7.16. Let B_t be 1-dimensional and let $f: \mathbf{R} \to \mathbf{R}$ be a bounded function. Prove that if $t < T$ then

$$E^x[f(B_T)|\mathcal{F}_t] = \frac{1}{\sqrt{2\pi(T-t)}} \int_{\mathbf{R}} f(x) \exp\left(-\frac{(x - B_t(\omega))^2}{2(T-t)}\right) dx\,. \tag{7.5.5}$$

(Compare with (7.5.4).)

By the Markov property,

$$E^x[f(B_T)|\mathcal{F}_t] = E^{B_t}[f(B_{T-t})]$$

$$= \frac{1}{\sqrt{2\pi(T-t)}} \int_R f(x) \exp\left(-\frac{(x-B_t)^2}{2(T-t)}\right) dx$$