BIOSTAT 704 - Homework 5

Austin Allen

March 8th, 2024

Problem 1

BE 9.21: Consider a random sample of size n from a Bernoulli distribution, $X_i \sim Bin(1, p)$.

Part (a): Find the CRLB for the variances of unbiased estimators of p.

Part (b): Find the CRLB for the variances of unbiased estimators of p(1 - p).

Part (c): Find a UMVUE of p.

BE 9.23: Let $X_1, ..., X_n$ be a random sample from a normal distribution, $N(0, \theta)$.

Part (a): Is the MLE, $\hat{\theta}$, and unbiased estimator of θ?

Part (b): Is $\hat{\theta}$ a UMVUE of θ?

BE 9.26: Consider a random sample of size n from a distribution with pdf $f(x;\theta) = 1/\theta$ if $0 < x \leq \theta$, and zero otherse; $0 < \theta$.

Part (a): Find the MLE $\hat{\theta}$.

Part (b): Find the MME $\tilde{\theta}$.

Part (c): Is $\hat{\theta}$ unbiased?

Part (d): Is $\tilde{\theta}$ unbiased?

Part (e): Compare the MSEs of $\hat{\theta}$ and $\tilde{\theta}$.

BE 9.31: Let $\hat{\theta}$ and $\tilde{\theta}$ be the MLE and MME, respectively, for θ in Exercise 26.

Part (a): Show that $\hat{\theta}$ is consistent.

Part (a): Show that $\tilde{\theta}$ is consistent.

BE 9.35: Find the asymptotic distribution of the MLE of p in Exercise 4(a).

Problem 2

Suppose that $X_1, ..., X_n$ is a random sample from $GAM(\theta, 2)$.

Part (a): Show that the MLE of θ is $\hat{\theta} = \sum_{i=1}^{n} \frac{X_i}{2n}$

Part (b): Is the MLE of θ simply consistent?

Problem 3

Consider again the MME and MLE of θ in BE Exercise 9.26. Suppose you wanted to compare their performance for estimating θ.

Part (a): Is it appropriate to compare their performance by computing their relative efficiency? If so, what is it and which estimator has better performance? If not, explain why not.

Part (b): Is it appropriate to compare their performance by computing their asymptotic relative efficiency? If so, what is it and which estimator has better performance? If not, explain why not.

Part (c): Could either of the metrics described in part (a) or (b) be used to compare the finite-sample performance of these estimators? If so, which one? If not, propose another metric that could be used.

Part (d): Could either of the metrics described in part (a) or (b) be used to compare the large-sample performance of these estimators? If so, which one? If not, propose another metric that could be used.

Problem 4.

Consider again the scenario given in BE Exercise 9.21. It can be shown that $\bar{X}_n$ is the MLE of p.

Part (a): What is the MLE of p(1 − p)?

Part (b): Is the MLE found in part (a) a UMVUE for p(1 − p)? Explain your reasoning.

Part (c): Does the result found in part (b) contradict the following result given on page 316 of the course text?

$$\tau(\hat{\theta})_{\text{MLE}} \sim^d N(\tau(\theta), \text{CRLB}(\tau(\theta)))$$

Explain your reasoning.

Note: The result above extends the result given on page 316 from θ to $\tau(\theta)$. This extension can be established using Cramer's Theorem.