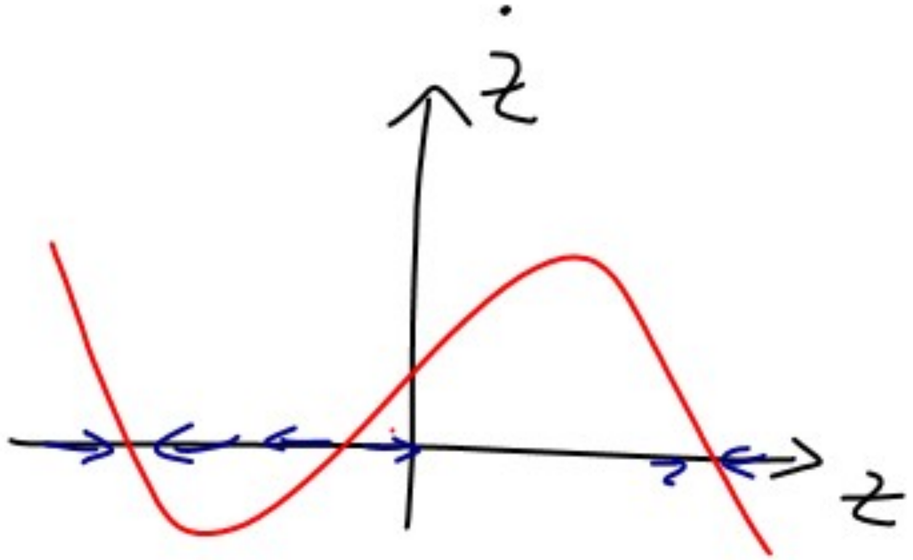
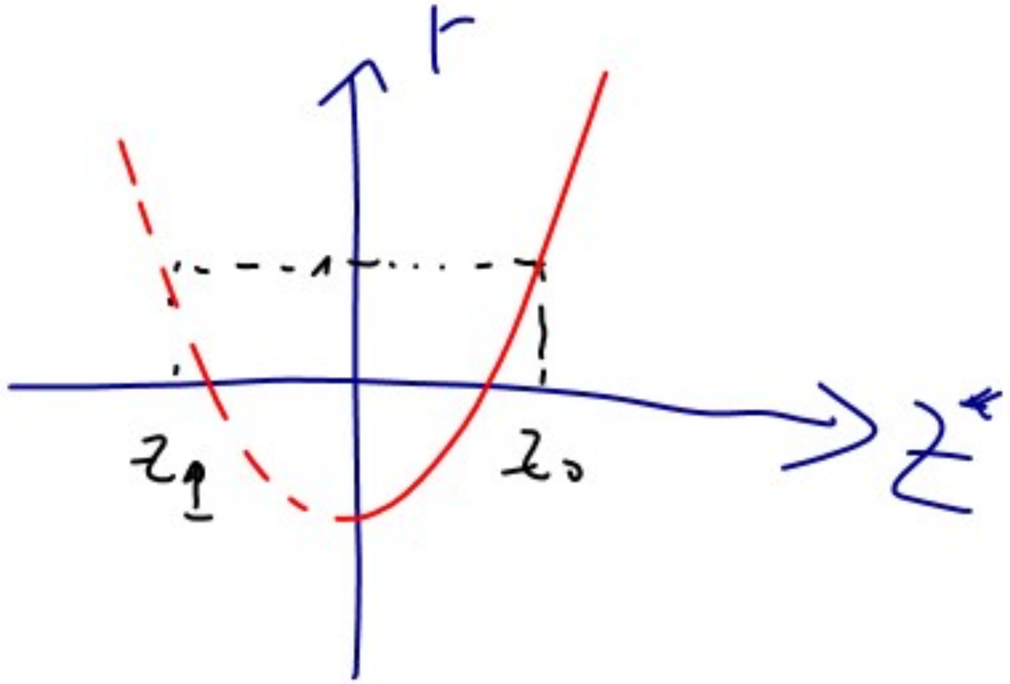
