Proposition 2.6. *For any fixed $t \in [0,T]$ and $u \in Dom(\delta)$ we have $\chi_{(0,t]}u \in Dom(\delta)$ and $\chi_{(t,T]}u \in Dom(\delta)$ and*

$$\int_0^t u(s)\delta W(s) = \int_0^T \chi_{(0,t]}(s)u(s)\delta W(s) \text{ and } \int_t^T u(s)\delta W(s) = \int_0^T \chi_{(t,T]}(s)u(s)\delta W(s),$$

with

$$\int_0^T u(s)\delta W(s) = \int_0^t u(s)\delta W(s) + \int_t^T u(s)\delta W(s).$$

Since $u \in Dom(\delta)$,

$$\mathbb{E}[\delta(u\chi_{(0,t]})^2] = \sum_{n=0}^{\infty} (n+1)! \, \| \tilde{f}_n \chi_{(0,t]} \|^2_{L^2([0,T]^{n+1})}$$

$$\leq \sum_{n=0}^{\infty} (n+1)! \, \| \tilde{f}_n \|^2_{L^2([0,T]^{n+1})} = \mathbb{E}[\delta(u)^2] < \infty$$

Similarly for $\chi_{(t,T]}$. Hence, $\chi_{(0,t]}u$ and $\chi_{(t,T]}u$ belong to $Dom(\delta)$.

Note that

$$\int_0^t u(s)\, \delta W(s) = \sum_{n=0}^{\infty} I_{n+1}(\tilde{f}_n \cdot \chi_{(0,t]}(s)) = \int_0^T \chi_{(0,t]}(s)\, u(s)\, \delta W(s)$$

and

$$\int_t^T u(s)\, \delta W(s) = \sum_{n=0}^{\infty} I_{n+1}(\tilde{f}_n \cdot \chi_{(t,T]}(s)) = \int_0^T \chi_{(t,T]}(s)\, u(s)\, \delta W(s)$$

Since the Skorohod integral is linear,

$$\int_0^T u(s)\,\delta W(s) = \int_0^T \left(\chi_{(0,t]} + \chi_{(t,T]}\right) u(s)\,\delta W(s)$$

$$= \int_0^T \chi_{(0,t]}(s)\,u(s)\,\delta W(s) + \int_0^T \chi_{(t,T]}(s)\,u(s)\,\delta W(s)$$

$$= \int_0^t u(s)\,\delta W(s) + \int_t^T u(s)\,\delta W(s)$$