G_0, a type 1 grammar G_1, a type 2 grammar G_2, & a type 3 grammar G_3, generating the formal language L_0, L_1, L_2 & L_3a respectively.

Choose the true statement,

(a) G_0 cannot generate all possible L_1

(b) G_1 cannot generate all possible L_2

(c) G_2 cannot generate all possible L_3

(d) G_3 cannot generate all the finite sets

08. We are given a PDA M_1 accepting L_1 & a PDA M_2 accepting L_2.

Choose the false statement.

(a) We can construct a PDA M_3 that accepts all the strings M_1 & M_2 accept.

(b) We cannot construct a PDA that accepts some strings accepted by both M_1 & M_2 .

(c) We cannot construct a PDA that accepts string $\overline{L}_1$. $\overline{L}_2$.

(d) We can construct a PDA that accepts all the strings not in L_1.

09. Consider the languages

i. $L_1 = \{a^n b^{2n} \mid n \geq 1\}$

ii. $L_2 = \{ca^n b^n \mid n \geq 1\}$

Choose the false statement.

(a) L_1 is accepted by some DPDA

(b) L_2 is accepted by some DPDA

(c) $L_1 \cup L_2$ is not accepted by any DPDA

(d) $L_1 \cup L_2$ is not accepted by some PDA

10. Consider the languages

i. $L_1 = \{ww^R \mid w \in \Sigma^*\}$

ii. $L_2 = \{w \not\subset w^R | w \in \Sigma^*\}$

iii. $L_3 = \{wxw^R | w,x \in \Sigma^*\}$

When $\Sigma = \{a, b\}$.

Choose the false statement.

(a) L_1 cannot be accepted by any DPDA & needs a PDA or LBA to accept it.

(b) L_2 can be accepted by a DPDA but not a finite automata.

(c) L_3 is a sample regular language $\{a, b\}+ \{a, b\} \{a, b\}+$ & can be accepted by some finite automata.

(d) All L_1, L_2 & L_3 are inherently ambiguous languages i.e. any CFG constructed for them will be necessarily ambiguous.

11. We are given the regular sets $R_1 = \varepsilon^*$ & R_2 = set of all strings over {a, b} where the 11^{th} symbol form the right end is a. We are also given an arbitrary CFL L_1 & an arbitrary DCFL L_2.

Choose the false statements.

(a) $L_1 = R_1$ or $L_1 = R_2$ are both undecidable

(b) $L_2 = R_1$ or $L_2 = R_2$ are both decidable

(c) $L_1 \cap L_2 \cap R_1$ has an emptiness problem that is undecidable.

(d) $R_1 = R_2$ is undecidable

12. Consider the following machine

(i) A PDA M_1 where the stack size cannot be more than a function f of the input size n.

(ii) A DPDA M_2 where the stack size bounded by some prime number.

(iii) A ODA M_3 where the maximum stack size should be m! for some integer m.

Choose the false statement

(a) M_1 accepts only regular sets

(b) M_2 accepts only regular sets

(c) M_3 can accepts CFL_s

(d) None of the above

13. Consider the language $L_1 = \{a^n b^{2n} \mid n \geq 1\}$, the homomorphism h([p) = a, h (q) = aa, Let $L_2 = h^{-1} (L_1)$ & $R = p^* q^*$.

Choose the false statement

(a) $L_1 \cap R = \{a^m b^m \mid m \geq 1 \}$

(b) L_2 & L_1 are both CFLs that are not regular.

(c) L_2 is not a CFL but is a CSL.

(d) None of the above.

Solutions

Level -1

01. Ans : (b) & (d)

Sol : 1. $a^* \cap b^* = \phi$ a CFL

2. The CFLs are not closed under intersection.

02. Ans : (b) & (d)

Sol : The DCFLs are a smaller class than the CFLs. The CFLs are closed under union but not under intersection. Every CFL is a CSL. Complement of CFL is recursive

03. Ans : (a)

Sol : (b) L_1 is a standard CSL

(c) L_2 is CFL

04. Ans : (d)

Sol : $S \rightarrow AB$

$A \xrightarrow{*} a^{+}s$

$B \xrightarrow{*} \{b^n c^n / n \geq 0\}$

Note that ε is not generated by the grammar

05. Ans : (d)

Sol : The CFLs are not closed under intersection or complement.

06. Ans : (d)

Sol : A CFL is not closed under intersection, or complement & hence difference

07. Ans : (b)

Sol : The language L= $\{ww^R / w \in \Sigma^*)$ is a CFL but not a DCFL.

08. Ans : (c)

09. Ans : (b)

Sol : The PDA are not the same for non determinism.

10. Ans : (b)

Sol : The DPDA & PDA are not equivalent. Non-determinism does not add any power to the finite automata or the TM.

11. Ans : (c)

Sol : The language is a CFL but not a DCFL.

12. Ans : (d)

Sol : Any CFL can be accepted by a PDA that can be accepted by final state. This has an equivalent model of acceptance by final state.

