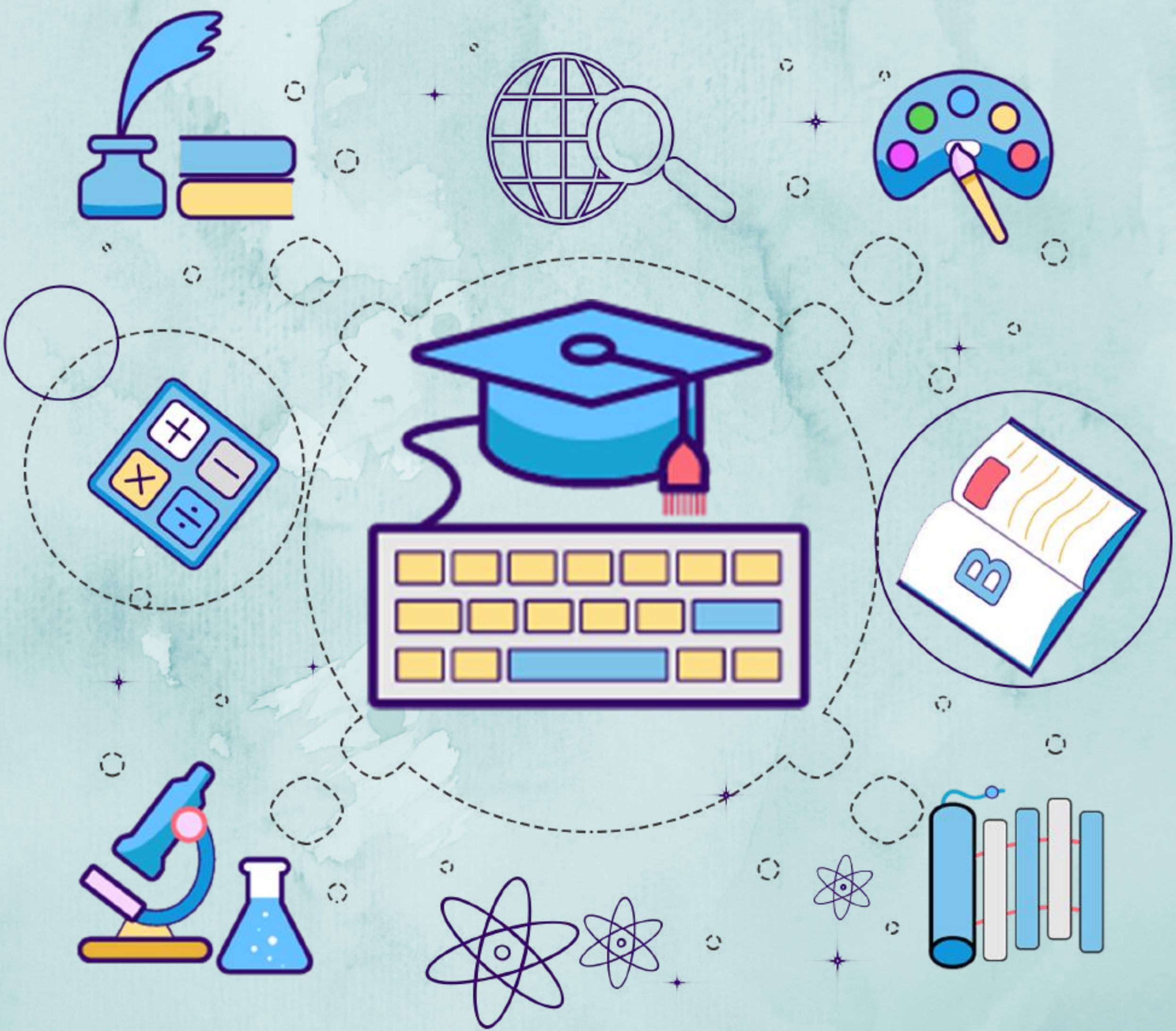


KTU STUDY MATERIALS

COMPUTER GRAPHICS AND IMAGE PROCESSING

CST 304

Module 4

CST304- COMPUTER GRAPHICS & IMAGE PROCESSING

MODULE 4

SYLLABUS

(Fundamentals of Digital Image Processing)

- **Introduction to Image processing and applications.**
- Image as 2D data.
- Image representation in Gray scale, Binary and Colour images.
- **Fundamental steps in image processing.**
- Components of image processing system.
- Coordinate conventions.
- Sampling and quantization.
- Spatial and Gray Level Resolution.
- Basic relationship between pixels- neighbourhood, adjacency, connectivity.
- Fundamentals of spatial domain-convolution operation.

What Is Digital Image Processing

- An **image** may be defined as a two-dimensional function, f(x, y), where x and y are **spatial (plane) coordinates**, and the amplitude of f at any pair of coordinates (x, y) is called the **intensity or gray level** of the image at that point.
- When x, y, and the amplitude values of f are **all finite, discrete quantities,** we call the image a **digital image.**
- The field of **digital image processing** refers to processing digital images by means of a digital computer.

- Digital image is composed of a finite number of elements, each of which has a particular location and value.
- These elements are referred to as picture elements, image elements, pels, and pixels.
- Pixel is the term most widely used to denote the elements of a digital image.

The Origins of Digital Image Processing

- **Digital image processing** is the use of a digital computer to process digital images through an algorithm.
- As a subcategory or field of digital signal processing, digital image processing has many advantages over analog image processing.
- It allows a much wider range of algorithms to be applied to the input data and can avoid problems such as the build-up of noise and distortion during processing.

- The generation and development of digital image processing are mainly affected by three factors:
- First, the development of computers.
- Second, the development of mathematics (especially the creation and improvement of discrete mathematics theory).
- Third, the demand for a wide range of applications in environment, agriculture, military, industry and medical science has increased.

Why do we need digital image processing?

Improvement of pictorial information for human perception.

- We want to enhance the quality of images , so that the images should have better look and quality .

Image processing for autonomous machine applications.

- Applications in industry such as quality control , assembly operations etc.

Efficient storage and transmission.

- If we want to store a particular image on your computer , this image will needs certain amount of disk space, process the image using certain image properties that the disk space used for storing is less.

Familiar with some terms..

Resolution

- It is the number of pixels (individual points of color) contained on a display monitor, expressed in terms of the number of pixels on the horizontal axis and the number on the vertical axis.
- The sharpness of the image on a display depends on the resolution and the size of the monitor.
- The same pixel resolution will be sharper on a **smaller monitor** and gradually lose sharpness on larger monitors because the same number of pixels are being spread out over a larger number of inches.

Pixels

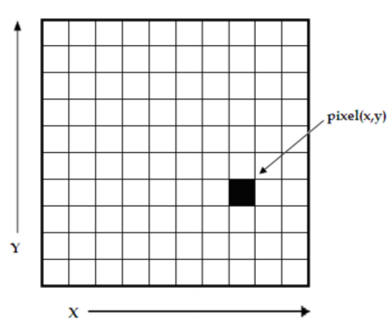
- Pixel, short for picture element, is the smallest unit in a graphic display or digital image.
- Computer displays are made up of a grid of pixels.

- Each pixel is made up of red, blue, and green lighting elements that are used in different combinations and intensities to make millions of different colors.

