CV Project 1

多媒體工程所
宋秉一
9957513

Contents

Figures

Hough Transform

To get accurate real number points for camera calibration, we determine the 2D-3D point pair using Hough transform and some simple UI.

The Hough transform is a voting algorithm to find geometric functions in the image. The basic idea is spanning a parameter space (known as Hough space. Figure 1) for any possible target functions in the image, and each point in Hough space presents a target function.

In our Project, we want to find lines and solve the intersection points by voting the Hough points using edge positions, and the line is considered to be a peak point in Hough space.

We define the line function and let r, θ to be two parameters for line. The r term is the perpendicular distance from edge point to line, and θ is the angle between x-axes to line.

$$\mathrm{L}(r,\theta) = ax + by + c = 0,$$
$$where\ a = -cot\theta, \qquad b = -1, \qquad c = r/sin\theta$$

The range of r is limited by the size of image, and θ consider to be the rotation angle of lines.

$$\begin{cases} 0 \le r \le \sqrt{width^2 + height^2} \\ 0 \le \theta \le 360 \end{cases}$$

Project all edge points to the all possible lines and compute the perpendicular distance $\mathrm{d}(\mathrm{p},\mathrm{L})$ from edge points to lines. If the distance is zero, we consider the point is on the line and vote to this line (Hough point). But in computer case, we could set a threshold to determine whether the point is close enough to this line.

$$\mathrm{d}(\mathrm{p},\mathrm{L}) = \frac{|ax + by + c|}{\sqrt{a^2 + b^2}}$$
$$\text{Vote L if } \mathrm{d}(\mathrm{p},\mathrm{L}) \le \text{Threshold}$$

After voting every edge points, we could get a complete Hough space of image. The lines are completely extracted by finding peaks in Hough space (Figure 2), and the intersection points could be computed by pairs of lines (Figure 3).

