$$\max_{\{X\}} \; f(X)$$

$$\text{s.a.} \quad g_k(X) = c_k \qquad k = 1, 2, \ldots, K$$

$$h_l(X) \leq d_l \qquad l = 1, 2, \ldots L$$

C.P.O. K-T

1. $$\frac{\partial f(X)}{\partial x_i} = \sum_{k=1}^{K} \lambda_k \frac{\partial g_k(X)}{\partial X_i} + \sum_{l=1}^{L} \gamma_l \frac{\partial h_l(X)}{\partial X_i}$$

2. $\gamma_k \geq 0$

3. $\gamma_h \left(h_l(X) - d_l \right) = 0$

$$\max_{\{x,y\}} \; x + y^{1/2}$$

$$\text{s.t.} \quad p_x x + p_y y = I$$

$$-x \leq 0$$

$$-y \leq 0$$

$$\frac{\partial\, x + y^{1/2}}{\partial x} = \lambda \frac{\partial(p_x x + p_y y - I)}{\partial x} + \gamma_x \frac{\partial(-x)}{\partial x} + \gamma_y \frac{\partial(-y)}{\partial x}$$

$$\frac{\partial\, x + y^{1/2}}{\partial y} = \lambda \frac{\partial(p_x x + p_y y - I)}{\partial y} + \gamma_x \frac{\partial(-x)}{\partial y} + \gamma_y \frac{\partial(-y)}{\partial y}$$

① $1 = \lambda P_x - \gamma_x$ ③ $\gamma_x \geq 0$ ⑤ $\gamma_x x = 0$ ⑦ $I = P_x x + P_y y$

② $\frac{1}{2y^{1/2}} = \lambda P_y - \gamma_y$ ④ $\gamma_y \geq 0$ ⑥ $\gamma_y y = 0$

Ⓐ $\gamma_x > 0, \ \gamma_y > 0$

$\downarrow$ ⑤ $\quad \downarrow$ ⑥

$x^* = 0 \quad y^* = 0$

$\Rightarrow \Leftarrow$ ⑦

Ⓑ $\gamma_x = 0, \ \gamma_y > 0$

$\downarrow$ ⑥

$y^* = 0$

$\Rightarrow \Leftarrow \frac{1}{2 \cdot 0}$ ②

Ⓒ $\gamma_x = 0, \ \gamma_y = 0$

⑤ $\downarrow \quad \downarrow$ ⑥

$x^* > 0 \quad y^* > 0$

① $1 = \lambda P_x$ ② $\frac{1}{2y^{1/2}} = \lambda P_y$

$\lambda = \frac{1}{P_x}$

$y^* = \left(\frac{P_x}{2 P_y}\right)^2$

---

⑦ $x^* = \frac{I - P_y y}{P_x} = \frac{I - P_y \cdot \left(P_x / 2P_y\right)^2}{P_x}$