For (a), (b), (c) we note that a PDA can make an infinite number of moves on ε input, & for each one of these moves, the store can grown in size. So in general the PDA needs an infinite push down store.

13. Ans : (c)

Sol : (c) if the vocabulary is {a} then L = (aa)* which is a regular set

(b) L is a standard CFL, the language of all palindromes is not a DCFL.

(d) G is unambiguous. G is a partially CFG. Ambiguity problem is for all CFGs.

(d) A L is not a DCFL, no LR (k) can generate it.

14. Ans : (d)

Sol : CFLs over a single alphabet are regular.

15. Ans : (a)

Sol : L is a standard CSL that is not a CFL (& hence not regular). This can be shown using the pumping Lemma for CFLs. Also it models the definition before use requirement in programming languages.

16. Ans : (c)

Sol : L is a CSL that is not a CFL. It cannot be shown to be a CFL by using pumping Lemma for CFLs. A LBA can be constructed by accepting L & so it is a CSL. L models the requirement of the same procedure to have the same number of parameters.

17. Ans : (d)

Sol : We can construct DPDA's to accept L_1 & L_2. We can construct a nondeterministic PDA to accept $L_1 \cup L_2$. However $L_1 \cup L_2$ will require non-determinism & cannot be a DCFL.

18. Ans : (d)

Sol: It can be shown that L_1 & L_2 are CFLs by constructing CFG's generating them. Ambiguity can be artificially added to them to make them generated by ambiguous CSG's.

It runs out that $L_1 \cup L_2$ is inherently ambiguous. As strings $\{a^n b^n c^n)$ will have two derivations.

19. Ans : (a)

Sol: The languages generated by linear grammars are a proper subset of the CFLs. The linear grammar S   aSc/b generates $\{a^n\, b\, c^n \mid n \geq 1\}$

All right linear grammars & all left linear grammars are trivially linear grammar. So all regular sets can be generated by linear grammars. So all regular expressions can be denoted by linear grammar.

Level – 1

01. Ans : (d)

Sol: The language is a CSL & hence a R.E. set accepted by a TM or its Vice versa.

02. Ans : (b)

Sol: satisfying the pumping Lemma give no conclusion.

$\therefore$ L is a regular set that satisfies the Pumping Lemma.

03. Ans : (d)

Sol : (a) not true as G_1 every CFL is not a DCFL. So every CFG L_1 does not have an equivalent LR(k) grammar G_1.

(b) LR(k) grammars are unambiguous, but G_1 being an arbitrary CFG it may be ambiguous.

(c) The concatenation of two CFLs is a CFL that need not be a DCFL.

(d) If L is a DCFL the L$ is LR(k) for some k.

04. Ans : (c)

Sol : The DPDA that is accepted by final state accepts DCFLs. If the prefix property is satisfied we can use DPDA's that is accepted by empty store.

(c) Standard theorem. If L is a DCFL then L$ can be generated by some LR (k) grammar.

05. Ans : (d)

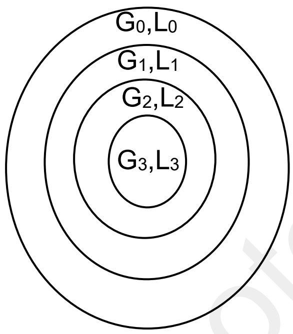
Sol. M_1 accepts all CFLs. M_2 accepts some CSLs. M_3 is a variation of the Turing machines.

06. Ans : (d)

Sol : Consider the grammar $S \rightarrow a$. It is in CNF, GNF & OG at the same time.

07. Ans : (d)

Sol :



08. Ans : (d)

Sol : (a) The union of two CFLs is a CFL.

(b) The intersection of two CFLs need not be a CFL.

(c) The complement of a CFL need not be a CFL & the CSLs are closed under concatenation.

(d) The CFLs are not closed under complement.

09. Ans : (c)

Sol: L_1 is a DCFL as a DPDA can be constructed to accept it. L_2 also can be accepted by an easily constructed DPDA. Since L_1 & L_2 are CFls. $L_1 \cup L_2$ is a CFL & can therefore be accepted by some PDA.

$L_1 \cup L_2$ is a DCFL as DPDA can use the presence of the first symbol 'c' to table the non determinism.

10. **Ans : (d)**

Sol : As (c) is a regular set it will have unambiguous left linear or right linear grammar generating it.

As L_2 is a DCFL it can be accepted by DPDA. L_1 is a standard CFL that is not a CSL.

11. **Ans : (d)**

Sol : It is decidable a DCFL L_2 is equivalent to a given regular set R. This is undecidable for DCFLs.

The intersection of two DCFLs can given all the valid computations of a turing machine & so is the same as the halting problem.

It is decidable if two regular sets are equivalent.

12. **Ans (c)**

Sol : For $M_1 \cup M_2$ the stack size is bounded. So the stack can be stored in the finite control. So $M_1 \cup M_2$ accept only regular sets.

