Laboratory Session 05 : May 21, 2024

Exercises due : June 3, 2024

Exercise 1

- given the following un-normalized posterior distribution

$$g(\theta \mid x) \propto \frac{1}{2}\exp -\frac{(\theta+3)^2}{2} + \frac{1}{2}\exp -\frac{(\theta-3)^2}{2}$$

- draw a Markov Chain from the posterior distribution using a Metropolis-Hastings algorithm
- use a Norm $(0, 1)$ as random-walk candidate density
- plot the sampled distribution
- analyze the chain with the `CODA` package and plot the chain autocorrelation
- try to use different burn-in cycles and thinning and plot the corresponding posterior distribution and the chain autocorrelation function. What are the best parameters ?

Exercise 2

- A set of measured data should follow, according to the physics model applied to them, a linear behavior. Data are the following:

Y	-7.821	-1.494	-15.444	-10.807	-13.735	-14.442	-15.892	-18.326
X	5	6	7	8	9	10	11	12

- perform a simple linear regression model running a Markov Chain Monte Carlo with JAGS, assuming that data follow the model:

```
Z[i] = a + b * X[i];
```

- and the likelihood of the measured data follow a Gaussian likelihood distribution:
`Y[i] ~dnorm(Z[i], c)`
- you can constrain the parameter a, b and c to the following intervals: $a \in [1, 10]$, $b \in [-1, 3]$ and $c \in [0.034, 4]$
- run JAGS experimenting with the burnin and number of iterations of the chain. Plot the evolution of the chains and the posterior distributions of a and b. Compute the 95% credibility interval for the parameters.
- using the obtained posterior distributions, compute the posterior distribution of $\sigma = 1/\sqrt{c}$

Exercise 3

- suppose we observe the following values

$$x = 2.06, 5.56, 7.93, 6.56, 205$$

- and we assume that the data come from a gaussian distribution with unknown mean m and variance s^2
- build a simple JAGS model and run a Markov Chain Monte Carlo to obtain the posterior distribution of the mean and variance.
- Assume uniform prior distributions for the parameters, `m ~dunif(-10, 10)` and `s ~dunif(0,50)`.
- compute also the posterior distribution for `m/s`

Exercise 4

- The data set that Edwin Hubble ised to show that galaxies are moving either away or towards us are given in the following table:

D	0.0032	0.0034	0.214	0.263	0.275
V	170	290	-130	-70	-185
D	0.275	0.45	0.5	0.5	0.63
V	-220	200	290	270	200
D	0.8	0.9	0.9	0.9	0.9
V	920	450	500	500	960
D	2	2	2	2	
V	500	850	800	1090	

- Using this data set define a JAGS model to fit data with the following:
  `V[i] ~dnorm(b * D[i], c)`
- where V represent the velocity in units of km/s, D is the observed distance (in units of parsec), and b and c are two parameters of the model
- assume whatever prior distribution you think is appropriate
- plot the evolution of the chains, the posterior distribution of the parameters and the 95% credibility interval