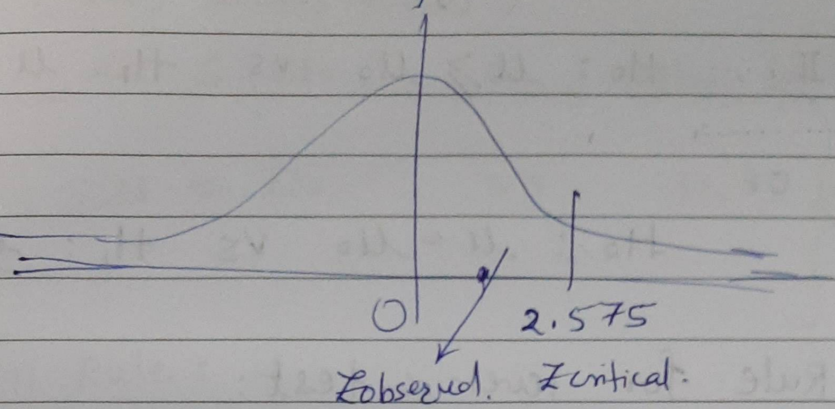
$\therefore \quad Z = 1.68 \not> Z_{\alpha/2} = 2.575$



So we fail to Reject Ho.

Correct level of significance (α) to be used in each situation depends on the severity of situation.
V.Imp
If rejecting Null (Ho) when it is True, gives catastrophic damage then you should choose smaller α.

case III: $\alpha = 0.1$.

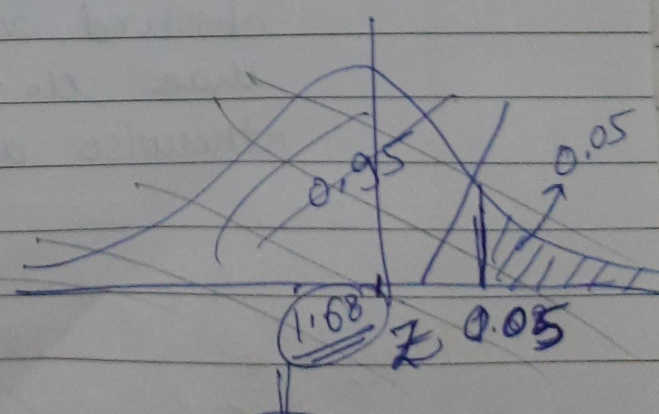
$Z = 1.68$.

$\Phi(Z_{\alpha/2}) = \Phi(Z_{0.05}) = P(Z \leq z) =$

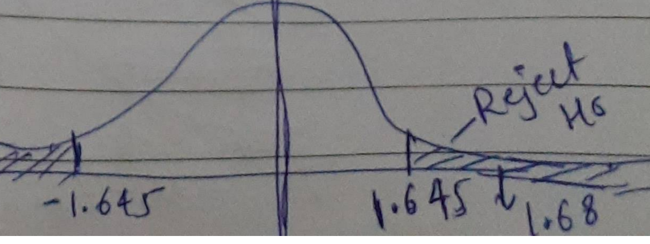
$\Phi(Z_{0.05}) = 0.95$

$\therefore Z_{0.05} = 1.645$.

$\therefore Z = 1.68 > Z_{\alpha/2} = 1.645$



$\therefore$ Reject Ho.



$\therefore$ Only 10% of times even if Ho is True, it is saying False.

Statistics → Non-parametric Statistical Inference
(predicting distribut^n based on sample)

Statistics → Parametric Statistical Inference
(You know the distribution, but you don't know the parameters so based on sample find out those parameters

Hypothesis ⇒ A statement about population parameter.

(Idea that can be tested)

Null hypothesis (H_0) ⇒ Assumed to be True. OR Strong belief

Alternative hypothesis (H_1) ⇒ claim or challenging belief something which you are trying to prove.

H_0 and H_1 are contradictory to each other.

Hypothesis → Simple : ~~$H_0: \mu \leq 2$ or $H_1: \mu \neq 2$~~

$H_0: \mu = 2$ vs $H_1: \mu \neq 2$ $H_0: \mu = 3$ vs $H_1: \mu > 3$

Hypothesis → Composite : does not specify populat^n dist^n.

* A hypothesis (Null or composite), when True, completely specifies the population distribution is called simple hypo & the one that does not is called composite hypothesis.

If you reject Null i.e. Accept alternative then you know $\mu \neq 2$ But you don't know what is Actual mean

Page

Both Null & Alternative hypo can be composite or both can be simple

both composite eg ~~$H_0:\mu$~~ $H_0: \mu > 3$ vs $H_1: \mu \leq 3$

both simple eg. $H_0: \mu = 3$ vs $H_1: \mu = 2$.

Method

① Find H_0 and H_1

② Collection of sample

$$\underbrace{(X_1, X_2, \ldots X_n)}_{\text{Sample}} = \underbrace{(x_1, x_2, \ldots x_n)}_{\text{Realized Value of sample}}$$

$$\bar{x}_n = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Small $\bar{x}_n$ is realized value of sample X_n (nth sample)

③ Decision Rule :-

divedes Sample space (Ω) → Critical / Rejection Region (C) (Reject H_0)

divedes Sample space (Ω) → Acceptance Region (C^*) (Accept H_0)

* If Obsezed sample falls in C then Reject Null Hypo
* If Obsezed sample falls in C^* then Accept Null hypo (H_0)

Date
Page

$P(H) = 1/2$.

Ex.1) whether the coin is ~~B.~~ Unbiased or biased towards the head with $P(H) = 3/4$.

→ ① $H_0 : p = 1/2$ vs. $H_1 : p = 3/4$.

99.99% of Naturally, coins are unbiased i.e. convention or strong belief $\therefore H_0 : p = 1/2$ & we are challenging that $P(H)$. $H_1 : p = 3/4$.

② Sample collection (drawing Sample).

Two samples drawn after 4 coin tosses. → X_1 = HHHT
→ X_2 = HTHT.

③

$\Omega = \{$ HHHH (1)
HHHT, HHTH, HTHH, THHH (4)
THHT, HHTT, HTHT, HTTH, THTH, TTHH, (6)
TTTH, TTHT, THTT, HTTT (4)
TTTT (1) $\}$.

④ ~~Decision Rule~~ X: No. of heads in tosses of coin.
Let X be RV and $X = \{x : x = 0, 1, 2, 3, 4\}$.

④ Decision Rule

$C = \{X = 0$ or $X = 1$ or $X = 3$ or $X = 4\}$ Reject Null

$C^* = \{X = 2\}$. Accept Null.

$\boxed{\Omega = C \cup C^* \text{ \& } C \cap C^* = \phi}$

from sample X_1, we reject Null hypo i.e. coin is biased towards head i.e. Accept ~~Null~~ Alternative.

from sample X_2, Accept Null

We are taking decision based on sample.

$\boxed{\text{Accept } H_0 \equiv \text{Fail to reject } H_0.}$

i.e. Based on sample, we don't have enough evidence to reject Ho i.e. we failed to reject Ho.

4-evening

Testing Procedure ⇒

θ is unknown if we want to test specific hypothesis about θ.

$H_0: \theta \in A$

vs

$H_1: \theta \in A^c$

eg. $N(\mu, \sigma^2)$

$H_0: \mu \leq 10$

vs

$H_1: \mu > 10$

$\therefore$ $A = \{\mu \in \mathbb{R} / \mu \leq 10\}$

$A^c = \{\mu \in \mathbb{R} / \mu \not> 10\}$.

1) Type-I Error :- when we reject Ho, based on the sample, even if it is True

(Reject Ho when Ho is True) i.e originally Null is True and still you are rejecting it.

