1.1) $$E(w) = -\frac{1}{N}\sum_{n=1}^{N} t^n \ln(y^n) + (1-t^n)\ln(1-y^n)$$

$$\frac{\partial E}{\partial w_i} = \frac{\partial}{\partial w_i} - \frac{1}{N}\sum_{n=1}^{N} t^n \ln(y^n) + (1-t^n)\ln(1-y^n)$$

$$= -\frac{1}{N}\sum_{n=1}^{N} t^n \frac{\partial}{\partial w_i}\ln(y^n) + (1-t^n)\frac{\partial}{\partial w_i}\ln(1-y^n)$$

$$= -\frac{1}{N}\sum_{n=1}^{N} t^n \frac{\partial y^n}{\partial w_i}\cdot\frac{1}{y^n} + (1-t^n)\frac{\partial}{\partial w_i}\cdot\frac{1-y^n}{1-y^n}$$

$$y^n = g_w^n(x) = \frac{1}{1+e^{-w^T x}}$$

$$\frac{\partial E}{\partial w_i} = -\frac{1}{N}\sum_{n=1}^{N} t^n \frac{1}{g_w^n(x)}\cdot\frac{\partial g_w^n(x)}{\partial w_i} + (1-t^n)\frac{1}{1-g_w^n(x)}\cdot\frac{\partial}{\partial w_i}(1-g_w^n(x))$$

$$= -\frac{1}{N}\sum_{n=1}^{N} t^n x_i^n(1-g_w^n(x)) + (1-t^n)(x_i^n g_w^n(x))$$

$$\Rightarrow \frac{\partial E^n}{\partial w_i} = -t^n x_i^n(1-g_w^n(x)) + (1-t^n)x_i^n g_w^n(x)$$

$$-\frac{\partial E^n}{\partial w_i} = (t^n - y^n)x_i^n$$

1.2 $$y_k^n = \frac{e^{a_k^n}}{\sum_{k'} e^{a_k^n}} \qquad a_k^n = w_k^T x^n$$

$$E^n = -\sum_{k=1}^{C} t_k^n \ln(y_k^n) = -\sum_{k=1}^{C} t_k^n \ln\left\{\frac{e^{a_k^n}}{\sum_{k'} e^{a_k^n}}\right\} =$$

$$-\sum_{k=1}^{C} t_k^n \left(\ln(e^{a_k^n}) - \ln(\textstyle\sum_{k'} e^{a_k^n}\right) =$$

$$-\sum_{k=1}^{C} t_k^n \left(w_k^T x^n - \ln(\textstyle\sum_{k'} e^{a_k^n}\right)$$

$$-\frac{\partial E^n}{\partial w_{kj}} = \frac{\partial}{\partial w_{kj}} \sum_{k=1}^{C} t_k^n \left(w_k^T x^n - \ln \textstyle\sum_{k'} e^{a_k^n}\right)$$

$$\frac{\partial E^n}{\partial w_{jk}} = \frac{\partial}{\partial w_{kj}} \left[-\sum_{k' \neq k} t_{k'}^n \left(w_{k'}^T x^n - \ln \textstyle\sum_{k''} e^{w_{k''}^T x^n}\right) - \right.$$

$$\left.\sum_{k'=k} t_{k'}^n \left(w_{k'}^T x^n - \ln \textstyle\sum_{k''} e^{w_{k''}^T x^n}\right)\right.$$

$$= \frac{\partial}{\partial w_{kj}} \left[-\sum_{k' \neq k} t_{k'}^n \left[w_{k'}^T x^n - \ln \sum_{k''} e^{w_{k''}^T x^n}\right] - t_k^n \left[w_k^T x^n - \ln \sum_{k''} e^{w_{k''}^T x^n}\right]\right]$$

$$= -\sum_{k' \neq k} t_{k'}^n \left[\frac{\partial}{\partial w_{kj}} w_{k'}^T x^n - \frac{\partial}{\partial w_{kj}} \ln \textstyle\sum_{k''} e^{w_{k''}^T x^n}\right]$$

$$-t_k^n \left[\frac{\partial}{\partial w_{kj}} w_k^T x^n - \frac{\partial}{\partial w_{kj}} \ln \textstyle\sum_{k''} e^{w_{k''}^T x^n}\right]$$

$$= \sum_{k' \neq k} t_{k'} \frac{e^{w_k^T x^n}}{\sum_{k''} e^{w_{k''}^T x^n}} x^n - t_k^n x^n + t_k \frac{e^{w_k^T x^n}}{\sum_{k''} e^{w_{k''}^T x^n}} x^n$$

1.2 $= \sum_{k=1}^{c} t_k y_k x^n - t_k x^n = \quad = y_k x^n \sum_{k=1}^{c} t_k - t_k x^n$

$= x^n [y_k - t_k]$