For M_3 the stack is not bounded. At the most we have to add the stack with dummy symbols so that the stack is m! for some m. So this is the general definition of a PDA and accept all the CFLs.

So for M_3 also the stack size is bounded by m!. Hence the memory is finite and can be stored in finite. Hence M_2 accepts only regular set.

13. **Ans : (c)**

Sol: L is a CFL. The CFLs are closed under homomorphism & inverse homomorphism, so h^{-1} $(L_1) = L_2$ is a CFL. L_2 has p's & q's jumbled up intersection with R arrange the p's before the q's. As a's, (bb)'s are the same in L_1 they are the same number of p's and q's in any string in L_2.

Chapter – 3

Turing Machines, Modifications, CSLs, Recursive & R.E. sets

Level – 1 Questions

01. To evaluate expressions in C without function calls, which of the following is mandatory?

    (a) one stack is enough

    (b) two stacks are must

    (c) unlimited number of stacks are required

    (d) a turing machine is required and it must be non deterministic in the general case

02. Some how I proved that P = NP then which of the following statements is true?

    (a) NP is closed under complement

    (b) NP – CoNP (complement of NP)

    (c) Both a and b are true

    (d) None of the above is true

03. Recursive sets are closed under

    (a) Kleene closure

    (b) Substitution

    (c) Homomorphism

    (d) Inverse homomorphism

04. Consider the following languages

    L_1 = {<M> : L(M) contains a word of length < 1000}

L_2 = {<M> : M) accepts some word within 1000 steps of computation}

(R$\longrightarrow$ Recursive, RE$\longrightarrow$ recursively enumerable)

Which of the following statements is true?

(a) $L_1 \in R$ and $L_2 \in RE$
(b) $L_1 \in R$ and $L_2 \in RE$
(c) $L_1 \in RE$ and $L_2 \notin RE$
(d) $L_1 \in RE$ and $L_2 \in RE$

05. The language generated by the grammar $S \rightarrow aSa|bSb|d$ is accepted by a

A. DPDA B. PDA

C. LBA D. Turing machine

(a) D>C>B>A (b) A>B>C>D

(c) B>C>D>A (d) C>D>B>A

06. The language consisting of an equal number of a's and b's can be accepted by a

A. DPDA B. PDA

C. turing machine D. LBA

(a) C > D > B > A (b) A > B > C > D

(c) B > C > D > A (d) D > C > B > A

07. If a 2DFA we supply an unlimited amount of ink so that it can write on its input tape, it become a

(a) DPDA (b) PDA

(c) LBA (d) Turing machine

08. A finite set that is accepted by a finite automata is always

(a) type 0 but not type 1

(b) type 1 but not type 2

(c) type 2 but not type 3

(d) always type 3

09. Which of the following languages are closed under complement?

(a) CFL

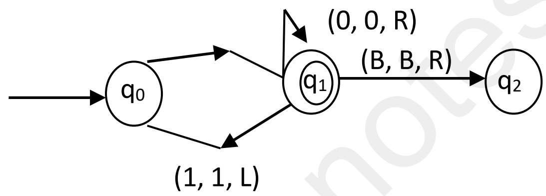
(b) CSL

(c) Recursive sets

(d) Recursively enumerated sets

Statement for linked answer Q 10 to Q 11 is given below :

Let M be the Turing Machine given below :



10. The languages accepted by M is

(a) $\{00^n | n \geq 0\}$

(b) $\{0^n10^n1 \mid n \geq 1\}$

(c) $\{0^n1\ 0^n1 \mid n \geq 1\}$

(d) $\{0^n10^n1 \mid n \geq 1\}$

11. The above language is

(a) Regular language (b) DCFL

(c) CFL (d) CSL

12. Choose the correct statement

(a) The recursive sets can be effectively enumerated

(b) The formal language can be effectively enumerated.

(c) All real numbers can be effectively enumerated

(d) The finite automata can be effectively enumerated

13. Choose the correct statement

(a) If any problem in P is closed under complement, then P = NP.

(b) If every problem in NP is closed under complement, then P = NP.

(c) If every problem in NP is NP – complete, then P = NP.

(d) If any problem in NP is not NP – complete, then P – NP.

14. Choose the undecidable problem.

(a) Equivalence of finite automata

(b) Equivalence of regular expression

(c) Emptiness problem of CSLs

(d) Emptiness problem of R.E sets

15. The following are not countable

(a) the subsets of all regular sets

(b) the subsets of all CFLs

(c) The subsets of all CSLs

(d) All of the above

16. The following problems are not decidable

(a) Whether a CFL is infinite or finite

(b) The membership problem of CSL's.

(c) The membership problem of recursive sets

(d) Whether a CFL consists of all strings over the terminal vocabulary.

17. The union of the classes of regular sets and context sensitive languages is

(a) not countably infinite

(b) countably infinite

(c) sometimes countably infinite

(d) none of the above

18. Let M_1 be a deterministic finite automata accepting a language L_1 & let M_2 be a non – deterministic finite automata accepting a language L_2.

