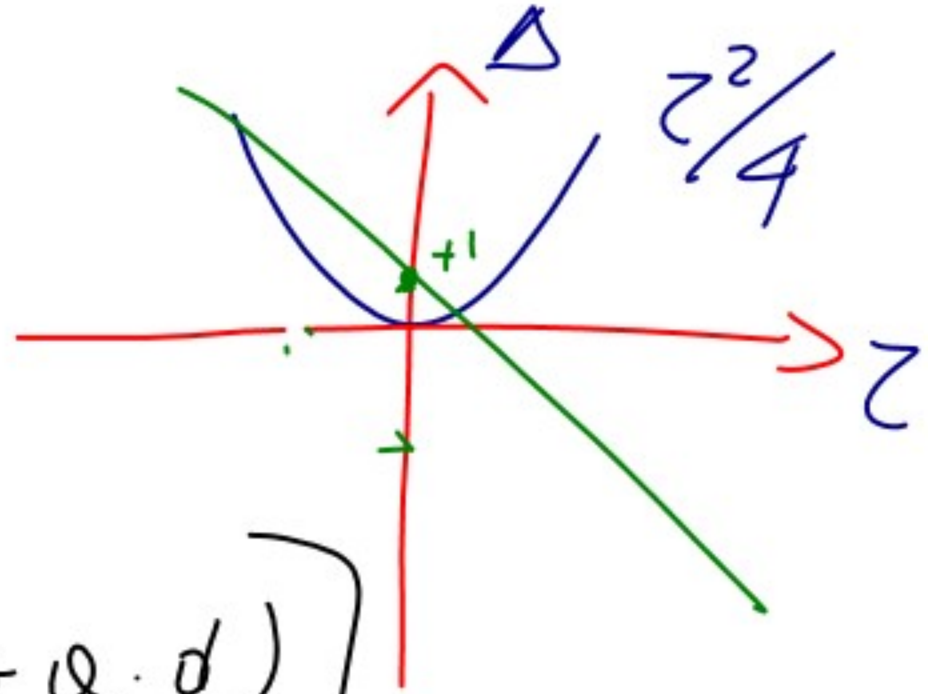
$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