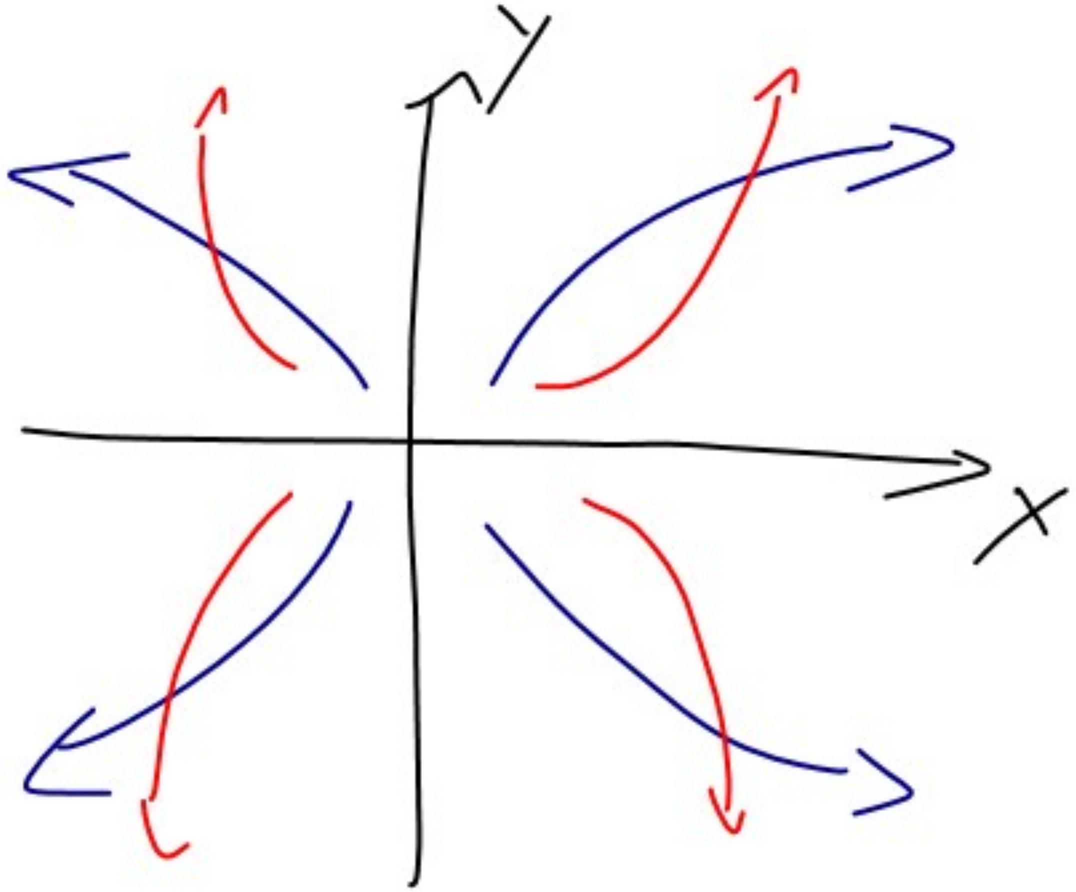


