

Let n be the dimension size of a symmetric rank r tensor. Define the following quantities Δ_r , which represent the number of unique elements in the upper hyper-triangle of a tensor of rank r :

$$\begin{aligned}
\Delta_0 &= 1 \\
\Delta_1 &= n \\
\Delta_2 &= \frac{n(n+1)}{2} \\
\Delta_3 &= \frac{n(n+1)(n+2)}{6} \\
&\dots \\
\Delta_r &= \frac{n(n+1)(n+2)\cdots(n+(r-1))}{r!}
\end{aligned}$$

We now can define the functions Z_r , which map an element's multidimensional index ($i \leq j \leq k \dots$) to the corresponding one-dimensional index in the vector containing the flattened upper hypertriangle of the tensor.

$$Z_0(n) = 0$$

$$Z_1(i, n) = Z_0(n - i) + \sum_{a=1}^i \Delta_0$$

$$Z_2(i, j, n) = Z_1(j - i, n - i) + \sum_{a=1}^i \Delta_1 - \sum_{b=1}^{a-1} \Delta_0$$

$$Z_3(i, j, k, n) = Z_2(j - i, k - i, n - i) + \sum_{a=1}^i \Delta_2 - \sum_{b=1}^{a-1} \Delta_1 + \sum_{c=1}^{b-1} \Delta_0$$

$$Z_4(i, j, k, l, n) = Z_3(j - i, k - i, l - i, n - i) + \sum_{a=1}^i \Delta_3 - \sum_{b=1}^{a-1} \Delta_2 + \sum_{c=1}^{b-1} \Delta_1 - \sum_{d=1}^{c-1} \Delta_0$$