2) Type-II Error :- when we accept Ho, based on sample, even if it is False

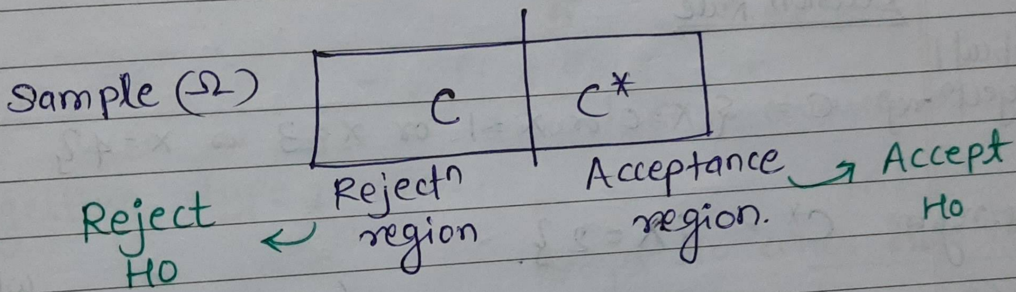
(Accept Ho when Ho is False)

||

H1 is True

		Decision	
		Accept H_0	Reject H_0
Truth	H_0 is True	✓	Type-I
	H_1 is True i.e. H_0 is False	Type-II	✓

eg. Sample $\underset{\sim}{X} = (x_1, x_2, \ldots x_n)$ is a random sample



$\therefore$ Prob. of Type-I error $= P(\underbrace{\text{Reject } H_0}_{\text{Sample} \in \text{Critical Region}} \text{ when } H_0 \text{ is True})$

$= P(\underset{\sim}{X} \in C \text{ when } \theta \in A)$

$\therefore$ Prob of Type-II error $= P(\text{Accept } H_0 \text{ when } H_0 \text{ is False})$

$\equiv H_1$ is True

$= P(\underset{\sim}{X} \in C^* \text{ when } \theta \in A^c)$

$= 1 - P(\underset{\sim}{X} \in C \text{ when } \theta \in A^c)$

Continuing prev. example.

Ho: $p = 1/2$ vs H₁: $p = 3/4$

and we are testing this Hyp. based on tossing the coin 4 times. ($\therefore n=4$)

$$\Omega = \{X / X = \{0,1,2,3,4\}\}.$$

<u>Decision Rule</u>

Critical / Rejectⁿ regⁿ $C = \{X=0 \text{ or } X=1 \text{ or } X=3 \text{ or } X=4\}$

Acc. region $C^* = \{X = 2\}$.

(Reject Ho even when Null Ho is True)

① Prob of type I error $= P(\underset{\sim}{X} \in C \text{ when } \theta \in A)$

$$= P(X=0 \cup X=1 \cup X=3 \cup X=4 \text{ when } p=1/2)$$

when Ho is True

$$\Rightarrow P(X=0)+P(X=1)+P(X=3)+P(X=4), \; p=1/2$$

$$\Rightarrow 1-P(X=2), \; p=1/2$$

Distribution of X is $X \sim Bin(n, p)$

i.e. $X \sim Bin(4, 1/2)$

②

$\therefore P(X=K) = {}^nC_K\, p^K (1-p)^{n-K}$

$P(X=x_2) = 4C \ldots$

$\therefore P(X=2) = 4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2$

$= \frac{4\times 3}{2} \times \frac{1}{4} \times \frac{1}{4} = 3/8.$

$\therefore$ Prob. of type 1 error $= 1 - P(X=2)$, $p = 1/2$ ($P(X=2) = 3/8$)

$= \frac{5}{8} \approx 0.625$

i.e. there is 5/8 or around 60% chance of getting type 1 error. i.e. H_1 is True

Accept H_0 when H_0 is False

(2) Prob. of Type 2 error $= P(X \in C^* \text{ when } \theta \in A^c)$

$X \sim Bin(n, p) \Rightarrow X \sim Bin(4, 3/4)$

$= P(X=2),\ X \sim Bin(4, 3/4)$

$= P(X=2),\ p = 3/4$

H_1 is True.

$=$

$P(X=2) = 4C_2 (3/4)^2 (1/4)^2 = \cancel{6}^{3} \times \frac{3^2}{8\,\cancel{4^2}} \times \frac{1}{4^2} \Rightarrow$

$= \frac{\cancel{4}\times 3}{2} \times \frac{9}{\cancel{4}\,16} \times \frac{1}{16} = \frac{3\times 9}{128} = \frac{9}{32}$

$= \frac{\cancel{4}\times 3}{2} \times \frac{3\times 3}{\cancel{4}\times 4} \times \frac{1}{4\times 4} = \frac{27}{128}$

8×16

* If some-one else designed the Test-2 with decision Rule.

$C = \{X=0, X=1, X=4\}$

$C^* = \{X=2, X=3\}$

*

(1) Prob of Type-1 error = P(Reject Ho when Ho is True)

$= P(X \in C \text{ when } \theta \in A)$

$= P(X=0)+P(X=1)+P(X=4)$ when $X \sim Bin(4, 1/2)$

$= 4C_0 (1/2)^0 (1/2)^4 + 4C_1 (1/2)^1 (1/2)^3 + 4C_4 (1/2)^4 (1/2)^0$

$= \frac{1}{2^4} + \frac{4}{2^4} + \frac{1}{2^4} = \frac{6}{2^4} = \frac{3}{8}.$

(2) Prob of Type-2 error = P(Accept Ho when Ho is false.)

$= P(X \in C^* \text{ when } \theta \in A^c)$

$= P(X=2)+P(X=3)$ when $X \sim Bin(4, 3/4)$, $p = 3/4$

$= 4C_2 (3/4)^2 (1/4)^2 + 4C_3 (3/4)^3 (1/4)^1$

$= 6 \times \frac{3^2}{4^4} + \frac{4 \times 3^3}{4^4} = \frac{45}{64}$

Type-I

Type

VImp

moto

	1st Test		2nd Test
Type-I	5/8	>	3/8
Type-II	9/32	<	45/64

If you try to reduce one error then other error will increase. So the tests cannot be compared.

V.Imp Conclusion ⇒ Consider set of all tests which has same type I error & among them find the one which has minimum type II error. That is the Best Test.

* Type I error is more dangerous than Type II error

θ is unknown & we want to test a specific hyp about θ.

5-morning

* How to mathematically define a Best Test.

observed sample

Prob. of Type I error = $P(\underset{\sim}{X} \in C \text{ if } \theta \in A)$ —— ①

Prob of Type II error = $P(\underset{\sim}{X} \in C^* \text{ if } \theta \in A^c)$ —— ②

$= 1 - P(\underset{\sim}{X} \in C \text{ if } \theta \in A^c)$

from ① & ②

Null

$P(\underset{\sim}{X} \in C)$ = Prob. of type I error, when $\theta \in A$

= 1 - Prob of type II error, when $\theta \in A^c$

Alternative

$P(\underset{\sim}{X} \in C^*)$ = Prob. of type II error when $\theta \in A^c$

* Power function :

Power fn of a hypothesis test with rejection/critical region C is defined as

$\beta(\theta) = P(\underset{\sim}{X} \in C)$ // probability of Rejection region.

Note ⇒ The ideal power of a best ~~Test~~ Test should have

$$\beta(\theta) = \begin{cases} 0, & \text{if } \theta \in A \quad (H_0 \text{ is True}) \\ 1, & \text{if } \theta \in A^c \quad (H_0 \text{ is false}) \end{cases}$$

Accep

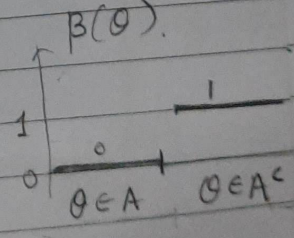
* If H_0 is true then the power f^n $\beta(\theta)$ should be zero i.e. the probability of rejection is zero and it means probability of accept is 1 so accept Null Hyp (H_0)

* If H_0 is false i.e. alternative is True (H_1 is True) then the power f^n $\beta(\theta)$ should be 1 i.e. probability of rejection is 1 which means reject the Null Hyp (H_0)

When H_0 is actually True, accept H_0
When H_0 is actually False, Reject H_0 } ideal power of Test

As the ideal power f^n doesn't exist in practical scenario, So a Good test is a test in which power f^n is close to zero when $\underbrace{\theta \in A}_{\text{Null is True}}$ and power f^n is close to 1 when $\underbrace{\theta \in A^c}_{\text{Null is False}}$

Ideally Power of a Test,



Prob of type I error $= P(X \in C$ if $\theta \in A)$
(Reject H_0 if H_0 is True) $= \beta(\theta)$ if $\theta \in A$
$= 0$

Prob of Type II error $= 1 - P(X \in C$ if $\theta \in A^c)$
(Accept H_0 if H_0 is false) $= 1 - \beta(\theta)$ if $\theta \in A^c$
$= 1 - 1$
$= 0$

why does the increasing f^n takes maximum value at end point ?

