11 June 2019

GRAPPA Student Seminar
Indirect dark matter searches

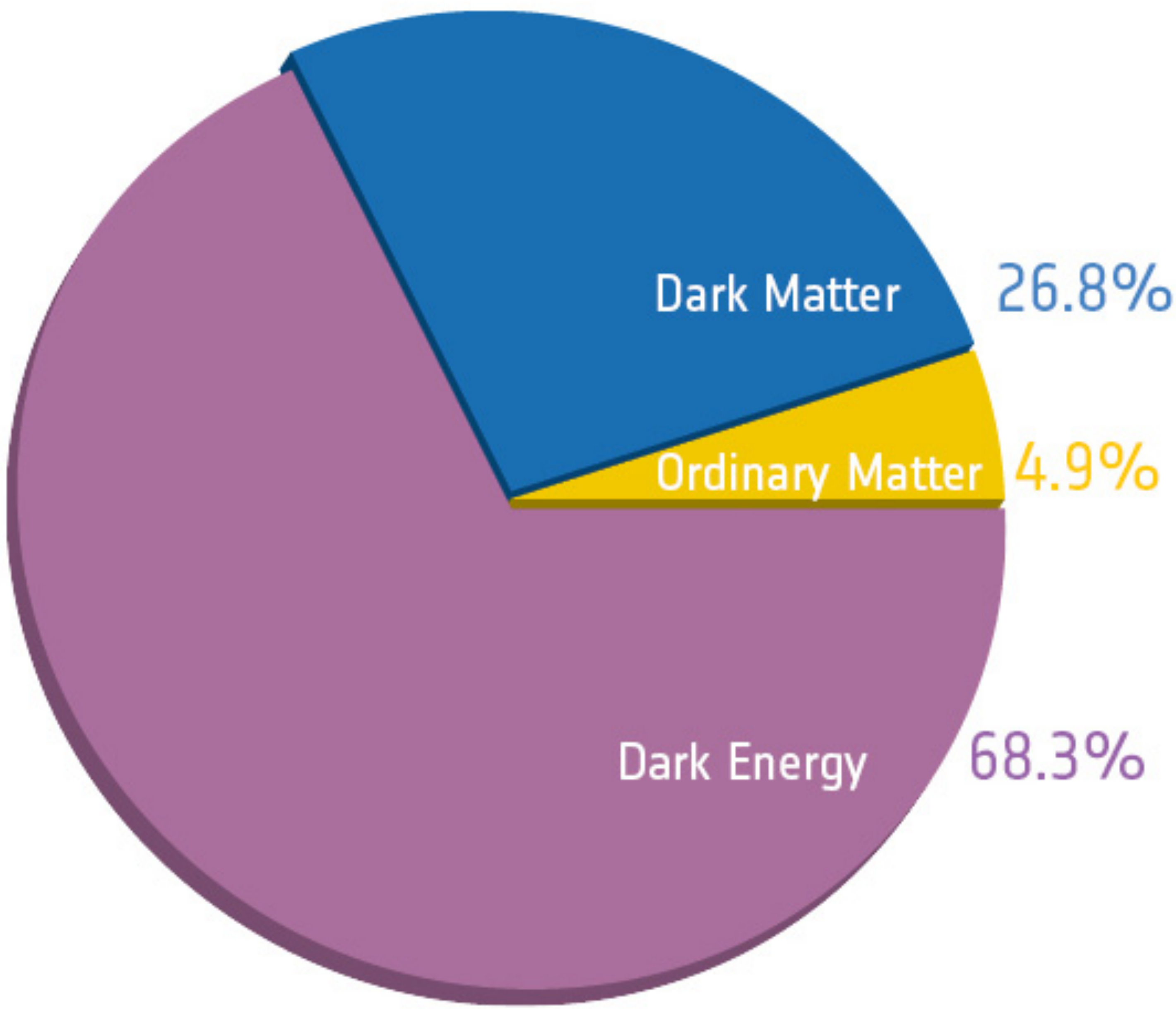
Shin'ichiro Ando

Practical notes

- Please submit your first coding assignments to “notebooks/” directory on GitHub repository
- Please start looking for and reading publications for your final presentation!