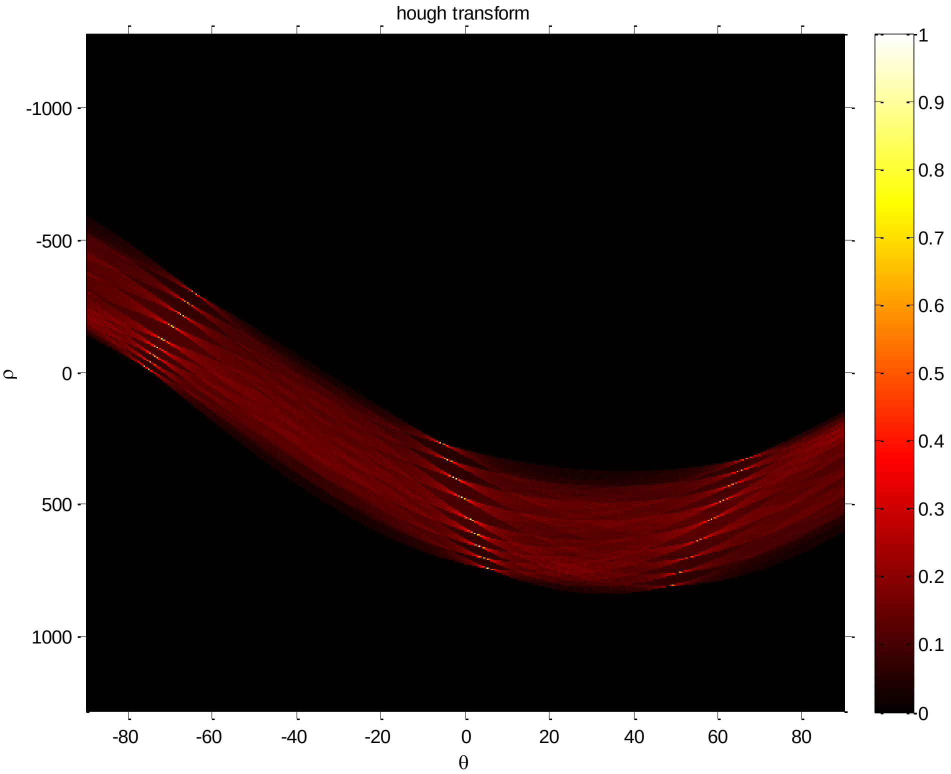

Figure 1.The Hough space

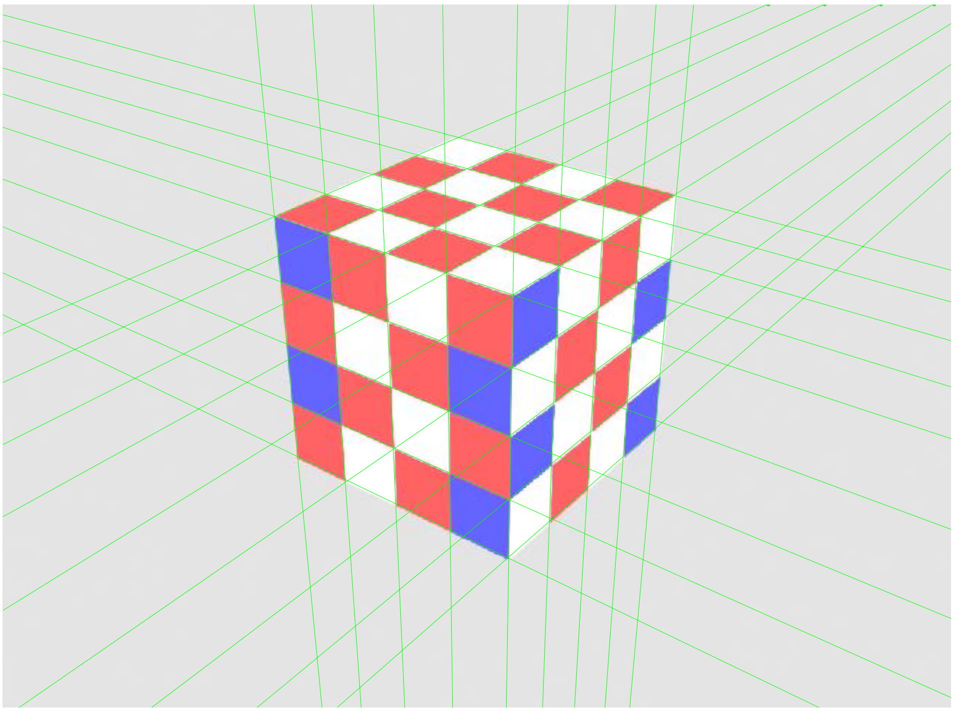

Figure 2.The extracted lines using Hough peaks

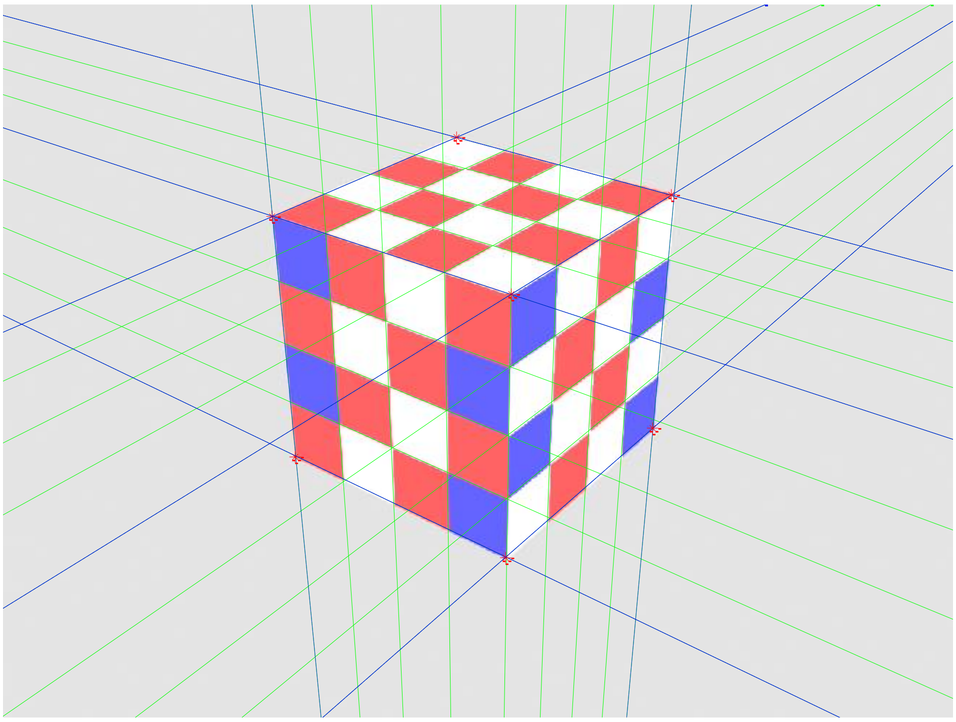

Figure 3.The intersection points from pairs of lines

Solve the Projection Matrix

$$sp = MP, where\ s \geq 0$$

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

where u, v, X, Y, Z are known as 2D to 3D correspondence point

$$\begin{cases} M_{11}\mathrm{X} + M_{12}Y + M_{13}Z + M_{14} = su \\ M_{21}\mathrm{X} + M_{22}Y + M_{23}Z + M_{24} = sv \\ M_{31} + M_{32} + M_{33} + M_{34} = s \end{cases}$$

$$\begin{cases} M_{11}\mathrm{X} + M_{12}Y + M_{13}Z + M_{14} - (M_{31} + M_{32} + M_{33} + M_{34})u = 0 \\ M_{21}\mathrm{X} + M_{22}Y + M_{23}Z + M_{24} - (M_{31} + M_{32} + M_{33} + M_{34})v = 0 \end{cases}$$

Since the degrees of freedom is 12 and 2 equations for each correspondence point, we need at least $n \geq 6$ pairs of correspondence point to solve M.

