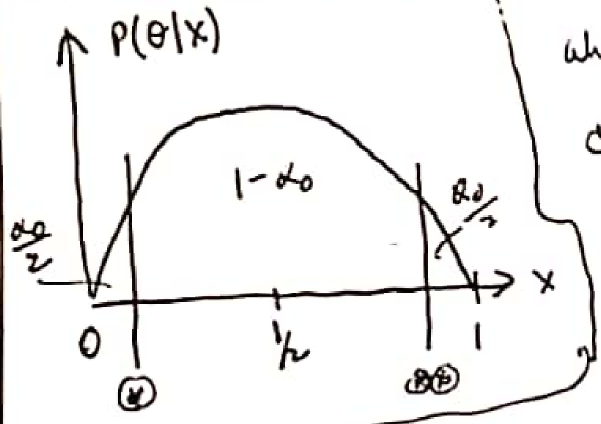
<u>Stat inference review</u> From lecture 02.

① Point estimation
② CI
③ Theory Test

(data comes in $x=1$, $n=2$)

$\underset{\text{Laplace Prior}}{P(\theta) = \text{Beta}(1,1)} \Rightarrow P(\theta|x) = \text{Beta}(2,2)$

From last class $X=1, n=2$ $\mathcal{F}: \text{Bin}(n, \theta)$



when $\alpha_0 = 0$ then the CR is the support.

$$CR_{\theta, 1-\alpha_0} := \left[Q\left[\theta|x, \frac{\alpha_0}{2}\right], Q\left[\theta|x, 1-\frac{\alpha_0}{2}\right] \right]$$

↗ ⊛ posterior Beta(2,2) ↗ ⊛⊛ posterior Beta(2,2)

Goal produce a 95% CR for theta.

$$= [\text{qbeta}(2.5\%, 2, 2), \underbrace{\text{qbeta}(97.5\%, 2, 2)] = [0.094}_{\text{computer}}, 0.906]$$

$P(\theta \in [.094, .906] \,|\, x) = 95\%$. This is a real probability statement. The CI approach cannot provide such a statement. CR is highly interpretable. Also the CR always has to be IN the parameter space. CR is a proper subset of Θ for $\alpha_0 > 0$) — not what happens in CI's.

Here is the 95% CI for this data:

$$CI_{\theta, 95\%} = \left[0.5 \pm 1.96\sqrt{\frac{(.5)(.5)}{2}}\right] = [-0.21, 1.21] = \text{not a proper subset of } \Theta = (0,1)$$

You have to believe in the Bayesian setup with the prior to get around the CI problem.

The above CR is technically a two sided CR. You can also create a one-sided CR. (ie. left or right sided).

$CR_{L,\theta,1-\alpha_0} := [\text{smallest value in } \Theta \text{ or } -\infty,\ \underset{\text{qbeta}(.95,2,2)}{Q[\theta|X,\ 1-\alpha_0]}]$

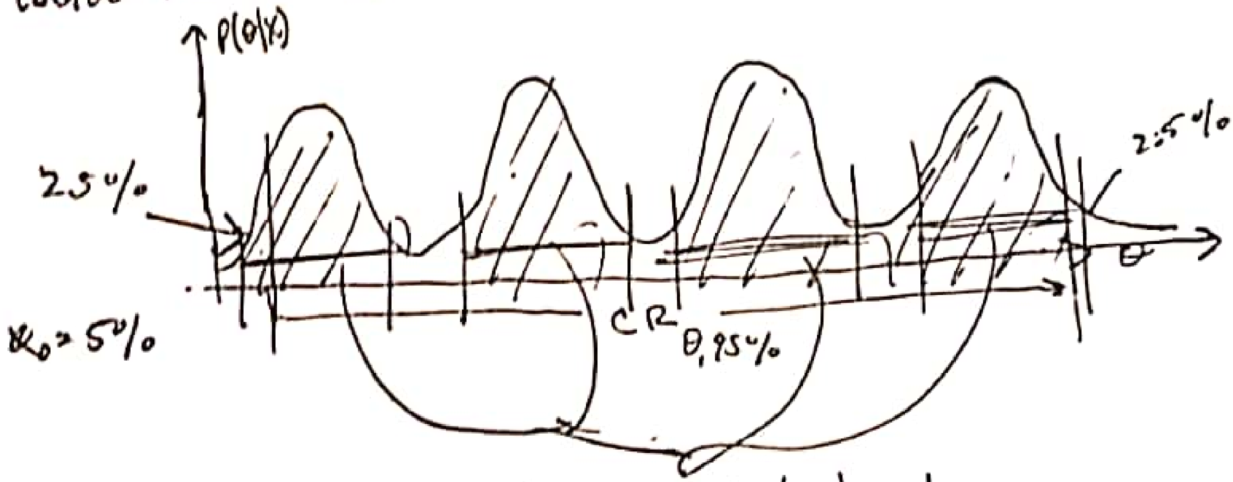
in our case (data set given) $= [0,\ 0.865] \Rightarrow P(\theta < .865\ |x) = 95\%$.

$CR_{R,\theta,1-\alpha_0} = [\underset{\text{qbeta}(.95,2,2)}{Q[\theta|X,\alpha_0]},\ \text{largest value in } \Theta \text{ or } \infty] = [.136, 1]$

$\Rightarrow P(\theta > .136\,|x) = 95\%$

You can use the posterior to take another approach, called high density region (HDR)

Consider the following posterior for θ:



4 Pieces of high density.
$HDR_{\theta,95\%}$

$P(\theta \in HDR_{\theta,95\%}) = 95\%$ but it has minimum width

Sometimes the CR = HDR (ex: unimodal posteriors).