$$\boxed{I \geq \frac{P_x^2}{4 P_y}}$$

$$④ \ \gamma_y = 0, \ \gamma_x > 0$$

$$\Downarrow \qquad \qquad \Downarrow ⑤$$

$$⑥ \ y^* > 0 \qquad x^* = 0$$

$$I = \cancel{p_x x}^{=0} + p_y \cdot y$$

$$\boxed{y^* = \frac{I}{p_y}}$$

$$① \quad 1 = \lambda p_x - \gamma_x$$

$$② \quad \frac{1}{2 y^{1/2}} = \lambda p_y$$

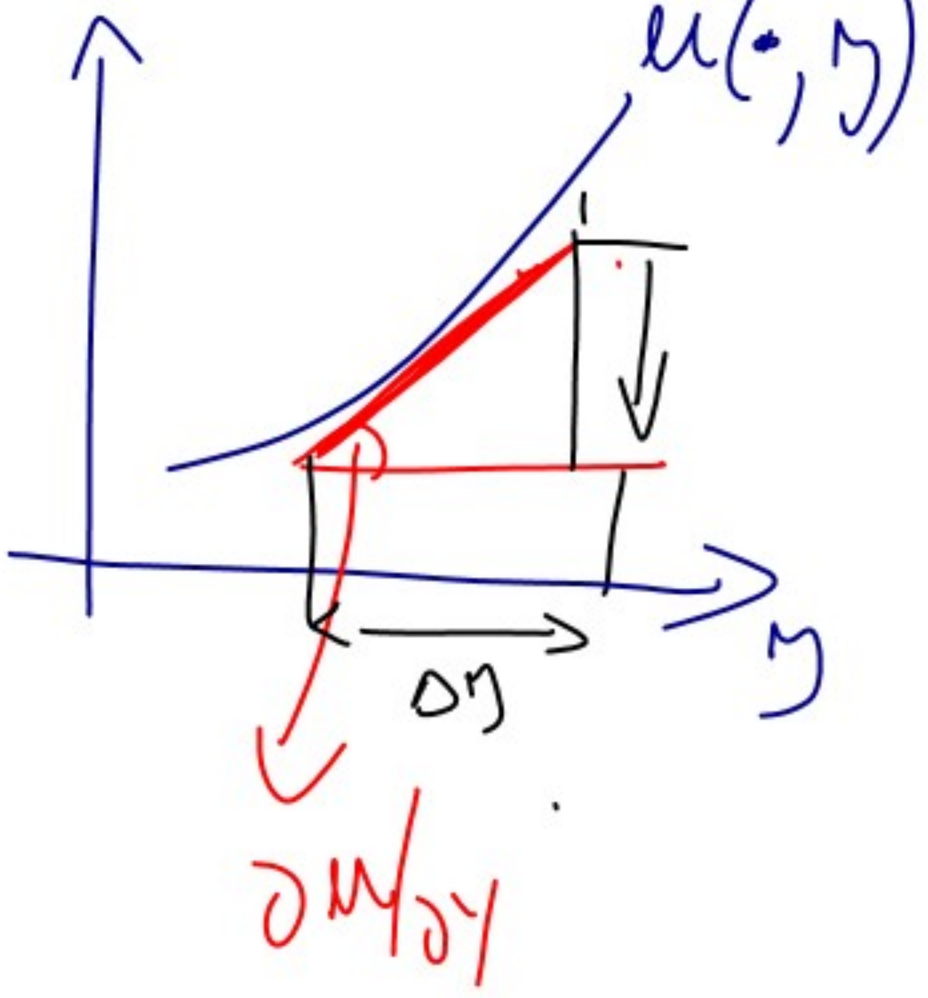
$$\lambda = \frac{1}{2 p_y \left(\frac{I}{p_y}\right)^{1/2}} = \frac{1}{2 p_y^{1/2} I^{1/2}}$$

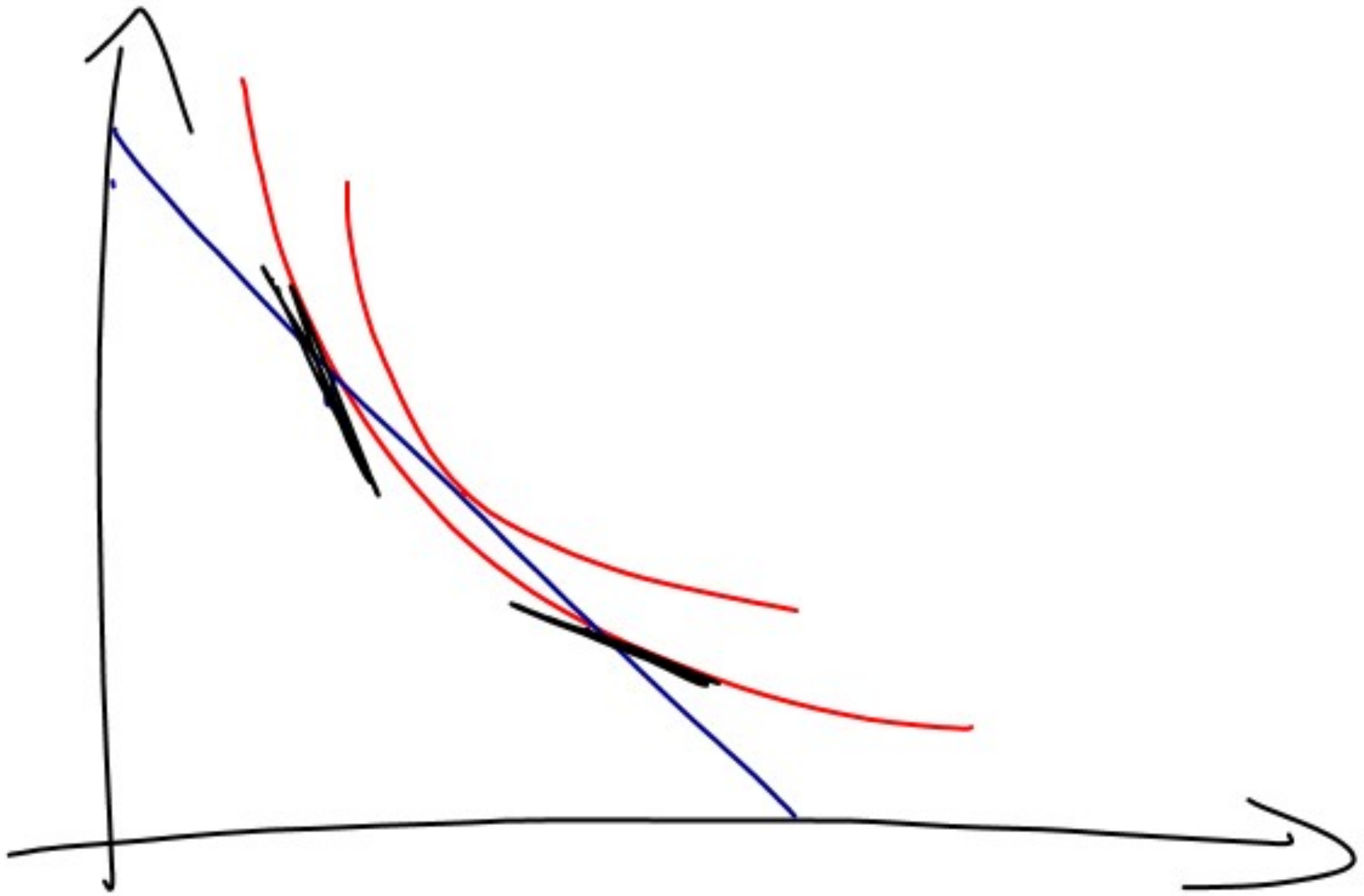
$$\gamma_x = \frac{p_x}{2 (p_y I)^{1/2}} - 1 > 0$$

