$$\max_{\{x,y\}} \; ax + by \quad \text{s.a.} \quad p_x x + p_y y = I$$

$$-x \le 0$$
$$-y \le 0$$

① $a = \lambda p_x - \gamma_x$ ③ $\gamma_x \ge 0$ ⑤ $\gamma_x x = 0$

② $b = \lambda p_y - \gamma_y$ ④ $\gamma_y \ge 0$ ⑥ $\gamma_y y = 0$

⑦ $x \ge 0$ ⑧ $y \ge 0$ ⑨ $p_x x + p_y y = I$

(c) $\gamma_x = 0, \gamma_y > 0$

$x > 0 \quad y^* = 0$

(9) $p_x x = I \quad x^* = I/p_x$

$\left(\frac{I}{p_x}, 0\right)$

① $a = \lambda p_x$

② $b = \lambda p_y - \gamma_y \qquad \gamma_y > 0$

$\to \lambda = a/p_x$

$\to b = a\frac{p_y}{p_x} - \gamma_y \Rightarrow \gamma_y = a\frac{p_y}{p_x} - b$

$a\frac{p_y}{p_x} - b > 0 \qquad \frac{p_y}{p_x} > \frac{b}{a} \qquad \frac{a}{p_x} > \frac{b}{p_y}$

$\gamma_x > 0, \gamma_y = 0$

$p_x = \lambda a - \gamma_x$

$p_y = \lambda b \rightarrow \lambda = \frac{p_y}{b}$

$p_x = \frac{p_y}{b} a - \gamma_x$

$\gamma_x = -\frac{p_y}{b} a + p_x > 0$

$\frac{a}{p_x} < \frac{b}{p_y}$

$h_x = 0 \quad h_y > 0$

$\min_{\{h_x, h_y\}} \; p_x h_x + p_y h_y$ s.t. $a h_x + b h_y = \bar{v}$

$-h_x \le 0$

$-h_y \le 0$

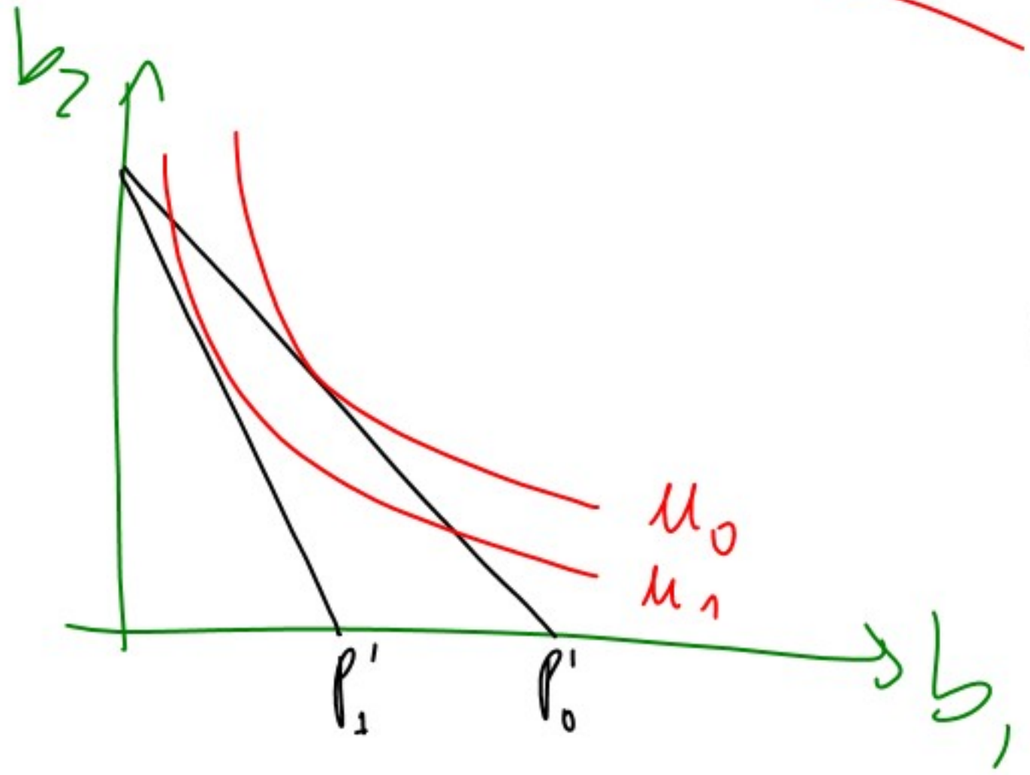
$$\frac{\partial (p_x h_x + p_y h_y)}{\partial h_y} = \lambda \frac{\partial}{\partial h_y} (a h_x + b h_y) - \gamma_x \frac{\partial h_x}{\partial h_y} - \gamma_y \frac{\partial h_x}{\partial h_y}$$

