Functions Z_r which map a multidimensional index ($i \leq j \leq k ...$) corresponding to the address of an element in the upper triangle of a symmetric tensor of rank r and dimension size n to the index corresponding to the same element in the **flattened upper-triangle vector**, which is the row-wise ravelled (flattened) upper triangle of the symmetric tensor. In other words, the functions map the r-dimensional upper triangular symmetric tensor elements to the 1-dimensional vector of unique symmetric tensor elements.

$$
\begin{aligned}
\Delta_0 &= 1 \\
\Delta_1 &= n \\
\Delta_2 &= \frac{n(n+1)}{2} \\
\Delta_3 &= \frac{n(n+1)(n+2)}{6} \\
\dots \\
\Delta_r &= \frac{n(n+1)(n+2)\cdots(n+(r-1)}{r!}
\end{aligned}
$$

$$
\begin{aligned}
Z_0(n) &= 0 \\
Z_1(i,n) &= Z_0(n-i) + \sum_{a=1}^{i} \Delta_0 \\
Z_2(i,j,n) &= Z_1(j-i, n-i) + \sum_{a=1}^{i} \Delta_1 - \sum_{b=1}^{a-1} \Delta_0 \\
Z_3(i,j,k,n) &= Z_2(j-i, k-i, n-i) + \sum_{a=1}^{i} \Delta_2 - \sum_{b=1}^{a-1} \Delta_1 + \sum_{c=1}^{b-1} \Delta_0 \\
Z_4(i,j,k,l,n) &= Z_3(j-i, k-i, l-i, n-i) + \sum_{a=1}^{i} \Delta_3 - \sum_{b=1}^{a-1} \Delta_2 + \sum_{c=1}^{b-1} \Delta_1 - \sum_{d=1}^{c-1} \Delta_0
\end{aligned}
$$