* **Size-α-test :-**

For $0 \le \alpha \le 1$, a test with power f^n $\beta(\theta)$ is called a size-α-test if

$$\sup_{\theta \in A} \beta(\theta) = \alpha .$$

(Ho is True) probability of Rejection

Maximum probability of Rejecting the Null hyp when it is True is α.

i.e α is Maximum Type I error

$\therefore$ Test with maximum Type I error is called as size-α-test.

eg $\alpha = 0.01$ then there is 1% chance that Even though Ho is True you will still reject Ho.

* **level-α-test / Significance level :-**

$$\sup_{\theta \in A} \beta(\theta) \le \alpha$$

Maximum probability of Rejecting Ho when it is True is atmost α.

i.e. Maximum Prob of Type I error is atmost α.

level-α test ⟶ P(Type I error) is maximum α or atmost α.

size-α-test ⟶ P(Type I error) is α

⇑

eg. Population has $X \sim Bin(5, \theta)$ ($n = 5$, $p = \theta$). Quality testing is done by 5 diff machines.

θ is the failure probability of the product on each machine. Company has strong belief that $\theta \leq 1/2$ & someone in the company claims that $\theta > 1/2$ i.e, product will fail the quality test ~~with~~ more than 50% chance that

→ ~~X is RV which counts no. of failures out of~~ 5 tests.

→ Test I decision Rule   Test II ~~decision Rule~~.

Reject H_0 if $X = 5$ Reject H_0 if $X = 3, 4, 5$

RV X: Number of quality test failures.

Test I ⇒

$$\beta_1(\theta) = P(\underset{\sim}{X} \in C)$$

$$= P(X = 5)$$

$$= {}^5C_5\, \theta^5 (1-\theta)^0$$

$$= \theta^5$$

$P(\theta \in C)$ $P(\theta \in C^*)$

$H_0: \theta \leq 1/2$ vs $H_1: \theta > 1/2$

and $X \sim Bin(5, \theta)$

$X \sim Bin(5, \theta)$

classmate

Date

Page

size-α-test for Test I $= \sup_{\theta \in A} \beta(\theta)$

Ho true

$= \sup_{\theta \leq 1/2} \beta(\theta)$

$= \sup_{\theta \leq 1/2} \theta^5$ increasing fn so taking max value

$= (1/2)^5 = 1/32$

Test II:

$\beta_2(\theta) = P(\underline{X} \in C)$

$= P(X=3) + P(X=4) + P(X=5)$

$= 5C_3\, \theta^3 (1-\theta)^2 + 5C_4 (\theta)^4 (1-\theta) + 5C_5 (\theta)^5 (1-\theta)^0$

$= 10 . \theta^3 (1-\theta)^2 + 5 (\theta)^4 (1-\theta)$

$= 5\theta^3 (1-\theta) [\, 2(1-\theta) + \theta \,]$

$= 5\theta^3 (1-\theta) [\, 2 - 2\theta + \theta \,]$

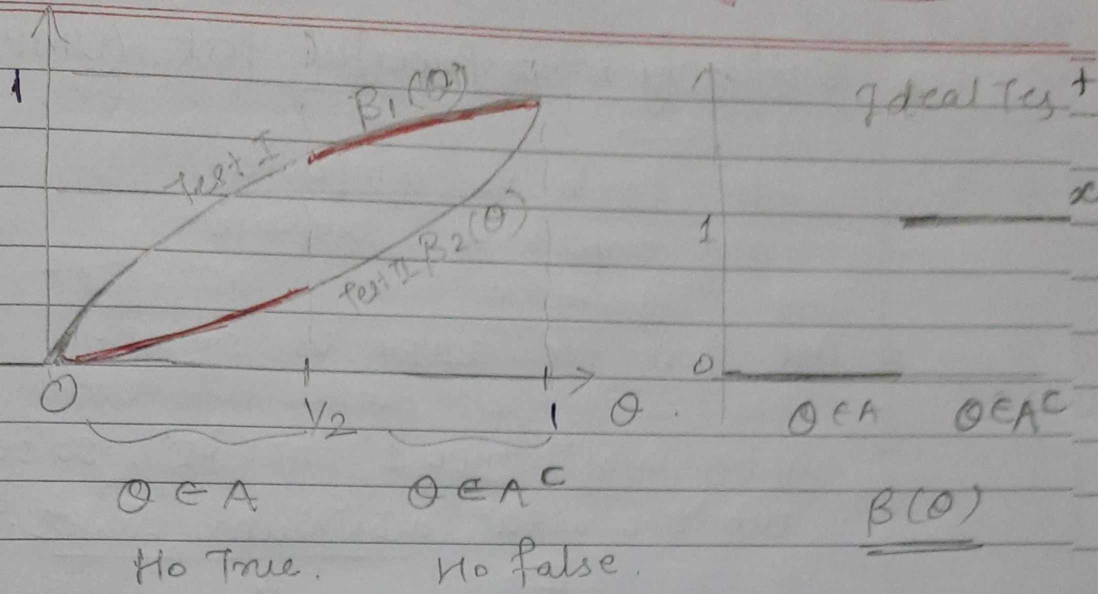
$= 5\theta^3 (1-\theta)(2-\theta) = 5\theta^3 (2 - \theta - 2\theta + \theta^2)$

$= 5\theta^3 (\theta^2 - 3\theta + 2)$

size-α-test for Test II $= \sup_{\theta \in A} \beta_2(\theta)$

$= \sup_{\theta \leq 1/2} (\quad)$

$=$



Test I has maximum Type I error
Test II has maximum Type II error.

$\therefore$ For Test II, as Type I error is less ~~as~~
so it is best

and for test I, type II error is less
so it is best.

* Power of a Test $\Rightarrow$

The power of a ~~tes~~ level-α-test with power f^n $\beta(\theta)$ for testing $H_0: \theta \in A$ vs $H_1: \theta \in A^c$ is defined as

$$\beta^* = \underbrace{\sup_{\theta \in A^c} \beta(\theta)}_{}$$

prob of Reject^n when Alternative is True i.e. H_0 is false

power

$*$ then maximum probability that will reject H_0.

$1 - \beta^*$ = min probability that when H_0 is false will accept it.

* Uniformly most Powerful Test (UMP) ⇒

for simple vs composite
or composite vs composite

* MP Test for simple vs simple.

& has max^m type 1 - error 'α', so select the one with maximum power.

when Ho false, Reject Ho

Let D be class of all level-α-tests for testing $H_0: \theta \in A$ vs $H_1: \theta \in A^c$. A test in class D with power f^n $\beta(\theta)$ is called UMP level-α-test if

$$\beta^* = \sup_{\theta \in A^c} \beta(\theta) \geq \sup_{\theta \in A^c} \beta'(\theta)$$

(β^*) If power of test is bigger than any other test

for any test with power f^n $\beta'(\theta)$

* Most powerful Test (MP) :- Particular case of UMP.

If A and A^c are singletone sets, then a level-α-test (UMP) is called MP test.

singletone : Simple vs simple hyp.

If hyp is simple vs composite OR
composite vs composite } UMP test.

