

$$\frac{\omega^2}{2} = W \quad W = 2$$

$$r(\theta) = \theta^{1/2}$$

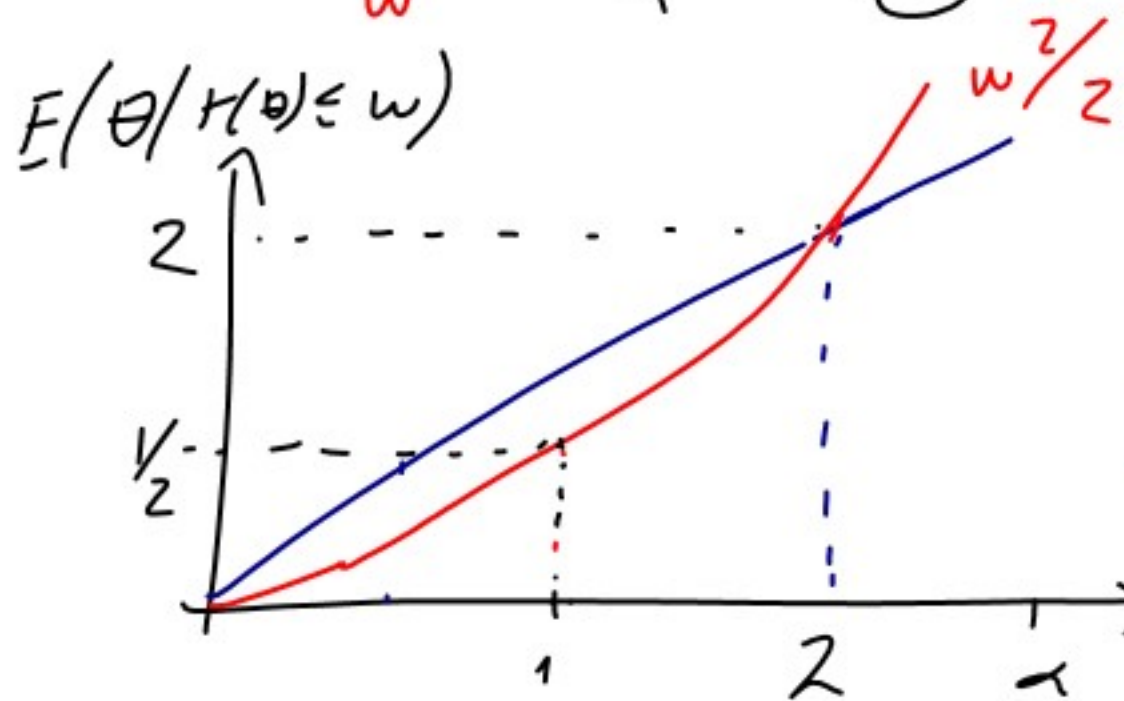
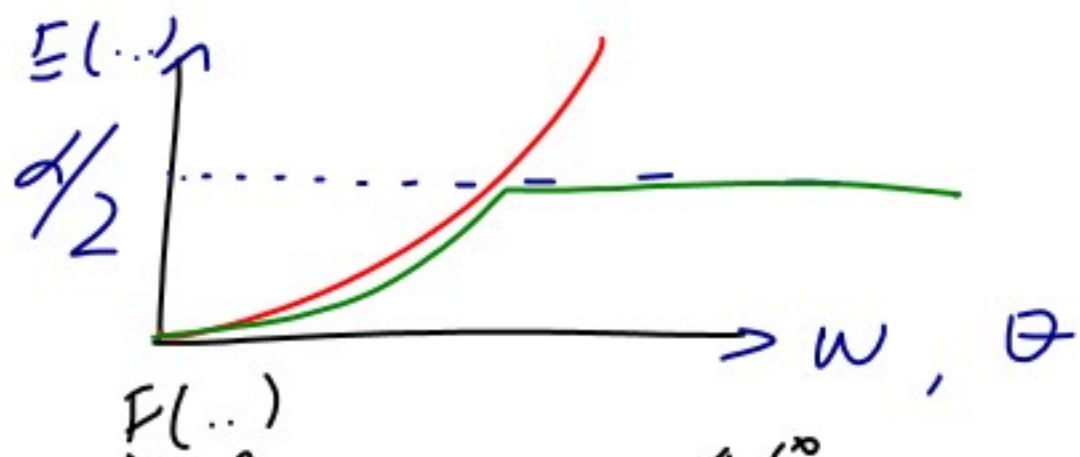
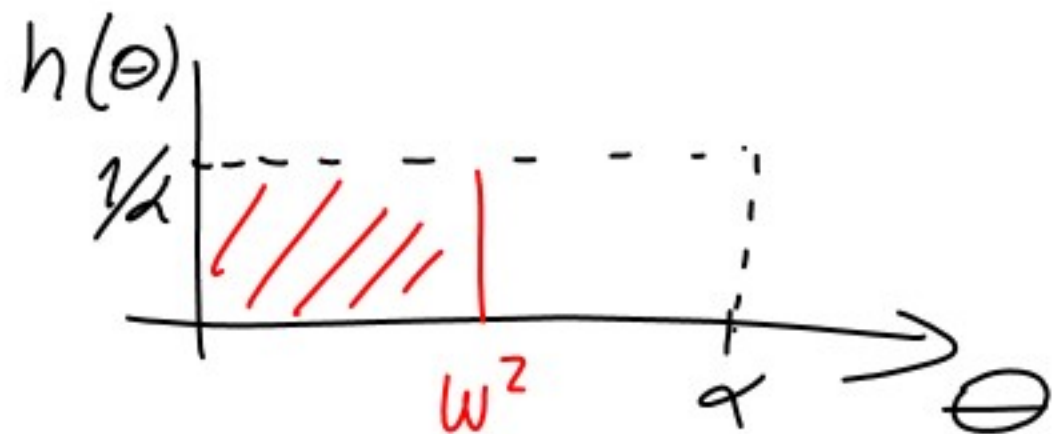
$$E(\theta | r(\theta) \leq \omega)$$