$$\frac{p_x}{2 (p_y I)^{1/2}} > 1 \qquad \frac{p_x^2}{4 p_y} > I$$

$$\max_{\{x,y\}} u(x,y) \quad \text{s.a.} \quad P_x x + P_y y = I$$

$$\left(\frac{1}{P_x}\right) \partial u/\partial x > \partial u/\partial y \cdot \left(\frac{-1}{P_y}\right)$$

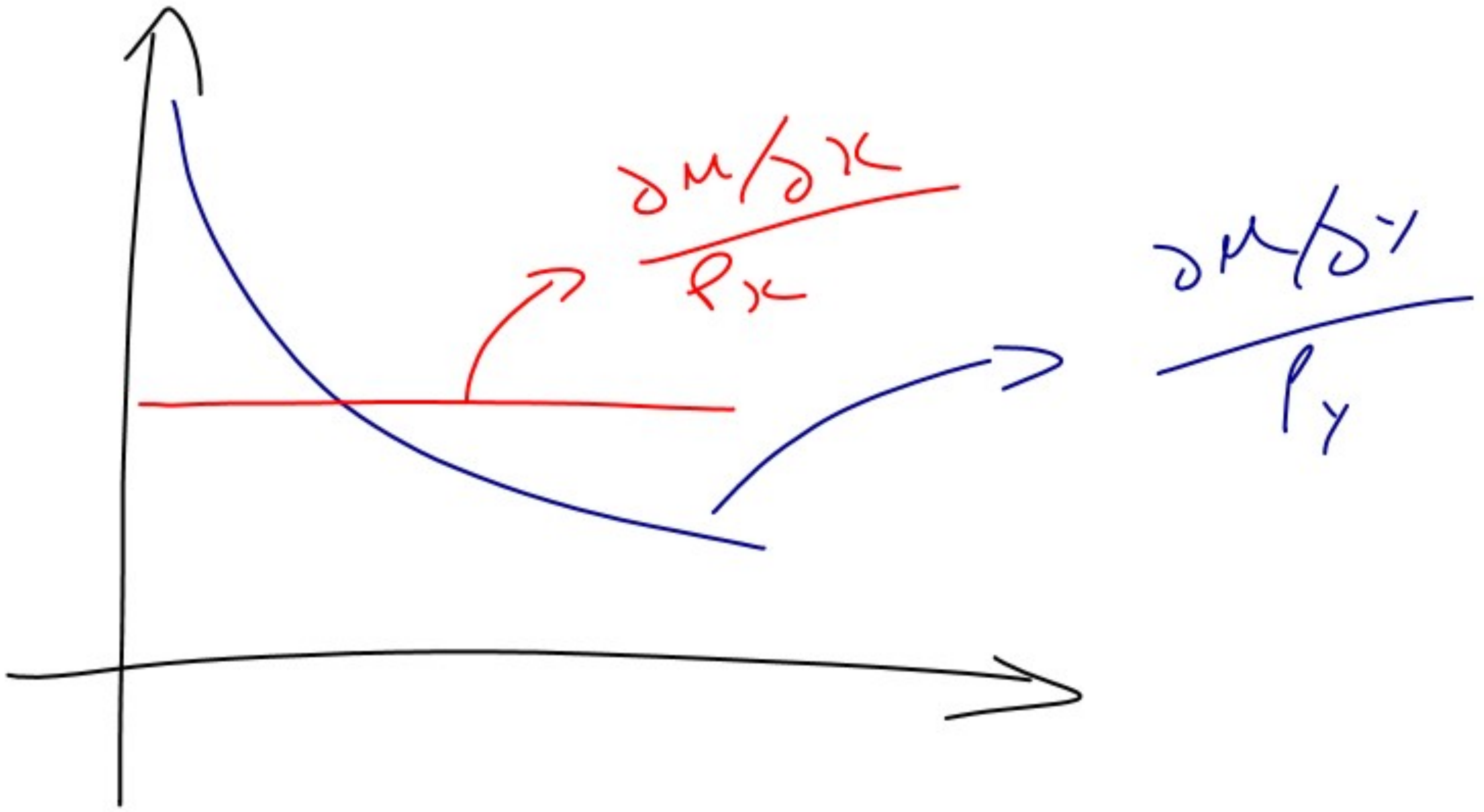




$$u(x,y) = x + y^{1/2}$$

$$\frac{\partial u/\partial x}{P_x} \qquad \frac{\partial u/\partial y}{P_y}$$

$$\frac{1}{P_y} \qquad \frac{1}{2 P_y y^{1/2}}$$



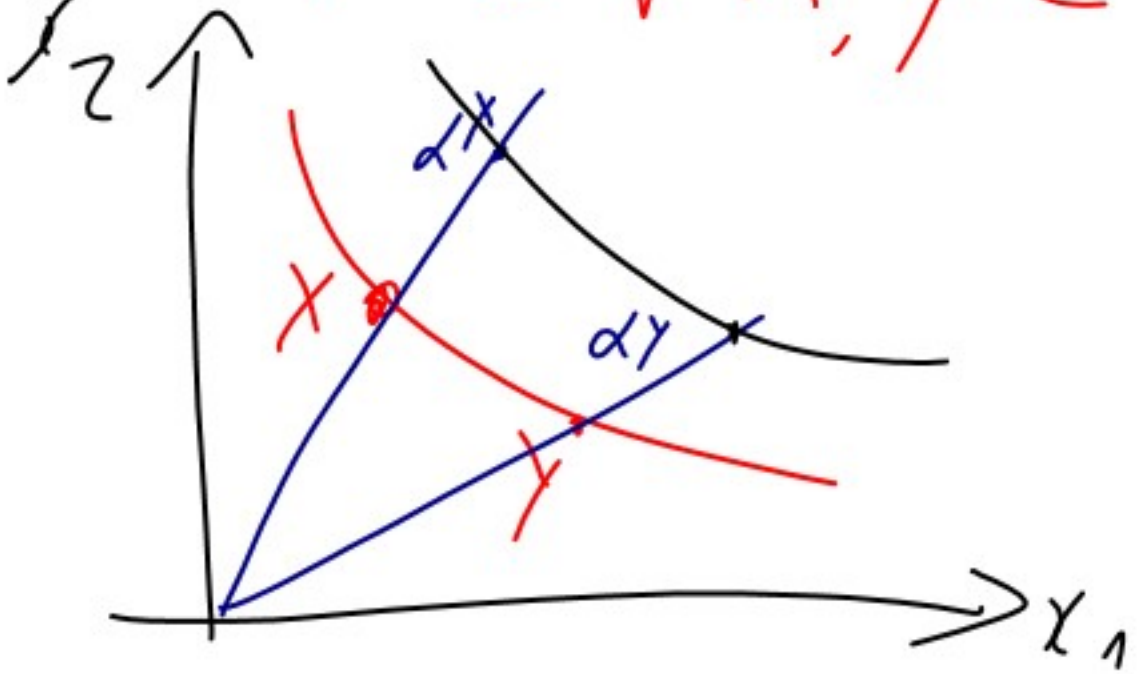
$\frac{\partial u/\partial x}{p_x}$
$\frac{\partial u/\partial y}{p_y}$

DOS TIPOS IMPORTANTES DE $u(\cdot)$

① HOMOTÉTICAS ?