$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1X_1 & -u_1Y_1 & -u_1Z_1 & -u_1 \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1X_1 & -v_1Y_1 & -v_1Z_1 & -v_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -u_nX_n & -u_nY_n & -u_nZ_n & -u_n \\ 0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -v_nX_n & -v_nY_n & -v_nZ_n & -v_n \end{bmatrix} \begin{bmatrix} M_{11} \\ M_{12} \\ M_{13} \\ M_{14} \\ M_{21} \\ M_{22} \\ M_{23} \\ M_{24} \\ M_{31} \\ M_{32} \\ M_{33} \\ M_{34} \end{bmatrix} = 0$$

$\Rightarrow Ax = 0, \text{where } A \text{ is } 2n - \text{by} - 12 \text{ matrix}$

Solve x using SVD.

$$A = U\Sigma V^T$$

$$x = 12^{th}\ column\ of\ V = [\mathrm{v}_1 \quad \mathrm{v}_2 \quad \cdots \quad \mathrm{v}_{12}]^T$$

$$M = \begin{bmatrix} \mathrm{v}_1 & \mathrm{v}_2 & \mathrm{v}_3 & \mathrm{v}_4 \\ \mathrm{v}_5 & \mathrm{v}_6 & \mathrm{v}_7 & \mathrm{v}_8 \\ \mathrm{v}_9 & \mathrm{v}_{10} & \mathrm{v}_{11} & \mathrm{v}_{12} \end{bmatrix}$$

Check ***s*** *term*

In a reasonable case of given image, the object must in front of camera. We have to modify M by checking s term. Project arbitrary 3D point on object using M, and update M by the following rules.

$$\begin{cases} M = M\ , where\ s \geq 0 \\ M = -M, where\ s < 0 \end{cases}$$

Camera Parameters

Extract the camera intrinsic parameter K and extrinsic parameter R , t from the projection matrix M.

$$sp = MP, where\ s \geq 0$$

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = K[R\ t]\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$where\ K = \begin{bmatrix} \alpha & -\alpha\ cot\theta & u_0 \\ 0 & \beta/sin\theta & v_0 \\ 0 & 0 & 1 \end{bmatrix}, R = [r_1 \quad r_2 \quad r_3]$$

Method 1

$$M = [A\ b] = \rho[KR\ Kt]$$

$$where\ A = [a_1 \quad a_2 \quad a_3]$$

$$\begin{cases} \rho = \dfrac{1}{|a_3|} \\ u_0 = \rho^2(a_1 \cdot a_3) \\ v_0 = \rho^2(a_2 \cdot a_3) \end{cases}$$

$$\begin{cases} cos\theta = -\dfrac{(a_1 \times a_3) \cdot (a_2 \times a_3)}{|a_1 \times a_3|\ |a_2 \times a_3|} \\ \alpha = \rho^2|a_1 \times a_3|sin\theta \\ \beta = \rho^2|a_2 \times a_3|sin\theta \end{cases}$$

$$\begin{cases} r_1 = \dfrac{(a_2 \times a_3)}{|a_2 \times a_3|} \\ r_2 = r_1 \times r_3 \\ r_3 = \rho a_3 \end{cases}$$

Method 2

$$M = [A\ b] = [\rho KR\ \rho Kt]\ , where\ \rho \neq 0$$

K is a 3-by-3 upper-triangle matrix

R is a 3-by-3 orthogonal matrix

$$A^{-1} = \rho(KR)^{-1} = \rho R^{-1}K^{-1}$$

$$A^{-1} = (R^{-1})(\rho K)^{-1} = Q\mathrm{r}$$

where Q is a orthogonal matrix, r is upper-triangle matrix

$$\begin{cases} R^{-1} = Q \Rightarrow R = Q^{-1}\ , det(R) = \det(Q) = 1 \\ \rho K = \mathrm{r}^{-1} \Rightarrow K = \dfrac{\mathrm{r}^{-1}}{\rho K(3,3)} \end{cases}$$

$$t = (\rho K)^{-1} * \rho Kt = \mathrm{r} * b$$

Projection matrix between two camera coordinate

Assume P_A is the 3D point in camera A coordinate and, P_B is the 3D point in camera B coordinate, and P is 3D point in the world coordinate. We can project any P_B to camera A coordinate using the projection matrix ${}^{A}_{B}M$ between two camera coordinate.

$$\begin{cases} P_A = {}^{A}_{W}R * P + t_A \\ P_B = {}^{B}_{W}R * P + t_B \end{cases}$$

$$\begin{cases} P = {}^{A}_{W}R^{-1} * (P_A - t_A) \\ P = {}^{B}_{W}R^{-1} * (P_B - t_B) \end{cases}$$

$${}^{A}_{W}R^{-1} * (P_A - t_A) = {}^{B}_{W}R^{-1} * (P_B - t_B)$$

$${}^{A}_{W}R^{-1} * P_A - {}^{A}_{W}R^{-1} * t_A = {}^{B}_{W}R^{-1} * P_B - {}^{B}_{W}R^{-1} * t_B$$

$${}^{A}_{W}R^{-1} * P_A = {}^{B}_{W}R^{-1} * P_B - {}^{B}_{W}R^{-1} * t_B + {}^{A}_{W}R^{-1} * t_A$$

$$P_A = {}^{A}_{W}R * {}^{B}_{W}R^{-1} * P_B - {}^{A}_{W}R * {}^{B}_{W}R^{-1} * t_B + {}^{A}_{W}R * {}^{A}_{W}R^{-1} * t_A$$

$$P_A = {}^{A}_{W}R * {}^{B}_{W}R^{-1} * P_B - {}^{A}_{W}R * {}^{B}_{W}R^{-1} * t_B + t_A$$

$$P_A = R * P_B + t$$

$$where \begin{cases} R = {}^{A}_{W}R * {}^{B}_{W}R^{-1} \\ t = -{}^{A}_{W}R * {}^{B}_{W}R^{-1} * t_B + t_A \end{cases}$$