If hyp is simple vs simple then we try to find MP level-α-test

Maximum Type-I error $= \alpha$

Tutorial 23

* Testing procedures for population having Normal distribution.

Test : Population having $N(\mu, \sigma^2)$

Target: What is μ & σ^2 under diff scenarios.

$(x_1, x_2, \ldots x_n)$ is Random Sample drawn from Popu^n

Case I : Known variance i.e. σ^2 is known.

Subcase I → Suppose, $\boxed{H_0 : \mu = \mu_0 \quad vs \quad H_1 : \mu \neq \mu_0}$ Subcase I

where μ_0 is given. Two sided test or Two tailed Test

→ Decision Rule (level-α-test)

Reject H_0 if

$$\underbrace{|Z|}_{Z \text{ observed}} = \left| \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \right| > \underbrace{Z_{\alpha/2}}_{Z \text{ critical}}$$

Mod cz its two tailed test.

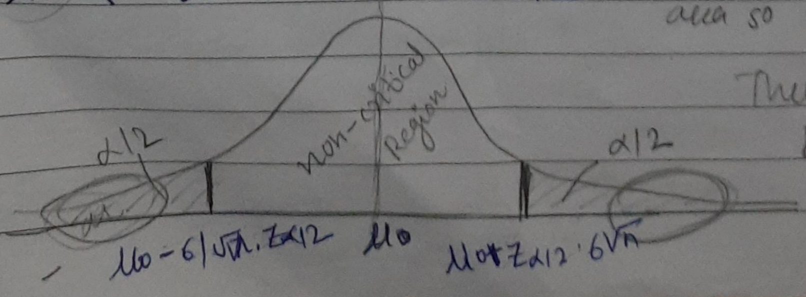
Sample mean

either $\bar{X} > \mu_0 + \sigma/\sqrt{n} \cdot Z_{\alpha/2}$

or $\bar{X} < \mu_0 - \sigma\sqrt{n} \cdot Z_{\alpha/2}$

observed $\bar{x}$ favoured the Alternative in area so reject H_0

Then reject H_0



0.005 5 1-0.005

Date
Page

Tutorial-Sheet 1

(23)

(6). $\boxed{A} \xrightarrow{\mu = 8} \boxed{B}$

$\mu + X$ $\longrightarrow$ $X \sim N(0, 4)$ Random Noise

$E(X) = 0$

value Recived at locat^n B

Strong belief | **Challenging belief**

$H_0 : \mu_0 = 8$ vs $H_1 : \mu_0 \neq 8$

i.e. on an avg Added random noise is actually 0

1) $\bar{x} = 9.5$ and $n = 5$.

$\bar{x}$: their avg of those $\leftarrow$ n: Same signal value is sent 5 times

$Var(X) = 4$ $\downarrow$ Random noise can vary by this quantity

$\mu + X \sim N(\mu, 4)$

2) $$Z = \frac{\bar{x} - \mu}{6/\sqrt{n}} = \frac{9.5 - 8}{2/\sqrt{5}} = \frac{1.5}{2/\sqrt{5}} = 1.68$$

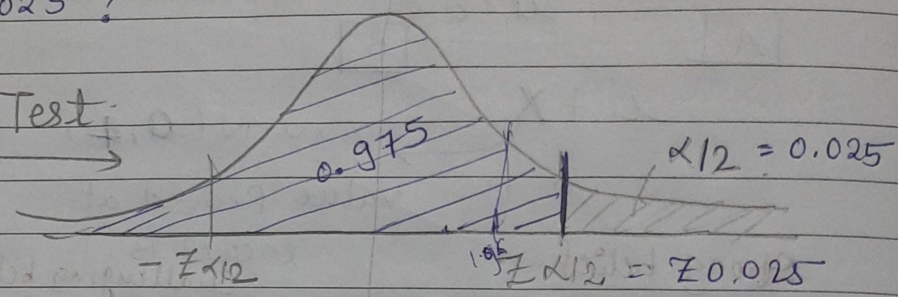
3) Assuming level-α-test OR significance level

Case I: $\alpha = 0.05$ i.e. 5% of the time even though the mean is 8, will reject it

$Z_{\alpha/2} = Z_{0.025}$ (From Normal table)

$Z_{0.025}$?

Two Sided Test



$\Phi(Z) = \cancel{\Phi(Z \leq 0.}$

$\Phi(z) = P(Z \leq z)$

$\Phi(\underbrace{Z_{0.025}}_{x}) = 0.975$ now find value from normal table. (Table is giving value of probabilities) Search for 0.975 & then get value of x

$Z_{0.025} = 1.96$

$\therefore Z_{\alpha/2} = 1.96$

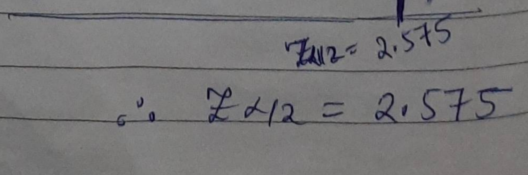
$Z = 1.68 \ngtr 1.96 = Z_{\alpha/2}$

i.e. Fail to Reject Ho. i.e. Accept Ho.

Case II : Significance level $\alpha = 0.01$

$Z = 1.68$

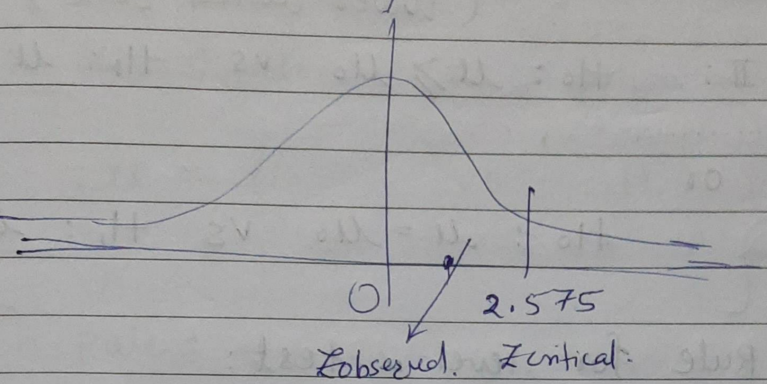
$Z_{\alpha/2} = Z_{0.005}$



$\Phi(z) = P(\underset{\sim}{Z \leq z})$

$\Phi(Z_{0.005}) = 0.995$ $\therefore Z_{\alpha/2} = 2.575$

$\therefore \; Z = 1.68 \not> Z_{\alpha/2} = 2.575$



So we fail to Reject H_0.

2.575 ~~2.580~~

Correct level of significance (α) to be used in each situation depends on the severity of situation.
Imp
If rejecting Null (H_0) when it is True, gives catastrophic damage then you should choose smaller α.

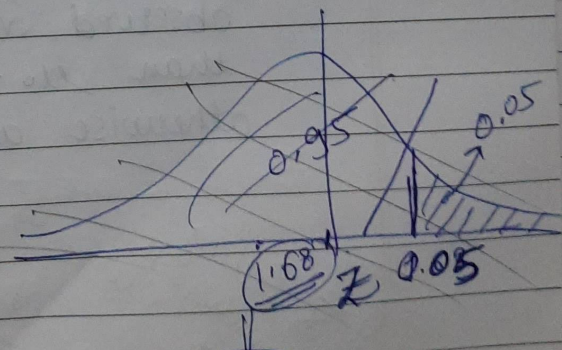
Case III: $\alpha = 0.1$.

$Z = 1.68$.

$\Phi(Z_{\alpha/2}) = \Phi(Z_{0.05}) = P(Z \le Z) =$

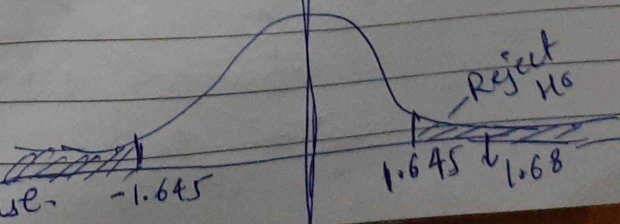
$\Phi(Z_{0.05}) = 0.95$

1.645 $\therefore Z_{0.05} = 1.645$.