$$\dot{z} = f(z)$$





$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

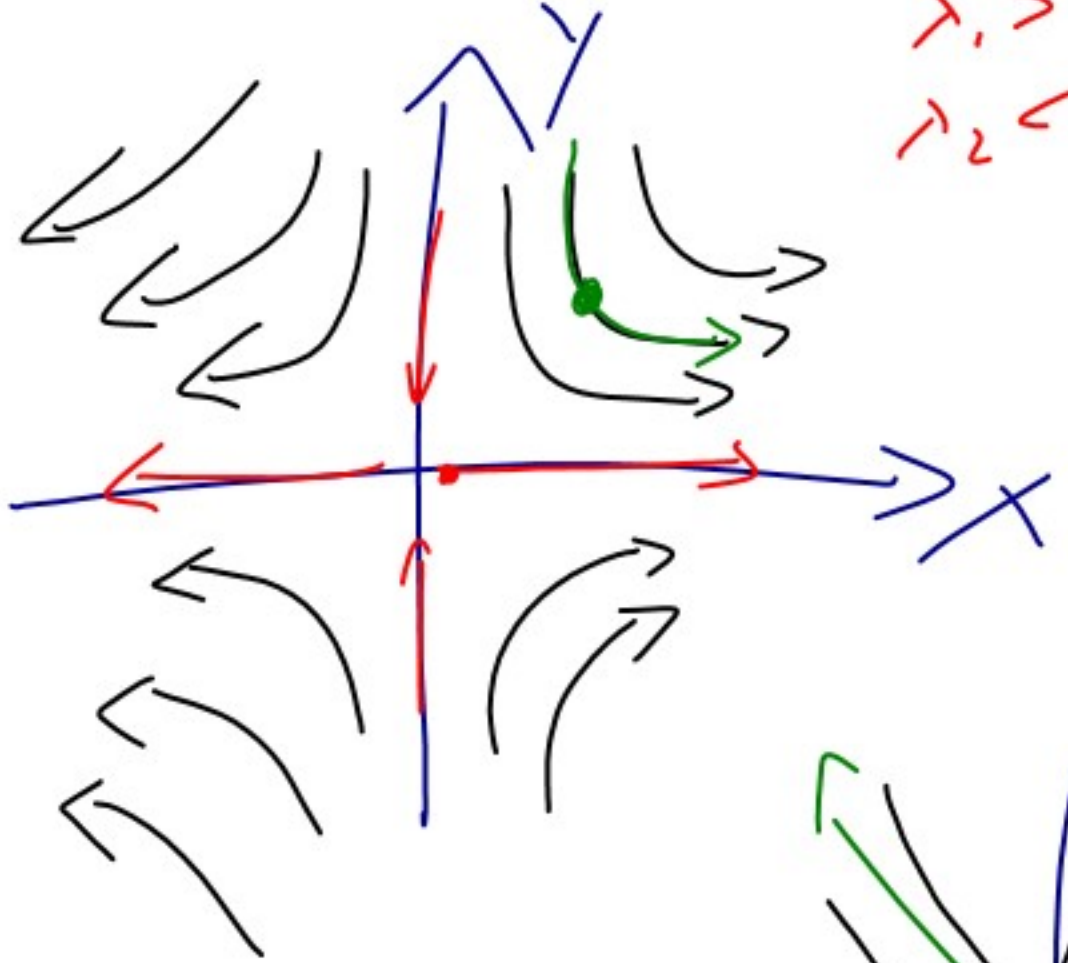
$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} \lambda_1 x \\ \lambda_2 y \end{pmatrix}$$

