

# Mathematical Biology

## A qualitative study of Lotka-Volterra Models

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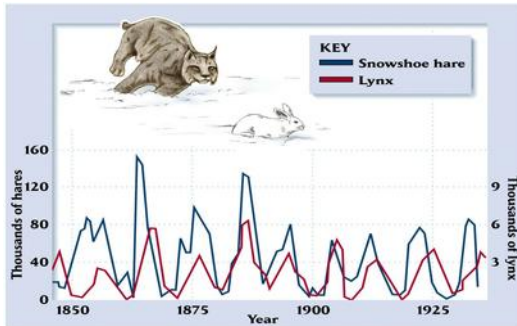
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# History

- Aim: describe the dynamics of population
- Historically started by the observation of Lynx and rabbit population



# History



# Lotka-Volterra Model

- Simplest model to describe interaction prey-predator
- $\dot{x} = ax - bxy$
- $\dot{y} = -cy + dxy$
- $x$  is the population prey,  $y$  is the population of the predator
- $a$ : Natural growth rate of prey in the absence of predation
- $b$ : Death rate due to predation
- $c$ : Natural death rate of predators in the absence of prey
- $d$ : Growth rate due to predation

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## Assumptions

- Unlimited supply for the prey
- Supply for the predators depends only on the prey
- No role played by the environment



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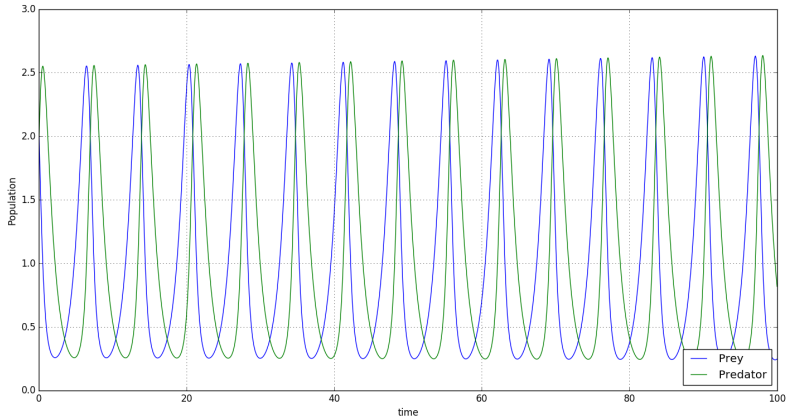
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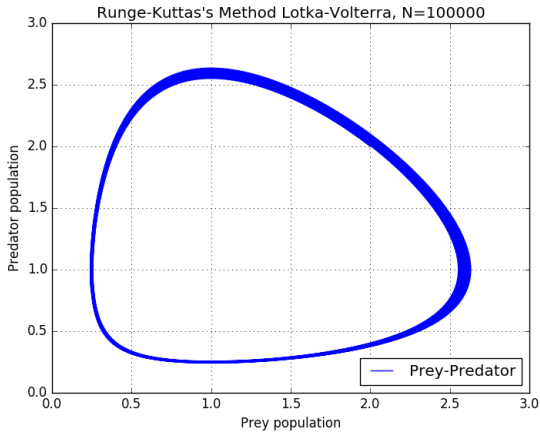
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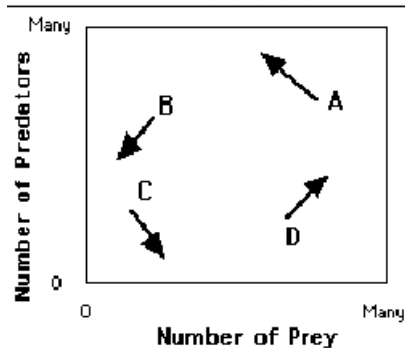
$$a=b=c=d=1$$