$\therefore Z = 1.68 > Z_{\alpha/2} = 1.645$

$\therefore$ Reject H_0.



$\therefore$ Only 10% of times even if H_0 is True, it is saying False.

$X_1, X_2, \ldots X_n \sim N(\mu, \sigma^2)$

Case II: σ^2 is known

(Lower tailed test)

Subcase II: $H_0: \mu \geq \mu_0$ vs $H_1: \mu < \mu_0$

One Sided Test

or

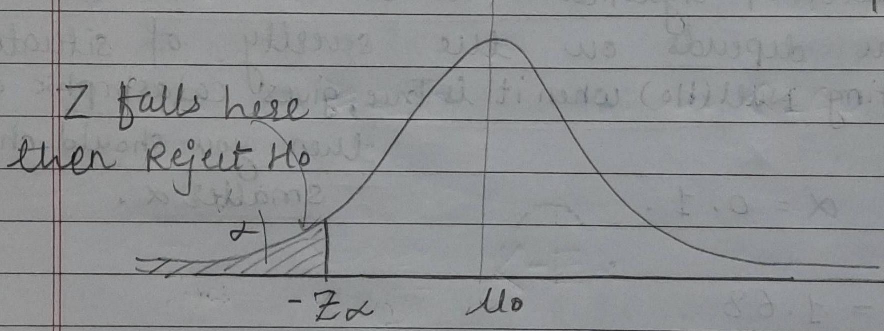
$H_0: \mu = \mu_0$ vs $H_1: \mu < \mu_0$

→ Decision Rule for level-α-test:

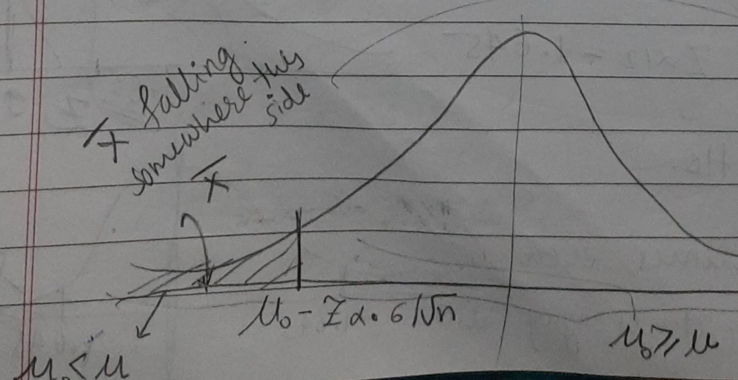
Reject H_0 if

$$Z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} < -Z_\alpha$$

one sided Test



$\Rightarrow \overline{X} < \mu_0 - Z_\alpha \cdot \sigma/\sqrt{n}$ i.e. If the observed value of sample mean is less than $\mu_0 - Z_\alpha \cdot \sigma/\sqrt{n}$ then you reject otherwise accept.



then Reject Ho. as our observed ($\overline{X}$) sample favoured Alternative i.e $\mu_0 < \mu$

Subcase III : (Upper tailed Test)

$H_0 : \mu \leq \mu_0$ vs $H_1 : \mu > \mu_0$.

or $H_0 : \mu = \mu_0$ vs $H_1 : \mu > \mu_0$.

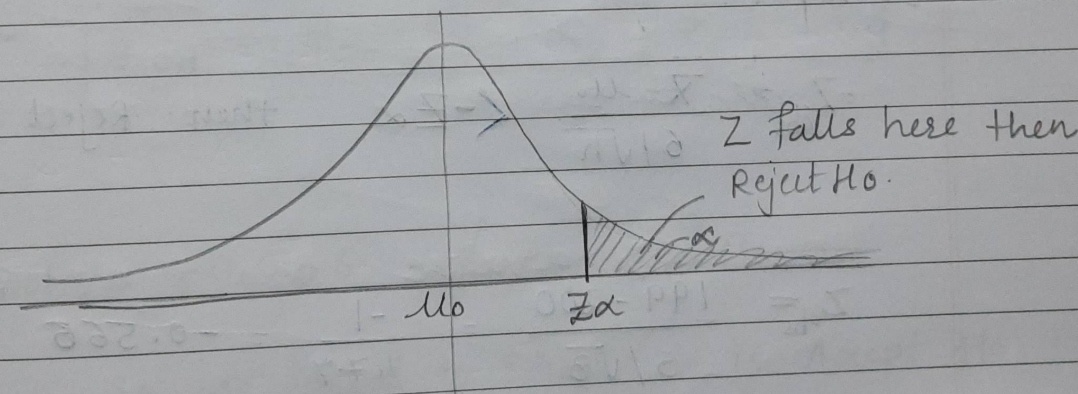
→ Decision Rule :

Reject H_0 if

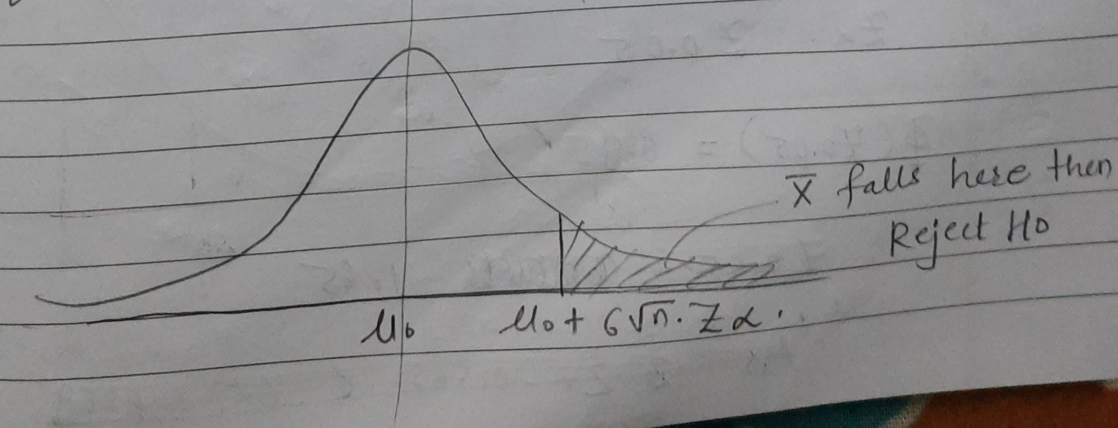
$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} > Z_\alpha$$

(upper tailed)

$$\Rightarrow \bar{X} > \mu_0 + \sigma/\sqrt{n} \cdot Z_\alpha$$



OR if observed mean $\bar{X} > \mu_0 + \sigma\sqrt{n} \cdot Z_\alpha$



(24) Breaking strength $\mu_0 \geq 200$ psi
is atleast 200 psi.

Std deviation $\sigma = 5$ psi

$n =$ ~~8~~ $n =$ ~~8~~ Sample size $= 8$

$\underline{X} = \{210, 195, 197, 199, 198, 202, 196, 195\}$.

$\Rightarrow$ $\overline{X} = 199$.

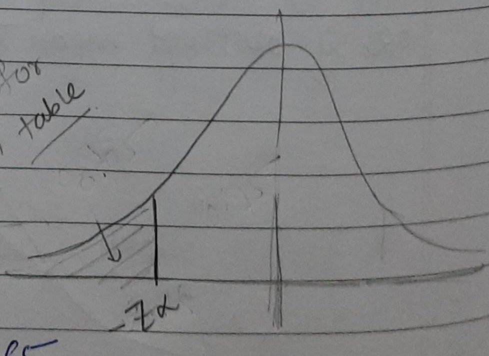
$\Rightarrow$ Test 1: 5% level of significance.