Choose the correct statement :

(a) L_1 is equal to L_2

(b) It is decidable if $L_1 = L_2$

(c) There is no algorithm to decide if L_1 is empty

(d) The intersection of L_1 & L_2 is always a finite set

19. Let M_1 be a deterministic push down automata accepting a DCFL L_1. Let G_2 be an LR(k) grammar generating the language L_2. Let M_2 be a two way deterministic finite automata (2DFA) accepting the language L_3.

Choose the correct statement.

(a) There is no algorithm to decide if $L_1 \cap L_2$ is empty.

(b) $L_1 = L_2$

(c) $L_2 = L_3$

(d) The problem of whether $L_1 = L_2$ is undecidable

20. Let M1 be a push down automata accepting a language L_1.

Choose the correct statement :

(a) $L_1 = \phi$ is decidable

(b) $L_1 = \sum^*$ is decidable

(c) L_1 is a finite set is not decidable

(d) L_1 is an infinite set is not decidable

21. Given an arbitrary C program C_1 and a JAVA program, J_2. Choose the correct statement.

(a) C_1 may not be an algorithm but J_2 is always an algorithm.

(b) C_1 may not be an algorithm but J_2 may not be an algorithm.

(c) Both C_1 & J_2 are algorithms always.

(d) neither C_1 nor J_2 necessarily an algorithm.

22. Consider an arbitrary C program which has some assignment statement a. Choose the correct statement.

(a) It is decidable whether a is executed.

(b) It is undecidable whether a is live or dead always

(c) It is decidable if a is repeatedly executed in some iteration.

(d) None of the above

23. Choose the correct statement

(a) The graph coloring problem is undecidable.

(b) The register optimization problem in compilers can be reduced to graph coloring problem is undecidable.

(c) The graph coloring problem with less than 10 nodes is intractable.

(d) The graph coloring problem is decidable and seemingly intractable.

24. A is given a recursive set L accepted by some LBA M & a homomorphism h. Choose the correct statement.

(a) It may not be possible to decide if L = ϕ but it is always possible to decide if h(L) = ϕ

(b) It is undecidable if h (L) is a R.E. set

(c) The emptiness problem of h(L) is the same as the halting problem of turing machine

27. A considered the sets of Turing machines describing all the Hamiltornian cycle problems. Choose the correct statement.

(a) The membership problem of S is decidable

(b) It is decidable if the complement of S is empty

(c) It is decidable if S is empty, finite or infinite

(d) It is decidable if S = Σ^*

Level - 2 Questions

01. A single tape turning machine M has two states q_0 and q_1, of which q_0 is the starting state. The tape alphabet of M is {0, 1, B} and its input alphabet is {0, 1}. The symbol B is the blank symbol used to indicate end of an input string. The transition function of M is described in the following table.

	0	1	B
q_0	q_0, 1, R	q_0, 1, R	Q_1, B, R
q_1	q_0, 1, R	q_0, 0, R	Halt

Which of the following statements is true about M?

(a) M accepts all strings that end with 0 only

(b) M accepts all strings that end with 1 only

(c) M accepts all strings in $(0+1)^+$ only

(d) M accepts all strings in $(0+1)^*\ 01\ (0+1)^* + 1^*0^*$

02. Match the following

Grammar	Classification
P.$S \rightarrow aS \mid bS \mid a$, $S \rightarrow A$, $A \rightarrow a$	1.Type 3 and right linear
Q.$S \rightarrow aAb \mid ab$, $C \rightarrow c/cC$ $S \rightarrow AC/abc$	2. Type 2 and not generating a regular set
R.$S \rightarrow LWaR$ $Wa \rightarrow Waaaa$ $WR \rightarrow W1\ R$ $aW1 \rightarrow W1a$ $LW1 \rightarrow LW$ $WR \rightarrow W2R$ $aW2R \rightarrow W2Raaaa$ $LW2R \rightarrow \in$	3. Type 0 and not generating a CFL
S.$S \rightarrow aSBC \mid aBC$ $CB \rightarrow BC$ $aB \rightarrow ab$ $bB \rightarrow bb$ $bC \rightarrow bc$ $cC \rightarrow cc$	4. Type 1 and not generating a CFL
	5. LR (k)
	6. Type 3 and left linear

Codes :

(a) P-1. Q=5. R-3, S-4

(b) P=6, Q-4, R-3, S-5

(c) P-1, Q-3, R-6, S-2

(d) P-1, Q-2, R-3, S-6

03. Choose the true statement.

(a) The regular sets, R.E. sets, CSLs, DCFLs and CFLs are closed under intersection

(b) The regular sets, recursive sets, DCFLs and CFLs are preserved under the operation of complement

(c) The regular sets are closed under intersection but the CFLs are not

(d) The CFLs are closed under complement but the regular sets and R.E. sets are not

Statement for Linked answer Q.04 and Q.05 is given below.