$\Longleftrightarrow \forall x, y \in X : x \sim y \Longleftrightarrow \alpha x \sim \alpha y$

$\forall \alpha > 0$



$\Longleftrightarrow u(x)$ HG.1 EN x

$u(\alpha x) = \alpha u(x)$

(2) CUASILINEALES:

$$x \in \mathbb{X} \subset \mathbb{R}^m_+$$

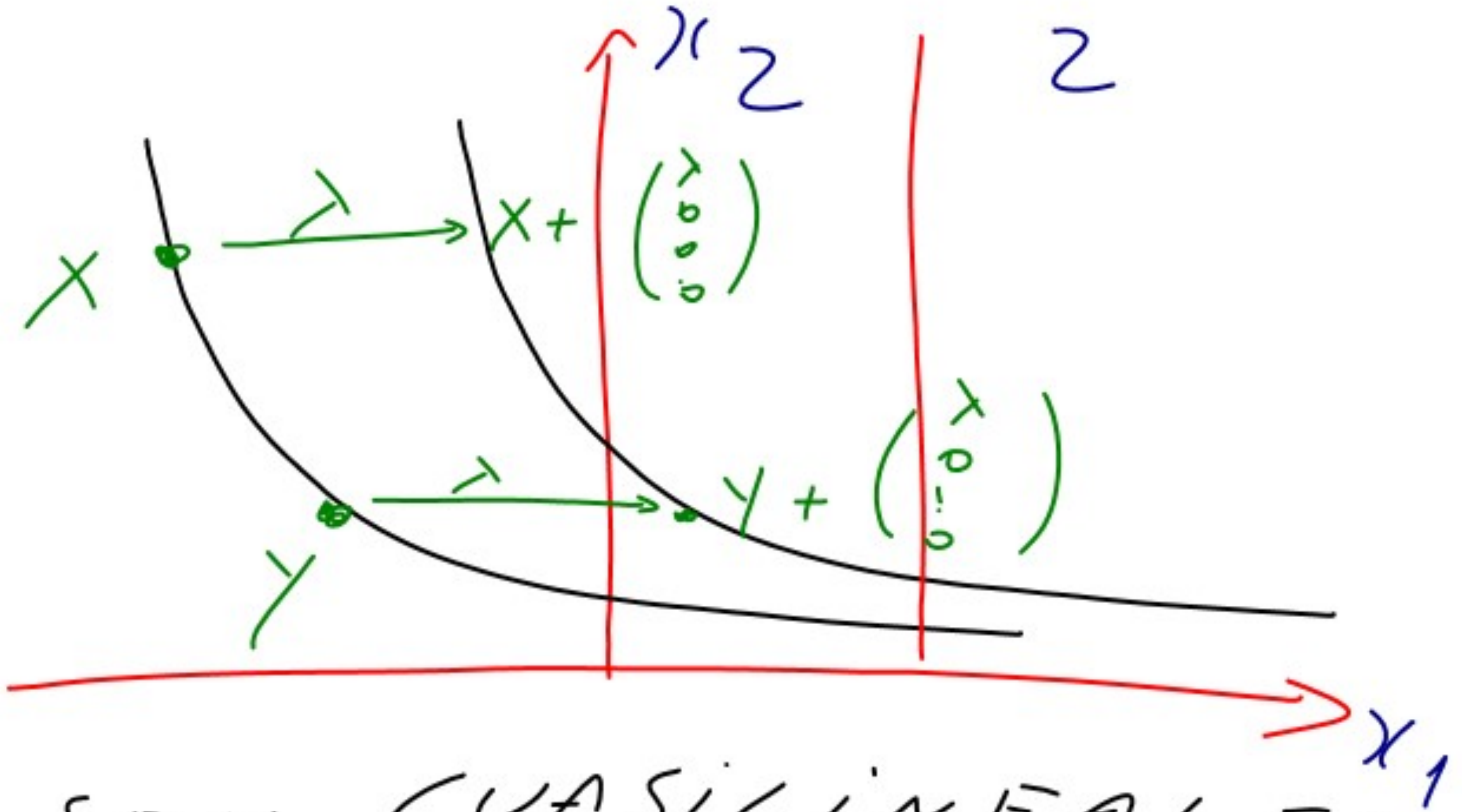
$$x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix}$$