$$\lambda_1'' = \frac{z}{2} \pm \frac{\sqrt{z^2 - 4\Delta}}{2}$$

$$\begin{bmatrix} a & -(a + d + 1 + a \cdot d) \\ -1 & d \end{bmatrix}$$

$$\Delta = \cancel{ad} - (a + d + 1 + \cancel{a \cdot d})$$

$$\Delta = -1 - z$$

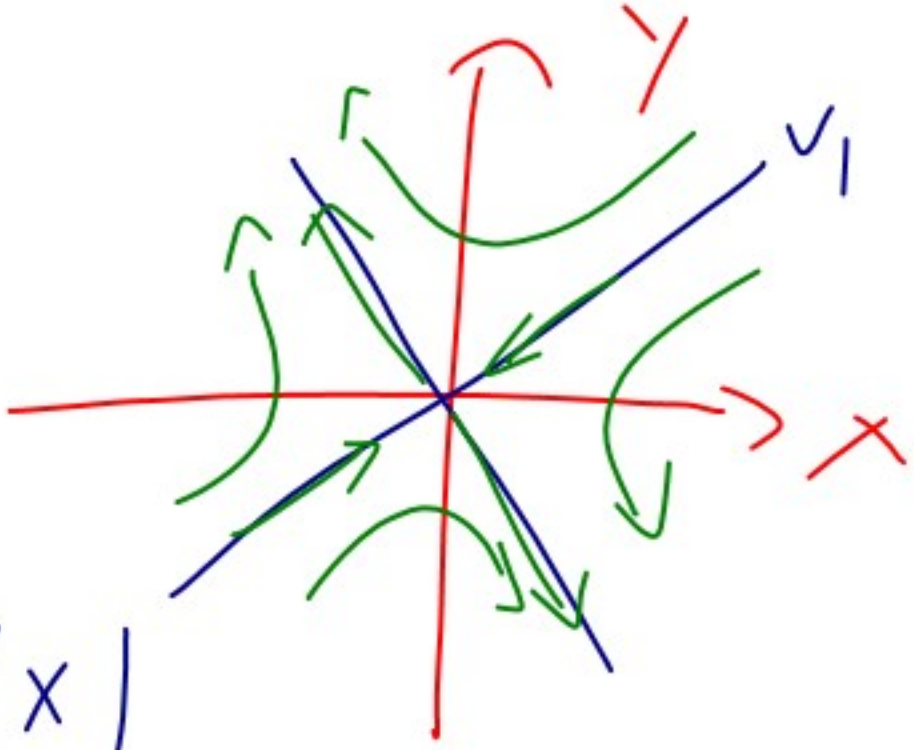
$\tau = 0$

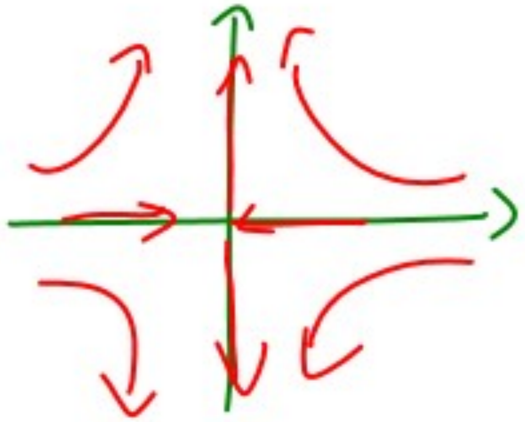
$\Delta = -1$

$$\lambda_i = \frac{0}{2} \pm \frac{\sqrt{0-4(-1)}}{2} = \pm 1$$

$V_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\quad V_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\lambda_1 = -1$ $\quad \lambda_2 = +1$

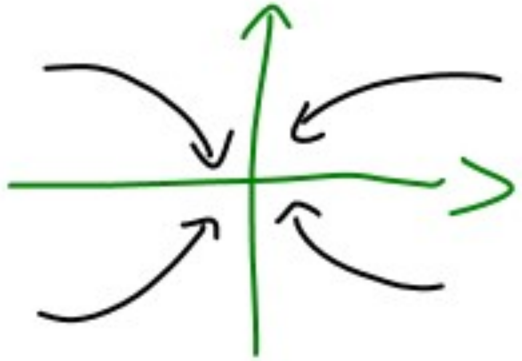


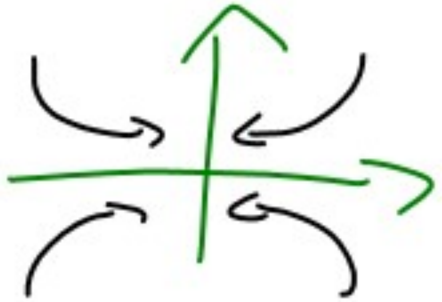


$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$\lambda_1, \lambda_2 \in \mathbb{R}_{--}$



