$$\succsim$$

$$\downarrow \quad \leftarrow \text{CONTINUIDAD}$$

$$u(\cdot)$$

$$\max_{\{x\}} u(x) \quad px \leq w \qquad\qquad \min_{\{h\}} p \cdot h \quad \text{s.a} \quad u(h) \geq \bar{u}$$

$$x(p,w) \qquad\qquad h(p,\bar{u})$$

$$h = \nabla e$$

$$v(p, e(p,\bar{u})) = \bar{u}$$

$$v(p,w) = u(x(p,w)) \longleftrightarrow e(p,\bar{u}) = p \cdot h(p,\bar{u})$$

$$e(p, v(p,w)) = w$$

EJEMPLOS

① COBB-DOUGLAS: $u(x,y) = x^{\alpha} y^{1-\alpha}$

WALRASIANA: $x(p,w) = \dfrac{\alpha w}{p_x}$ $\quad y(p,w) = \dfrac{(1-\alpha) w}{p_y}$

$$v(p,w) = u\big(x(p,w), y(p,w)\big) = \left(\frac{\alpha w}{p_x}\right)^{\alpha} \left(\frac{(1-\alpha) w}{p_y}\right)^{1-\alpha}$$

$$\rightarrow \quad v(p,w) = w \left(\frac{\alpha}{p_x}\right)^{\alpha} \left(\frac{1-\alpha}{p_y}\right)^{1-\alpha}$$

$$v(p, e(p,\bar{u})) = \boxed{e(p,\bar{u}) \left(\frac{\alpha}{p_x}\right)^{\alpha} \left(\frac{1-\alpha}{p_y}\right)^{1-\alpha} = \bar{u}}$$

$$e(p,\bar{u}) = \bar{u} \left(\frac{p_x}{\alpha}\right)^{\alpha} \left(\frac{p_y}{1-\alpha}\right)^{1-\alpha}$$

$$e(p,\bar{u}) = \bar{u}\left(\frac{p_x}{\alpha}\right)^{\alpha}\left(\frac{p_y}{1-\alpha}\right)^{1-\alpha}$$

$$h_x(p,\bar{u}) = \frac{\partial e(p,\bar{u})}{\partial p_x} = \bar{u}\,\alpha\left(\frac{p_x}{\alpha}\right)^{\alpha-1}\frac{1}{\alpha}\left(\frac{p_y}{1-\alpha}\right)^{1-\alpha}$$

$$= \bar{u}\left(\frac{\alpha}{p_x}\frac{p_y}{1-\alpha}\right)^{1-\alpha}$$

$$h_y(p,\bar{u}) = \bar{u}\left(\frac{1-\alpha}{p_y}\frac{p_x}{\alpha}\right)^{\alpha}$$

LEONTIEF

$$u(x,y) = \min\{x,y\}$$

$$\min_{\{h_x, h_y\}} \; p_x h_x + p_y h_y \quad \text{s.a.} \quad \min\{h_x, h_y\} \geq \bar{u}$$

$$h_x = h_y = \bar{u}$$

$$h_x(p,\bar{u}) = h_y(p,\bar{u}) = \bar{u}$$

$$e(p,\bar{u}) = h_x(p,\bar{u})\, p_x + h_y(p,\bar{u})\, p_y$$

$$= \bar{u}\,(p_x + p_y)$$

$$w = v(p,w)\,(p_x + p_y)$$

$$v(p,w) = \frac{w}{p_x + p_y}$$

$$x(p,w) = -\frac{\partial v / \partial p_x}{\partial v / \partial w} = -\frac{\partial \frac{w}{p_x+p_y} / \partial p_x}{\partial \frac{w}{p_x+p_y} / \partial w} = \frac{w}{p_x+p_y}$$

$$y(p,w) = -\frac{\partial v / \partial p_y}{\partial v / \partial w} = \frac{w}{p_x+p_y}$$

PRODUCTOR

$$q = z_1^{\alpha} z_2^{\beta}$$

$$\alpha, \beta > 0$$

$$\min_{\{z_1, z_2\}} z_1 p_1 + z_2 p_2 \quad \text{s.a.} \quad q = z_1^{\alpha} z_2^{\beta}$$

$$\mathcal{L} = -z_1 p_1 - z_2 p_2 - \lambda\left(q - z_1^{\alpha} z_2^{\beta}\right)$$

$$\left.\begin{aligned} p_1 &= \lambda \alpha z_1^{\alpha-1} z_2^{\beta} \\ p_2 &= \lambda \beta z_1^{\alpha} z_2^{\beta-1} \\ q &= z_1^{\alpha} z_2^{\beta} \end{aligned}\right\} \rightarrow \frac{p_1}{p_2} = \frac{\alpha}{\beta}\frac{z_2}{z_1} \rightarrow z_1 = \frac{p_2}{p_1}\frac{\alpha}{\beta} z_2$$

$$q = \left(\frac{p_2}{p_1}\frac{\alpha}{\beta}\right)^{\alpha} z_2^{\alpha+\beta}$$

$$\rightarrow z_2 = q^{\frac{1}{\alpha+\beta}} \left(\frac{p_1}{p_2}\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}}$$

$$z_1 = q^{\frac{1}{\alpha+\beta}} \left(\frac{p_2}{p_1}\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}}$$

