Øksendal 3.5
   Prove that
$$M_t = B_t^2 - t$$
is an $\mathcal{F}_t$-martingale.

What we need to show is:
1. M_t is $\mathcal{F}_t$-measurable
2. $E[|M_t|] < \infty$
3. $E[M_t | \mathcal{F}_s] = m_s$, $s \le t$

The first is immediate. Since B_t^2 and t are $\mathcal{F}_t$-measurable, m_t also is.
   Now notice that
$$E[|M_t|] = E[|B_t^2 - t|] \le E[|B_t^2|] + E[|t|]$$
$$< \infty$$

Finally,
$$E[M_t | \mathcal{F}_s] = E[B_t^2 - t | \mathcal{F}_s]$$
$$= E[(B_t - B_s + B_s)^2 | \mathcal{F}_s] - E[t | \mathcal{F}_s]$$
$$= E[(B_t - B_s)^2 | \mathcal{F}_s] + 2E[(B_t - B_s)B_s | \mathcal{F}_s] + E[B_s^2 | \mathcal{F}_s] - E[t | \mathcal{F}_s]$$
$$= t - s + B_s^2 - t = B_s^2 - s = m_s$$
for $s < t$ as desired.