Gray scale

- Grayscale is a group of shades without any visible color.
- On a monitor, each pixel of a grayscale display carries an amount of light, ranging from the weakest amount of light, or black, to the strongest amount of light, or white.
- Grayscale only contains brightness information, not color.
- On a computer display, images are composed of pixels, which are comprised of one red, one green and one blue dot.
- Each of these dots has its own brightness level as well and, therefore, can be converted to grayscale.
- A grayscale image is one with all color information removed

Image as 2D data



- A digital image is a 2D array of pixels.
- Each pixel is characterized by its (x, y) coordinates and its value.
- Digital images are characterized by matrix size, pixel depth and resolution.
- The matrix size is determined from the number of the columns (m) and the number of rows (n) of the image matrix (m × n).
- The size of a matrix is selected by the operator. Generally, as the matrix dimension increases the resolution is getting better .
- Pixel or bit depth refers to the number of bits per pixel that represent the color levels of each pixel in an image.

Image representation in Gray scale, Binary and Color images

The binary image

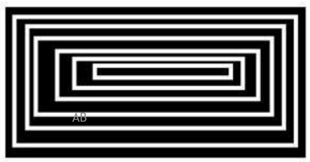
- The binary image as it name states, contain only two pixel values. 0 and 1.
- Here 0 refers to black color and 1 refers to white color.
- It is also known as Monochrome.

Black and white image

- The resulting image that is formed hence consist of only black and white color and thus can also be called as Black and White image.
- **No gray level**
- One of the interesting this about this binary image that there is no gray level in it.
- Only two colors that are black and white are found in it.

Format

- Binary images have a format of PBM (Portable bit map)



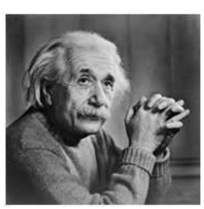
2, 3, 4,5, 6 bit color format

- The images with a color format of 2, 3, 4, 5 and 6 bit are not widely used today. They were used in old times for old TV displays, or monitor displays.
- But each of these colors have more then two gray levels, and hence has gray color unlike the binary image.
- In a 2 bit 4, in a 3 bit 8, in a 4 bit 16, in a 5 bit 32, in a 6 bit 64 different colors are present.

8 bit color format

- 8 bit color format is one of the most famous image format.
- It has 256 different shades of colors in it.
- It is commonly known as Grayscale image.
- The range of the colors in 8 bit vary from 0-255.
- Where 0 stands for black, and 255 stands for white, and 127 stands for gray color.
- This format was used initially by early models of the operating systems UNIX and the early color Macintoshes.

A grayscale image of Einstein is shown below:



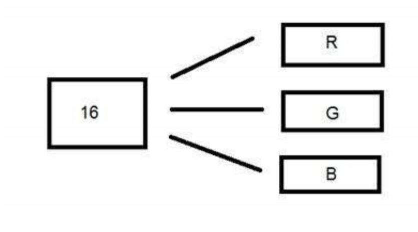
Format

- The format of these images are PGM (Portable Gray Map).
- This format is not supported by default from windows.
- In order to see gray scale image, you need to have an image viewer or image processing toolbox such as Matlab.

16 bit color format

- It is a color image format.
- It has 65,536 different colors in it.
- It is also known as High color format.
- It has been used by Microsoft in their systems that support more then 8 bit color format.
- Now in this 16 bit format and the next format we are going to discuss which is a 24 bit format are both color format.
- The distribution of color in a color image is not as simple as it was in grayscale image.

- A 16 bit format is actually divided into three further formats which are Red , Green and Blue. The famous (RGB) format.
- It is pictorially represented in the image below.



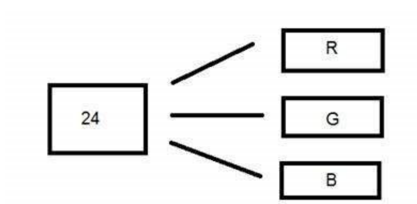
- Now the question arises, that how would you distribute 16 into three. If you do it like this,
- 5 bits for R, 5 bits for G, 5 bits for B
- Then there is one bit remains in the end.
- So the distribution of 16 bit has been done like this.
- 5 bits for R, 6 bits for G, 5 bits for B.
- The additional bit that was left behind is added into the green bit. Because green is the color which is most soothing to eyes in all of these three colors.
- Note this is distribution is not followed by all the systems.
- Some have introduced an alpha channel in the 16 bit.

- Another distribution of 16 bit format is like this:
- 4 bits for R, 4 bits for G, 4 bits for B, 4 bits for alpha channel.
- Or some distribute it like this
- 5 bits for R, 5 bits for G, 5 bits for B, 1 bits for alpha channel.

24 bit color format

- 24 bit color format also known as true color format.
- Like 16 bit color format, in a 24 bit color format, the 24 bits are again distributed in three different formats of Red, Green and Blue.

Since 24 is equally divided on 8, so it has been distributed equally between three different color channels.
Their distribution is like this.
8 bits for R, 8 bits for G, 8 bits for B.



- Behind a 24 bit image.
- Unlike a 8 bit gray scale image, which has one matrix behind it, a 24 bit image has three different matrices of R, G, B.

Format

It is the most common used format. Its format is PPM (Portable pixMap) which is supported by Linux operating system. The famous windows has its own format for it which is BMP (Bitmap).

Applications of Digital Image Processing

The main purpose of the DIP is divided into following 5 groups:

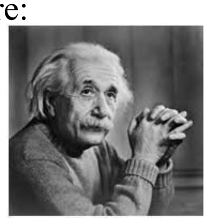
1. **Visualization:** The objects which are not visible, they are observed.
2. **Image sharpening and restoration:** It is used for better image resolution.
3. **Image retrieval:** An image of interest can be seen
4. **Measurement of pattern:** In an image, all the objects are measured.
5. **Image Recognition:** Each object in an image can be distinguished.

Applications of Digital Image Processing

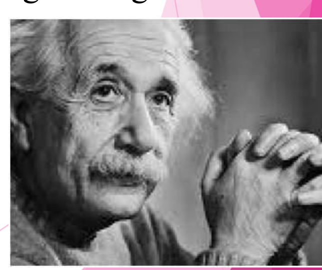
- Image sharpening and restoration
- Medical field
- Remote sensing
- Transmission and encoding
- Machine/Robot vision
- Color processing
- Pattern recognition
- Video processing
- Microscopic Imaging

Image sharpening and restoration

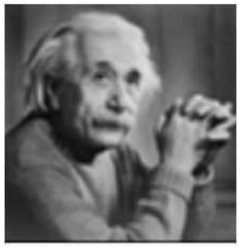
- **Image sharpening and restoration** refers here to process images that have been captured from the modern camera to make them a better image or to manipulate those images in way to achieve desired result.
- It refers to do what Photoshop usually does.
- This includes Zooming, blurring , sharpening , gray scale to color conversion, detecting edges and vice versa , Image retrieval and Image recognition. The common examples are:
- The original image



The zoomed image

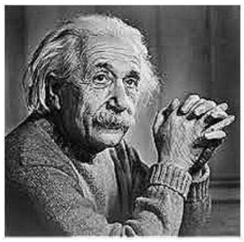


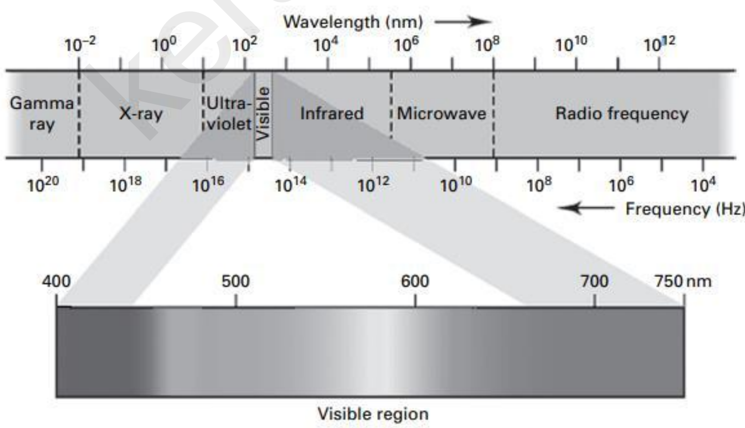
- Blurr image



Edges

Sharp image





The electromagnetic spectrum arranged according to the energy of the photons, or the frequency of the waves. See also color plate.