① $p_x = \lambda a - \gamma_x$ ③ $\gamma_x \ge 0$ ⑤ $\gamma_x h_x = 0$

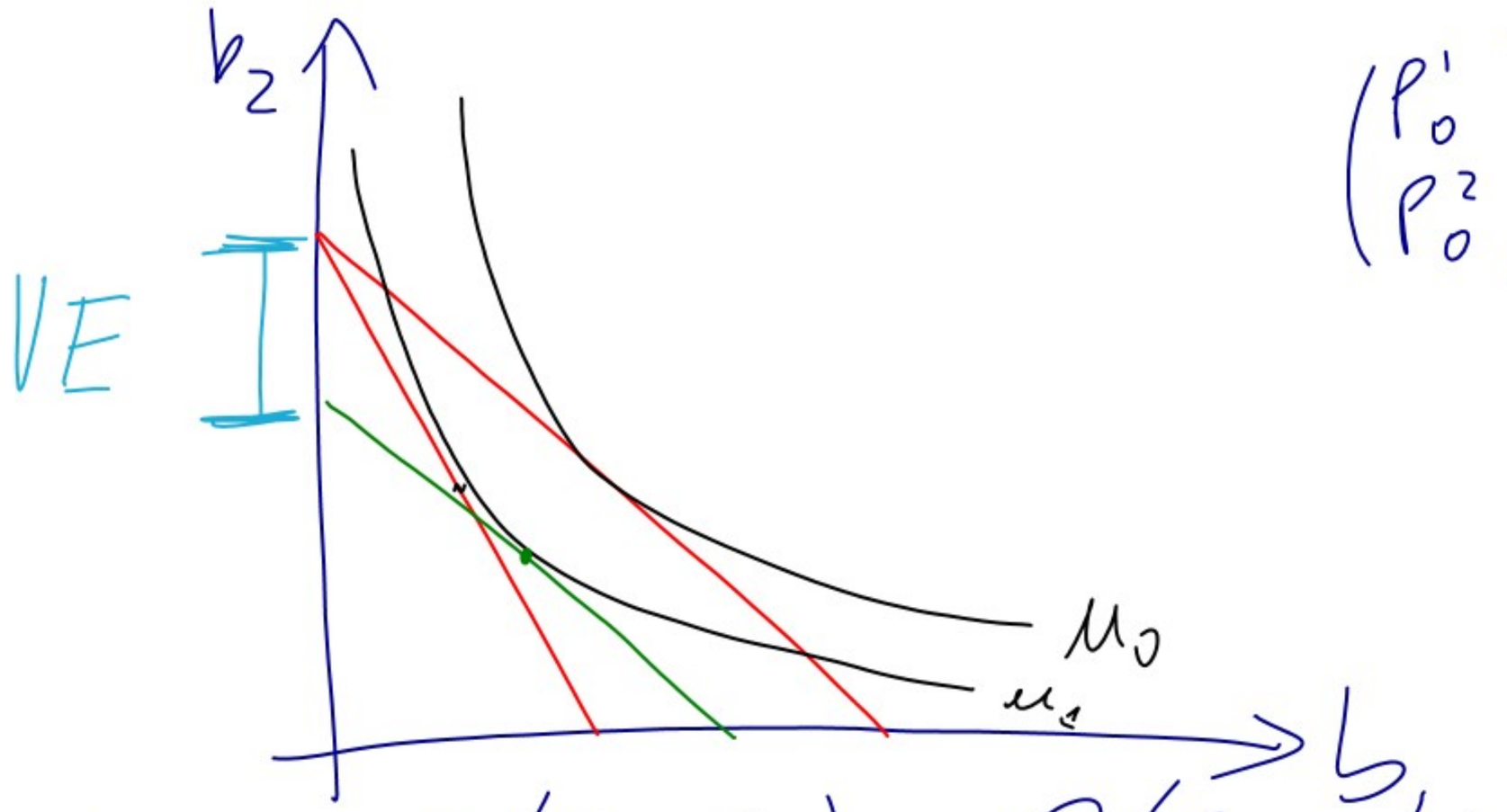
② $p_y = \lambda b - \gamma_y$ ④ $\gamma_y \ge 0$ ⑥ $\gamma_y h_y = 0$

⑦ $\bar{v} = a h_x + b h_y$ ⑧ $h_x \ge 0$ ⑨ $h_y \ge 0$

$$u(x(p_1, I)) - u(x(p_0, I))$$



$$p_1' > p_0'$$



$$\begin{pmatrix} p_0^1 \\ p_0^2 \end{pmatrix} \rightarrow \begin{pmatrix} p_1^1 \\ p_0^2 \end{pmatrix}$$

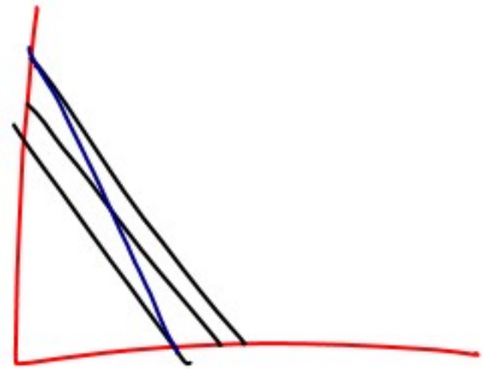
$$p_1^1 > p_0^1$$

$$e(p, v(p, I)) = I$$

$$VE = e(p_0, u_1) - e(p_0, u_0)$$

$$VF = \mathcal{C}(\rho_0, \mu_1) - I$$

$$VF = \mathcal{C}(\rho_0, \mathcal{N}(\rho_f, I)) - I$$



$$X \in \mathbb{R}^m$$

$$\max_{X} f(X) \quad \text{s.a.} \quad g_k(X) = c_k \quad k = 1, 2, \dots K$$

$$h_\ell(X) \le d_\ell \quad \ell = 1, 2, \dots L$$

$$\frac{\partial f(X)}{\partial x_i} = \sum_{k=1}^{K} \lambda_k \frac{\partial g_k}{\partial x_i} + \sum_{\ell=1}^{L} \gamma_\ell \frac{\partial h_\ell(X)}{\partial x_i} \qquad i = 1, 2, \dots m$$

$$\gamma_\ell \ge 0$$

$$\gamma_\ell \left(h_\ell(X) - d_\ell \right) = 0$$