Using homogeneous form

$$P_A = {}^{A}_{B}M \begin{bmatrix} P_B \\ 1 \end{bmatrix} = [R \quad t] * \begin{bmatrix} P_B \\ 1 \end{bmatrix}$$

$${}^{A}_{B}M = [R \quad t]$$

Implementation of "A Flexible New Technique for Camera Calibration"

$$Homography\ between\ the\ model\ plane\ and\ its\ image$$

$$sp = HP, where\ s \geq 0$$

$$\begin{bmatrix} su \\ sv \\ s \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

where u, v, X, Y are known as 2D to 3D plane correspondence point

$$\begin{cases} H_{11}\mathrm{X} + H_{12}Y + H_{13} = su \\ H_{21}\mathrm{X} + H_{22}Y + H_{23} = sv \\ H_{31} + H_{32} + H_{33} = s \end{cases}$$

$$\begin{cases} H_{11}X + H_{12}Y + H_{13} - (H_{31} + H_{32} + H_{33})u = 0 \\ H_{21}X + H_{22}Y + H_{23} - (H_{31} + H_{32} + H_{33})v = 0 \end{cases}$$

Since the degrees of freedom is 8 and 2 equations for each correspondence point, we need at least $n \geq 4$ pairs of correspondence point to solve H.

$$\begin{bmatrix} X_1 & Y_1 & 1 & 0 & 0 & 0 & -u_1X_1 & -u_1Y_1 & -u_1 \\ 0 & 0 & 0 & X_1 & Y_1 & 1 & -v_1X_1 & -v_1Y_1 & -v_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ X_n & Y_n & 1 & 0 & 0 & 0 & -u_nX_n & -u_nY_n & -u_n \\ 0 & 0 & 0 & X_n & Y_n & 1 & -v_nX_n & -v_nY_n & -v_n \end{bmatrix} \begin{bmatrix} H_{11} \\ H_{12} \\ H_{13} \\ H_{21} \\ H_{22} \\ H_{23} \\ H_{31} \\ H_{32} \\ H_{33} \end{bmatrix} = 0$$

$$\Rightarrow Ax = 0, \text{where } A \text{ is } 2n - \text{by} - 9 \text{ matrix}$$

Solve x using SVD.

$$A = U\Sigma V^T$$

$$x = 9^{th}\ column\ of\ V = [\mathrm{v}_1 \quad \mathrm{v}_2 \quad \cdots \quad \mathrm{v}_9]^T$$

$$H = \begin{bmatrix} \mathrm{v}_1 & \mathrm{v}_2 & \mathrm{v}_3 \\ \mathrm{v}_4 & \mathrm{v}_5 & \mathrm{v}_6 \\ \mathrm{v}_7 & \mathrm{v}_8 & \mathrm{v}_9 \end{bmatrix}$$

Check $\boldsymbol{s}$ *term*

Project arbitrary 3D point on object using H, and update H by the following rules.

$$\begin{cases} H = H\ , where\ s \geq 0 \\ H = -H, where\ s < 0 \end{cases}$$

Intrinsic parameters

$$B = K^{-T}K^{-1} = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{12} & B_{22} & B_{23} \\ B_{13} & B_{23} & B_{33} \end{bmatrix}$$

$$\mathrm{b} = [B_{11} \quad B_{12} \quad B_{22} \quad B_{13} \quad B_{23} \quad B_{33}]^T$$

$$\mathrm{v}_{ij}^k = [h_{1i}^k * h_{1j}^k \quad h_{1i}^k * h_{2j}^k + h_{2i}^k * h_{1j}^k \quad h_{2i}^k * h_{2j}^k \quad h_{3i}^k * h_{1j}^k + h_{1i}^k * h_{3j}^k \quad h_{3i}^k * h_{2j}^k + h_{2i}^k * h_{3j}^k \quad h_{3i}^k * h_{3j}^k]$$

$$Vb = \begin{bmatrix} \mathrm{v}_{12}^1 \\ \mathrm{v}_{11}^1 - \mathrm{v}_{22}^1 \\ \mathrm{v}_{12}^2 \\ \mathrm{v}_{11}^2 - \mathrm{v}_{22}^2 \\ \vdots \\ \mathrm{v}_{12}^n \\ \mathrm{v}_{11}^n - \mathrm{v}_{22}^n \end{bmatrix} b = 0$$

Solve b by SVD and we get the B. Now we could compute the camera intrinsic parameter using B.