Medical field

- The common applications of **DIP** in the field of medical is
- Gamma ray imaging
- CT scan
- X Ray Imaging
- Medical CT
- UV imaging
- Segmentation and texture analysis, which is used for cancer and other disorder identifications.

Medical diagnostic imaging

- Projection radiography and x-ray computed tomography (CT) using transmission of x-rays through the body.
- Digital subtraction angiography (DSA) produces enhanced images of the blood vessels by subtracting "pre-contrast" and "post-contrast" images.
- Mammography produces images of the soft tissue in the breast

- Nuclear medicine using emission of gamma rays from radiotracers injected into the body; includes planar scintigraphy and emission computed tomography (SPECT and PET)

- Ultrasound imaging using reflection of ultrasonic waves within the body Magnetic resonance imaging (MRI) using the precession of spin systems in a large magnetic field; including functional MRI (fMRI)

UV imaging

- In the field of remote sensing , the area of the earth is scanned by a satellite or from a very high ground and then it is analyzed to obtain information about it.

- One particular application of digital image processing in the field of remote sensing is to detect **infrastructure damages caused by an earthquake.**

- As it takes longer time to grasp damage, even if serious damages are focused on.

- Since the area effected by the earthquake is sometimes so wide , that it not possible to examine it with human eye in order to estimate damages.

- Even if it is , then it is very hectic and time consuming procedure. So a solution to this is found in digital image processing.

- An image of the effected area is captured from the above ground and then it is analyzed to detect the various types of damage done by the earthquake

The key steps include in the analysis are

•The extraction of edges

•Analysis and enhancement of various types of edges

Transmission and encoding

- The very first image that has been transmitted over the wire was from London to New York via a submarine cable.
- The picture that was sent is shown below.

- The picture that was sent took three hours to reach from one place to another.

Now just imagine , that today we are able to see live video feed , or live cctv footage from one continent to another with just a delay of seconds.

- It means that a lot of work has been done in this field too.

This field doesnot only focus on transmission , but also on encoding.

- Many different formats have been developed for high or low bandwith to encode photos and then stream it over the internet or e.t.c.

Machine/Robot vision

- Apart form the many challenges that a robot face today , one of the biggest challenge still is to increase the **vision of the robot.**
- Make robot able to see things , identify them , identify the hurdles e.t.c.
- Much work has been contributed by this field and a complete other field of computer vision has been introduced to work on it.

Hurdle detection

- Hurdle detection is one of the common task that has been done through image processing, by identifying different type of objects in the image and then calculating the distance between robot and hurdles.

- **Color processing**
- Color processing includes processing of colored images and different color spaces that are used. For example RGB color model , YCbCr, HSV.
- It also involves studying transmission , storage , and encoding of these color images.

- **Pattern recognition**
- Pattern recognition involves study from image processing and from various other fields that includes machine learning (a branch of artificial intelligence).
- In pattern recognition , image processing is used for identifying the objects in an images and then machine learning is used to train the system for the change in pattern.
- Pattern recognition is used in **computer aided diagnosis** , **recognition of handwriting , recognition of images e.t.c**

- **Video processing**
- A video is nothing but just the very fast movement of pictures.
- The quality of the video depends on the number of frames/pictures per minute and the quality of each frame being used.
- Video processing involves noise reduction , detail enhancement , motion detection , frame rate conversion , aspect ratio conversion , color space conversion e.t.c.

Components of image processing system

Components of an Image Processing System

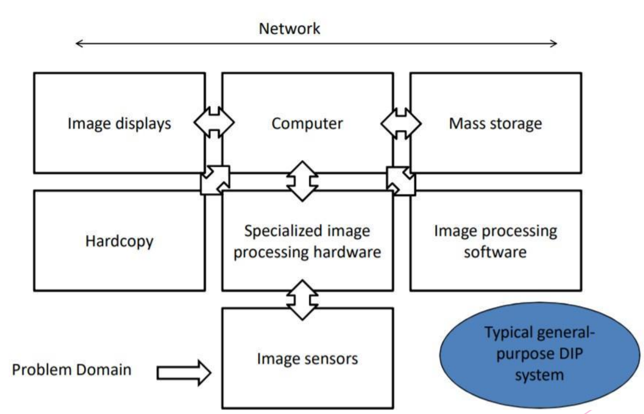


Image Sensors:

- Image sensors senses the intensity, amplitude, co-ordinates and other features of the images and passes the result to the image processing hardware.
- It includes the problem domain.

Image Processing Hardware:

- Image processing hardware is the dedicated hardware that is used to process the instructions obtained from the image sensors.
- It passes the result to general purpose computer.
- Usually consists of the digitizer, mentioned before, plus hardware that performs other primitive operations, such as an arithmetic logic unit (ALU), which performs arithmetic and logical operations in parallel on entire images.

- This type of hardware sometimes is called a front end subsystem, and its most distinguishing characteristic is speed.
- In other words, this unit performs functions that require fast data throughputs that the typical main computer cannot handle.

Computer:

- Computer used in the image processing system is the general purpose computer that is used by us in our daily life.
- The computer in an image processing system is a general-purpose computer andcan range from a PC to a supercomputer.
- In dedicated applications, sometimes specially designed computers are used to achieve a required level of performance.

Image Processing Software:

- Image processing software is the software that includes all the mechanisms and algorithms that are used in image processing system.

Mass Storage:

- Mass storage stores the pixels of the images during the processing.
- Mass storage capability is a must in a image processing applications.
- Digital storage for image processing applications falls into three principal categories:
- Short-term storage for use during processing.
- On line storage for relatively fast recall

- Archival storage, characterized by infrequent access
- One method of providing short-term storage is computer memory. Another is by specialized boards, called frame buffers, that store one or more images and can be accessed rapidly.
- On-line storage generally takes the form of magnetic disks and optical-media storage. The key factor characterizing on-line storage is frequent access to the stored data.

Hard Copy Device:

- Once the image is processed then it is stored in the hard copy device.
- It can be a pen drive or any external ROM device.

Image Displays

- The displays in use today are mainly color (preferably flat screen) TV monitors.
- Monitors are driven by the outputs of the image and graphics display cards that are an integral part of a computer system.