Ram obtains a turing machine that can accept the empty set L_n. Shyam obtains a turing machine that can accept the set of all strings over the input alphabet L_m. Thomas studies L_n to classify it in the Chomsky hierarchy. Rahim construct a turing machine to accept L_1 the complement of L_n.

04. Choose the correct statement :

(a) Thomas will not be able to determine if L_n is a regular set.

(b) Thomas will not be able to determine all strings in L_n as L_n is a recursive set.

(c) Shyam will not be able to determine if L_m is the same as L_n as the equivalence of turing machines is undecidable.

(d) Shyam will not be able to determine if L_m is a subset of L_n as the equivalence problem of turing machines and the containment problem is undecidable.

05. Choose the correct statement :

(a) Rahim will not be able to determine L_1 as L_1 is not R.E.

(b) Rahim will not be able to determine L_1 as L_1 is recursive.

(c) Rahim will not be able to determine L_1 as L_1 is R.E.

(d) Rahim will not be able to determine L_1 as L_n is a finite.

06. Consider the complexity classes P, NP, PSPACE, EXPTIME, then which of the following relationship is true?

(a) $P \subset NP \subset PSPACE \subset EXPTIME$

(b) $P \subset NP \subset EXPTIME \subset PSPCE$

(c) $P \subset PSPACE \subset NP \subset EXPTIME$

(d) $NP \subset EXPTIME \subset P \subset PSPACE$

07. Which of the following statement is false

(a) A Turing machine is more powerful than a finite state machine because it has halt state.

(b) A finite state machine can be assumed to be a Turing machine of finite tape length, with rewinding capability and unidirectional tape movement.

(c) both (a) and (b)

(d) none of the above

08. Choose the false statement

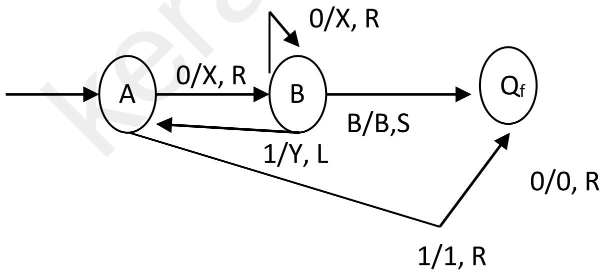
(a) Every CSL can be accepted by a finite automata if we allow an unbounded number of states.

(b) The complement of the language $L = \{ww | w \in \{a, b\}^*\}$ is a CFL.

(c) The complement of the language $L = (a^n \; b^n c^n | \; n \geq 1\}$ is a CFL

(d) Every regular expression does not denote a regular set

Common Data for Q09 and Q10 is given below.



09. The above TM accepts

(a) 0^* b) 1^*

(c) $(0+1)^+$ (d) $(0+1)^*$

10. The language accepted by the above TM is

(a) finite

(b) recursive set

(c) CSL but not CFL

(d) CFL but not regular

Common Data for Q11 & Q12 is given below.

Consider a language L_1. L_1 = {0, 1)* if another language L_2 is a CFL.

L_1 = empty set if another language L_2 is not a CFL.

L_2 = a language that is R.E. and its complement is R.E.

11. L_1 is

(a) regular and not finite

(b) CFL and not regular

(c) CSL and not a CFL

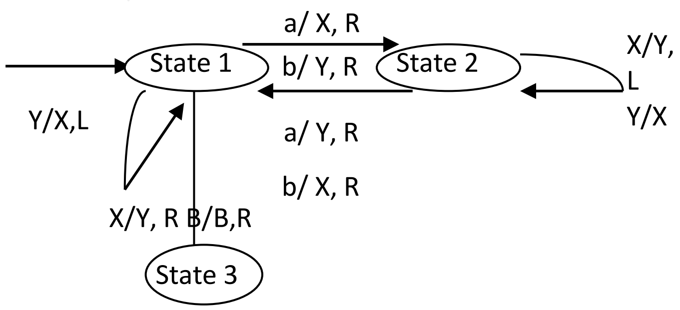
(d) r.e and not recursive

12. L_2 is

(a) empty (b) regular

(c) CFL (d) r.e

13. Consider the following Turing machine M :



Assume that initially tape contains the strings over the alphabet {a, b} and can end of the string is identified with the help of continuous blank symbols after the given string.

The above Turing machine M accepts

(a) All strings of a's and b's including ε.

(b) All strings of a's and b's including ε.

(c) All odd length strings over {a, b}.

(d) All even length strings over {a, b}.

14. Choose the correct statement in relation to a given instance P of a post corresponding problem.

(a) P can be simulated by the intersection of two deterministic push down automata.

(b) P can be simulated by the intersection of two regular sets.

(c) P cannot be reduced to the emptiness problem of context sensitive languages (CSLs)

(d) P can be simulated by the intersection of two languages L_1 & L_2, where L_1 is generated by an LR(k) grammar & L_2 is a language for which a description as a regular expression R exists.

15. Consider a language L_1 generated by a CSG G_1 & a language L_2 accepted by a non-deterministic LBA M_2. Choose the correct statement.

