FUNCIONES C.E.S.

I

CONSTANT ELASTICITY OF SUSTITUTION

$$u(x,y) = A\left(a x^{\rho} + (1-a) y^{\rho}\right)^{1/\rho} \qquad 0 \le a \le 1$$

LA ELASTICIDAD DE SUSTITUCIÓN ES:

$$s = \frac{d \ln(y/x)}{d \ln(f_x/f_y)}$$ PARA UNA FUNCIÓN $f(x,y)$

APLIQUEMOS LA DEFINICIÓN A $u(x,y)$

$$u_x = \frac{\partial u}{\partial x} = A\frac{1}{\rho}\left(a x^{\rho} + (1-a) y^{\rho}\right)^{\frac{1}{\rho}-1} a \rho x^{\rho-1}$$

$$u_y = \frac{\partial u}{\partial y} = A\frac{1}{\rho}\left(a x^{\rho} + (1-a) y^{\rho}\right)^{\frac{1}{\rho}-1} (1-a) \rho y^{\rho-1}$$

$$\frac{u_y}{u_x} = \frac{(1-a)}{a}\left(\frac{y}{x}\right)^{\rho-1} \Rightarrow \frac{u_x}{u_y} = \left(\frac{a}{1-a}\right)\left(\frac{y}{x}\right)^{1-\rho}$$

$$\left(\frac{u_x}{u_y}\right)^{\frac{1}{1-\rho}}\left(\frac{1-a}{a}\right)^{\frac{1}{1-\rho}} = \frac{y}{x}$$

$$\ln\frac{y}{x} = \left(\frac{1}{1-\rho}\right)\ln\left(\frac{u_x}{u_y}\right) + \frac{1}{1-\rho}\ln\left(\frac{1-a}{a}\right)$$

$$\frac{\partial \ln(y/x)}{\partial \ln(u_x/u_y)} = \frac{1}{1-\rho} = s$$

AHORA, DEMOSTREMOS Q:

$$CES \rightarrow A\left(a x^{\rho} + (1-a) y^{\rho}\right)^{1/\rho} = u(x,y)$$

$\rho = 1$	$\rho \to 0$	$\rho \to -\infty$
$u(x,y) = A(a x + (1-a) y)$	$u(x,y) = x^{a} y^{1-a}$	$u(x,y) = \min\{x, y\}$
(LINEAL)	COBB-DOUGLAS	LEONTIEFF
$s = \infty$	$s = 1$	$s = 0$

$\boxed{\rho \to 0}$ NOTEMOS QUE

II

$$\lim_{\rho\to 0}\left(\alpha x^{\rho}+(1-\alpha)y^{\rho}\right)^{1/\rho} = EXP\left[\underbrace{\lim_{\rho\to 0} \ln\left(\alpha x^{\rho}+(1-\alpha)y^{\rho}\right)^{1/\rho}}\right]$$

$$= \lim_{\rho\to 0} \frac{\ln\left(\alpha x^{\rho}+(1-\alpha)y^{\rho}\right)}{\rho} = \frac{\ln 1}{\rho\to 0} = \frac{0}{0} \quad \text{(INDETERMINACIÓN)}$$

L'HOPITAL:

$$= \lim_{\rho\to 0} \frac{\frac{\partial \ln\left(\alpha x^{\rho}+(1-\alpha)y^{\rho}\right)}{\partial \rho}}{\frac{\partial \rho}{\partial \rho}} = \lim_{\rho\to 0} \frac{\frac{\alpha x^{\rho}\ln x+(1-\alpha)y^{\rho}\ln y}{\alpha x^{\rho}+(1-\alpha)y^{\rho}}}{1}$$

$$= \alpha\ln x+(1-\alpha)\ln y$$

$\left[\text{NOTAR: } \frac{\partial \alpha x^{\rho}}{\partial \rho} = \frac{\partial}{\partial \rho}\alpha e^{\ln x\cdot\rho} = \alpha \ln x\cdot x^{\rho}\right]$

$$\Rightarrow \lim_{\rho\to 0}\left(\alpha x^{\rho}+(1-\alpha)y^{\rho}\right)^{1/\rho} = EXP\left[\alpha\ln x+(1-\alpha)\ln y\right]$$

$$= x^{\alpha}y^{(1-\alpha)}$$

SUPONGAMOS $0\leq x\leq y$

$\boxed{\rho\to -\infty}$

$$\lim_{\rho\to -\infty}\left(\alpha x^{\rho}+(1-\alpha)y^{\rho}\right)^{1/\rho} = \lim_{\rho\to -\infty}\frac{x}{x}\left(\alpha x^{\rho}+(1-\alpha)y^{\rho}\right)^{1/\rho}$$

$$= \lim_{\rho\to -\infty} x\left(\alpha\left(\frac{x}{x}\right)^{\rho}+(1-\alpha)\left(\frac{y}{x}\right)^{\rho}\right)^{1/\rho}$$

$$= EXP\left[\lim_{\rho\to -\infty}\ln x + \underbrace{\lim_{\rho\to -\infty}\frac{1}{\rho}\ln\left(\alpha+(1-\alpha)\left(\frac{y}{x}\right)^{\rho}\right)}\right]$$

$\cancel{= EXP\{\ln x +}$

$$\frac{\lim_{\rho\to -\infty}\ln\left(\alpha+(1-\alpha)\left(\frac{y}{x}\right)^{\rho}\right)}{\lim_{\rho\to -\infty}\rho} = \frac{\ln(\alpha)}{\lim_{\rho\to -\infty}\rho} = 0$$

NOTAR: SI $x<y \Rightarrow \lim_{\rho\to -\infty}\left(\frac{y}{x}\right)^{\rho} = 0$

SI $x=y \Rightarrow \lim_{\rho\to -\infty}\left(\frac{y}{x}\right)^{\rho} = 1$