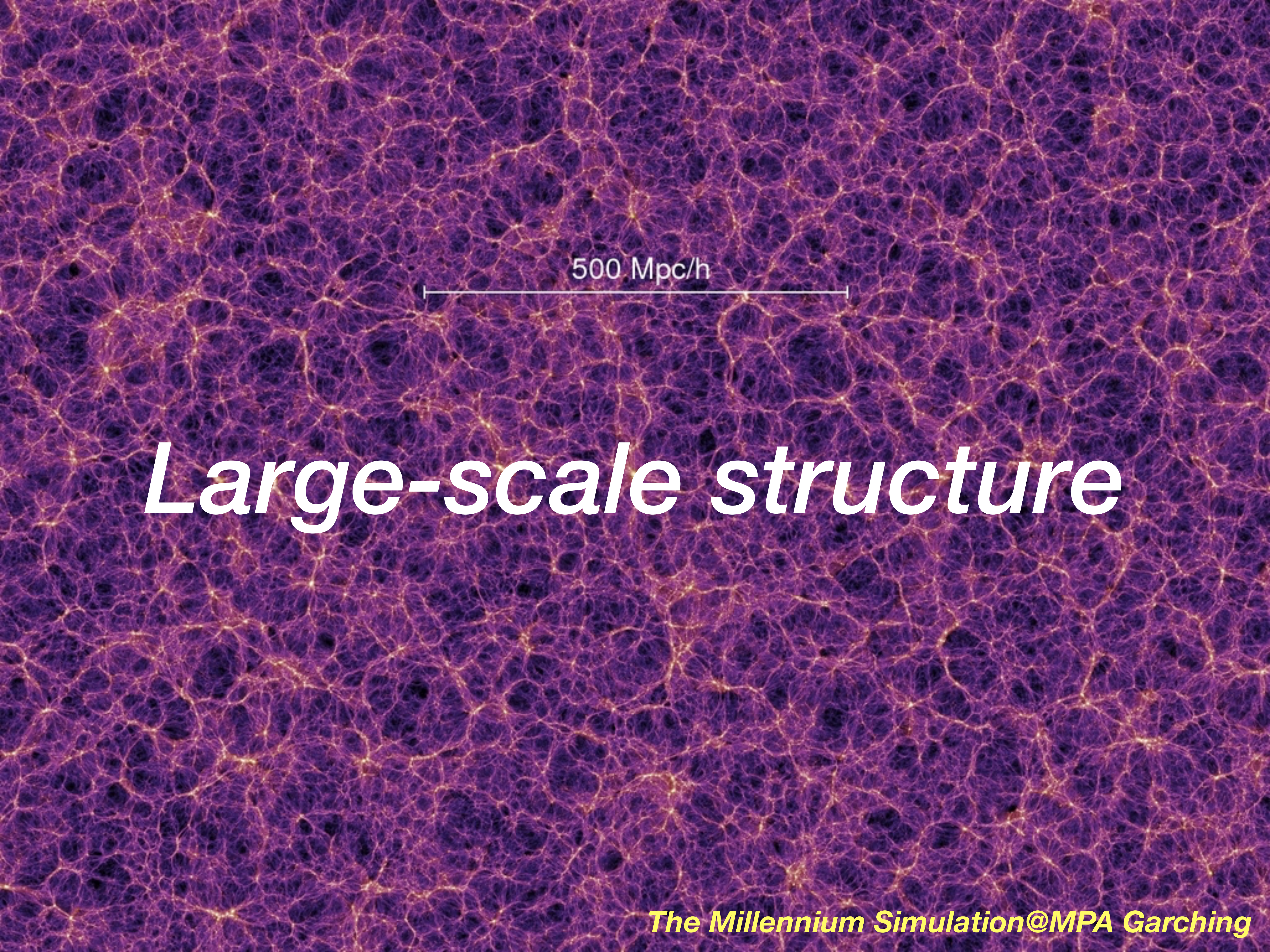
Constituent of the Universe

Planck 2015



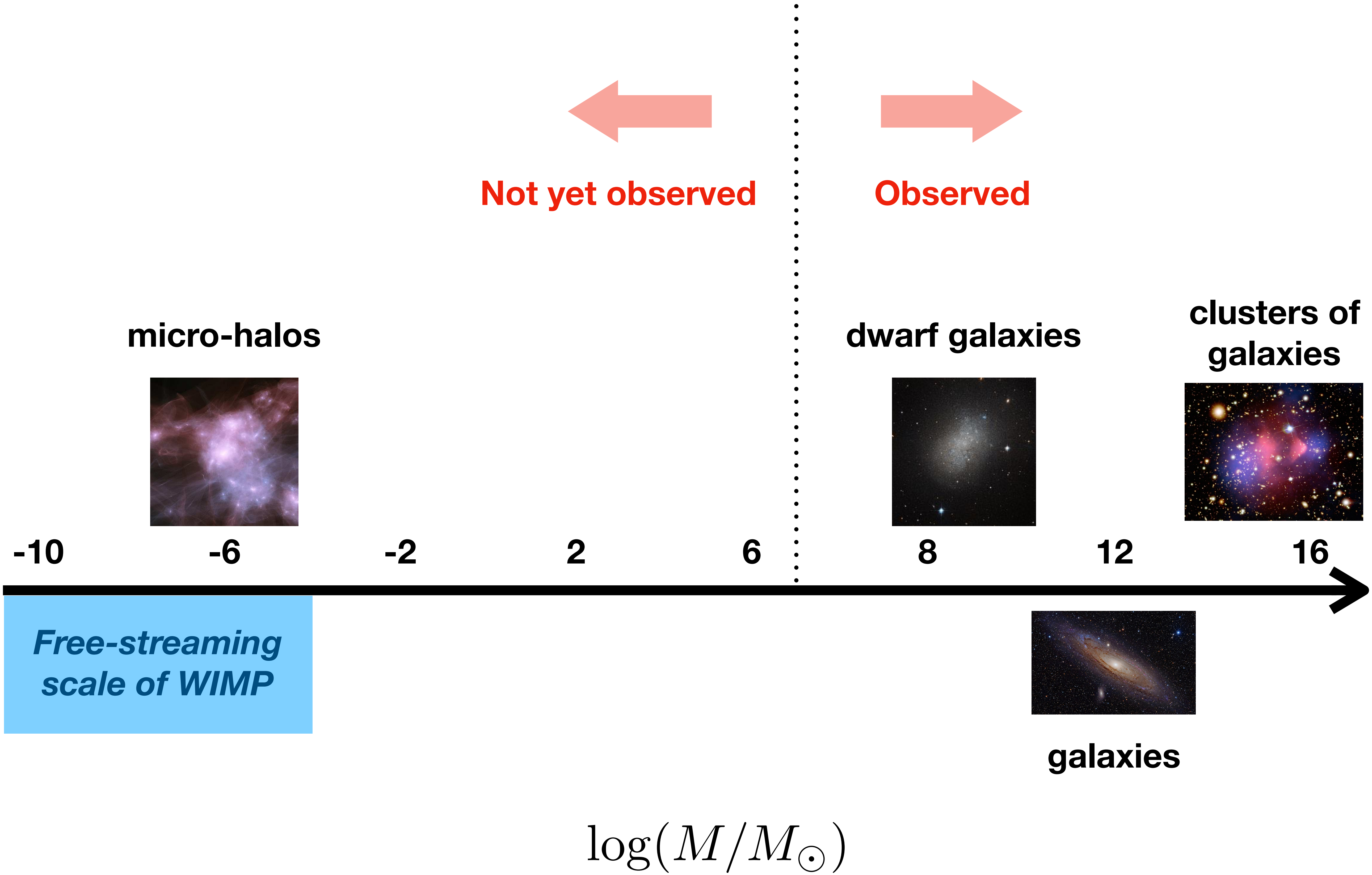
- CMB, galaxy power spectrum, weak lensing, supernova Ia, etc.
- 27% of the total energy / 85% of the total matter is made of dark matter
- Properties of dark matter
  - **Collisionless**
  - **Non-baryonic**
  - **Doesn't interact with photons**
  - **Cold** (or warm; hot dark matter erases too many structures)

Large-scale structure



The Millennium Simulation@MPA Garching

Dark matter: Origin of all the structures



Dark matter: Origin of all the structures

- How do dark matter structures form? — *Spherical collapse model*
- What is abundance, mass distribution, etc.? — *Halo mass function*
- Impact on dark matter annihilation in halos — *Indirect dark matter searches*

