Index issues

underground mathematics

Problem

 \sqrt{x} or $x^{\frac{1}{2}}$? Whichever we prefer, we need to be comfortable working with indices and roots in either form.

Below is a series of equations, for which you should think about the following questions.



For what values of x and y do these statements make sense?

What are the possible values for the expressions?

(1)
$$(xy)^{\frac{1}{2}} = x^{\frac{1}{2}}y^{\frac{1}{2}}$$

(2)
$$(xy)^{\frac{5}{3}} = x^{\frac{5}{3}}y^{\frac{5}{3}}$$

(3)
$$(xy)^{\frac{2}{3}} = x^{\frac{2}{3}}y^{\frac{2}{3}}$$

(4)
$$(xy)^{-\frac{1}{2}} = x^{-\frac{1}{2}}y^{-\frac{1}{2}}$$

(5)
$$(xy)^{\frac{1}{2}} = x^{\frac{1}{3}}y^{\frac{2}{3}}$$

(6)
$$(xy)^{-2} = x^2y^2$$

(7)
$$\left(\frac{x}{y}\right)^{-\frac{1}{3}} = x^{-\frac{1}{3}}y^{\frac{1}{3}}$$

(8)
$$\left(\frac{x}{y} - \frac{1}{y}\right)^{\frac{1}{2}} = y^{-\frac{1}{2}}(x-1)^{\frac{1}{2}}$$