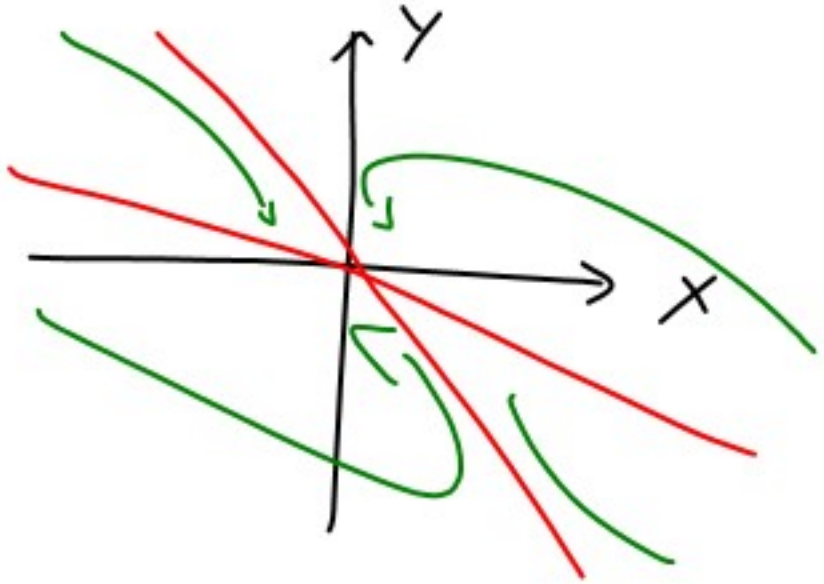


$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} z \\ w \end{pmatrix} = P^{-1} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = A \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} \dot{z} \\ \dot{w} \end{pmatrix} \leftarrow P^{-1} \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = P^{-1} P \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} P^{-1} \begin{pmatrix} x \\ y \end{pmatrix}$$



$$\begin{pmatrix} \dot{z} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} z \\ w \end{pmatrix}$$

$$\begin{pmatrix} z \\ w \end{pmatrix} = \begin{pmatrix} z_0 e^{\lambda_1 t} \\ w_0 e^{\lambda_2 t} \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = P \begin{pmatrix} z \\ w \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = P \begin{pmatrix} z \\ w \end{pmatrix} \qquad \boxed{\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = A \begin{pmatrix} x \\ y \end{pmatrix}} \qquad \lambda v = A v$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} v_1 & v_2 \end{pmatrix} \begin{pmatrix} z_0 e^{\lambda_1 t} \\ w_0 e^{\lambda_2 t} \end{pmatrix} = z_0 e^{\lambda_1 t} v_1 + w_0 e^{\lambda_2 t} v_2$$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = z_0 \lambda_1 e^{\lambda_1 t} v_1 + w_0 \lambda_2 e^{\lambda_2 t} v_2 = z_0 e^{\lambda_1 t} A v_1 + w_0 e^{\lambda_2 t} A v_2$$

$$\cancel{= A \left(z_0 e^{\lambda_1 t} \quad w_0 e^{\lambda_2 t} \right) = A \begin{pmatrix} x \\ y \end{pmatrix}}$$

$$z_0 e^{\lambda_1 t} V_1 + w_0 e^{\lambda_2 t} V_2 = \begin{pmatrix} V_1 & V_2 \end{pmatrix} \begin{pmatrix} z_0 e^{\lambda_1 t} \\ w_0 e^{\lambda_2 t} \end{pmatrix}$$

$$= P \begin{pmatrix} z_0 e^{\lambda_1 t} \\ w_0 e^{\lambda_2 t} \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

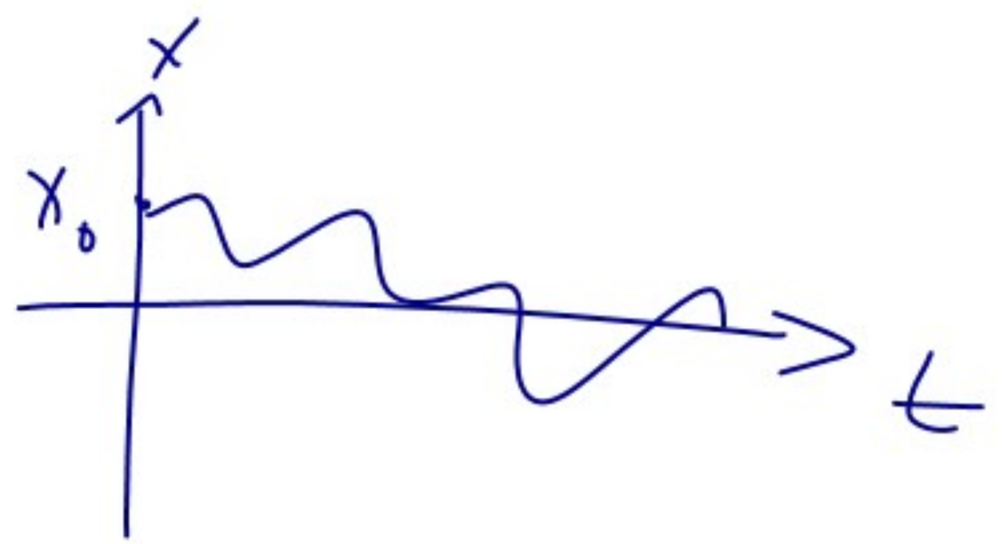
$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = z_0 e^{\lambda_1 t} A V_1 + w_0 e^{\lambda_2 t} A V_2 = A \left(z_0 e^{\lambda_1 t} V_1 + w_0 e^{\lambda_2 t} V_2 \right)$$