Networking

- It is almost a default function in any computer system, in use today. Because of the large amount of data inherent in image processing applications the key consideration in image transmission is bandwidth.
- In dedicated networks, this typically is not a problem, but communications with remote sites via the internet are not always as efficient

Fundamental Steps in Digital Image Processing:

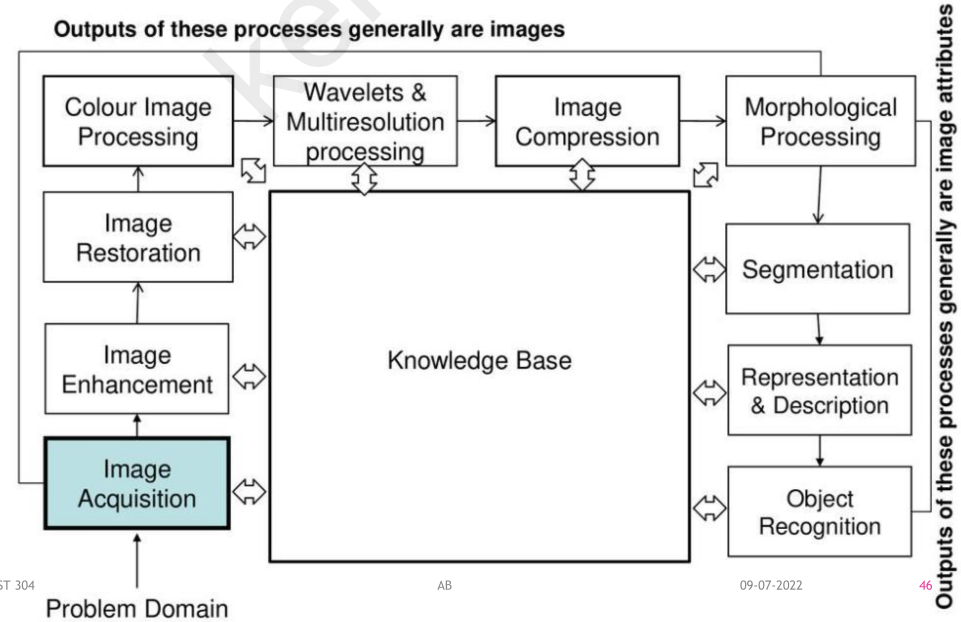


Image Acquisition

- This is the first fundamental steps in digital image processing.
- Image is captured by a sensor(example camera) and digitized if the output of the camera or sensor is not already in digital form using analog to digital converter.
- Generally, the image acquisition stage involves pre-processing, such as scaling etc.
- The general aim of any image acquisition is to transform an **optical image (real-world data) into an array of numerical data** which could be later manipulated on a computer.
- Image acquisition is achieved by suitable cameras.

- We use different cameras for different applications.
- If we need an **X-ray image**, we use a camera (film) that is sensitive to X-rays.
- If we want an **infrared image,** we use cameras that are sensitive to infrared radiation.
- For normal images (family pictures, etc.), we use cameras that are sensitive to the **visual spectrum.**

Image Enhancement

- Image enhancement is among the simplest and most appealing areas of digital image processing.
- The process of manipulating an image so the result is more suitable than the original for **specific applications.**
- The idea behind enhancement techniques is to bring out details that are hidden or simple to highlight certain features of interest in an image.
- Such as, changing brightness & contrast etc.

Image Restoration

- Image restoration is an area that also deals with improving the appearance of an image.
- However, unlike enhancement, which is **subjective**(human subjective preferences regarding what constitutes a good enhancement result), image restoration is **objective,** in the sense that restoration techniques tend to be based on mathematical or probabilistic models of image degradation.

Color Image Processing

- Color image processing is a famous area because it has increased the use of digital images on the internet.
- This includes color modeling, processing in a digital domain, etc....

Wavelets and Multi-Resolution Processing

- In this stage, an image is represented in various degrees of resolution.
- Multiresolution analysis provides information on both the spatial and frequency domain.
- Wavelets are a more general way to represent and analyze multiresolution images

Compression

- Compression is a technique which is used for reducing the requirement of storing an image.
- It is a very important stage because it is very necessary to compress data for internet use.

Morphological Processing

- This stage deals with tools which are used for extracting the components of the image, which is useful in the representation and description of shape.

Segmentation

- In this stage, an image is a partitioned into its objects.
- Segmentation is the most difficult tasks in DIP.
- It is a process which takes a lot of time for the successful solution of imaging problems which requires objects to identify individually.

Representation and Description

- Representation and description follow the output of the segmentation stage.
- The output is a raw pixel data which has all points of the region itself.
- To transform the raw data, representation is the only solution.
- Whereas description is used for extracting information's to differentiate one class of objects from another.

Object recognition

- In this stage, the label is assigned to the object, which is based on descriptors.

Knowledge Base

- Knowledge is the last stage in DIP.
- In this stage, important information of the image is located, which limits the searching processes.
- The knowledge base is very complex when the image database has a high-resolution satellite.

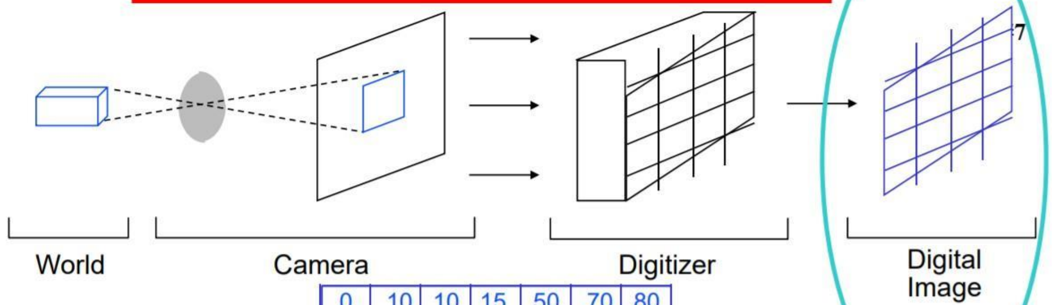
Digital image representations

There are two types of images digital images and analog images.

- **Analog images** are the type of images that we, as humans, look at.
- They include such things as photographs, paintings, TV images, and all of our medical images recorded on film or displayed on various display devices, like computer monitors.
- What we see in an analog image is various levels of brightness (or film density) and colors.
- It is generally continuous and not broken into many small individual pieces.

- **Digital images** are recorded as many numbers.
- The image is divided into a matrix or array of small picture elements, or pixels.
- Each pixel is represented by a numerical value.
- The advantage of digital images is that they can be processed, in many ways, by computer systems.

Digital Images



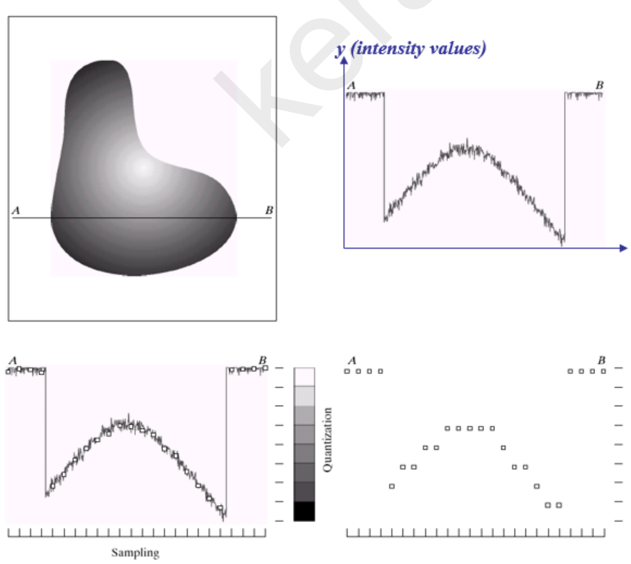
0	10	10	15	50	70	80
0	0	100	120	125	130	130
0	35	100	150	150	80	50
0	15	70	100	10	20	20
0	15	70	0	0	0	15
5	15	50	120	110	130	110
5	10	20	50	50	20	250

PIXEL
(picture element)

Typically:
0 = black
255 = white

Video Lecturers on Digital Image Processing

Gholamreza Anbarjafari, PhD

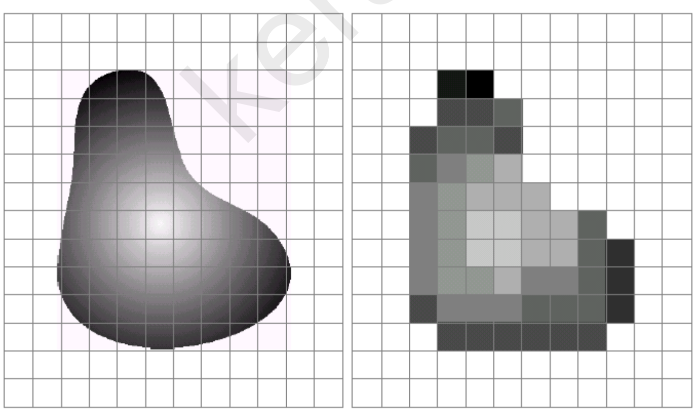


Generating a digital image. (a) Continuous image. (b) A scaling line from A to B in the continuous image, used to illustrate the concepts of sampling and quantization. (c) sampling and quantization. (d) Digital scan line.

