BIOSTAT 704 - Homework 2

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Problem 1

Let $Y_1, ..., Y_n$ be a random sample where $Y_i \sim \text{GAM}(\theta, \kappa)$.

- Y_i represents the amount of rainfall accumulated in a reservoir over a year
- $\mu = E(YPi) = 132$
- $\sigma^2 = Var(Y_i) = 26.4 \implies \sigma = 5.14$
- $Y_n = 1/n \sum_{i=1}^{n} X_i$
  - Note: I'm not sure if this was to be $Y_n = 1/n \sum_{i=1}^{n} Y_i$

Part (a): Give an approximation of $P(\bar{Y} > 200)$ for $n = 500$

To calculate this probability, we're going to rely on the central limit theorem. We will let $Z = \frac{\bar{Y}_n - \mu}{\sigma/\sqrt{n}} = \frac{\bar{Y}_n - 132}{5.14/\sqrt{500}}$.

$$
\begin{aligned}
P(\bar{Y}_n \geq 200) &= P(\frac{\bar{Y}_n - 132}{0.23} \geq \frac{200 - 132}{0.23}) \\
&= 1 - P(\frac{\bar{Y}_n - 132}{0.23} < \frac{200 - 132}{0.23}) \\
&= 1 - P(Z < 295.9) \\
&= 0
\end{aligned}
$$

Part (b): What is the true distribution of $\bar{Y}_n$?

I decided to use the MGF method to deterimine this distribution:

$$
\begin{aligned}
M_{Y_i}(t) &= E(exp(Y_i t)) \\
&= (\frac{1}{1 - \theta t})^{\kappa} \\
M_{\bar{Y}_n}(t) &= E(exp(\bar{Y}_n t)) \\
&= E(exp((1/n) \sum Y_i t)) \\
&= E(exp((t/n) \sum Y_i)) \\
&= E(\prod exp((Y_i t)/n)) \\
&= \prod E(exp((Y_i t)/n))) \\
&= \prod M_{Y_i}(t/n) \\
&= (\frac{1}{1 - \theta t/n})^{n\kappa}
\end{aligned}
$$