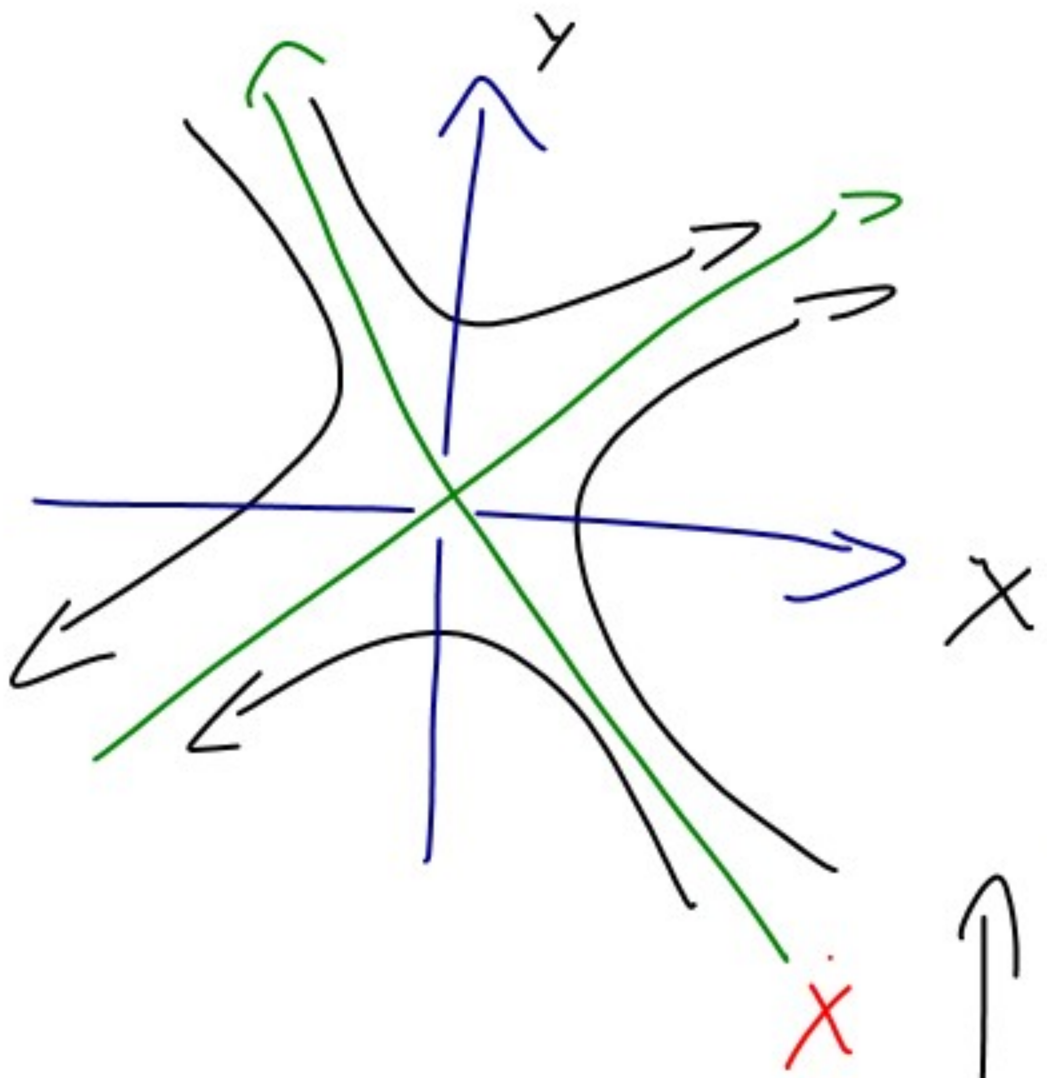
$$\dot{x} = \lambda_1 x \rightarrow x_t = x_0 e^{\lambda_1 t}$$