- The basic idea behind sampling and quantization is illustrated in figure.
- (a) shows a continuous image, f(x, y), that we want to convert to digital form.
- An image may be continuous with respect to the x- and y-coordinates, and also in amplitude.
- To convert it to digital form, we have to sample the function in both coordinates and in amplitude.
- Digitizing the coordinate values is called **sampling.**
- Digitizing the amplitude values is called **quantization.**

- The one-dimensional function shown in (b) is a plot of amplitude (gray level) values of the continuous image along the line segment AB in (a).
- The random variations are due to image noise.
- To sample this function, we take equally spaced samples along line AB, as shown in (c).
- The location of each sample is given by a vertical tick mark in the bottom part of the figure.

- In order to form a digital function, the gray-level values also must be converted (quantized) into discrete quantities.
- The right side of (c) shows the gray-level scale divided into eight discrete levels, ranging from black to white.
- The vertical tick marks indicate the specific value assigned to each of the eight gray levels.
- The continuous gray levels are quantized simply by assigning one of the eight discrete gray levels to each sample



a b

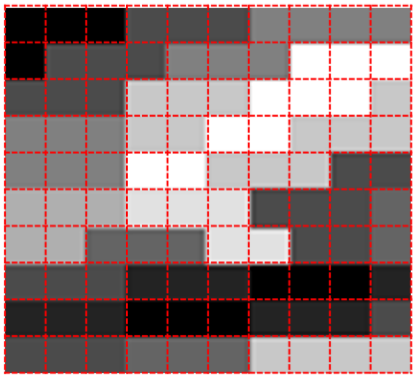
FIGURE 2.17 (a) Continuos image projected onto a sensor array. (b) Result of image sampling and quantization.

a b

(a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.

Video Lecturers on Digital Image Processing

Gholamreza Anbarjafari, PhD



0	0	0	75	75	75	128	128	128	128
0	75	75	75	128	128	128	255	255	255
75	75	75	200	200	200	255	255	255	200
128	128	128	200	200	255	255	200	200	200
128	128	128	255	255	200	200	200	75	75
175	175	175	225	225	225	75	75	75	100
175	175	100	100	100	225	225	75	75	100
75	75	75	35	35	35	0	0	0	35
35	35	35	0	0	0	35	35	35	75
75	75	75	100	100	100	200	200	200	200

Sampling	Quantization
Digitization of co-ordinate values.	Digitization of amplitude values.
x-axis(time) – discretized.	x-axis(time) – continuous.
y-axis(amplitude) – continuous.	y-axis(amplitude) – discretized.
Sampling is done prior to the quantization process.	Quantizatin is done after the sampling process.
It determines the spatial resolution of the digitized images.	It determines the number of grey levels in the digitized images.
It reduces c.c. to a series of tent poles over a time.	It reduces c.c. to a continuous series of stair steps.
A single amplitude value is selected from different values of the time interval to represent it.	Values representing the time intervals are rounded off to create a defined set of possible amplitude values.

Representing Digital Images

- The result of sampling and quantization is a matrix of real numbers.
- Assume that an image f(x, y) is sampled so that the resulting digital image has M rows and N columns.
- The values of the coordinates (x, y) now become discrete quantities.
- The values of the coordinates at the origin are (x, y)=(0, 0).
- The next coordinate values along the first row of the image are represented as (x, y)=(0, 1).

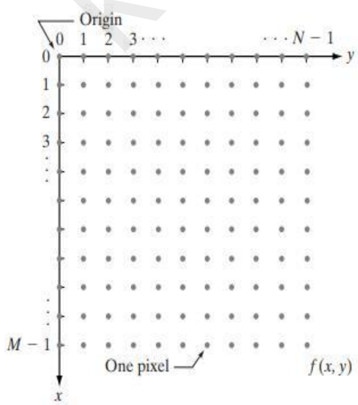


FIGURE 2.18 Coordinate convention used in this book to represent digital images.

The notation introduced in the preceding paragraph allows us to write the complete $M \times N$ digital image in the following compact matrix form:

$$f(x, y) = \begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0, N-1) \\ f(1,0) & f(1,1) & \cdots & f(1, N-1) \\ \vdots & \vdots & & \vdots \\ f(M-1,0) & f(M-1,1) & \cdots & f(M-1, N-1) \end{bmatrix}. \qquad (2.4\text{-}1)$$

The right side of this equation is by definition a digital image. Each element of this matrix array is called an image element, picture element, pixel, or pel.
The terms image and pixel will be used throughout the rest of our discussions to denote a digital image and its element.

- In some discussions, it is advantageous to use a more traditional matrix notation to denote a digital image and its elements:

$$\mathbf{A} = \begin{bmatrix} a_{0,0} & a_{0,1} & \cdots & a_{0,N-1} \\ a_{1,0} & a_{1,1} & \cdots & a_{1,N-1} \\ \vdots & \vdots & & \vdots \\ a_{M-1,0} & a_{M-1,1} & \cdots & a_{M-1,N-1} \end{bmatrix}. \qquad (2.4\text{-}2)$$

- Clearly, aij=f(x=i, y=j)=f(i, j), so Eqs. (2.4-1) and (2.4-2) are identical matrices.

Spatial and Gray-Level Resolution

Image resolution

- Image resolution can be defined in many ways.
- In pixel resolution, the term resolution refers to **the total number of count of pixels in an digital image**.
- If an image has M rows and N columns, then its resolution can be defined as M X N.
- If we define resolution as the total number of pixels, then pixel resolution can be defined with set of two numbers.

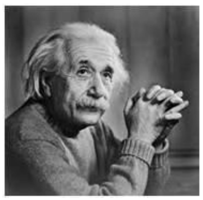
- The first number the width of the picture, or the pixels across columns, and the second number is height of the picture, or the pixels across its width.
- We can say that the higher is the pixel resolution, the higher is the quality of the image.
- We can define pixel resolution of an image as 4500 X 5500.

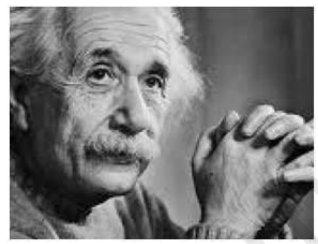
Spatial resolution

- Spatial resolution states that the clarity of an image cannot be determined by the pixel resolution.
- The number of pixels in an image does not matter.
- **Spatial resolution** can be defined as the smallest discernible detail in an image.
- Or in other way we can define spatial resolution as the number of independent pixels values per inch.

- In short what spatial resolution refers to is that we cannot compare two different types of images to see that which one is clear or which one is not.
- If we have to compare the two images, to see which one is more clear or which has more spatial resolution, we have to compare two images of the same size.

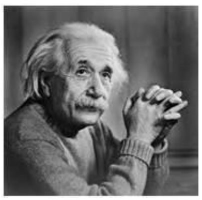
For example:
You cannot compare these two images to see the clarity of the image.