Spherical collapse model

- Deriving two **magic numbers** *analytically*

  - Over-density of virialized halos compared with background

    $$\Delta_{\mathrm{vir}} = 18\pi^2$$

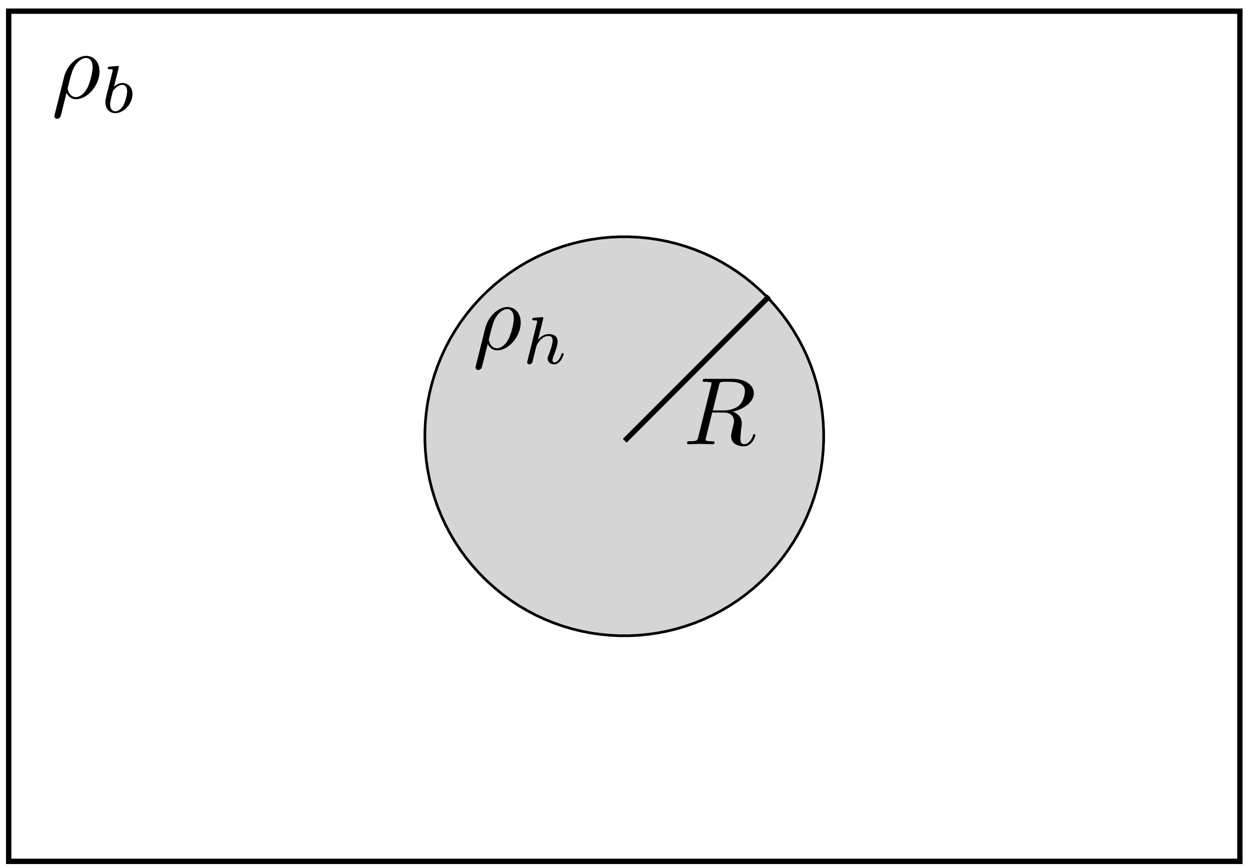
    Useful for simulations to find halos

