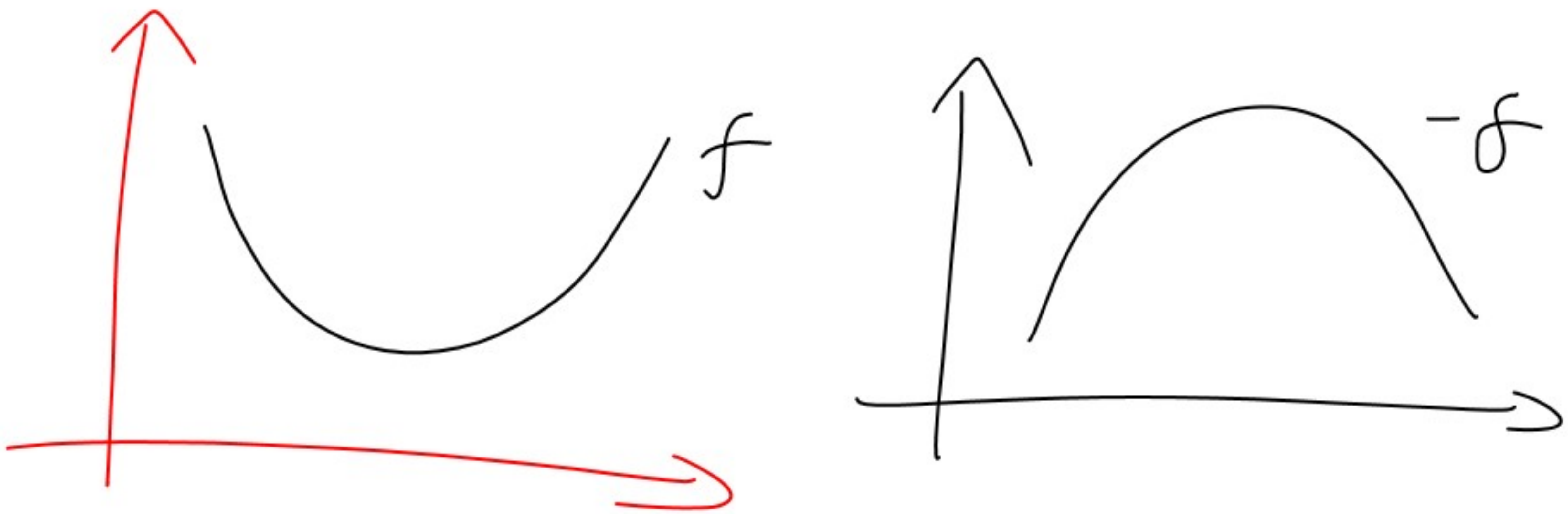
$$\begin{tikzcd}
 & \succsim \arrow[d] & \\
 & u(\cdot) \arrow[dl] \arrow[dr] & \\
\max\limits_{\{x\}} u(x) \text{ s.a. } px \le w \arrow[d] & & \min\limits_{\{h\}} p \cdot h \text{ s.a. } u(h) \ge \bar{u} \arrow[d] \\
x(p,w) \arrow[d] & & h(p,\bar{u}) \arrow[d] \\
v(p,w) & & e(p,\bar{u})
\end{tikzcd}$$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1. COBB-DOUGLAS

$$\hat{u}(x) = \prod_{i=1}^{n} x_i^{\alpha_i} \qquad \alpha_i \geq 0 \qquad \sum_i \alpha_i = 1$$

$$u(x) = \ln \hat{u}(x)$$

$$u(x) = \sum_{i=1}^{n} \alpha_i \ln(x_i)$$

$$\max_{\{x\}} \sum_{i=1}^{n} \alpha_i \ln(x_i)$$

$$\text{s.a.} \sum_{i=1}^{n} x_i p_i = w$$

$$\boxed{\mathcal{L} = \sum_{i=1}^{n} \alpha_i \ln(x_i) + \lambda\left(w - \sum_i p_i x_i\right)}$$

$$\lambda = \frac{\partial v(p,w)}{\partial w}$$

$$\lambda = \frac{\partial u(x(p,w))}{\partial w}$$

$$\tilde{u}(x) = \left(\prod_i x_i^{\hat{\alpha}_i}\right)^{\frac{1}{\sum_i \hat{\alpha}_i}} = \prod_i x_i^{\hat{\alpha}_i / \sum_i \hat{\alpha}_i} \to \alpha_i$$