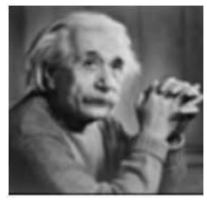




- Although both images are of the same person, but that is not the condition we are judging on.
- The picture on the left is zoomed out picture of Einstein with dimensions of 227 x 222.
- Whereas the picture on the right side has the dimensions of 980 X 749 and also it is a zoomed image.
- We cannot compare them to see that which one is more clear.
- Remember the factor of zoom does not matter in this condition, the only thing that matters is that these two pictures are not equal.

- So in order to measure spatial resolution , the pictures below would server the purpose.





- Now you can compare these two pictures.
- Both the pictures has same dimensions which are of 227 X 222.
- Now when you compare them, you will see that the picture on the left side has more spatial resolution or it is more clear then the picture on the right side.
- That is because the picture on the right is a blurred image.

- **Measuring spatial resolution**
- Since the spatial resolution refers to clarity, so for different devices, different measure has been made to measure it.

Dots per inch

- Dots per inch or DPI is usually used in monitors.

Lines per inch

- Lines per inch or LPI is usually used in laser printers.

Pixel per inch

- Pixel per inch or PPI is measure for different devices such as tablets , Mobile phones e.t.c.

1250 dpi
300 dpi
150 dpi

High Spatial Resolution | Medium Spatial Resolution | Low Spatial Resolution

Gray level resolution

- **Gray level resolution** refers to the predictable or deterministic change in the shades or levels of gray in an image.
- In short gray level resolution is equal to the number of bits per pixel.

BPP

- The number of different colors in an image is depends on the depth of color or bits per pixel.

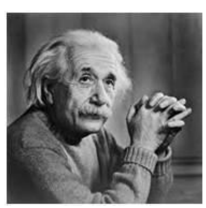
Mathematically

- The mathematical relation that can be established between gray level resolution and bits per pixel can be given as.

$$L = 2^k$$

bpp

- In this equation L refers to number of gray levels.
- It can also be defined as the shades of gray.
- k refers to bpp or bits per pixel.
- So the 2 raise to the power of bits per pixel is equal to the gray level resolution.
- For example:



The above image of Einstein is an gray scale image.
Means it is an image with 8 bits per pixel or 8bpp.
Now if were to calculate the gray level resolution, here how we gonna do it.

$L = 2^k$

Where k = 8

$L = 2^8$

L = 256.

It means it gray level resolution is 256.
Or in other way we can say that this image has 256 different shades of gray.
The more is the bits per pixel of an image, the more is its gray level resolution.

Finding bpp from Gray level resolution

- You can also find the bits per pixels from the given gray level resolution. For this, we just have to twist the formula a little.

$L = 2^k$

- Now if we were to find the bits per pixel or in this case k, we will simply change it like this.

K = log base 2(L)

For example:

- If you are given an image of 256 levels. What is the bits per pixel required for it.
- Putting 256 in the equation, we get.
- K = log base 2 (256)
- K = 8.
- So the answer is 8 bits per pixel.

Basic relationship between pixels- neighborhood, adjacency, connectivity.

We consider several important relationships between pixels in a digital image.

1. Neighbours of a Pixel
2. Adjacency, Connectivity, Regions, and Boundaries
3. Distance Measures
4. Image Operations on a Pixel Basis

Neighbors of a Pixel

- Any pixel p(x, y) has two vertical and two horizontal neighbors, specified by [(x+1, y), (x-1, y), (x, y+1), (x, y-1)]
- This set of pixels are known the **4-neighbors** of P, and is denoted by **N4(P).**
- All of them are at a unit distance from P.
- The four **diagonal neighbors** of p(x,y) are given by, [(x+1, y+1), (x+1, y-1), (x-1, y+1), (x-1 ,y-1)]
- This set is denoted by **ND (P).**
- The points **S** are together known as 8-neighbors of the point P, denoted by **N8(P).**

Adjacency

- Let V be the set of gray level values used to define adjacency . In a binary image V= {0,1}.
- In a gray scale image , V contains more elements.
- In the adjacency of pixels with a range of possible gray levels values from 0 to 255, set V could be any subset of these 256 values.
- Two pixels are linked if they are neighbors and their gray levels satisfy few detailed pattern of similarity.
- For instance, in a binary image two pixels are connected if they are 4-neighbors and have same value (0/1).

3 types of adjacency

4 adjacency

- Two pixels p and q with values from V are 4-adjacent if q is in the set **N4(P).**
- Example V= { 1,2,3}
- Here **N4(P) ={4,3,1,3}**
- **q=2**
- not belongs to N4(P).
- **Here p and q are not 4-adjacent**

1	4	5
1	2 p	3
1	3	2 q

8 adjacency

- Two pixels p and q with values from V are 8-adjacent if q is in the set **N8(P).**
- Mixed adjacency is a modification of 8-adjacency "introduced to eliminate the ambiguities that often arise when 8- adjacency is used. (eliminate multiple path connection)
- Example V= { 1,2,3}
- Here **N8(P) ={4,1,3,5}**
- **q=5**
- not belongs to V .
- Here p and q are not 8-adjacent

1	2 p	3
1	4	5 q
3	2	1

m adjacency(mixed adjacency)

- Two pixels p and q with values from V are m-adjacent if,
- q is in **N4(P) or**

q is in ND (P) and the set N4(P) intersection N4(q) has no pixels whose values are from V.

V= {1,2,3}

N4(P)= {0,1,3}

P not belongs to N4(P)

ND (P)= {0,3,2,4}

Q=2 condition true.

N4(q)={0,1,3} false ..

3	**0**	**2 q**	**1**
1	2 p	3	4
4	1	0	3
1	1	3	2

Find the shortest 4-, 8- and m-path between p and q ∀

(i) V= {0,1}

(ii) V= {1,2}

Given image,

```
3 1 2 1 (q)
2 2 0 2
1 2 1 1
1 0 1 1
(p)
```

V= {0,1}

4-path

```
    3 1 2 1 (q)
    2 2 0→2
    1 2 1 1
(p) 1→0→1 1
```

$N_4(0) = \{2\}$ which is not a member of V

So there is no 4 path

```
    3 1 2 1 (q)
    2 2 0 2
    1→2 1 1
(p) 1 0 1 1
```

$N_4(1) = \{2\}$ which is not a member of V

8 path

3 1 2 1 (q)
2 2 0 2
1 2 1 1
1→0 1 1
(p)

path = 4

3 1 2 1 (q)
2 2 0 2
1 2 1→1
1→0 1 1
(p)

path = 5

So shortest path = 4

m path

3 1 2 1 (q)
2 2 0 2
1 2 1 1
1→0→1 1
(p)

1) q is in $N_4(p)$ or

2) q is in $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ has no pixels whose values are from V

At the above point condition 1 is false so check condition 2.

$N_D(0) = \{1\}$

3 1 2 1 (q)
2 2 0 2
1 2 1 1
1→0→1 1
(p)

$N_4(p) = \{2, 2\}$

$N_4(q) = \{2, 2\}$

$N_4(p) \cap N_4(q) = \{2, 2\}$ no elements from V.

So path = 5

3 1 2 1
2 2 0 2
1 2 1 1
1→0→1 1

path = 5.

V = {1, 2}

3 1→2→1 (q)
2 2 0 2
1→2 1 1
1 0 1 1
(p)

4-path

4 path = 6.

3 1 2 1 (q)
2 2 0 2
1 2 1 1
(p) 1 0 1 1

4 path = 6.

Shortest path = 6.

