Problem 2.2. Prove the linearity of the Skorohod integral.

Let $u, v \in Dom(\delta)$. Then

$$\delta(u) = \sum_{n=0}^{\infty} I_{n+1}(\hat{f}_n), \qquad \delta(v) = \sum_{n=0}^{\infty} I_{n+1}(\hat{g}_n)$$

By the Problem 1.2,

$$\delta(u) + \delta(v) = \sum_{n=0}^{\infty} I_{n+1}(h_n)$$

where h_n is such that $h_n = \hat{f}_n + \hat{g}_n$ for all n.

Since the sum of symmetric functions is symmetric, we have that h_n is symmetric. Thus,

$$\delta(u) + \delta(v) = \delta(u+v)$$

Now let $u \in Dom(\delta)$ and λ be a scalar. By

Remark 2.3. By (1.17) a stochastic process u belongs to $Dom(\delta)$ if and only if

$$E[\delta(u)^2] = \sum_{n=0}^{\infty} (n+1)! \|\widetilde{f}_n\|^2_{L^2([0,T]^{n+1})} < \infty \,. \tag{2.4}$$

we know that $\lambda u \in Dom(\delta)$. In fact,

$$\delta(\lambda u) = \sum_{n=0}^{\infty} I_{n+1}(\lambda \hat{f}_n) = \lambda \sum_{n=0}^{\infty} I_{n+1}(\hat{f}_n) = \lambda\, \delta(u)$$