$$= E(\theta | \theta^{1/2} \leq \omega) \quad \theta \leq r^{-1}(\omega)$$

$$= E(\theta | \theta \leq \omega^2)$$

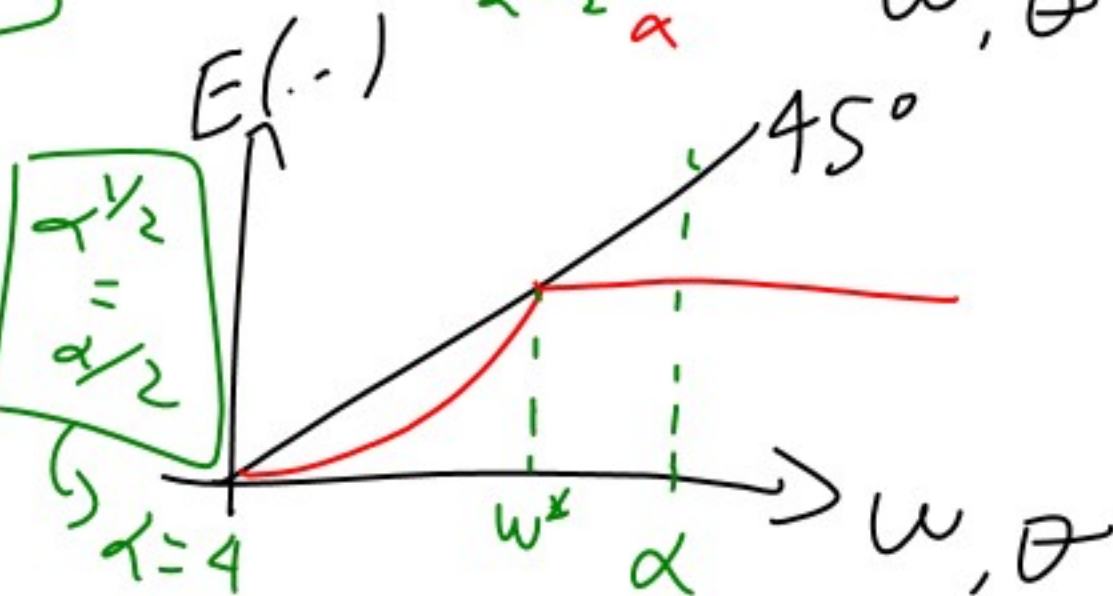
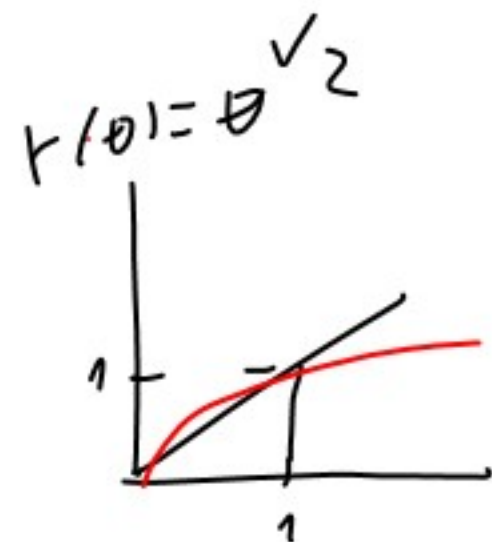
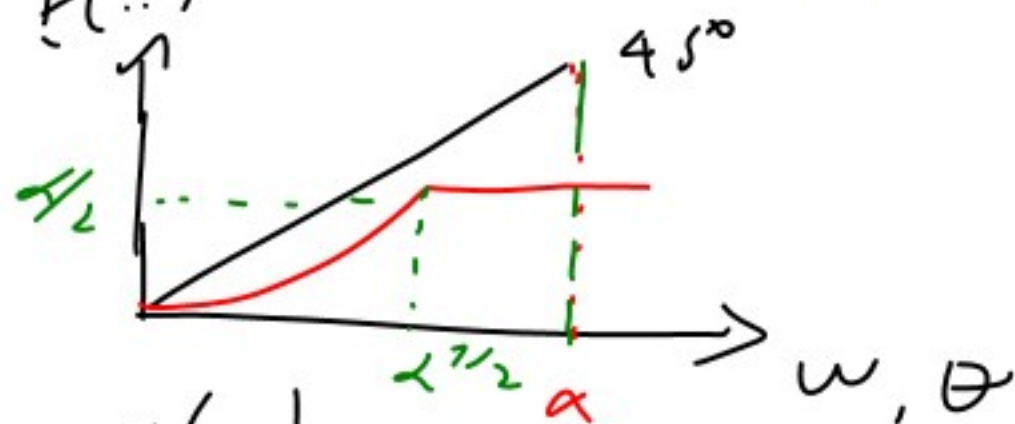
$$= \frac{\int_0^{\omega^2} \frac{1}{\alpha} \theta d\theta}{\int_0^{\omega^2} \frac{1}{\alpha} d\theta}$$

