A. Problems to Submit

To receive full credit, you must provide detailed arguments for each of these problems.

1. BE Exercises: 9.2, 9.4, 9.6, 9.7
2. Consider a random sample of size 2 from the distribution given below.

$$f(x;\theta) = \begin{cases} \frac{1-\theta}{3} & \text{if } x = 0 \\ \frac{1}{3} & \text{if } x = 1 \\ \frac{1+\theta}{3} & \text{if } x = 2 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Suppose $\theta \in \{-1, 0, 1\}$. What is the MLE of θ?
   (b) Now suppose that $\theta \in [-1, 1]$. What is the likelihood function?

B. Additional Practice Problems

Problems in part B are optional. You do not need to submit your solutions to these problems.

1. BE Exercises: 9.1, 9.3, 9.8
2. Suppose that $G_1, \ldots, G_n$ is an iid sample from a Bernoulli distribution with parameter $\pi \in (0, 1)$. Suppose that $Y_1, \ldots, Y_n$ is another iid sample such that the distribution of Y_i conditional on $G_i = g$ is $N(\mu_g, \sigma^2)$. Further, suppose that G_i denotes the class (0 or 1) patient i belongs to, and Y_i is their outcome (e.g., blood pressure).
   (a) What is the likelihood function if only $Y_1 = y_1, \ldots, Y_n = y_n$ (i.e., only outcomes) were observed?
   (b) What is the likelihood function if $(Y_1, G_1) = (y_1, g_1), \ldots, (Y_n, G_n) = (y_n, g_n)$ (i.e., both outcomes and class membership) were observed?

C. Advanced Problems

You do not need to submit your solutions to theproblems in part C.

1. Consider again **Example [9.5]** from the course notes. Using the Intermediate Value Theorem and the Mean Value Theorem, prove that a unique root of a function's first derivative is a global maximum if the second derivative evaluated at the root is negative.
2. BE Examples: 9.2.8, 9.2.12, 9.2.13