Hwb 2.5

$P(X_* < 17000 \mid X_1, X_2, \sigma^2 = 1000^2) = \text{pnorm}(17000, 18600, 1224.8)$

$P(X_* \mid X, \sigma^2) = N\left(\bar{x}, \frac{\sigma^2}{n} + \sigma^2\right)$ $\cancel{\neq} N(\bar{x}, \sigma^2)$

$= N\left(18600, \frac{1000^2}{2} + 1000^2\right) = N(18600, 1{,}500{,}000) = N(18600, 1224.8^2)$

$P\left(\frac{X_* - 18600}{1224.8} < \frac{17000 - 18600}{1224.8}\right) = P(Z < -1.31) \approx 10\%$

$P(\theta \mid X, \sigma^2) = N\left(\bar{x}, \frac{\sigma^2}{n}\right)$

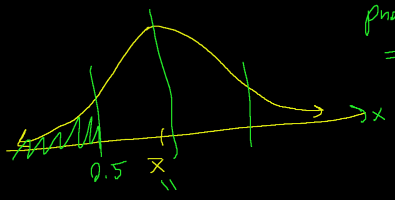
$n_0 = 2 \qquad y_1 = 0,\ y_2 = 1$

$P(\sigma^2 \mid \theta) = \text{InvGamma}\left(\frac{2}{2}, \frac{2\sigma_0^2}{2}\right) = \text{InvGamma}(1, \sigma_0^2)$

$\sigma_0^2 = \frac{1}{n_0} \sum (y_i - \theta)^2 = \frac{1}{2}\left((0-\theta)^2 + (1-\theta)^2\right)$

$= \frac{1}{2}\left(\theta^2 + 1 - 2\theta + \theta^2\right)$

$= \frac{1}{2}\left(1 - 2\theta + 2\theta^2\right)$



$\text{pnorm}(0.5, \bar{x}, \sigma/\sqrt{n}) = \int_{-\infty}^{0.5} f(x)\,dx$

$P(X_* \mid X, \sigma^2) = N\left(\bar{x}, \frac{\sigma^2}{n} + \sigma^2\right)$

$= N\left(13.9, \frac{5^2}{20} + 5^2\right) = N(13.9, 5.12^2)$

$P(\theta, \sigma^2 \mid X) = N(\theta, \sigma^2) = \frac{1}{\sqrt{2\pi}\,\sigma^2} e^{-\frac{1}{2\sigma^2}(x-\theta)^2}$

$\propto (\sigma^2)^{-\frac{1}{2}} e^{-\frac{1}{2\sigma^2}(x-\theta)^2}$

Hwb 3.1

a) $X_1, \dots, X_{21} \overset{iid}{\sim} N(\theta, \sigma^2),\ \theta = 12$

b) $\hat{\sigma}^2_{MLE} = \frac{1}{21} \sum_{i=1}^{21} (X_i - 12)^2 = 0.0814$

c) $P(\sigma^2 \mid \theta) \propto (\sigma^2)^{-1} = \text{InvGamma}(0, 0)$

$\Rightarrow P(\sigma^2 \mid \theta, x) = \boxed{\text{InvGamma}\left(\frac{n}{2}, \frac{n\hat{\sigma}^2_{MLE}}{2}\right)} = \text{InvGamma}(11.5, 0.85)$

d) ______

e) $H_a: \sigma^2 > 0.35^2 \Rightarrow H_0: \sigma^2 \le 0.35^2$

$P_{val} = P(H_0 \mid X, \theta) = P(\sigma^2 \le 0.35^2 \mid x, \theta)$

$= \text{pinvgamma}(0.35^2, 11.5, 0.85)$

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$P(\sigma^2 \mid \theta, x) = \text{InvGamma}\left(\frac{3}{2}, \frac{3\hat{\sigma}^2_{MLE}}{2}\right)$

$= \text{InvGamma}\left(1.5, \frac{40.7}{2}\right)$

$\hat{\sigma}^2_{MMSE} = \frac{40.7}{3-2} = 40.7$

$\hat{\sigma}^2_{MAP} = \frac{40.7}{3+2} = 8.14$

$\hat{\sigma}^2_{MMAE} = \text{qinvgamma}(0.5, 1.5, 20.4)$

$T_{21}(12, \sqrt{0.0814})$

$PI_{X_*, 95\%} = \left[Q[2.5\%, X_* \mid X],\ Q[97.5\%, X_* \mid X]\right]$

$= \left[\text{qt.scaled}(.025, 21, 12, \sqrt{.0814}),\ \text{———}\right]$