$$v_0 = \frac{B_{12}B_{13} - B_{11}B_{23}}{B_{11}B_{22} - B_{12}B_{12}}$$

$$\lambda = B_{33} - \frac{[B_{13}B_{13} + v_0(B_{12}B_{13} - B_{11}B_{23})]}{B_{11}}$$

$$\alpha = \sqrt{\frac{\lambda}{B_{11}}}$$

$$\beta = \sqrt{\frac{\lambda B_{11}}{B_{11}B_{22} - B_{12}B_{12}}}$$

$$\gamma = -\frac{B_{12}\alpha^2\beta}{\lambda}$$

$$u_0 = \frac{\gamma v_0}{\beta} - \frac{B_{13}\alpha^2}{\lambda}$$

$$K = \begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

Extrinsic parameters

$$R = [r_1 \quad r_2 \quad r_3]$$

$$\lambda = \frac{1}{\|K^{-1}H(:,1)\|} = \frac{1}{\|K^{-1}H(:,2)\|}$$

$$r_1 = \lambda K^{-1}H(:,1)$$

$$r_2 = \lambda K^{-1}H(:,2)$$

$$r_3 = r_1 \times r_2$$

$$t = \lambda K^{-1}H(:,3)$$

Since the rotation matrix R is not a perfect orthogonal matrix, we need to make a refinement of R by SVD.

$$R = U\Sigma V^T$$

$$\mathrm{R}_{new} = UV^T$$

Experiment Results

Calibration points and test points

The calibration 2D points are extracted from image 5 and image 7 using Hough transform, and compute the camera parameters $K[R\ t]$ for each image. The test 2D points are extracted from image 5 and image 7 using Hough transform, and set the correspondent 3D positions manually.

