**Problem 5.2.** Prove equation (5.22), that is, that

$$\frac{d}{dt}W(t) = \dot{W}(t),$$

where the derivative exists in $(\mathcal{S})^*$.

We saw (Example 5.3) that

$$W(t) = \sum_k \left( \int_0^t e_k(y)\, dy \right) H_{\varepsilon(k)}$$

and recall that

$$\dot{W}(t) := \sum_k e_k(t) H_{\varepsilon(k)}$$

Therefore,

$$\frac{d}{dt} W(t) = \frac{d}{dt} \sum_k \left( \int_0^t e_k(y)\, dy \right) H_{\varepsilon(k)}$$

$$= \sum_k e_k(t) H_{\varepsilon(k)}$$