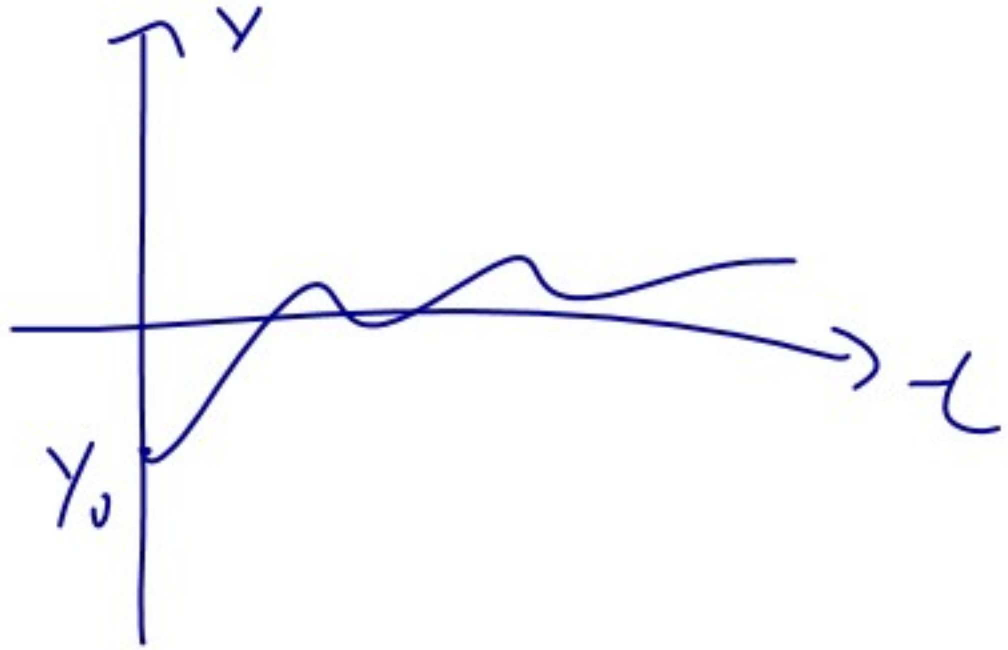
$$= A \boxed{\begin{pmatrix} V_1 & V_2 \end{pmatrix} \begin{pmatrix} z_0 e^{\lambda_1 t} \\ w_0 e^{\lambda_2 t} \end{pmatrix}}$$

$$= A P \begin{pmatrix} z \\ w \end{pmatrix} = A \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} a & c \\ b & d \end{pmatrix}\begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} a \cdot m + c \cdot n \\ b\,m + d \cdot n \end{pmatrix}$$

$$= m\begin{pmatrix} a \\ b \end{pmatrix} + n\begin{pmatrix} c \\ d \end{pmatrix}$$


x
x_0
t

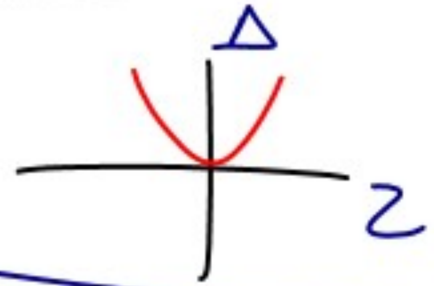

y
y_0
t

$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = z_0 V_1 + w_0 V_2 = \begin{pmatrix} V_1 & V_2 \end{pmatrix} \begin{pmatrix} z_0 \\ w_0 \end{pmatrix}$$

$\checkmark$?

