# 4.45

Friday, 24 September 2021 13:33

**Exercício 4.45** *Seja $U$ um subespaço de $\mathcal{P}_3(\mathbb{R})$. Considere a transformação linear $T : U \longrightarrow \mathcal{P}_2(\mathbb{R})$ dada por: $T(p(x)) = p'(x) + (x+1)p(0)$. Seja*

$$[T]_\gamma^\beta = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 1 & 2 \end{bmatrix} \qquad \text{onde} \qquad \beta = \{\, x - x^2 + x^3,\ 1 + x + x^2 \,\}.$$

*(a) Determine $[p(x)]_\beta$ sabendo que $[T(p(x))]_\gamma = \begin{bmatrix} 3 \\ 5 \\ 4 \end{bmatrix}$.*

*(b) Se $\gamma = \{\, x+2,\ p_1(x),\ p_2(x) \,\}$, determine $3p_1(x) + 3p_2(x)$.*

a) $[p(x)]_\beta$

- $[T(p(x))]_\gamma = [T]_\gamma^\beta \cdot [p(x)]_\beta$

$$\begin{bmatrix} 3 \\ 5 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \Rightarrow \begin{cases} c_1 + c_2 = 3 \\ 2c_1 + c_2 = 5 \\ c_1 + 2c_2 = 4 \end{cases} \Leftrightarrow \quad c_1 = 3 - c_2 \quad c_2 = 1 \quad c_1 = 2$$

$\therefore [p(x)]_\beta = (2, 1)$

b) $3p_1(x) + 3p_2(x)$

1º Vamos escrever $T(x - x^2 + x^3)$ e $T(1 + x + x^2)$ na base $\gamma$.

Igualando à definição de $T(p(x))$ teremos o que precisamos.

$$[T(x - x^2 + x^3)]_\gamma = [T]_\gamma^\beta \cdot [x - x^2 + x^3]_\beta$$

$$= \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

Com isso,

$$T(x-x^2+x^3) = 1\underbrace{(x+2)}_{\gamma_1}+2\underbrace{p_1(x)}_{\gamma_2}+\underbrace{p_2(x)}_{\gamma_3}$$

Por outro lado, pela def de T:

$$T(x-x^2+x^3) = 1-2x+3x^2+(x+1)\cdot 0 = 1-2x+3x^2$$

Para

$$[T(1+x+x^2)]_\gamma = [T]_\gamma^\beta \cdot [1+x+x^2]_\beta = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$T(1+x+x^2) = 1(x+2)+1\,p_1(x)+2\,p_2(x).$$

$$T(1+x+x^2) = 1+2x+(x+1)\cdot 1 = 2+3x$$

Obtemos as seguintes igualdades:

$$1-2x+3x^2 = 1(x+2)+2p_1(x)+p_2(x)$$

$$2+3x = 1(x+2)+1\,p_1(x)+2\,p_2(x)$$

Somando:

$$3+x+3x^2 = 2(x+2)+3p_1(x)+3p_2(x)$$

Portanto,

$$3p_1(x)+3p_2(x) = 3+x+3x^2-2x-4$$

$$= -1-x+3x^2$$