3 1 2 1 (q)   V = {1, 2}
2 2 0 2
1 2 1 1
(p) 1 0 1 1

8 path = 4

3 1 2 1 (q)
2 2 0 2
1 2 1 1
1 0 1 1
(p)

8 path = 4

Shortest path = 4

m path

V = {1, 2}

3 1 2 1 (q)
2 2 0 2
1 2 1 1
(p) 1 0 1 1

1) q is in $N_4(p)$ or
2) q is in $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ has no pixels whose values are from

3 1 2 1 (q)
2 2 0 2
1 2 1 1
1 0 1 1
(p)

m path = 6.

3 1 2 1 (q)
2 2 0 2
1 2 1 1
1 0 1 1
(p)

m path = 6

So Shortest path = 6.

Connectivity

- Let S represent a subset of pixels in an image . Two pixels p and q are said to be connected in S if there exists path between consisting entirely of pixels in S.
- For any pixel p in S, the set of pixels that are connected to it in S is called a connected component of S. if it only has one connected component the set is called a connected set.
- Example in the pixel arrangement with set V={20,32,40,50}
- The centre pixel 50 has the connected component C= {20,32,40}

10	20	32
40	50	70
100	45	60

REGION AND BOUNDARIES

- **Region**

  Let *R* be a subset of pixels in an image, we call *R* a region of the image if *R* is a connected set.

- **Boundary**

  The *boundary* (also called *border* or *contour*) of a region *R* is the set of pixels in the region that have one or more neighbors that are not in *R*.

Distance measures

If we have 2 pixels: p and q respectively

p with (x,y)

q with (s,t)

- **Euclidean distance between p and q:**

  $De(p,q) = [(x-s)^2 + (y-t)^2]^{1/2}$

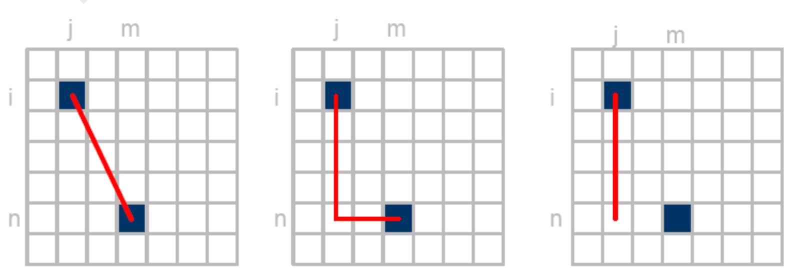
- ***D4* distance (also called *city-block distance*):**

  $D4(p,q) = |x-s| + |y-t|$

- ***D8* distance (also called *chessboard distance*) :**

  $D8(p,q) = \max (|x-s| , |y-t|)$

Distance measures



Euclidean Distance	City Block Distance	Chessboard Distance
$=\sqrt{(i-n)^2 + (j-m)^2}$	$= \lvert i-n\rvert + \lvert j-m\rvert$	$= \max[\ \lvert i-n\rvert, \lvert j-m\rvert\ ]$

Example:

Compute the distance between the two pixels using the three distances :

q:(1,1)

P: (2,2)

Euclidian distance : $((1-2)^2+(1-2)^2)^{1/2}$ = sqrt(2).

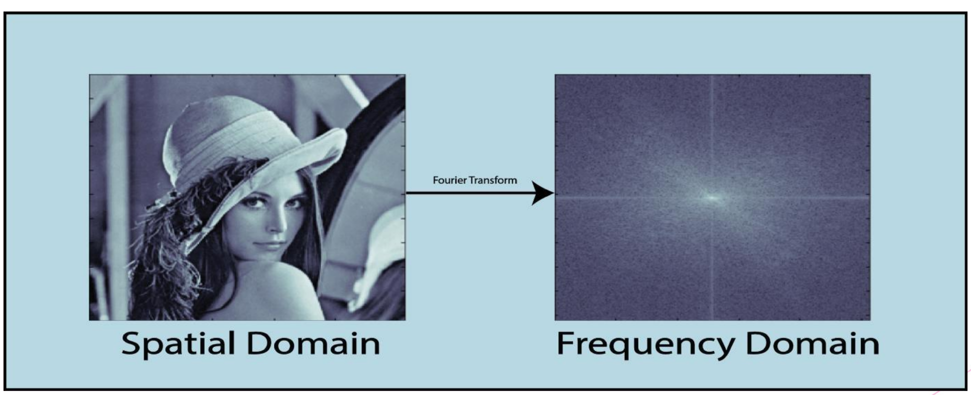
D4(City Block distance): |1-2| +|1-2| =2

D8(chessboard distance) : max(|1-2|,|1-2|)= 1

(because it is one of the 8-neighbors)

	1	2	3
1	q		
2		p	
3			

Fundamentals of spatial domain-convolution operation



Spatial Domain

- An image can be represented in the form of a 2D matrix where each element of the matrix represents pixel intensity.
- This state of 2D matrices that depict the intensity distribution of an image is called Spatial Domain.

Frequency Domain

- In frequency-domain methods are based on Fourier Transform of an image.
- Roughly, the term frequency in an image tells about the rate of change of pixel values.

Fundamental of spatial filtering

- The name filter is borrowed from frequency domain processing where filtering refers to accepting (passing) or rejecting certain frequency components.
- Filtering used to modify or enhancing an image.

Filters in frequency domain:

- Low pass filter: passes low frequencies: used for smoothing (blurring) on the image.
- High pass filter: passes high frequencies: used for sharpening the image.
- Band pass: passes frequencies within a band.
- Band reject: reject frequencies within a band.

- Many image enhancement techniques are based on spatial operations which are performed on local neighbourhood of an input pixels.

Filters in spatial domain:

- Spatial filters used different masks which are also known as kernels, templates or windows.
- There is a one-to-one correspondence between linear spatial filters and filters in frequency domains.

- **Spatial filters** can be classified into two types based on the basis nature of responses.
- Linear and nonlinear filtering. (Frequency domain filters just for linear filtering).
- Filtering creates a new pixel with coordinates equal to the coordinates of the center of the neighborhood, and whose value is the result of the filtering operations.
- If the operation performed on the image pixels is linear then the filter is called a linear spatial filter other wise the filter is non linear.

Spatial filtering

- The subimage is called a filter, mask, kernel, template, or window, with the first three terms being the most prevalent terminology.
- The values in a filter subimage are referred to as coefficients, rather than pixels.
- The concept of filtering has its roots in the use of the Fourier transform for signal processing in the so-called frequency domain.

- Here ,we are interested in filtering operations that are performed directly on the pixels of an image.
- We use the term spatial filtering to differentiate this type of process from the more traditional frequency domain filtering.

- Consider linear spatial filtering using a 3*3 neighborhood.
- At any point (x, y) in the image, the response **g(x, y)** of the filter is the sum of products of the filter coefficients and the image pixels values.

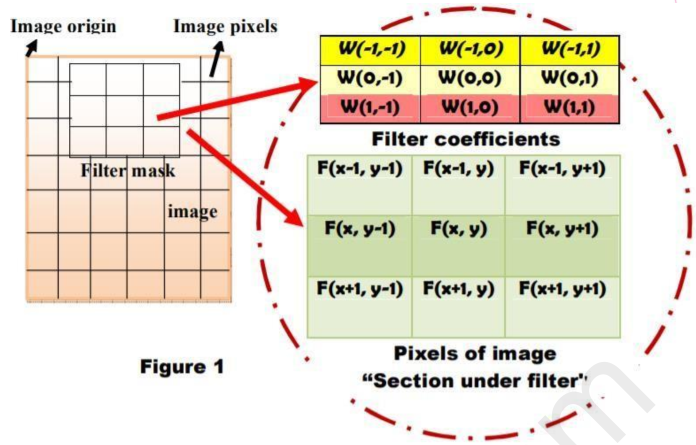


Figure 1

Spatial Correlation and convolution

$$g(x,y) = w(-1,-1)f(x-1,y-1) + w(-1,0)f(x-1,y) + \ldots + w(0,0)\,f(x,y) + \ldots + w(1,1)\,f(x+1,y+1)$$

Observe that the center coefficient of the filter, $w(0,0)$ aligns with the pixel at location(x,y).