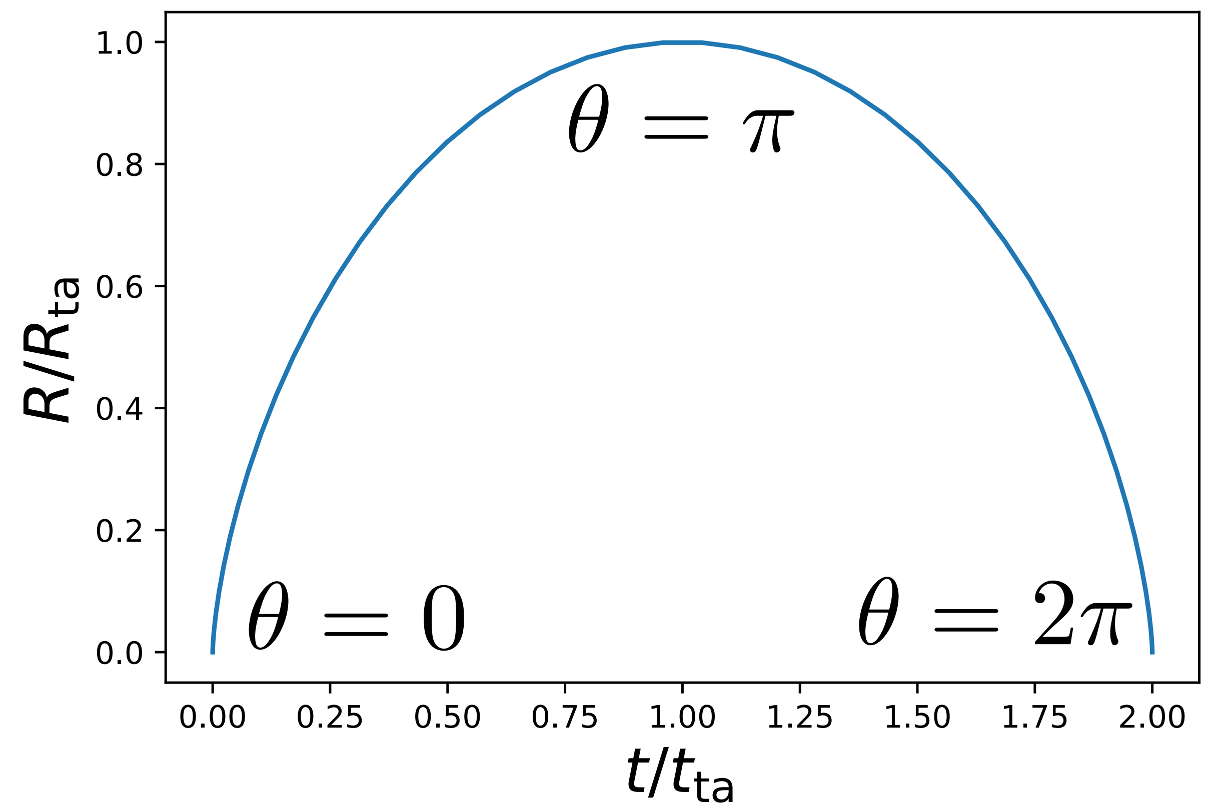
  - Linear extrapolation of over-density for halos that *just* collapsed

    $$\delta_c = 1.686$$

    Useful for analytic calculations to estimate number of halos

Spherical collapse model





Parameterized solution
(cf., expanding *closed* Universe)

$$t = \frac{A^3}{\sqrt{GM}}(\theta - \sin\theta)$$

$$R = A^2(1 - \cos\theta)$$

Spherical collapse model

When do halos virialize?