x_1 NUMERARIO

$\succsim$ SON CUASILINEALES

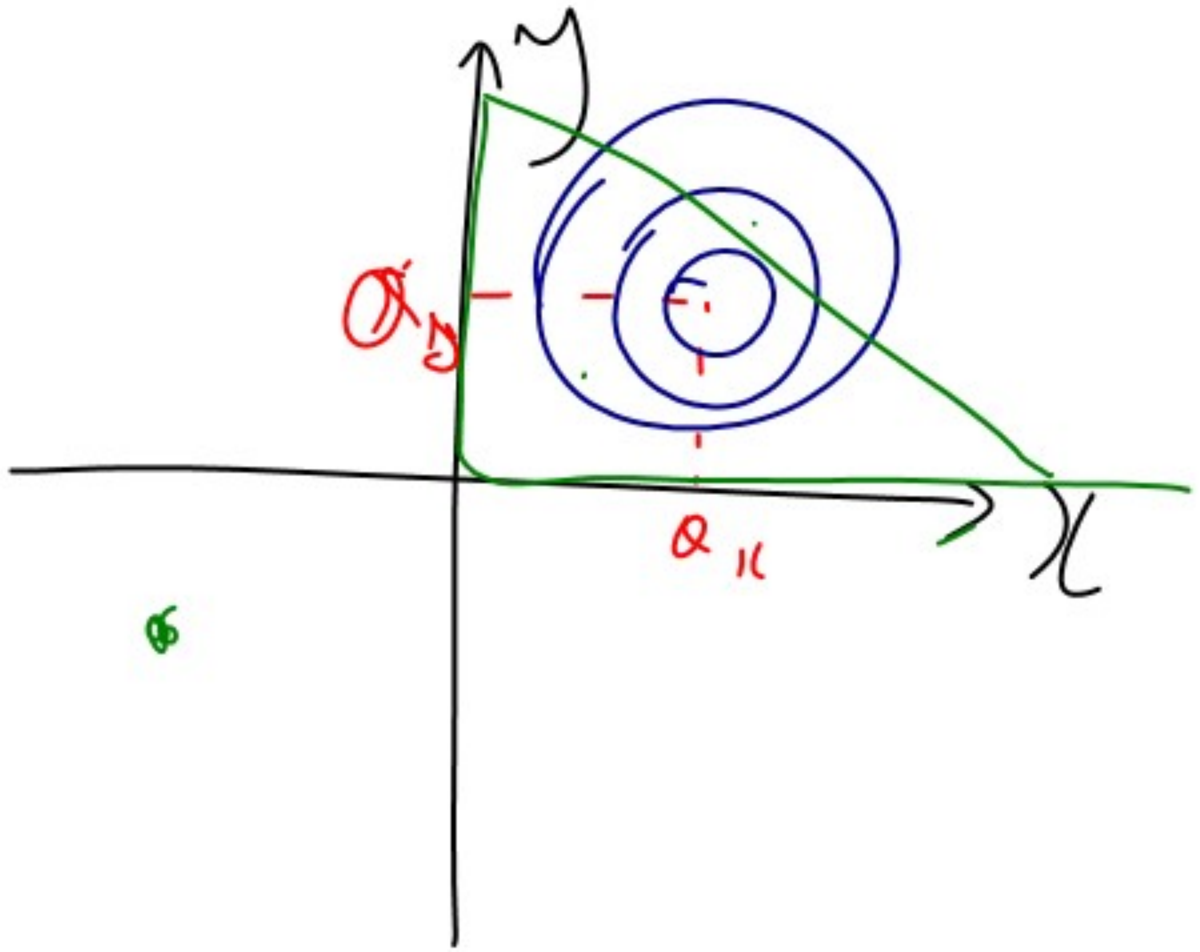
$\Updownarrow$

$$x \sim y \iff \begin{pmatrix} x_1+\lambda \\ x_2 \\ \vdots \\ x_m \end{pmatrix} \sim \begin{pmatrix} y_1+\lambda \\ y_2 \\ \vdots \\ y_m \end{pmatrix}$$



SI $\succsim$ SON CUASILINEALES $\Rightarrow \exists\, u$:

$$u(x_1, x_2, \dots, x_n) = x_1 + \phi(x_2, \dots, x_n)$$



$$f = (y - a_y)^2 + (x - a_x)^2$$

$$\min_{\{x,y\}} f(x,y)$$

s.a. $x \geq 0$

$y \geq 0$

$x + y \leq c$