(a) It is undecidable whether L_1 is empty, finite, infinite of the same as Σ^*.

(b) It is undecidable if a given strings w is generated by G_1 or accepted by G_1 or accepted by M_2.

(c) It is decidable if $L_1 = L_2$ or $L_1 \leq L_2$.

(d) It is decicable if L_1 or L_2 is accepted by some two way nondeterministic finite automata (2NFA)M_3.

16. Consider a language L_1 accepted by deterministic Turing Machine M_1 & a language L_2 accepted by a two push down tape machine M_2. Chose the correct statement.

(a) The membership problem of L_2 & $L_1 \cap L_2$ is decidable.

(b) The equivalence problem of L_1 & $L_{2,}$ viz. L_1 viz. $L_1 = L_2$ can be resolved by some algorithm.

(c) It is possible that $L_1 = L_2 = \Sigma^{*.}$

(d) It is decidable if L_1 & L_1 are regular sets.

17. Consider a CFG C1 generating syntactically correct C programs & a context free grammar J_1 generating syntactically correct JAVA programs.

Choose the correct statement.

(a) It is decidable if C_1 & J_1 decidable all and only algorithms

(b) It is undecidable if C_1 is ambiguous

(c) It is decidable if J_1 is ambiguous

(d) Both C_1 & J_1 have equivalent LR(k) grammars.

18. A takes two arbitrary DPDA M_1 & $M_{2.}$ He wants to determine if M_1 & M_2 accept some strings in common.

(a) A has to check for all strings of length ≤ 10

(b) A has to check for all strings of length $\leq 2^{10}$

(c) A has to check for all strings of length $\leq 2^{10}$

(d) A can never determine if M_1 & M_2 accept some string in common.

19. A is given a software package S_1. To test it exhaustively he generates test data using a form counter machine. Choose the correct statement.

(a) By generating enough test data, S_1 can be exhaustively tested, so it is a decidable problem.

(b) Exhaustive testing is an undecidable problem.

(c) Though exhaustive testing is undecidable we can use enough test data to see if that S_1 contains < 10 bugs.

(d) none of the above.

20. A argues that as the membership problem of regular sets is decidable, if we form the infinite union of regular sets we should decide the membership problem. Choose the correct statement.

(a) A's statement is always true

(b) A's statement is always false

(c) A' s statement is never true for any particular choice of regular sets.

(d) none of the above

21. A is given a R.E. set L_1 accepted by some Turing Machine M_1. He is also given that L_1 is a R.E. set that is not recursive.

(a) $\overline{L_1}$ is always decidable

(b) $\overline{L_1}$ is partially decidable

(c) $\overline{L_1}$ is undecidable

(d) $\overline{L_1}$ can be accepted by the finite union of 1000 counter machines

22. A is given a non- deterministic Turing machine M_1 that accepts a language L_1 & a deterministic Turing machine accepting a language L_2. Choose the correct statement.

(a) As any nondeterministic Turing machine has an equivalent deterministic Turing machine $L_1 = L_2$

(b) If M_2 is obtained by replacing the non determinism on M_1 by determinism then the time complexity of L_2 is polynomial in the size of the input always.

(c) $L_1 = \varphi$ may be undecidable, but $L_2 = \varphi$ is always decidable if h (L_1) = L_2.

(d) Both L_1 & L_2 suffer from the halting problem.

23. A is given languages L_1, L_2, L_3, L_4 & L_s; which are a R.E. set, a CSL, a CFL and a regular set. He is also given an ε-free homomorphism h. Choose the correct answer.

(a) $h(L_1)$, $h(L_2)$, $h(L_3)$, $h(L_4)$ & $h(L_{56})$, are regular sets & so their emptiness problem is decidable.

(b) $h(L_1)$, $h(L_2)$, $h^{-1}(L_3)$, $h^{-1}(L_4)$,& $h^{-1}(L_5)$), all are CFLs and so their equivalence problem is decidable for any pair of languages.

(c) $h^{-1}(h(L_1))$, $h^{-1}(h(L_2))$, $h^{-1}(h(L_3))$, $h^{-1}(h(L_4))$,& $h^{-1}(h(L_5))$, are all CSLs or recursive sets and so the problem of whether they are pairwise disjoint is decidable.

(d) The membership problem of L_1 is undecidable but the membership problem of L_2, L_3, L_4 & L_5 are undecidable.

Solutions

Level -1

01. Ans (b)

Sol : To evaluate we need a turing machine. A stack can only parse it.

02. Ans (c)

Sol : P is closed under complement, If NP is closed under complement then P = NP. If complement of P = NP then P = NP

03. Ans (d)

Sol : ε cannot be in any recursive set of (a), (b) & (c) are excluded.

04. Ans (d)

Sol : L_1 cannot be decided in a finite time, so it is R.E. L_2 is simple we run the TM for 1000 steps. So L_2 is recursive.