Y	L	K
Y_1	L_1	K_1
Y_2	L_2	K_2
Y_3	L_3	K_3
$\vdots$		
Y_N	L_N	K_3

$$Y = A K^{\gamma_2} L^{\gamma_1}$$

$$\ln Y_i = \gamma_0 + \gamma_1 \ln L_i + \gamma_2 \ln K_i + \varepsilon_i$$

$$\boxed{H_0 : \quad \gamma_1 + \gamma_2 = 1}$$

$$z_2 = y^{\frac{1}{\alpha+\beta}} \left(\frac{p_1}{p_2}\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}}$$

$$z_1 = y^{\frac{1}{\alpha+\beta}} \left(\frac{p_2}{p_1}\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}}$$

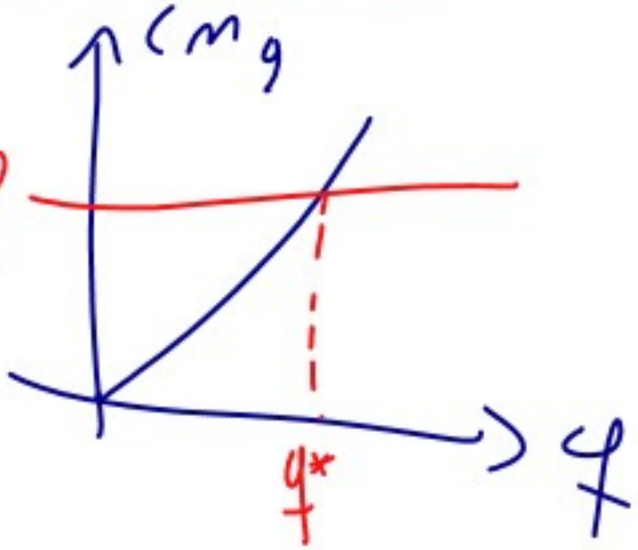
$$CT(y) = \underbrace{\left[p_2 \left(\frac{p_1}{p_2}\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}} + p_1 \left(\frac{p_2}{p_1}\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}}\right]}_{\phi(p_1,p_2,\alpha,\beta)} y^{\frac{1}{\alpha+\beta}}$$

$$CT(y) = \phi(p_1,p_2,\alpha,\beta)\, y^{\frac{1}{\alpha+\beta}}$$

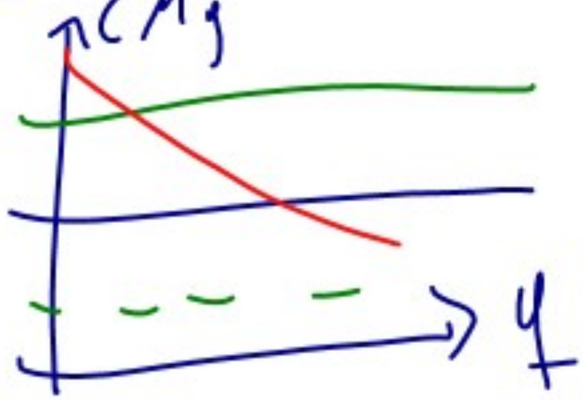
$$\boxed{CM_g(y) = \frac{1}{\alpha+\beta}\,\phi\, y^{\frac{1-(\alpha+\beta)}{\alpha+\beta}}}$$

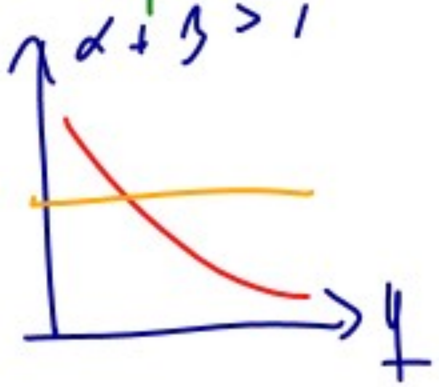
$$\frac{dCM_g}{dy} = \frac{1}{\alpha+\beta}\left(\frac{1-(\alpha+\beta)}{\alpha+\beta}\right) * y^{\frac{1-2(\alpha+\beta)}{\alpha+\beta}}$$

$\alpha+\beta<1$



$\alpha+\beta=1$





$$\pi(q) = -CT(q) + q\,P$$

$$\underset{\{q\}}{\mathrm{MAX}}\ \pi(q)$$

c. p. o. $\dfrac{\partial \pi}{\partial q} = 0 \Rightarrow P = CM_y(q)$