$\alpha = 0.05$

$H_0: \mu_0 \geq 200$ $\quad$ $H_1: \mu_0 < 200$ $\quad$ lower tailed test

$Z_{obs} = \dfrac{\overline{X} - \mu_0}{\sigma / \sqrt{n}} < -Z_\alpha$ then Reject H_0.

$Z_{obs} = \dfrac{199 - 200}{5/\sqrt{8}} = \dfrac{-1}{1.77} = -0.565$



0.045
+0.005
1.050

$Z_\alpha = Z_{0.05}$ $\quad$ now search for 0.95 in table

$\Phi(Z_{0.05}) = 0.95$
$(1-0.05)$

1.6
0.045
0.64

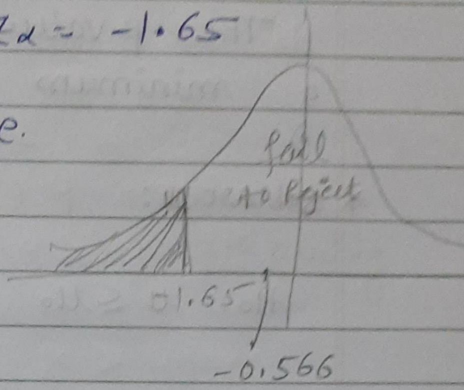
$\therefore Z_{0.05} = 1.65$

~~As $Z = -0.565 < Z_\alpha = 0.8289$~~ $\quad$ ~~Reject H_0~~

$\therefore \quad -Z_\alpha = -1.65$

$Z_{obs} = -0.566 \not< -Z_\alpha = -1.65$

So we failed to Reject Ho i.e. we should accept Ho.



Test II : $\alpha = 0.1$.

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = -0.566.$$

$\Phi(Z_{0.1}) = 0.9$

$\therefore \boxed{Z_{0.1} = 1.9}$ ~~or 539~~. $\therefore \quad -Z_\alpha = -1.9$.

$Z_{obs} = -0.566 \not< -Z_\alpha = -1.9$

$\therefore$ So we failed to Reject Ho i.e. Accept Ho.

i.e. Company's claim about the mean breaking strength is atleast 200 psi is correct.

* P-value (Tells what significance level (α) to be chosen)

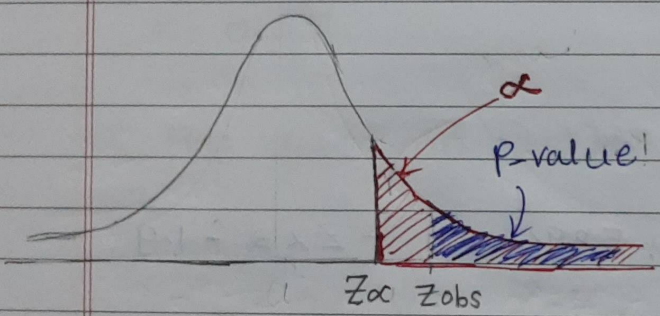
The p-value of a test is the probability of minimum Rejection Region.

*** Subcase III :**

$H_0: \mu \leq \mu_0$ vs $H_1: \mu > \mu_0$.

If $Z_{obs} > Z_\alpha$ then Reject H_0.

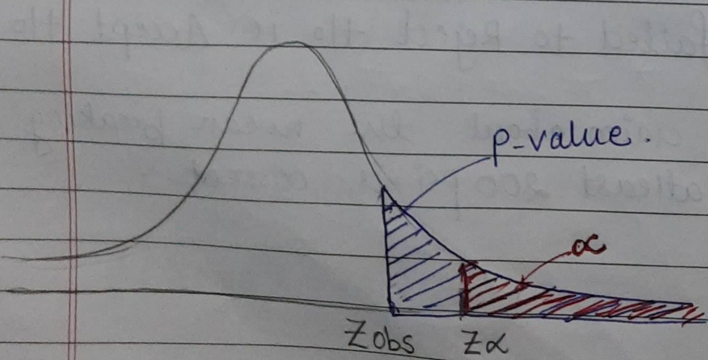
p-value $= P(Z > Z_{obs})$



$Z_{obs} = \dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}}$

$Z_{obs} > Z_\alpha$ then Reject H_0, here Z_{obs} is lying in Rejection region so Reject Null Hyp.

Here, $\alpha >$ p-value, then Reject H_0



Here, $\alpha \leq$ p-value, then Fail to Reject H_0 i.e. Accept H_0.

For Subcase III

P_value = $P(Z > Z_{obs})$

$\alpha >$ P_value, Reject Ho

$\alpha \leq$ P_value, Accept Ho.

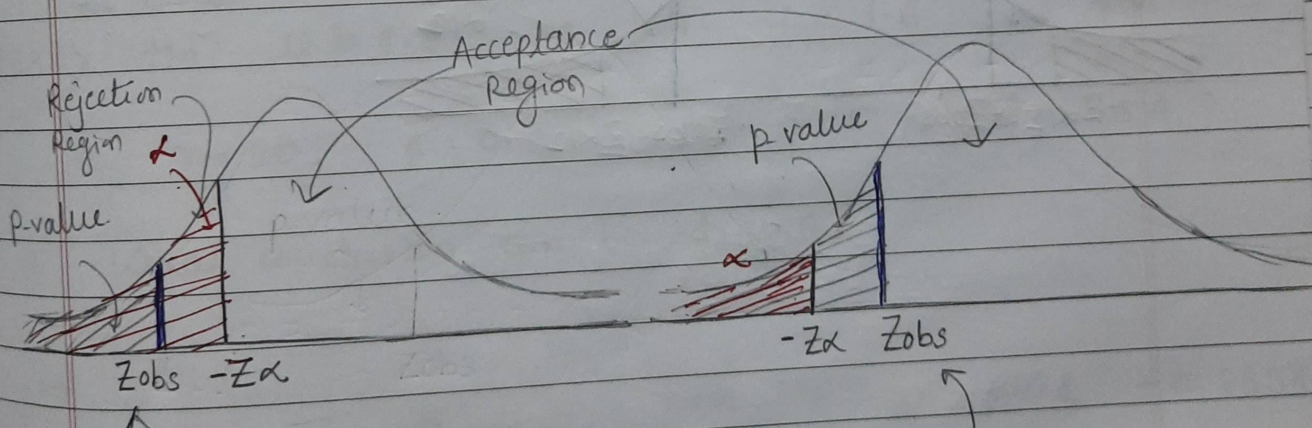
7th evening

* Subcase II :

$H_0: \mu \geq \mu_0$. $H_1: \mu < \mu_0$

If $Z_{obs} < -Z_\alpha$ then Reject Ho.

P_value = $P(Z < Z_{obs})$



P_value $< \alpha$, then Reject Ho

P_value $\geq \alpha$, then Accept Ho.

Subcase C: $H_0: \sigma^2 = \sigma_0^2$ vs $H_1: \sigma^2 \neq \sigma_0^2$

Two sided Test

or

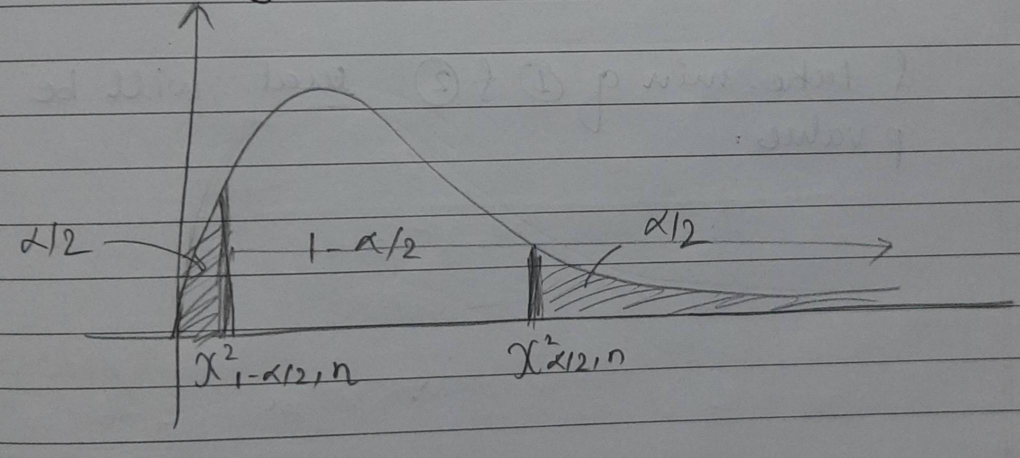
$H_0: \sigma = \sigma_0$ vs $H_1: \sigma \neq \sigma_0$