05. Ans (a)

Sol : The given grammar is a DCFL & hence DPDA can accept It. It is the language of palindrome with ‘d’ in the center. Every DCFL is accepted by a DPDA. A DPDA can be simulated by a PDA. A PDA can be simulated by an LBA. An LBA can be simulated by a TM.

06. Ans (a)

Sol : The language is a DCFL & hence can be accepted by a DPDA. Hence by a PDA. Hence by a LBA. Hence by a TM

07. Ans (c)

Sol : A LBA is nothing, but a 2 DFA with an unlimited amount of ink.

08. Ans (d)

Sol : A finite set is any way accepted by a FA.

09. Ans (b) & (c)

Sol : The class of CSLs & recursive sets are closed under complement.

10. Ans (c)

Sol : The string must end with a 0. The 1 takes it back to q_0& hence we take 0^n10^n.

11. Ans (a)

Sol : The language is a regular set & all the answers in a way are correct. (a) is the tightest & closet answer.

12. Ans (d)

Sol : The class of finite automata can be put into a one to one correspondence with the integer.

13. Ans (b)

Sol : P is closed under complement. If the class NP is closed under complement than P = NP.

14. **Ans (d)**

15. **Ans (d)**

Sol : The subsets of $\sum^*$ are CSLs, CFLs, regular sets etc and all are the formal languages & hence are uncountable.

16. **Ans (d)**

Sol : L = $\sum^*$ is not decidable for CFLs.

17. **Ans (b)**

Sol : The union of two countable infinite sets is countable infinite.

18. **Ans (b)**

Sol : (a) Since L_1 & L_2 may be accepted by two different finite automata, they may be different finite automata, they may be different regular sets. So it is not true that L_1 is always the same as L_2.

(b) The equivalence problem of regular sets is decidable. So this is a true statement.

(c) The emptiness problem of regular sets is decidable. For the algorithm just check strings of length 0 to number of states of the minimal DFA accepting the language. If any one of them is accepted then the regular set is not empty. So this is a false statement.

(d) For a counter example consider $L_1 = L_2 = \sum^*$. Then $L_1 \cap L_2 = \sum^*$ is not a finite set. So this is a false statement.

19. **Ans (a)**

Sol : (a) The intersection of two DCFLs is empty is the same as the halting problem of Turing machines and so is undecidable. So this is a true statement.

(b) L_1 & L_2 may be different DCFLs. So this is a false statement.

(c) L_2 need not be a regular set always. So $L_2 = L_3$ is false.

(d) It is decidable if a DCFL L_1 is equal to a given regular set $L_{3.}$ So this is false.

20. **Ans (a)**

Sol : Two CFLs & the invalid computation of Turing machine can be described by a regular set. So this is undecidable. So (b) is a false statement.

(c) It is decidable if a CFL is finite. Construct the graph of the (CFG) grammar generation L_1. If it has no cycles the L_1 is finite. So (c) is a false statement.

(d) It is decidable if a CFL is infinite. If the graph of the reduced CFG generating $L_{1.}$

(a) The emptiness problem of CFG's is decidable. Construct the reduced grammar generating L_1; if it vanishes then L_1 is empty. So (a) is a true statement.

(b) It is undecidable for a CFL L_1, $L_1 = \Sigma^{*.}$ This follows from the fast that the valid computations of a Turing machine can be given by the intersection of a cycle then L_1 is infinite. So (b) is a false statement.

21. **Ans (d)**

Sol : Both C_1 & J_2 are procedure which can realise all the R.E. Sets. So neither C_1 nor J_2 is necessarily an algorithm.

22. **Ans (b)**

Sol : (a) Whether 'a' is ever executed reduces the halting problem of Turing machines & so is undecidable.

(b) Whether 'a' is live or dead reduces the hating problem of turing machine

(c) It is undecidable if C contains a repeated iteration.

23. **Ans (d)**

Sol : (a) The graph coloring problem is NP – complete problem which is

Perhaps intractable but always decidable as there exists an algorithm of exponential complexity for the problem.

(b) Register optimization can be resolved by the graph coloring problem.

(c) If we have a finite number of nodes then a simple polynomial algorithm suffices for graph coloring & so it is intractable

(d) The graph coloring problem has an algorithm of less than or equal to exponential complexity.

24. Ans : (c)

Sol: If L is recursive set h(L) will be in general R.E set. So for context sensitive language & recursive sets we have to use ε- free homomorphism in closure properties.

25. Ans : (d)

Sol: (a) M_1 M_2 and M_3 describe all the R.E. sets and so the halting problem is undecidable.

(b) The equivalence problem of R.E. sets is undecidable
(c) It is undecidable if a turing machine accepts all the strings over the terminal alphabet.
(d) M_3 can realise a universal turing machine & so can simulate any turing machine.

26. Ans : (d)

Sol: M_1 & M_2 are just variations of the standard model of the turing machine & accept all the R.E. sets & nothing else.