$$= \sum_i x_i^{\alpha_i} \qquad \sum_i \alpha_i = \underline{1}$$

$$u(x_1, x_2) = \left(x_1^{\alpha_1} x_2^{\alpha_2}\right)^{\frac{1}{\alpha_1+\alpha_2}} = x_1^{\alpha_1/\alpha_1+\alpha_2} x_2^{\frac{\alpha_2}{\alpha_1+\alpha_2}}$$

$$= x_1^{\beta_1} x_2^{\beta_2} \qquad \beta_1 + \beta_2 = 1$$

C.P.O

$$\frac{\partial \mathcal{L}}{\partial x_i} = 0 \qquad i = 1, 2, \dots, m \qquad \frac{\alpha_i}{x_i} = \lambda p_i$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \qquad \sum_i \alpha_i = \sum_i \lambda p_i x_i \qquad \sum_i p_i x_i = w$$

$$\sum_i \alpha_i = \lambda \sum_i p_i x_i$$

$$\boxed{x_i = \frac{\alpha_i}{\lambda p_i} = \frac{\alpha_i}{\sum_i \alpha_i} \cdot \frac{w}{p_i}}$$

$$\lambda = \sum_i \alpha_i / w$$

PROBLEMA MINIMIZACIÓN GASTO

$$\max_{\{h\}} \; -p \cdot h$$

$$\text{s.a.} \quad u(h) \underset{=}{\geq} \bar{u}$$

$$\mathcal{L} = -\sum_i p_i h_i - \lambda\left(\bar{u} - \sum_i \alpha_i \ln h_i\right)$$

C.P.O

$$\frac{\partial \mathcal{L}}{\partial h_i} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0$$

$$\boxed{\begin{aligned} p_i &= \lambda \frac{\alpha_i}{h_i} \\ \bar{u} &= \sum_i \alpha_i \ln(h_i) \end{aligned}}$$

$$p_i = \frac{\lambda \alpha_i}{h_i} \qquad i=1,2,\dots,m$$

$$\left[\bar{u} = \sum_i \alpha_i \ln(h_i)\right.$$

$$\Rightarrow h_i = \frac{\lambda \alpha_i}{p_i}$$

$$\sum_i \alpha_i \ln(h_i) = \sum_i \left[\ln\lambda + \ln(\alpha_i) - \ln(p_i)\right]\alpha_i$$

$$\bar{u} = \sum_i \alpha_i \left[\ln\lambda + \ln\alpha_i - \ln p_i\right]$$

$$\bar{u} = \sum_i \alpha_i \left[\ln\lambda + \ln\alpha_i - \ln P_i \right]$$

$$\bar{u} = \ln\lambda \sum_i \alpha_i + \sum_i \alpha_i \ln\alpha_i - \sum_i \alpha_i \ln P_i$$

$$\ln\lambda = \frac{1}{\sum_i \alpha_i} \left[\bar{u} - \sum_i \alpha_i \ln\alpha_i + \sum_i \alpha_i \ln P_i \right]$$

$$\lambda = e^{\ln\lambda} = \exp\left\{ \overbrace{\frac{1}{\sum_i \alpha_i}}^{1} \left[\bar{u} - \sum_i \alpha_i \ln\alpha_i + \sum_i \alpha_i \ln P_i \right] \right\}$$

$$\lambda = EXP\left[\bar{u} - \sum_i \alpha_i \ln \alpha_i + \sum_i \alpha_i \ln P_i\right]$$

$$\lambda = \frac{e^{\bar{u}} \prod_{i=1}^{n} P_i^{\alpha_i}}{\prod_{i=1}^{n} \alpha_i^{\alpha_i}} = e^{\bar{u}} \prod_{i=1}^{n} \left(\frac{P_i}{\alpha_i}\right)^{\alpha_i}$$

$$\boxed{P_i = \lambda \frac{\alpha_i}{h_i}}$$

$$h_i(P, \bar{u}) = e^{\bar{u}} \left[\prod_{j=1}^{n} \left(\frac{P_j}{\alpha_j}\right)^{\alpha_j}\right] \left(\frac{\alpha_i}{P_i}\right)$$

② LEONTIEF: $u(x,y) = \min\{x,y\}$

$$\max_{\{(x,y)\}} \min\{x,y\} \quad \text{s.a.} \quad P_x x + P_y y = w$$

$x = y$

$$P_x x + P_y x = w$$

$$\boxed{x = \frac{w}{P_x + P_y} = y}$$