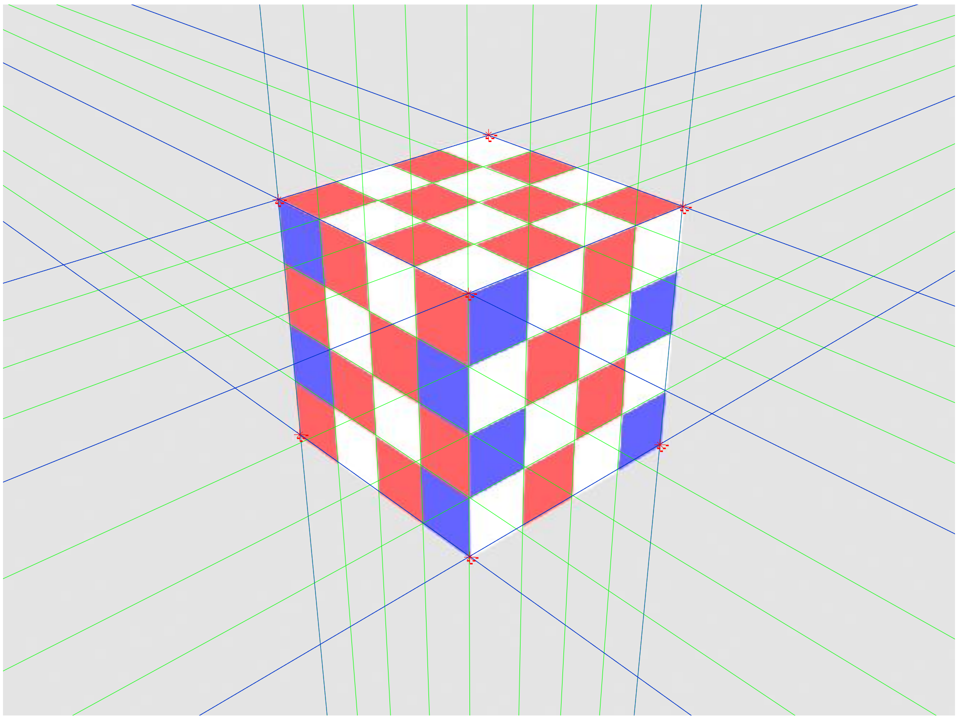

Figure 4.The calibration 2D points in image 5 extracted from Hough transform

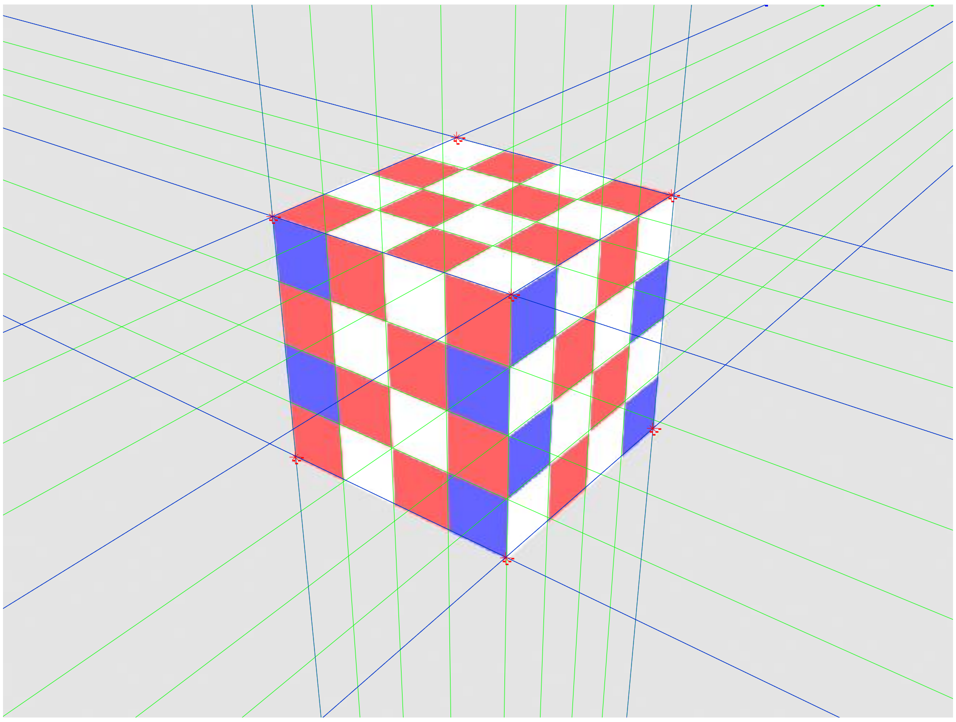

Figure 5.The calibration 2D points in image 7 extracted from Hough transform

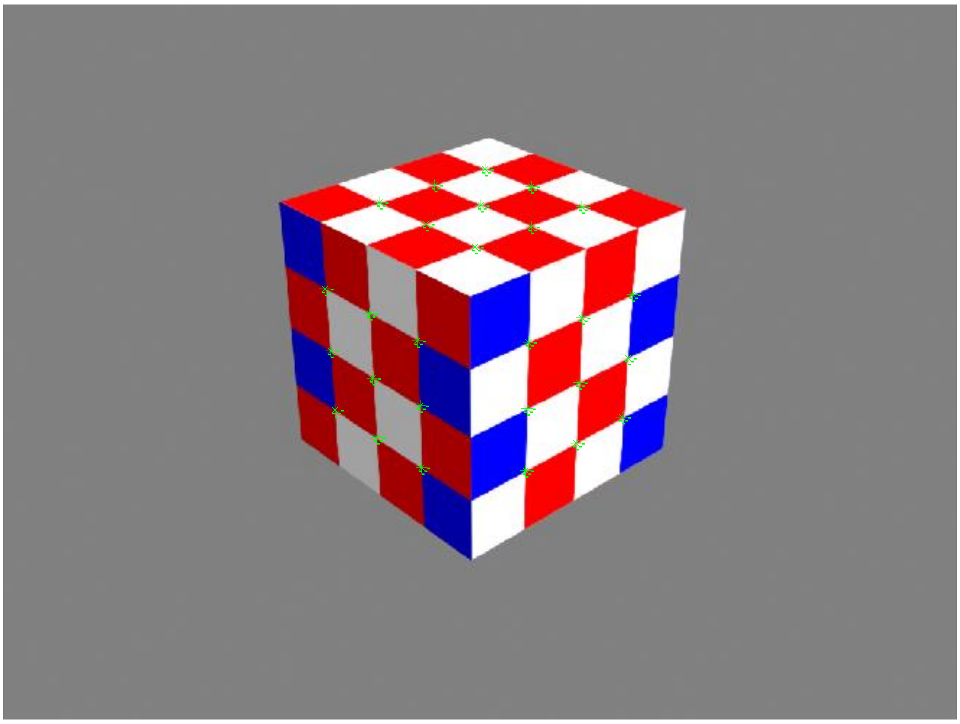

Figure 6.The test 2D points in image 5 extracted from Hough transform

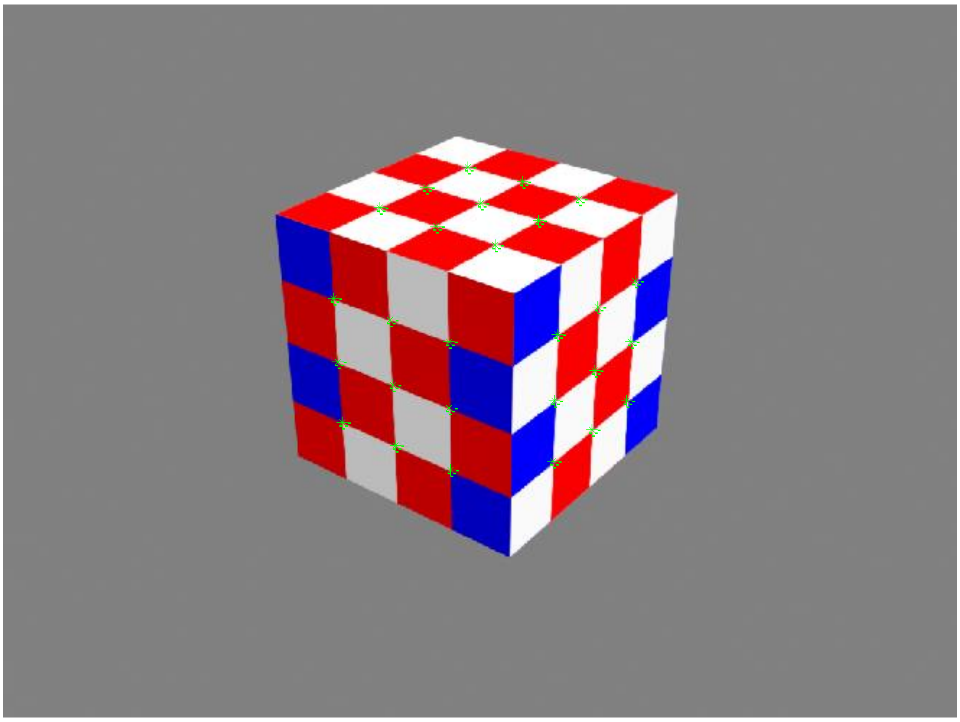

Figure 7.The test 2D points in image 7 extracted from Hough transform

The projection matrix

image	M
5	$\begin{bmatrix} 0.0151 & 0.0032 & -0.0063 & 0.8468 \\ -0.0023 & 0.0157 & -0.0027 & 0.5315 \\ 0.0000 & 0.0000 & 0.0000 & 0.0017 \end{bmatrix}$
7	$\begin{bmatrix} 0.0128 & 0.0028 & -0.0085 & 0.8674 \\ -0.0026 & 0.0144 & -0.0022 & 0.4972 \\ 0.00000 & 0.0000 & 0.0000 & 0.0016 \end{bmatrix}$

Method 1

image	K	R	t
5	$\begin{bmatrix} 1235.2 & -0.0000 & 530.2 \\ 0 & 1233.4 & 391.1 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0.7311 & -0.0071 & -0.6822 \\ -0.3402 & 0.8630 & -0.3735 \\ 0.5914 & 0.5052 & 0.6285 \end{bmatrix}$	$\begin{bmatrix} -3.2105 \\ -8.4418 \\ 135.9800 \end{bmatrix}$
7	$\begin{bmatrix} 1264.6 & -0.0000 & 575.1 \\ 0 & 1270.1 & 368.2 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0.5714 & -0.0216 & -0.8204 \\ -0.3904 & 0.8721 & -0.2949 \\ 0.7218 & 0.4888 & 0.4899 \end{bmatrix}$	$\begin{bmatrix} -3.1258 \\ -6.0771 \\ 141.4774 \end{bmatrix}$

image	average projection error(pixel)
5	0.5417
7	0.9758

Method 2

image	K	R	t