Disadvantages of HDR ① it can be non-contiguous i.e. (in pieces)
② Computationally intense
③ no L or R intervals

Bayesian Hypothesis testing. We can immediately compute the following

Bayesian p-val $:= P(H_0|x)$, $P(H_a|x)$

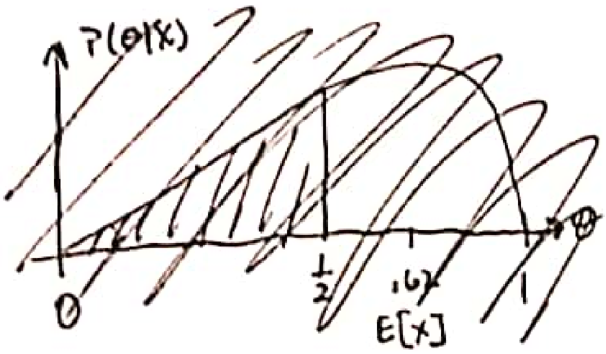
If $P(H_0|x) < \alpha_0 \Rightarrow$ reject H_0 → gives you the probability your theory is true.

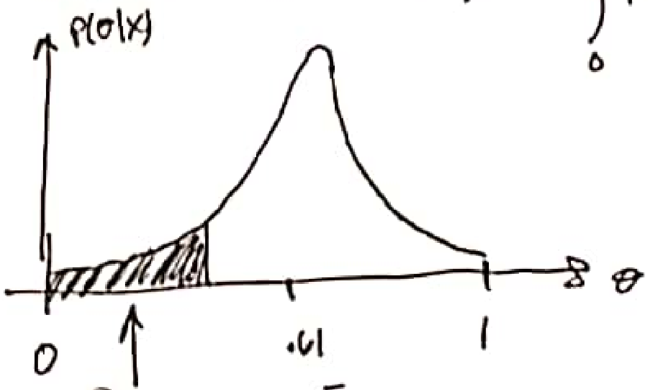
← threshold of sufficient evidence

Let's reiterate the hypothesis testing example from lec 03 $n=100$ flips of a coin where $x=61$ are heads. Test if coin is unfairly weighted towards H. at 5% significance level.

$H_a : \theta > 0.5 \Rightarrow H_0 : \theta \leq 0.5$ Assume $P(\theta) = \text{Beta}(1,1)$

$p(\theta|x) = \text{Beta}(62, 40)$ compute $P(H_0|x) = P(\theta \leq .5|x) = \int_0^{.5} p(\theta|x)\, d\theta$





Probability null hypothesis is true $= P(H_0|x) = \int_0^{.5} p(\theta|x)\,d\theta = \int_0^{.5} \frac{1}{B(62,40)} \theta^{61}(1-\theta)^{39}\,d\theta$

$= \text{pbeta}(0.5, 62, 40) = 0.014 = 1.4\% =$ probability null hypothesis is true

Trash this! Conclude the coin is unfair.

Uber driver does 200 rides and gets 37 non-5-star ratings. If his true proportion of ~~non~~ 5-star ratings is less than 25%, then uber policy is to fire the driver. Prove he should be fired (or not) at 5% sig level

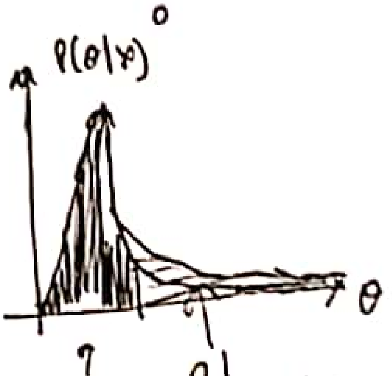
$H_a: \theta > 25\% \Rightarrow H_0\ \theta \le 25\%$ (innocent until proven guilty.)

$P(\theta) = \text{Beta}(1,1)$ $n = 200$ $X = 37$ $F: \text{Binomial}(n, \theta)$

$P(\theta|x) = \text{Beta}(38, 164)$, $P(H_0|x) = P(.25|x) = \int_{.25}^{1} \text{Beta}(38,164)\, d\theta =$

$P(\theta \le .25|x)$

$= \int_{0}^{.25} \text{Beta}(38,164)\, d\theta = \text{pbeta}(.25, 38, 164)$



$\int_{.25}^{1} \text{Beta}(38,164)\, d\theta$

$\text{pbeta}(.25, 38, 164) = .987 \approx 98.7\%$ retain H_0 do not fire.

probability

Lets test the coin again. Flip 100 times get 43 heads. Test if the coin is unfair, at 5% significance level.

$n=100$, $x=43$ $p(\theta) = \text{Beta}(1,1)$

$p(\theta|x) = \text{Beta}(44, 58)$ $H_a \Rightarrow \theta \neq .5$

$H_0 \Rightarrow \theta = .5$

$P(H_0|x) = P(\theta = .5 \mid x) = 0 \rightarrow$ Reject H_0 always?? Yes.

Something is wrong.

Using this approach, two-sided tests we always rejected, (if the posterior is cts). Does this make sense. This does make sense. Any infinitely precise theory of theta is wrong in the real world. A coin is never exactly $\frac{1}{2}$ likely to flip heads. So we need to slightly reframe our hypotheses using a notion of ~~"[illegible]~~ "margin of equivalence" called δ (delta).

$$H_a : \theta \notin [\theta_0 \pm \delta] \Rightarrow H_0 : \theta \in [\theta_0 \pm \delta]$$

Describe what fair means to you.