$$= \frac{\frac{1}{2} \theta^2 \Big|_0^{\omega^2}}{\omega^2} = \frac{1}{2} \omega^2$$



$$\alpha^{1/2} > \alpha/2$$

$$4 > \alpha$$



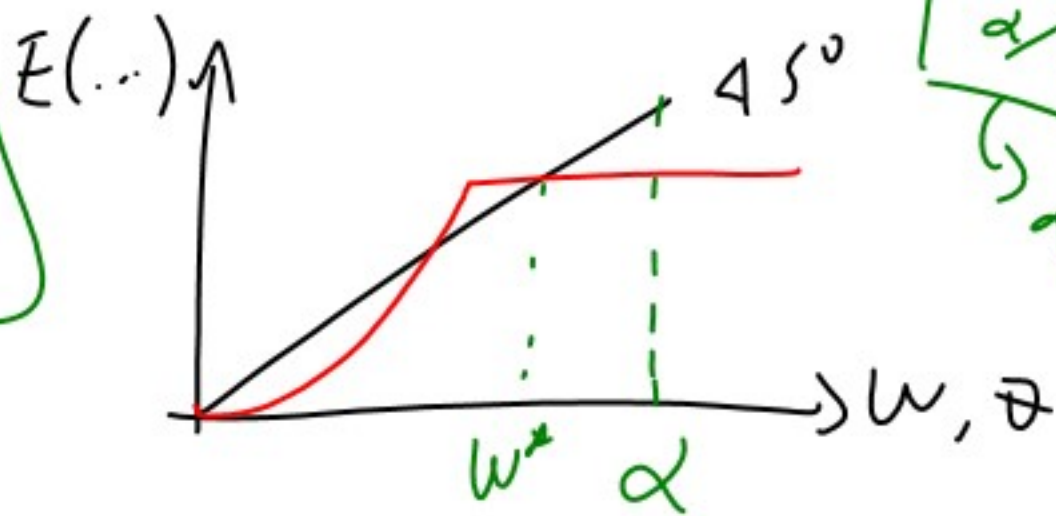
$$\alpha^{1/2} = \alpha/2$$

$$\alpha = 4$$

$$\frac{\omega^2}{2} = \frac{\alpha}{2}$$

$$\hat{\omega} = \alpha^{1/2}$$

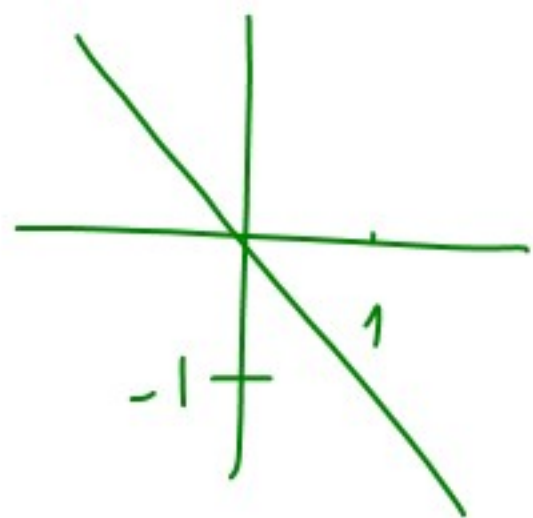
$$E(\theta | r(\theta) \leq \alpha^{1/2}) = \alpha/2$$