General mask of size $m * n$:

Assume that

$$m = 2a + 1$$
$$and$$
$$n = 2b + 1$$

(where a, b are **positive integers**).

(Odd filters)

In general, linear spatial filtering of an image of size $M * N$ with a filter of size $m * n$ is given by the expression:

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t).\,f(x+s,y+t)$$

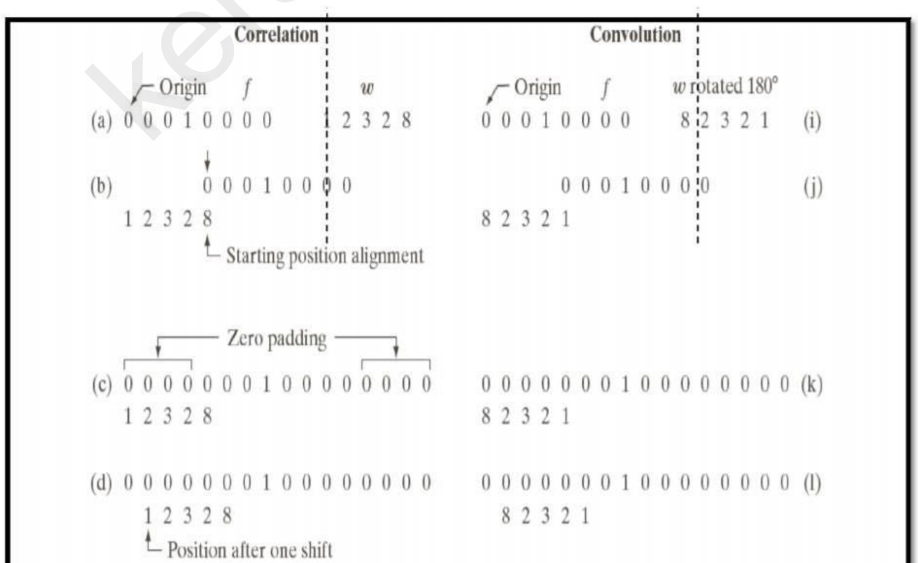
Where x and y are varied so that each pixel in w visits every pixel in f.

Spatial correlation and convolution

- **Correlation:** the process of moving a filter mask over the image and computing the sum of products at each location.

- **Convolution:** the same process as correlation, except that the filter is first rotated by 180 degree.

Example: 1-D illustration: (figure 2)
Assume that ***f*** is a 1-D function, and ***w*** is a filter



(e) 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0   0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 (m)
1 2 3 2 8   8 2 3 2 1
Position after four shifts

(f) 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0   0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 (n)
1 2 3 2 8   8 2 3 2 1
Final position

Full correlation result   Full convolution result
(g) 0 0 0 8 2 3 2 1 0 0 0 0   0 0 0 1 2 3 2 8 0 0 0 0 (o)

Cropped correlation result   Cropped convolution result
(h) 0 8 2 3 2 1 0 0   0 1 2 3 2 8 0 0 (p)

FIGURE 3.29 Illustration of 1-D correlation and convolution of a filter with a discrete unit impulse. Note that correlation and convolution are functions of *displacement*.

- 00

Notes:

- There are parts of the functions (images) that do not overlap (the solution of this problem is pad f with enough 0s on either side to allow each pixel in w to visit every pixel in f.
- If the filter is of size m, we need (m-1) 0s on either side of f.
- The first value of correlation is the sum of products of f and w for the initial position (Figure 2.c). (The sum of product =0) this corresponds to a displacement x =0
- To obtain the second value of correlation, we shift w are pixel location to the right (displacement x=1) and compute the sum of products (result =0).
- The first nonzero is when x=3, in this case the 8 in w overlaps the 1 in f and the result of correlation is 8.
- The full correlation result (figure 2.g) -12 values of x
- To work with correlation arrays that are the same size as f, in this case, we can crop the full correlation to the size of the original function. (Figure 2.h).

- The result of correlation is a copy of w, but rotated by 180 degree.
- The correlation with a function with a discrete unit impulse yields a rotated version of the function at the location of the impulse.
- The convolution with a function with a discrete unit impulse yields a copy of that function at the location of the impulse

Correlation and convolution with images

- With a filter of size ***m*n***, we pad the image with a minimum of ***m-1*** rows of 0s at the top and the bottom, and ***n-1*** columns of 0s on the left and right.
- If the filter mask is ***symmetric***, correlation and convolution yield the same result.

Summary:

- **Correlation** of a filter $w(x,y)$ of size $m * n$ with an image $f(x,y)$ denoted as

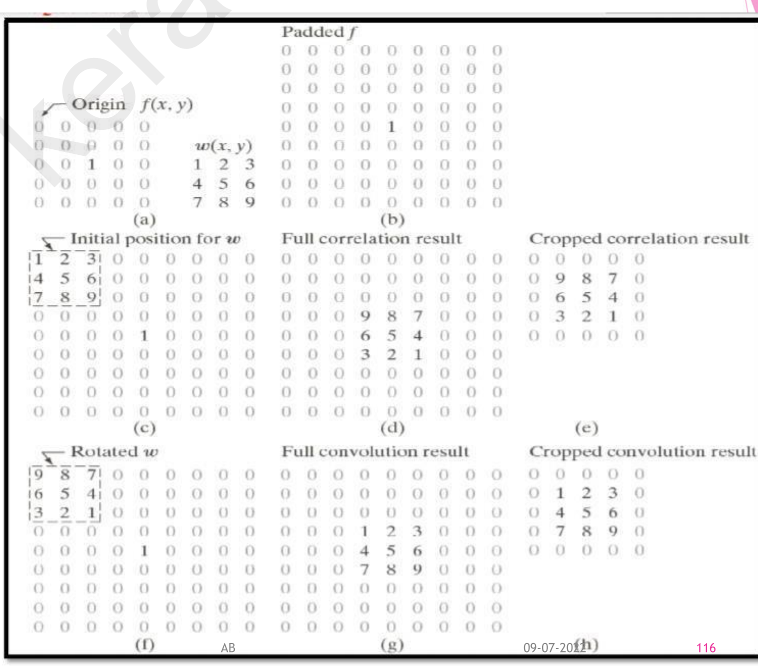
$$W(x,y) \circ f(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

- In similar manner, the **convolution** of $w(x,y)$ and $f(x,y)$ denoted by $w(x,y) * f(x,y)$ is given by:

$$W(x,y) * f(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x-s, y-t)$$

Where the minus sign on the right flip (rotate by 180^0)

(We can flip and shift either f or w)



Vector Representation of linear filtering:

- **Correlation**

$$R = w_1 z_1 + w_2 z_2 + \cdots + w_{mn} z_{mn} = \sum_{k=1}^{mn} w_k z_k = \boldsymbol{w}^T Z$$

- ✓ R- the response of a mask
- ✓ W_k– the coefficients of an m ∗ n filter
- ✓ Z_k – the corresponding image intensities encompassed by the filter

- **Convolution**

We simply rotate the mask by 180^0

Example: The general 3*3 mask equation:

$$R = w_1 z_1 + w_2 z_2 + \cdots + w_9 z_9 = \sum_{k=1}^{9} w_k z_k = \boldsymbol{w}^T Z$$

Where:

W and Z are g-dimensional vectors (mask and image)