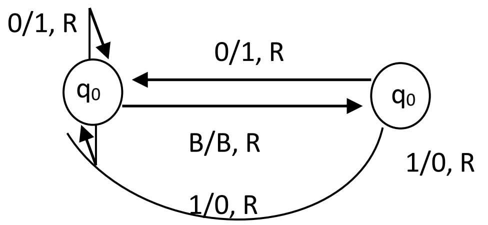
27. Ans : (c)

Sol: By Rice's Theorem all non-trivial properties of R.E. sets are undecidable. The (c) is a trivial property as there are an infinite number of graphs which have the Hamiltonian cycle.

Level – 2

01. Ans : (a)

Sol :



1) ε as input – m/c gets into an infinite loop. So (d) is not a correct choice.

2) 0 as input – accept

02. Ans : (d)

Sol : S generates $\{a^n\, b^n\, c^n \,/\, n, m \geq 1\}$

03. Ans : (c)

Sol : The DCFLs are not closed under intersection. The CFLs are not closed under intersection. The regular sets are closed under intersection.

04. Ans : (a)

Sol : (a) Regularly is decidable

(c) Membership of R.E. is decidable

05. Ans : (d)

Sol : $L_1 = \overline{L_n} = \sum^* \;\&\; L_{n = \varphi}$

06. Ans : (a)

Sol : It is generally accepted that (a) is the true case

07. Ans : (c)

Sol : Any automata has a halt state. A TM is not a FA.

08. Ans : (d)

Sol : The regular expression denotes regular sets.

09. Ans : (c)

Sol : ε is not accepted by the TM

10. Ans : (b)

Sol : The TM halts so it accepts a recursive set.

11. Ans : (a)

Sol : $L_1 = (0+1)^*$ is a regular & infinite set by itself.

12. Ans : (d)

Sol : L_2 can be any language.

13. Ans : (d)

Sol : ε is accepted & so in 'aa'. Use the methods of elimination

14. Ans : (a)

Sol : **(a)** The intersection of two DCFLs gives the valid computations of a Turing Machine which corresponds to solutions to the PCP problem P. So this is true.

(b) The intersection of two regular sets is regular. For a regular set we have a FA M accepting it. The halting problem of M is decidable. So this is a false statement.

15. Ans : (a)

Sol : (a) A CSL can be obtained from the intersection of two CFLs, as the Boolen closured two CFLs are the CSLs. Now the intersection of two CFLs give the valid computations of a Turing Machine & so the given choice reduces to the halting problem which is undecidable. So (a) is ture statement.

(b) The CSLs are automatically recursive sets, so their membership problem is decidable. So this is a false statement.

(c) The equivalence problem of two CFLs is undecidable, so this gives that the equivalence problem of CFLs is undecidable. So this is a false statement.

(d) It is undecidable if a CFL is regular, so this is undecidable.

16. Ans : (c)

Sol : (a) L_1 is recursive set & L_2 is R.E. set. The membership problem of L_2 is the halting problem of TMs & so undecidable. So this is a false statement.

(b) As L_1 is recursive set for which the halting problem is decidable & L_2 is a R.E. set for which the halting problem is undecidable, this is a false statement.

(c) It is undecidable if $L_1 = \phi$. So it is undecidable if $L_1 = \sum^*$. L_1 is a R.E. set. However some L_1 & L_2 may be $\sum^*$. So this is true.

(d) It is undecidable if recursive sets and R.E. sets are regular. So this is a false statement.

17. Ans : (b)

Sol : (a) C_1 & J_1 generate all C & JAVA programs which are procedures & not algorithms.

(b) It if undecidable if a CFG C_1 is ambiguous

(c) It is undecidable if a cfg J_2 is ambiguous

(d) Both C_1 & J_1 deal with CFG's. All CFG's do not have LR(k) equivalents.

18. Ans : (d)

Sol : The intersection of two DCFLs can give all the valid computations of a turing machine & so is undecidable.

19. Ans : (b)

Sol : Exhaustive testing reduce to the halting problem of turing machine. So we can never decide if S_1 will ever halt.

20. Ans : (a)

Sol : (a) The infinite union of regular sets can yield any formal languages including undecidable once.

(b) If we choose all regular sets as $\{\varepsilon\}$ then Lavanya's statement is true.

(c) If we choose all regular sets as if then Lavanya's statement is true.

21. Ans : (c)

Sol : As the R.E. sets are not closed under complement, $\overline{L}_1$ cannot be accepted by a Turing machine. A multi counter Turing machine is just a variation of the standard model of a Turing machine.

22. Ans : (d)

Sol : (a) M_1 & M_2 may be machine generating different R.E. sets

(b) Normally removing non-determinism leads to an exponential blown-up in the time complexity.

(c) The emptiness problem of L_1 and L_2 are undecidable.

(d) Both M_2 & M_1 can be corrected to the standard model of turing machines.

23. Ans : (d)

Sol : By definition recursive sets have a decidable membership problem. The membership of R.E. sets is undecidable are closed under ε- free homomorphism & so we use the standard results