$$4 < \alpha$$

$$\alpha^{1/2} < \alpha/2$$

$$\lambda_1 = \lambda_2 = \lambda$$



$$A = (Q_1, Q_2) \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix} (Q_1, Q_2)^{-1}$$

$$A (Q_1, Q_2) = (Q_1, Q_2) \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}$$

$$(A Q_1, A Q_2) = (\lambda Q_1, Q_1 + \lambda Q_2)$$

$$(A - \lambda I) Q_2 = Q_1$$

$$A Q_1 = \lambda Q_1$$

$$A Q_2 = Q_1 + \lambda Q_2$$

$$\begin{array}{l} (A - \lambda I) Q_1 = 0 \\ (A - \lambda I)^2 Q_1 = 0 \end{array}$$



$$\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$$

$$C = 1 + 1$$

$$C = 2$$

$$\Delta = (1)(1) - (2)(-2)$$

$$\Delta = 5$$

$$\lambda_{1,2} = \frac{C}{2} \pm \frac{\sqrt{C^2 - 4\Delta}}{2}$$

$$\lambda_{1,2} = \frac{2}{2} \pm \frac{\sqrt{4 - 20}}{2}$$

$$\lambda_{1,2} = 1 \pm 2i$$

$$\lambda_1 = 1 + 2i$$

$$\lambda_2 = 1 - 2i$$

$$V_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + i \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$\left[ \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1-\lambda & -2 \\ 2 & 1-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\lambda_1 = 1 + 2i$$

$$\lambda_2 = 1 - 2i$$

$$\begin{bmatrix} 1-2i & -2 \\ 2 & 1-2i \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -2i & -2 \\ 2 & -2i \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$-2x_1i - 2x_2 = 0$$

$$x_1i + x_2 = 0$$

$$x_1 = 1$$

$$x_2 = -i$$

$$V_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + i \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\lambda_1 = 1 + 2i \quad \lambda_2 = 1 - 2i$$

$$v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + i \begin{pmatrix} 0 \\ -1 \end{pmatrix} \quad v_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + i \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \alpha_1 \left[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} + i \begin{pmatrix} 0 \\ -1 \end{pmatrix} \right] e^{(1+2i)t} + \alpha_2 \left[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} + i \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right] e^{(1-2i)t}$$

$$= e^t \left\{ \alpha_1 \left[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} + i \begin{pmatrix} 0 \\ -1 \end{pmatrix} \right] [\cos(2t) + i \sin(2t)] + \alpha_2 \left[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} + i \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right] [\cos(2t) - i \sin(2t)] \right\}$$

$$= e^t \left\{ (\alpha_1 + \alpha_2) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cos(2t) - (\alpha_1 - \alpha_2) \begin{pmatrix} 0 \\ -1 \end{pmatrix} \sin(2t) \right\} + i \left\{ \begin{pmatrix} 0 \\ -1 \end{pmatrix} \cos(2t) (\alpha_1 - \alpha_2) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sin(2t) (\alpha_1 + \alpha_2) \right\}$$

$$= e^t \left\{ \hat{a} (\alpha_1 + \alpha_2) \left[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cos(2t) - \begin{pmatrix} 0 \\ -1 \end{pmatrix} \sin(2t) \right] + i (\alpha_1 - \alpha_2) \left[ \begin{pmatrix} 0 \\ -1 \end{pmatrix} \cos(2t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sin(2t) \right] \right\}$$

$$= e^t \left\{ \hat{a} \left[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cos(2t) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sin(2t) \right] + \hat{b} \left[ \begin{pmatrix} 0 \\ -1 \end{pmatrix} \cos(2t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sin(2t) \right] \right\}$$

$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} = e^0 \left\{ \hat{a} \left[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cos(0) - \begin{pmatrix} 0 \\ -1 \end{pmatrix} \sin(0) \right] + \hat{b} \left[ \begin{pmatrix} 0 \\ -1 \end{pmatrix} \cos(0) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sin(0) \right] \right\}$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \hat{a} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \hat{b} \begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} \Rightarrow \begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$