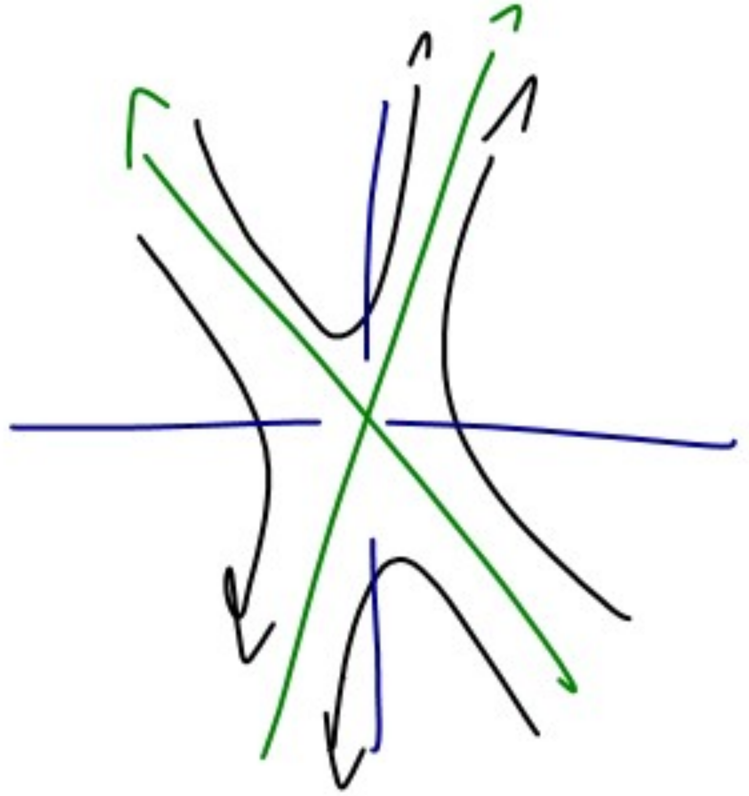
$$y_t = y_0 e^{\lambda_2 t}$$

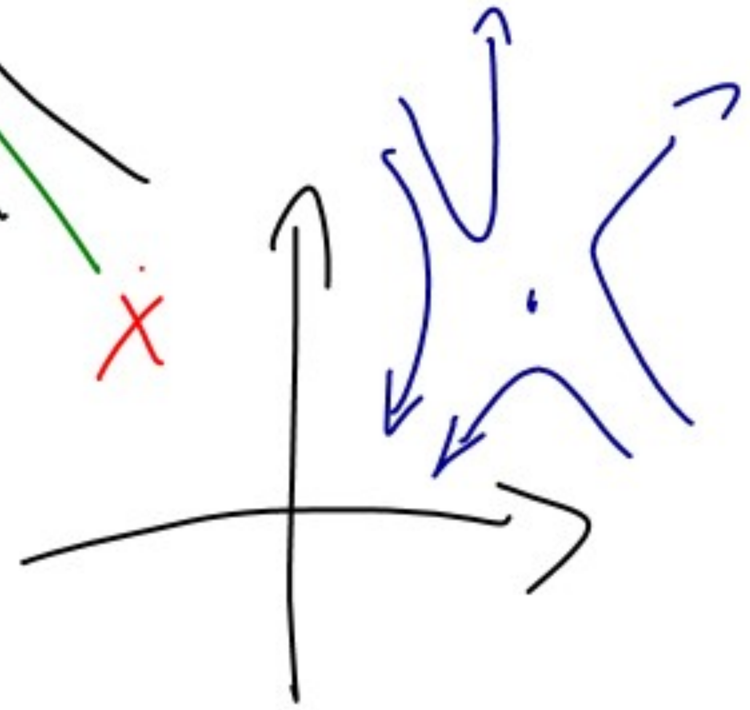
$\lambda_1 > 0$

$\lambda_2 < 0$









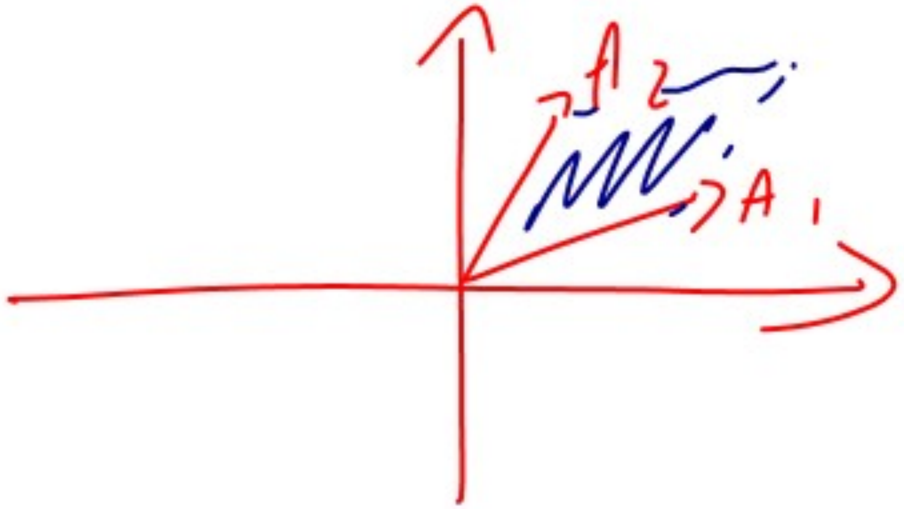
$$\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix} = \begin{pmatrix} a & b \\ c & d\end{pmatrix}\begin{pmatrix} x\\ y\end{pmatrix}$$

$$(A_1 \ A_2)$$

$$AW = \lambda W \qquad W \neq 0$$

$$|A - \lambda I| = 0$$

$$(A - \lambda I)W = \vec{0}$$



$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$|A - \lambda I| = \left| \begin{pmatrix} a & b \\ c & d \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right| = 0$$

