5.17

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Exercício 5.17 *Considere o espaço vetorial real* $\mathbb{M}_{2\times 3}(\mathbb{R})$ *com o produto interno usual*

$$\langle A\,,\,B\,\rangle \;=\; tr(B^tA)\,.$$

Dadas as matrizes

$$A \;=\; \begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 1 \end{bmatrix} \quad , \quad B \;=\; \begin{bmatrix} 1 & -2 & 1 \\ 1 & 0 & 3 \end{bmatrix} \qquad e \qquad C \;=\; \begin{bmatrix} 2 & -3 & 2 \\ 1 & 0 & -1 \end{bmatrix},$$

determine

$$\langle\, A\,,\,B\,\rangle \quad , \quad \langle\, A + B\,,\,C\,\rangle \quad , \quad \|\,A\,\|_2 \quad , \quad \|\,B\,\|_2 \qquad e \qquad \cos(\theta)\,,$$

onde θ *é o ângulo entre as matrizes* A *e* B.

$\langle A, B \rangle$

$$B^tA = \begin{bmatrix} 1 & 1 \\ -2 & 0 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & -4 \\ -2 & 6 & 5 \end{bmatrix}$$

$$\therefore\ tr(B^tA) = 5 = \langle A, B \rangle$$

$\langle A+B, C \rangle$

$$\begin{bmatrix} 2 & 1 \\ -3 & 0 \\ 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 & -2 & 3 \\ 0 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 10 \\ -6 & 6 & -9 \\ 4 & -6 & 2 \end{bmatrix}$$

$$\therefore\ tr(C^t(A+B)) = 12 = \langle A+B, C \rangle$$

$$\|A\|_2 = \langle A, A \rangle^{1/2} = (tr(A^tA))^{1/2}$$

$$\left[tr\left(\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 1 \end{bmatrix} \right) \right]^{1/2} = \left[tr\left(\begin{bmatrix} 2 & -2 & 1 \\ -2 & 4 & 2 \\ 1 & 2 & 5 \end{bmatrix} \right) \right]^{1/2} = \sqrt{11}$$

$$\|B\|_2 = \langle B, B \rangle^{1/2} = (tr(B^tB))^{1/2}$$

$$\left[\left(1 \quad 1 \right) \left(1 \quad -2 \quad 1 \right) \right]^{1/2} \left[\quad \left(2 \quad -2 \quad 4 \right) \right]^{1/2}$$

$$\left[\operatorname{tr} \begin{pmatrix} -2 & 0 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 3 \end{pmatrix} \right] = \left[\operatorname{tr} \begin{pmatrix} -2 & 4 & -2 \\ 4 & -2 & 10 \end{pmatrix} \right] = \sqrt{16} = 4$$

$$\cos \theta = \frac{\langle A, B \rangle}{\|A\| \|B\|} = \frac{5}{4\sqrt{11}}$$