5.14

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Exercício 5.14 *Sejam* $a_1, a_2, \cdots, a_n$ *reais quaisquer. Mostre que*

$$\left(\frac{a_1 + a_2 + \cdots + a_n}{n}\right)^2 \leq \frac{(a_1^2 + a_2^2 + \cdots + a_n^2)}{n},$$

utilizando a desigualdade de Cauchy–Schwarz em $\mathbb{R}^n$.

Utilizando o produto interno usual e

$$u = (a_1, a_2, \dots, a_n), \quad v = (1/n, 1/n, \dots, 1/n)$$

temos que:

$$\langle u, v \rangle = \sum_{i=1}^{n} a_i \cdot \frac{1}{n} = \frac{1}{n} \cdot \sum_{i=1}^{n} a_i$$

$$\langle u, u \rangle = \sum_{i=1}^{n} a_i^2$$

$$\langle v, v \rangle = \sum_{i=1}^{n} \frac{1}{n^2} = \frac{1}{n}$$

Pela desigualdade,

$$|\langle u, v \rangle|^2 \leq \langle u, u \rangle \langle v, v \rangle \iff \left(\frac{1}{n} \sum_{i=1}^{n} a_i\right)^2 \leq \left(\sum_{i=1}^{n} a_i^2\right) \cdot \frac{1}{n}$$

Ou seja,

$$\left(\frac{a_1 + a_2 + \dots + a_n}{n}\right)^2 \leq \frac{a_1^2 + a_2^2 + \dots + a_n^2}{n} \qquad \square$$