BIOSTAT 704 - Homework 6

Austin Allen

Problem 1

BE Exercise 10.5

Let $X_1, ..., X_n$ be a random sample from a gamma distribution, $X_i \sim GAM(\theta, 2)$. Show that $S = \sum_{i=1}^{n} X - i$ is sufficient for θ

Part (a): by using equation (10.2.1)

Part (b): by the factorization criterion of equation (10.2.3)

BE Exercise 10.7

Let $X_1, ..., X_n$ be independent with $X_i \sim NB(r_i, p)$. Find a sufficient statistic for p.

BE Exercise 10.16

For the random variables $X_1, ..., X_n$ in exercise 6, find the MLE of p by maximizing the pdf of the sufficient statistic. Is this the same as the usual MLE? Explain why this result is expected.

Problem 2

BE Exercise 10.11

Consider a random sample of size n from a uniform distribution, $X_i \sim UNIF(\theta_1, \theta_2)$.

Part (a): Show that $X_{1:n}$ is a sufficient statistic for θ_1 if θ_2 is known.

Part (b): Show that $X_{1:n}$ and $X_{n:n}$ are jointly sufficient for θ_1 and θ_2.

BE Exercise 10.12

Let $X_1, ..., X_n$ be a random sample from a two parameter exponential distribution, $X_i \sim EXP(\theta, \eta)$. Show that $X_{1:n}$ and $\bar{X}_n$ are jointly sufficient for θ and η.

BE Exercise 10.20

Show that the following families of distributions belong to the regular exponential class, and for each cae use this information to find complete sufficient statistics based on a random sample $X_1, ..., X_n$.

Part (a): $BIN(1,p); 0, p < 1$ **Part (b):** $POI(\mu); \mu > 0$ **Part (c):** $NB(r,p); r$ known,$0 < p < 1$ **Part (d):** $N(\mu, \sigma^2); -\infty < \mu < \infty, \sigma^2 > 0$ **Part (e):** $EXP(\theta); \theta > 0$ **Part (f):** $GAM(\theta, \kappa); \theta > 0, \kappa > 0$ **Part (g):** $BETA(\theta_1, \theta_2); \theta_1 > 0, \theta_2 > 0$ **Part (h):** $WEI(\theta, \beta); \beta$ known, $\theta > 0$

BE Exercise 10.25

Consider a random sample of size n from a distribution with pdf $f(x;\theta) = \theta x^{\theta-1}$ if $0 < x < 1$ and zero otherwise; $\theta > 0$.

Part (a): Find the UMBUE of $1/\theta$. *Hint*: $E[-lnX] = 1/\theta$.

Part (b): Find the UMVUE of θ.

Problem 3

Let $X_1, ..., X_n$ be a random sample from the Normal distribution $N(, 1).

Part (a): Find a complete and sufficient statistic for θ.

Part (b): Show that $\bar{X}_n^2 - \frac{1}{n}$ is a UMVUE for θ^2.