$$\tau = a + d$$

$$\Delta = ad - cb$$

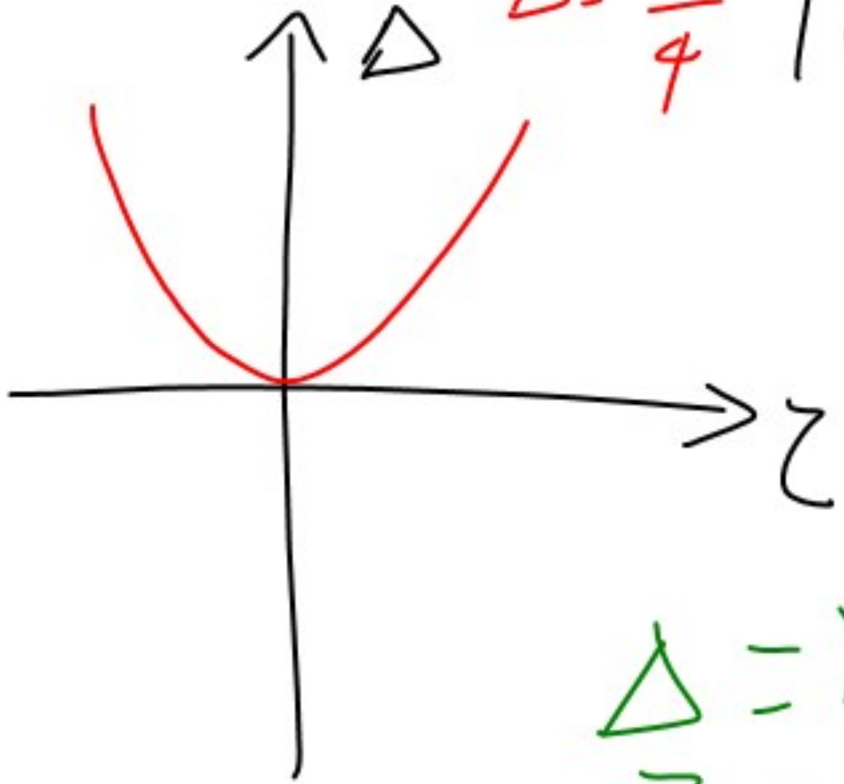
$$\begin{vmatrix} a - \lambda & b \\ c & d - \lambda \end{vmatrix} = 0 = (a-\lambda)(d-\lambda) - cb$$

$$0 = \lambda^2 - (a+d)\lambda + (ad - cb)$$

$$0 = \lambda^2 - \tau\lambda + \Delta$$

$$\lambda_{1,2} = \frac{\tau}{2} \pm \frac{1}{2}\sqrt{\tau^2 - 4\Delta}$$

$$\Delta = \frac{\tau^2}{4}$$



$$\Delta = \lambda_1 \lambda_2$$

$$\tau = \lambda_1 + \lambda_2$$

① $\lambda_1, \lambda_2 \in \mathbb{R}, \ \lambda_1 \neq \lambda_2$

$$A = P\begin{pmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_m \end{pmatrix} P^{-1}$$

$$\dot{\vec{X}} = A\vec{X} \qquad \dot{\vec{X}} = P\Lambda P^{-1}\vec{X}$$

$$\underbrace{P^{-1}\dot{\vec{X}}}_{\dot{\vec{Z}}} = \Lambda \underbrace{P^{-1}\vec{X}}_{\vec{Z}}$$

$$\dot{\vec{Z}} = \begin{pmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_m \end{pmatrix}\vec{Z} \qquad \vec{Z} = \begin{pmatrix} z_1^0 e^{\lambda_1 t} \\ z_2^0 e^{\lambda_2 t} \\ \vdots \\ z_m^0 e^{\lambda_n t} \end{pmatrix}$$

$$Z = P^{-1} X$$

$$X = PZ = P \begin{pmatrix} z_0^1 e^{\lambda_1 t} \\ \vdots \\ z_0^m e^{\lambda_m t} \end{pmatrix} = \begin{pmatrix} p_1 & p_2 & \cdots & p_m \end{pmatrix} \begin{pmatrix} z_0^1 e^{\lambda_1 t} \\ \vdots \\ z_0^m e^{\lambda_m t} \end{pmatrix}$$

$$X_t = \sum_{i=1}^{m} p_i z_0^i e^{\lambda_i t}$$

$$X_{t_0} = X(t = t_0)$$

$$X_{t_0} = \sum_{i=1}^{m} p_i z_0^i e^{\lambda_i t_0} = \begin{pmatrix} p_1 & p_2 & \cdots & p_m \end{pmatrix} \begin{pmatrix} z_0^1 e^{\lambda_1 t_0} \\ \vdots \\ z_0^m e^{\lambda_m t_0} \end{pmatrix}$$