* Decision Rule:

either $$W = \sum_{i=1}^{n} \left(\frac{x_i - \mu}{\sigma}\right)^2 > \chi^2_{\alpha/2, n}$$

or $$W = \sum_{i=1}^{n} \left(\frac{x_i - \mu}{\sigma}\right)^2 < \chi^2_{1-\alpha/2, n}.$$

then Reject H_0 ~~else~~ otherwise accept it.



$$\text{P-value} = 2 \min\left\{ P(W \leq W_{obs}),\ 1 - P(W \leq W_{obs}) \right\}$$

Normal distⁿ is symmetric, so p-value $= P(Z > |Z_{obs}|)$ but Chi-sqʳ is right skewed distribution.

~~P(W≤α~~

$P(W \leq W_{obs})$ if W_{obs} is on LHS.

$P(W \geq W_{obs}) = 1 - P(W \leq W_{obs})$ if W_{obs} in on RHS

Case IV: μ is unknown

$W^* = \sum_{i=1}^{n} \left(\frac{x_i - \bar{x}}{\sigma}\right)^2 \sim \chi^2_{(n-1)}$ for unknown mean

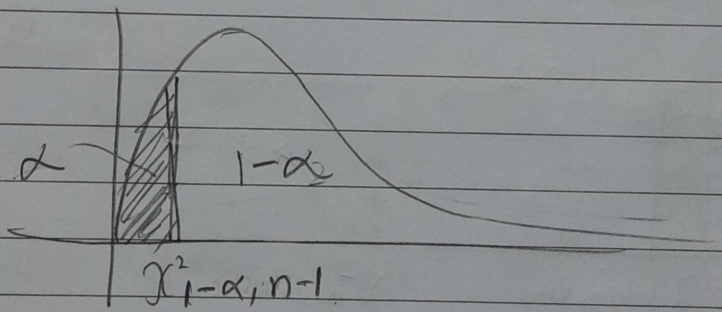
Subcase A: $H_0: \sigma^2 \geq \sigma_0^2$ vs $H_1: \sigma^2 < \sigma_0^2$

Decision Rule $\Rightarrow$

Reject H_0 if

$$W^* = \frac{(n-1)s^2}{\sigma_0^2} = \sum_{i=1}^{n} \left(\frac{x_i - \bar{x}}{\sigma_0}\right)^2 < \chi^2_{1-\alpha,\, n-1}$$

p-value $= P(W^* \leq W^*_{obs})$

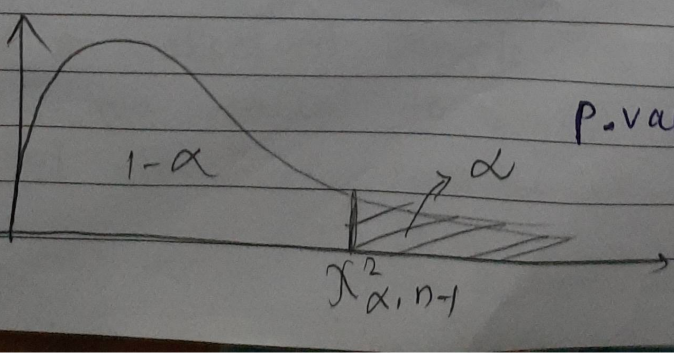


Subcase B: $H_0: \sigma^2 \leq \sigma_0^2$ vs $H_1: \sigma^2 > \sigma_0^2$

Decision Rule $\Rightarrow$

Reject H_0 if

$$W^* = \frac{(n-1)s^2}{\sigma_0^2} = \sum_{i=1}^{n} \left(\frac{x_i - \bar{x}}{\sigma_0}\right)^2 > \chi^2_{\alpha,\, n-1}$$



p-value $= P(W^* > W^*_{obs})$

Subcase C: $H_0: \sigma^2 = \sigma_0^2$ vs $H_1: \sigma^2 \neq \sigma_0^2$

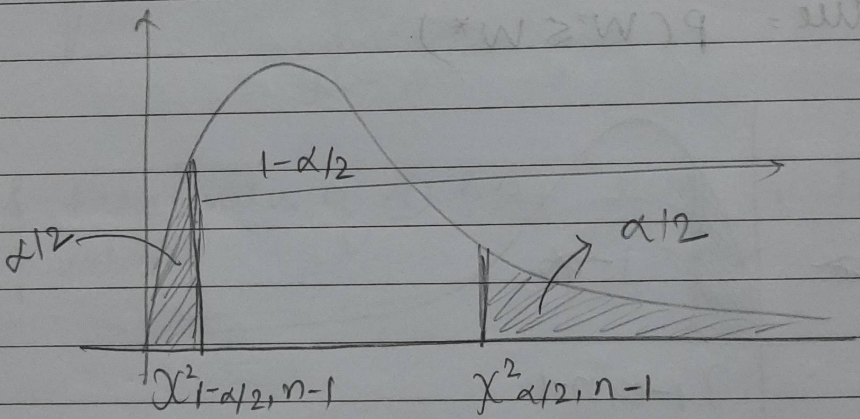
Two sided test

Decision Rule ⇒

Reject H_0 if

either $W^* = \frac{(n-1)s^2}{\sigma_0^2} > \chi^2_{\alpha/2, n-1}$

or $W^* = \frac{(n-1)s^2}{\sigma_0^2} < \chi^2_{1-\alpha/2, n-1}$



$$\text{P-value} = 2\min\left\{ P(W^* \leq W^*_{obs}),\ 1 - P(W^* \leq W^*_{obs}) \right\}$$

$Z \leftarrow \sigma$ unknown μ known $\rightarrow W$

$T \leftarrow \sigma^2$ known μ unknown $\rightarrow W^*$

9th sve.

Tutorial 1 Ex.16)

~~$H_0: \sigma_0 = 0.0003$~~ Let σ^2 is the diameter variance,

$H_0: \sigma^2 \leq 0.0002$ vs $H_1: \sigma^2 > 0.0002$

Strong belief

~~$H_0: \sigma^2 \geq 0.0002$~~ ~~$H_1: \sigma^2 < 0.0002$~~

$n = 10$, sample variance $= s^2 = 0.0003$

$\alpha = 0.05$.

→ μ is unknown & lower tailed test.

$$W^*_{obs} = \frac{(n-1)s^2}{\sigma^2} > \chi^2_{\alpha, n-1}$$

$$W^* = \frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$$

$$W^*_{obs} = \frac{9 \times 0.0003}{(0.0002)}$$

$$W^* = 13.5$$

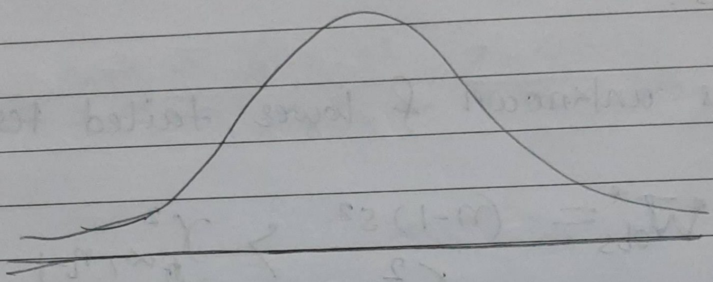
$$\chi^2_{\alpha, n-1} = \chi^2_{0.05, 9} = 16.919.$$

$$W^*_{obs} = 13.5 \not> \chi^2_{\alpha, n-1} = 16.919$$

So failed to Reject Null hypo.