5	$\begin{bmatrix} 1235.2 & -2.0000 & 530.2 \\ 0 & 1233.4 & 391.1 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0.7311 & -0.0071 & -0.6822 \\ -0.3402 & 0.8630 & -0.3735 \\ 0.5914 & 0.5052 & 0.6285 \end{bmatrix}$	$\begin{bmatrix} -3.2243 \\ -8.4418 \\ 135.9800 \end{bmatrix}$
7	$\begin{bmatrix} 1264.6 & -4.4000 & 575.1 \\ 0 & 1270.1 & 368.2 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0.5714 & -0.0216 & -0.8204 \\ -0.3904 & 0.8721 & -0.2949 \\ 0.7218 & 0.4888 & 0.4899 \end{bmatrix}$	$\begin{bmatrix} -3.1470 \\ -6.0771 \\ 141.4774 \end{bmatrix}$

image	average projection error(pixel)
5	0.5707
7	1.0325

Two Camera Projection

P_5 is the testing 3D point in camera 5 coordinate, and P_7 is the testing 3D point in camera 7 coordinate. Compute the projection error between projected P_5 in camera 7 coordinate and P_7.

$$M_{75} = \begin{bmatrix} 0.9776 & 0.0933 & -0.1886 & 26.4387 \\ -0.0904 & 0.9956 & 0.0243 & -1.2730 \\ 0.1900 & -0.0067 & 0.9818 & 8.5345 \end{bmatrix}$$

Total projection error = $norm(P_7 - M_{75}P_5) = 8.5463e - 014$

$$Homography$$

image	K	R	t
5	$\begin{bmatrix} 1276.0 & -32.3 & 525.5 \\ 0 & 1218.7 & 404.1 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0.7131 & 0.0144 & -0.7010 \\ -0.3571 & 0.8678 & -0.3455 \\ 0.6033 & 0.4967 & 0.6239 \end{bmatrix}$	$\begin{bmatrix} -2.8764 \\ -10.2416 \\ 138.5293 \end{bmatrix}$
7	$\begin{bmatrix} 1359.6 & -172.0 & 593.0 \\ 0 & 1304.3 & 342.2 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0.5180 & 0.0764 & -0.8520 \\ -0.4073 & 0.8979 & -0.1671 \\ 0.7522 & 0.4336 & 0.4962 \end{bmatrix}$	$\begin{bmatrix} -5.6007 \\ -3.5371 \\ 157.0293 \end{bmatrix}$