VALORES PROPIOS REALES REPETIDOS

$$\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix} = A\begin{pmatrix}x\\ y\end{pmatrix} \qquad z^2 - 4\Delta = 0$$



$$\lambda_1 = \frac{z}{2}$$

NO: $A = P\begin{pmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{pmatrix}P^{-1}$

$$A = \begin{pmatrix}Q_1 & Q_2\end{pmatrix}\begin{pmatrix}\lambda & 1\\ 0 & \lambda\end{pmatrix}\begin{pmatrix}Q_1 & Q_2\end{pmatrix}^{-1}$$

$$\begin{pmatrix}AQ_1 & AQ_2\end{pmatrix} = \begin{pmatrix}\lambda Q_1 & Q_1 + \lambda Q_2\end{pmatrix}$$

$$A\begin{pmatrix}Q_1 & Q_2\end{pmatrix} = \begin{pmatrix}Q_1 & Q_2\end{pmatrix}\begin{pmatrix}\lambda & 1\\ 0 & \lambda\end{pmatrix}\cancel{\begin{pmatrix}Q_1 & Q_2\end{pmatrix}^{-1}\begin{pmatrix}Q_1 & Q_2\end{pmatrix}}$$

$$\boxed{\begin{aligned} AQ_1 &= \lambda Q_1\\ AQ_2 &= Q_1 + \lambda Q_2\end{aligned}}$$

$$AQ_1 = \lambda Q_1$$

$$AQ_2 = Q_1 + \lambda Q_2$$

$$AQ_1 = \lambda I Q_1$$

$$AQ_1 - \lambda I Q_1 = 0$$

$$(A - \lambda I) Q_1$$

$$\cancel{A - \lambda}$$

$$\boxed{(A - \lambda I) Q_1 = 0}$$

$$(A - \lambda I) Q_2 = Q_1$$

$$(A - \lambda I)^2 Q_2 = (A - \lambda I) Q_1 = 0$$

$$\boxed{(A - \lambda I)^2 Q_2 = 0}$$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = A \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} Q_1 & Q_2 \end{pmatrix} \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix} \begin{pmatrix} Q_1 & Q_2 \end{pmatrix}^{-1} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} Q_1 & Q_2 \end{pmatrix}^{-1} \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix}}_{\begin{pmatrix} \dot{f} \\ \dot{g} \end{pmatrix}} = \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix} \underbrace{\begin{pmatrix} Q_1 & Q_2 \end{pmatrix}^{-1} \begin{pmatrix} x \\ y \end{pmatrix}}_{\begin{pmatrix} f \\ g \end{pmatrix}}$$

$$\begin{pmatrix} \dot{f} \\ \dot{g} \end{pmatrix} = \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix} \begin{pmatrix} f \\ g \end{pmatrix}$$

$$\begin{pmatrix} \dot f \\ \dot g \end{pmatrix} = \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix} \begin{pmatrix} f \\ g \end{pmatrix}$$

$$\dot f = \lambda f + g \longrightarrow \dot f = \lambda f + g_0 e^{\lambda t}$$

$$\boxed{\dot z + \alpha(t) z = \beta(t)}$$

$$\dot g = \lambda g \longrightarrow \frac{dg}{dt} = \lambda g$$

$$\int \frac{dg}{g} = \int \lambda \, dt$$

$$\ln g = \lambda t + c$$

$$g = g_0 e^{\lambda t}$$