① t_0

$$\begin{pmatrix} z_1^0 e^{\lambda_1 t_0} \\ \vdots \\ z_m^0 e^{\lambda_m t_0} \end{pmatrix} = P^{-1} X_{t_0}$$

② $\lambda_1 = \lambda_2 \in \mathbb{R}$

$$A(Q_1\ Q_2) = (Q_1\ Q_2)\begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}(Q_1\ Q_2)^{-1}(Q_1\ Q_2)$$

$$A(Q_1\ Q_2) = (Q_1\ Q_2)\begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}$$

$$(AQ_1\ \ AQ_2) = (\lambda Q_1\ \ Q_1 + \lambda Q_2)$$

$$\begin{cases} AQ_1 = \lambda Q_1 \\ AQ_2 = Q_1 + \lambda Q_2 \\ (A - \lambda I)Q_1 = 0 \\ (A - \lambda I)^2 Q_2 = 0 \end{cases}$$

Q_1

$$\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix} = A\begin{pmatrix}x\\ y\end{pmatrix} \qquad \begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix} = \begin{pmatrix}Q_1 & Q_2\end{pmatrix}\begin{pmatrix}\lambda & 1\\ 0 & \lambda\end{pmatrix}\underbrace{\begin{pmatrix}Q_1 & Q_2\end{pmatrix}^{-1}\begin{pmatrix}x\\ y\end{pmatrix}}_{\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \begin{pmatrix}Q_1, & Q_2\end{pmatrix}^{-1}\begin{pmatrix}x\\ y\end{pmatrix}}$$

$$\begin{pmatrix}Q_1, & Q_2\end{pmatrix}^{-1}\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix} = \begin{pmatrix}\lambda & 1\\ 0 & \lambda\end{pmatrix}\begin{pmatrix}Q_1 & Q_2\end{pmatrix}^{-1}\begin{pmatrix}x\\ y\end{pmatrix}$$

$$\begin{pmatrix}\dot{\alpha}\\ \dot{\beta}\end{pmatrix} = \begin{pmatrix}\lambda & 1\\ 0 & \lambda\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} \qquad \begin{cases}\dot{\alpha} = \lambda\alpha + \beta \to \dot{\alpha} = \lambda\alpha + \beta_0 e^{\lambda t}\\ \dot{\beta} = \lambda\beta \longrightarrow \beta_t = \beta_0 e^{\lambda t}\end{cases}$$

$$\dot{\alpha} = \lambda\alpha + B_0 e^{\lambda t} \qquad \alpha = e^{\lambda t}(B_0 t + C)$$

$$\alpha = a b$$

$$\dot{a} \text{ SOL HOMO.}$$

$$\dot{a} = \lambda a \rightarrow a = e^{\lambda t}$$

$$\dot{b} = B_0$$

$$\int dB = \int B_0\, dt$$

$$b = B_0 t + C$$

$$\dot{\alpha} = \dot{a} b + \dot{b} a$$

$$\dot{a} b + \dot{b} a = \lambda a b + B_0 e^{\lambda t}$$

$$b\underbrace{(\dot{a} - \lambda a)}_{=0} + \dot{b} a = B_0 e^{\lambda t} \rightarrow \dot{b} = B_0 e^{\lambda t} a^{-1}$$

$$\dot{b} = B_0 e^{\lambda t} e^{-\lambda t} = B_0$$

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} e^{\lambda t}(B_0 t + C) \\ B_0 e^{\lambda t} \end{pmatrix}$$

$$\boxed{\begin{pmatrix} x \\ y \end{pmatrix} = e^{\lambda t}\left[CQ + B_0 (t Q_1 + Q_2) \right]}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = (Q_1 \quad Q_2)\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = (Q_1 \quad Q_2)\begin{pmatrix} B_0 t + C \\ B_0 \end{pmatrix} e^{\lambda t}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = e^{\lambda t}\left[B_0 t Q_1 + C Q_1 + B_0 Q_2 \right] e^{\lambda t}$$