$$\underset{\{x\}}{\mathrm{MAX}} \quad f(x) \qquad \text{s.t.} \quad g_j(x) = c_j \qquad j = 1, 2, \ldots J$$

$$h_\ell(x) \leq d_\ell \qquad \ell = 1, 2, \ldots L$$

$$\frac{\partial f(x^*)}{\partial x_i} = \sum_{j=1}^{J} \lambda_j \frac{\partial g_j(x^*)}{\partial x_i} + \sum_{\ell=1}^{L} \gamma_\ell \frac{\partial h_\ell(x^*)}{\partial x_i} \qquad i = 1, \ldots N$$

$$\gamma_\ell \geq 0 \quad , \gamma_\ell \left(h_\ell(x^*) - d_\ell \right) = 0$$

$$-\frac{\alpha W}{P_x^2} = \bar{u}\left(\frac{\alpha}{1-\alpha}\frac{P_y}{}\right)^{1-\alpha} P_x^{\alpha-2}(\alpha-1) - \frac{\alpha W}{P_x}\cdot\frac{\alpha}{P_x}$$

$$-\frac{\alpha W}{P_x^2} = \bar{u}$$

$$u(x,y) = x^{\alpha} y^{1-\alpha}$$

$$x^* = \frac{\alpha W}{P_x}$$

$$y^* = \frac{(1-\alpha) W}{P_y}$$

$$v = W\left(\frac{\alpha}{P_x}\right)^{\alpha}\left(\frac{1-\alpha}{P_y}\right)^{1-\alpha}$$

$$e = \bar{u}\left(\frac{P_x}{\alpha}\right)^{\alpha}\left(\frac{P_y}{1-\alpha}\right)^{1-\alpha} \rightarrow$$

$$h_x = \bar{u}\,\cancel{\alpha}\,\frac{P_x^{\alpha-1}}{\alpha^{\alpha-1}}\left(\frac{P_y}{1-\alpha}\right)^{1-\alpha}$$

$$h_x = \bar{u}\left(\frac{\alpha}{P_x}\frac{P_y}{1-\alpha}\right)^{1-\alpha}$$

$$h_y = \bar{u}\left(\frac{1-\alpha}{P_y}\frac{P_x}{\alpha}\right)^{\alpha}$$

$$-\frac{\alpha W}{P_x^2} = \bar{u}\left(\frac{\alpha \quad P_M}{1-\alpha}\right)^{1-\alpha} P_x^{\alpha-2}(\alpha-1) - \frac{\alpha W}{P_x} \cdot \frac{\alpha}{P_x}$$

$$x^* = \frac{\alpha}{\alpha+\beta}\frac{w}{P_x}$$

$$-\frac{\alpha W}{\cancel{P_x^2}} = \bar{u}\left(\frac{P_M}{1-\alpha}\right)^{1-\alpha} \alpha \left(\frac{P_x}{\alpha}\right)^{\alpha}\left(\frac{\alpha-1}{\cancel{P_x^2}}\right) - \frac{\alpha^2 W}{\cancel{P_x^2}}$$

$$\alpha^2 W - \alpha W = \cancel{\alpha} W \cancel{(\alpha-1)} = \bar{u}\left(\frac{P_M}{1-\alpha}\right)^{1-\alpha}\left(\frac{P_x}{\alpha}\right)^{\alpha} \cancel{\alpha} \cancel{(\alpha-1)}$$

$$W = \bar{u}\left(\frac{P_M}{1-\alpha}\right)^{1-\alpha}\left(\frac{P_x}{\alpha}\right)^{\alpha}$$

$$\frac{\partial h_x}{\partial p_x} - \frac{\partial x}{\partial w} x = \frac{\partial x}{\partial p_x}$$

$$\bar{u}\left(\frac{\alpha \quad p_y}{1-\alpha}\right)^{1-\alpha} p_x^{\alpha-2}(\alpha-1)$$