A. Problems to Submit

To receive full credit, you must provide detailed written solutions for each of these problems.

1. Let $Y_1, Y_2, \ldots, Y_n$ be a random sample such that $Y_i \sim \text{GAM}(\theta, \kappa)$. The random variable Y_i represents the amount of rainfall accumulated over a year in a reservoir, with $\mu = \mathbb{E}(Y_i) = 132$ and $\sigma^2 = \mathbb{V}(Y_i) = 26.4$. Consider $\bar{Y}_n = \dfrac{1}{n}\displaystyle\sum_{i=1}^{n} X_i$.

   (a) Give an approximation of $P(\bar{Y}_n \geq 200)$ for $n = 500$.

   (b) What is the true distribution of $\bar{Y}_n$?

   (c) Does $\bar{Y}_n$ converges in probability? If so, what is its limit?

   (d) Calculate the true probability $P(|\bar{Y}_n - \mu| \leq 0.4)$ for $n = 20$.
   Explain what $|\bar{Y}_n - \mu| \leq 0.4$ means, in terms of the amount of rainfall, and interpret the above result.

   (e) Find $P(|\bar{Y}_n - \mu| \leq 0.4)$ when $n = 25, 50, 100, 200$, and 300. What pattern do you observe among the values for $P(|\bar{Y}_n - \mu| \leq 0.4)$ for the various values of n?

   (f) Now suppose $\sigma^2 = 79.3$, re-calculate the probabilities in part (c). Do the results change drastically? How do you explain such a trend?

   (g) Compared to $P(|\bar{Y}_n - \mu| \leq 0.4)$, will the probability $P(|\bar{Y}_n - \mu| \leq \varepsilon)$ increase or decrease, for $\varepsilon < 0.2$, $n = 50$ and $\sigma^2 = 26.4$. Justify your answer.

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2. Suppose that X_n follows a binomial distribution with parameter (n, π) where $\pi \in (0, 1)$. Define $Y_n = \frac{1}{n} X_n$ for each n.

   (a) Find the asymptotic normal distribution of Y_n.

   (b) Find the asymptotic normal distribution of $\ln(Y_n)$.

   In each case, clearly indicate which theorems you have used to establish the result.

   Hint: First, write Y_n as a sample mean of a random sample that you will specify.

3. BE Exercise: 7.15
   Hint: For 7.15 (e), please do use BE Theorem 7.5.1.

4. BE Exercise: 7.16

B. Additional Practice Problems

1. Let $X_n \sim$ Exponential$(1/n)$. Show that $X_n \xrightarrow{p} 0$ using Theorem 7.6.1 **AND** an exact expression for the related probability P_n.
2. BE Exercise 7.13
3. BE Exercises: 7.14, 7.17
4. Let $X_1, \ldots, X_n$ be a random sample from a Poisson(μ) distribution.
   (a) Show that $\bar{X}_n \xrightarrow{p} \mu$
   (b) Show that $\bar{X}_n \xrightarrow{d} \mu$
   (c) Show that $\sqrt{n}(\bar{X}_n - \mu)/\sqrt{\mu} \xrightarrow{d} N(0,1)$
   (d) Show that $\sqrt{n}(\bar{X}_n - \mu)/\sqrt{\bar{X}_n} \xrightarrow{d} N(0,1)$

   In each case, clearly indicate which theorems you have used to establish the result.