Based on observed sample we may conclude that we failed to Reject H0 i.e., we accept H0. i.e. companies claim is correct

$$P\text{-value} = P(W^* > W^*_{obs})$$
$$= P(W^* > 13.5)$$
$$= 1 - P(W^* \leq 13.5)$$
$$= 1 - \int_0^{13.5} f_w(w)\, dw.$$



Case II unknown variance

Ex. 21) $\mu \Rightarrow$ mean $= 5.7$

$SD = 0.5$ mg.

$n = 10$

$\alpha = 0.05$

$H_0: \sigma^2 \geq 0.5$ $H_1: \sigma^2 < 0.5$

Strong belief . challenging

5.728, 5.731, 5.722, 5.719, 5.727, 5.724, 5.726, 5.718, 5.723, 5.722

$\rightarrow$ $H_0: \sigma = 0.5$ vs $H_1: \sigma < 0.5$

Case III: Subcase II

$$W = \sum_{i=1}^{n} \left(\frac{x_i - \mu}{\sigma_0} \right)^2 \sim \chi^2_n$$

Date
Page

$$W_{obs} = \sum_{i=1}^{10} \left(\frac{x_i - \mu}{\sigma}\right)^2$$

$$= \sum_{i=1}^{10} \left(\frac{x_i - 5.7}{0.5}\right)^2$$

$$= \left(\frac{5.728 - 5.7}{0.5}\right)^2 + \left(\frac{0.031}{0.5}\right)^2$$

$$= \frac{1}{(0.5)^2}\Big((0.028)^2 + (0.031)^2 + (0.022)^2 + (0.019)^2 + (0.027)^2 + (0.024)^2 + (0.026)^2 + (0.018)^2 + (0.023)^2 + (0.022)^2\Big)$$

$$= 0.00054. \text{ check}$$

$$\chi^2_{1-\alpha, n} = \chi^2_{0.95, 10} = 3.940$$

$$W_{obs} = 0.00054 < \chi^2_{1-\alpha, n} = 3.940.$$

So we Reject H_0. ie, ~~the~~ company should can work on producing new drug.

$Var = \sigma^2$
$SD = \sqrt{Var}$

$\frac{(n-1)s^2}{\sigma^2}$

Ex.20) $\sigma_0^2 = 5000$
$n = 26$, $s^2 = 7200$
$\alpha = 0.02$
Max^m type I error is 2%

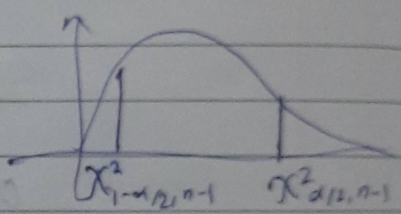
$H_0: \sigma^2 = 5000$ $H_1: \sigma^2 \neq 5000$ challenging claim

μ unknown $W^* = \frac{(n-1)s^2}{\sigma_0^2} \sim \chi^2_{\alpha/2, n-1}$

or $W^* = \frac{(n-1)s^2}{\sigma_0^2} \sim \chi^2_{1-\alpha/2, n-1}$

ans: $H_0: \sigma^2 = 5000$ vs. $H_1: \sigma^2 \neq 5000$

$$W^* = \frac{(n-1)s^2}{\sigma_0^2} \sim \chi^2_{n-1}$$



$$W^*_{obs} = \frac{25 \times 7200}{5000} = 36$$

i) $\chi^2_{\alpha/2, n-1} = \chi^2_{0.01, 25} = 44.314$

ii) $\chi^2_{1-\alpha/2, n-1} = \chi^2_{0.99, 25} = 11.524$

either $W^*_{obs} = 36 \not> \chi^2_{\alpha/2, n-1} = 44.314$

or $W^*_{obs} = 36 \not< \chi^2_{1-\alpha/2, n-1} = 11.524$

So we fail to Reject Null ie Accept Null hyp ie. the claim is correct ie var of lifetime of battery is 5000 $(hr)^2$

Indepe

$$P\text{-value} = 2 \min\left\{ P(W^* \leq W^*_{obs}),\ 1 - P(W^* \leq W^*_{obs}) \right\}$$

$$= 2 * \min\left\{ P(W^* \leq 36),\ 1 - P(W^* \leq 36) \right\}$$

$$= 2 * \min\{$$

10th morning

* Testing of the means of two Normal populations.

Independent

$\{X_1, X_2, \ldots X_n\} \sim N(\mu_x, \sigma_x^2) \leftarrow$ 1st population

$\{Y_1, Y_2, \ldots Y_n\} \sim N(\mu_y, \sigma_y^2) \leftarrow$ 2nd population

Thm:- Let $\{x_1, x_2, \ldots x_n\}$ and $\{Y_1, Y_2, \ldots Y_n\}$ be two independent samples, drawn from two different normal populations $N(\mu_x, \sigma_x^2)$ and $N(\mu_y, \sigma_y^2)$ respectively. then

Tut 2)

Ques Ex. 1) lake A : 11.5, 10.8, 11.6, 9.4, 12.4, 11.4, 12.2, 11, 10.6, 10.8

lake B : 11.8, 12.6, 12.2, 12.5, 11.7, 12.1, 10.4, 12.6

$\sigma_x^2 = 0.09, \quad \sigma_y^2 = 0.16, \quad \alpha = 0.05, \quad n = 10, \; m = 8$

Could you reject the claim that the two lakes are equally contaminated?

→

claim — challenging claim

$H_0: \mu_x = \mu_y$ vs $H_1: \mu_x \neq \mu_y$.

μ_x = mean PCB concentration of fishes at lake A

μ_y = —— " —— lake B.

$$Z_{obs} = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{\sigma_x^2}{n} + \frac{\sigma_y^2}{m}}}$$

$$\bar{X} = \frac{1}{10}(11.5 + 10.8 + \dots + 10.8) = 11.17$$

$$\bar{Y} = \frac{1}{8}(11.8 + 12.6 + 12.2 + \dots + 12.6) = 11.98$$

$$Z_{obs} = \frac{11.17 - 11.98}{\sqrt{\frac{0.09}{10} + \frac{0.16}{8}}} = \frac{-0.81}{0.029} = -27.93$$

Date
Page

$P(Z \leq Z)$

~~$Z_{\alpha/2} = Z_{0.025} = \phi(0.995) = 0.8289$~~

$Z_{\alpha/2} = Z_{0.025} = \phi(0.975) = 1.96$

~~$Z_{obs} = 27.93 > Z_{\alpha/2} = 0.8289\ 1.96$~~

$\therefore$ ~~We failed to Reject Ho ie Accept Null (Ho)~~

$|Z_{obs}| = 27.93 > Z_{\alpha/2} = 1.96 \therefore$

$\therefore$ We Reject Ho.

p-value $\Rightarrow 2P(Z > |Z_{obs}|) = 2 * P(Z > 27.93)$

$= 2 * P(Z \leq$

Tut 2
Case III :-

Ex.157 Sample A: $\bar{X} = \mu_x = 1309$, $n = 9$, $S_x = 420$

Sample B: $\bar{Y} = \mu_y = 1205$, $m = 16$, $S_y = 390$

a) Assuming population variances are same $\sigma_x^2 = \sigma_y^2$

→ $H_0: \mu_x = \mu_y$ $\qquad H_1: \mu_x \neq \mu_y$

$$S_p^2 = \frac{(n-1) S_x^2 + (m-1) S_y^2}{n+m-2}$$

$$S_p^2 = \frac{8 \times (420)^2 + 15 \times (390)^2}{9+16-2}$$

$$S_p^2 = 160552$$

μ_x: Mean lifetime of lighbulb from Batch A
μ_y: — " — Batch B

$\alpha = 0.05$ (You choose)

$$|T^*| = \left| \frac{\bar{X} - \bar{Y}}{\sqrt{S_p^2 (1/m + 1/n)}} \right| > t_{\alpha/2,\, n+m-2}$$