image	average projection error(pixel)
5	5.1673
7	13.1241

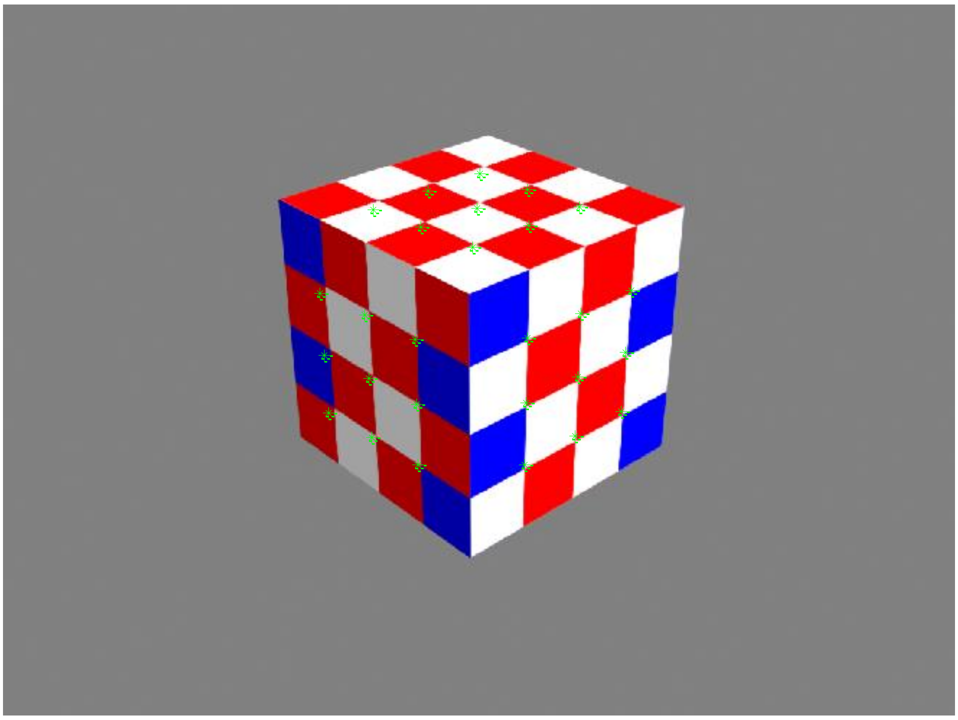

Figure 8.The projected test points in image 5

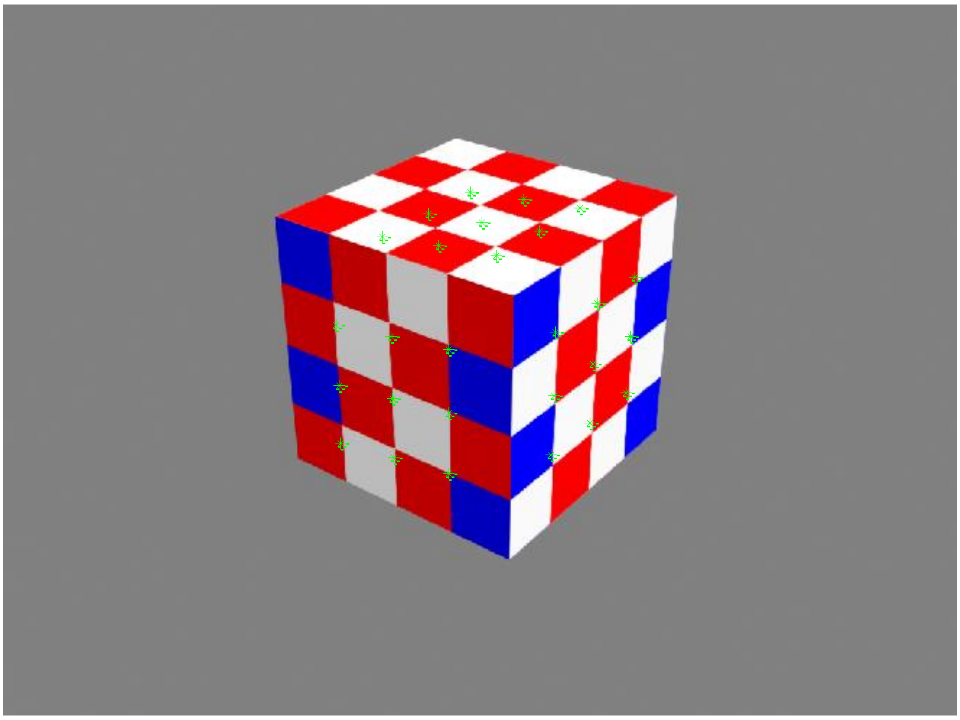

Figure 9.The projected test points in image 7

Discussions

The accuracy of camera calibration is deeply affected by calibration points we extracted from images. To improve the calibration point selection, we use Hough transform to get the real number points from pairs of lines.

With the given 2D-3D correspondence points, the camera parameters are well-estimated and very robust. But in most case, we even don't know the accuracy 3D points for calibration. The homography provide a novel think that only need 3D plane instead of 3D point position to recover the camera parameters. Even with only 1 or 2 3D planes, this method is also work with a less constrains (no skew, camera center is on image center).

The re-projection result shows that the camera intrinsic parameter is not perfectly estimated with homography method. With over constrained equations, the re-projection result seems no better than the original projection result.