$a=b=c=d=1$ ; phase-space

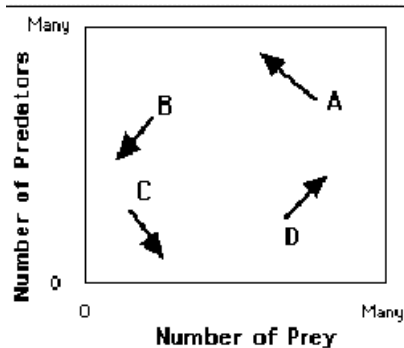


# Interpretation



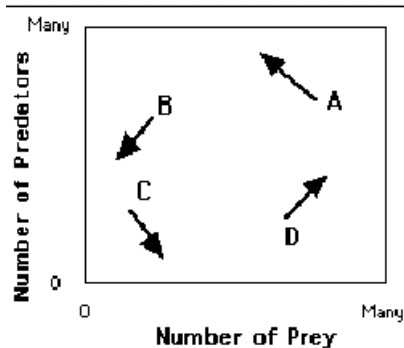
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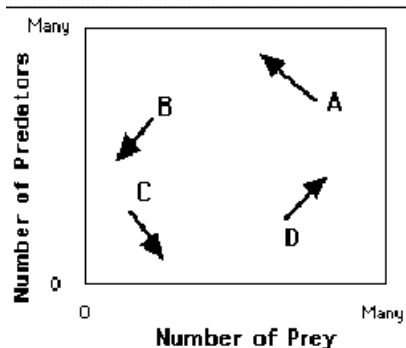
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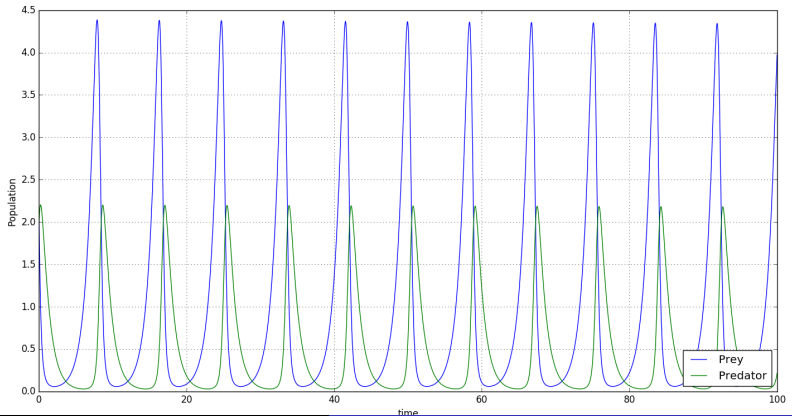


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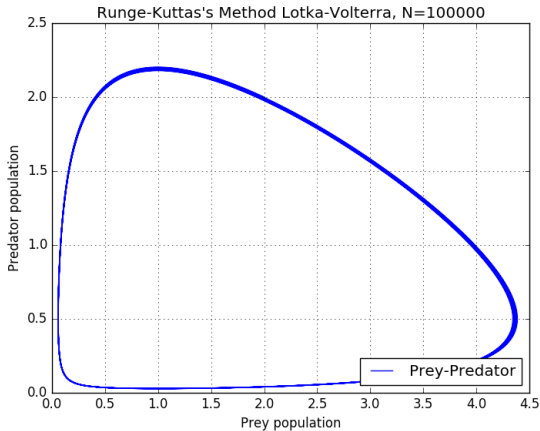


$$a=c=d=1, b=2$$

Increased death rate due to predation

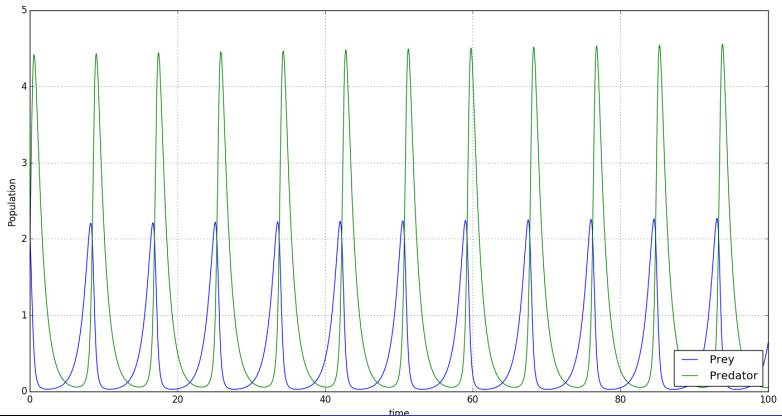


$a=c=d=1$ ,  $b=2$ ; phase-space

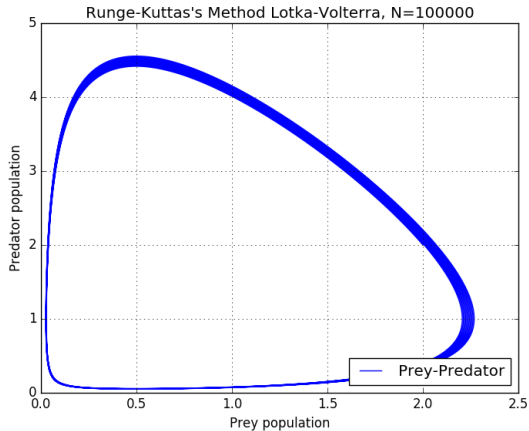


$$a=b=c=1, d=2$$

Growth rate due to predation doubled



$a=c=d=1$ ,  $b=2$ ; phase-space



# Problems

- Model does not consider competition among preys and predators
- Unlimited supplies
- Only two interacting species
- Model does not consider extinction situation