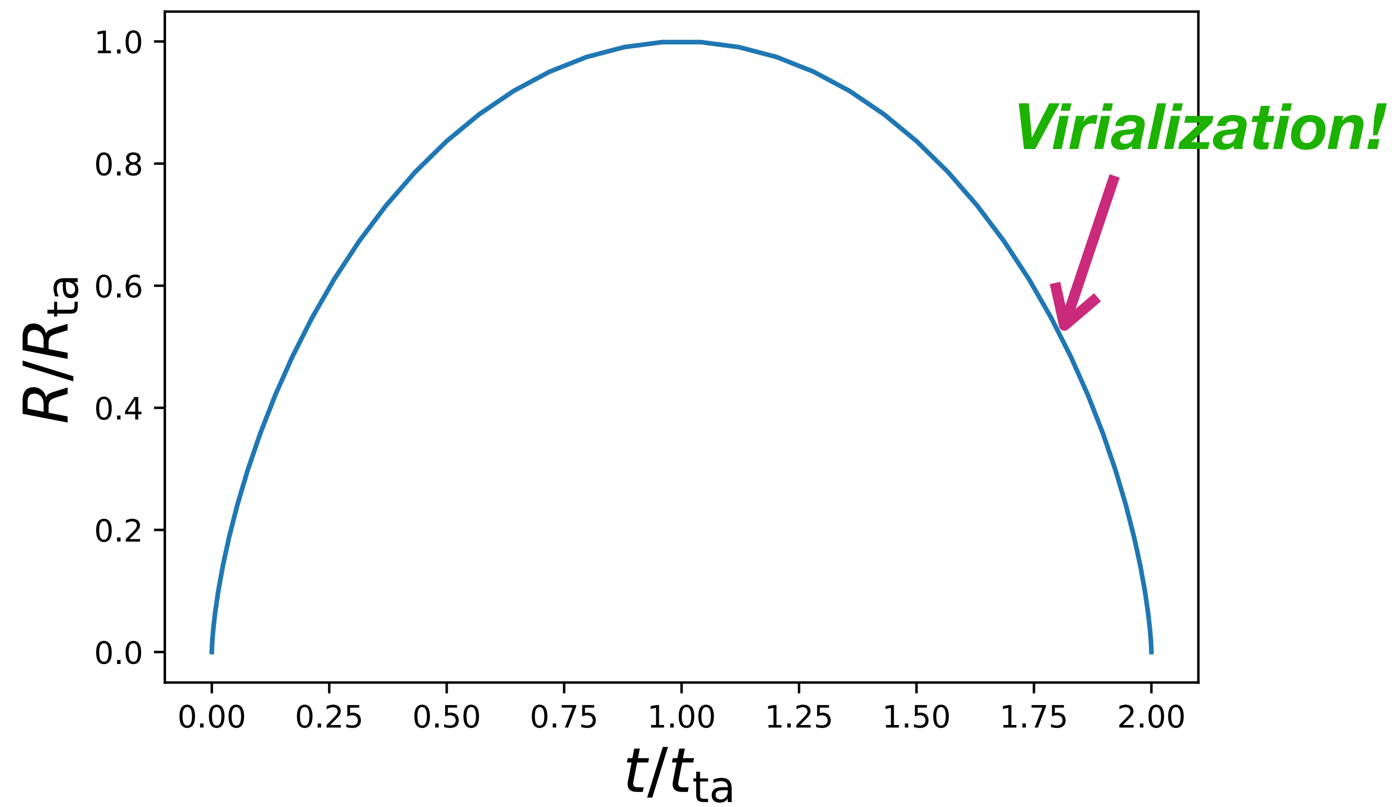
Virial theorem:

$$2K_{\mathrm{vir}} + U_{\mathrm{vir}} = 0$$

(for $1/R$ potential)

Total energy conservation:

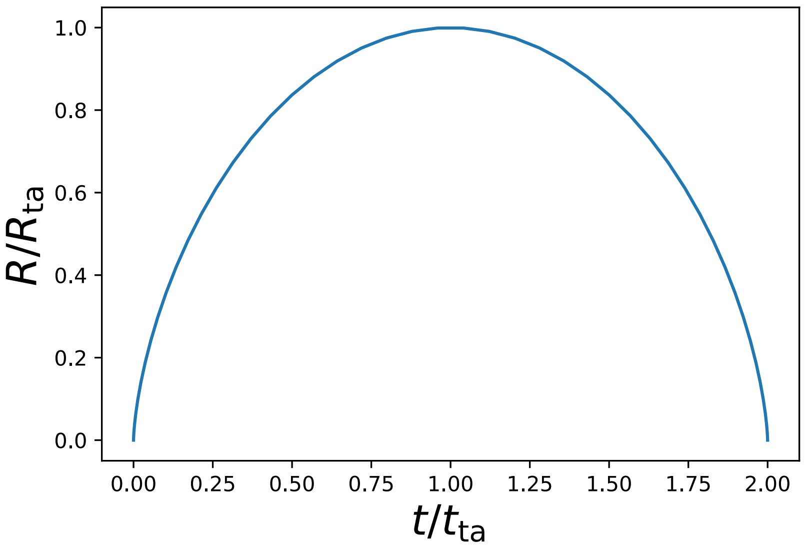
$$K_{\mathrm{vir}} + U_{\mathrm{vir}} = U_{\mathrm{ta}}$$



$$U_{\mathrm{vir}} = 2U_{\mathrm{ta}}$$

$$\boxed{R_{\mathrm{vir}} = \frac{R_{\mathrm{ta}}}{2}}$$

Spherical collapse model



$$t = \frac{A^3}{\sqrt{GM}}(\theta - \sin\theta)$$

$$R = A^2(1 - \cos\theta)$$

$$R_{\rm vir} = \frac{R_{\rm ta}}{2}$$

How dense is a virialized halo compared with background?

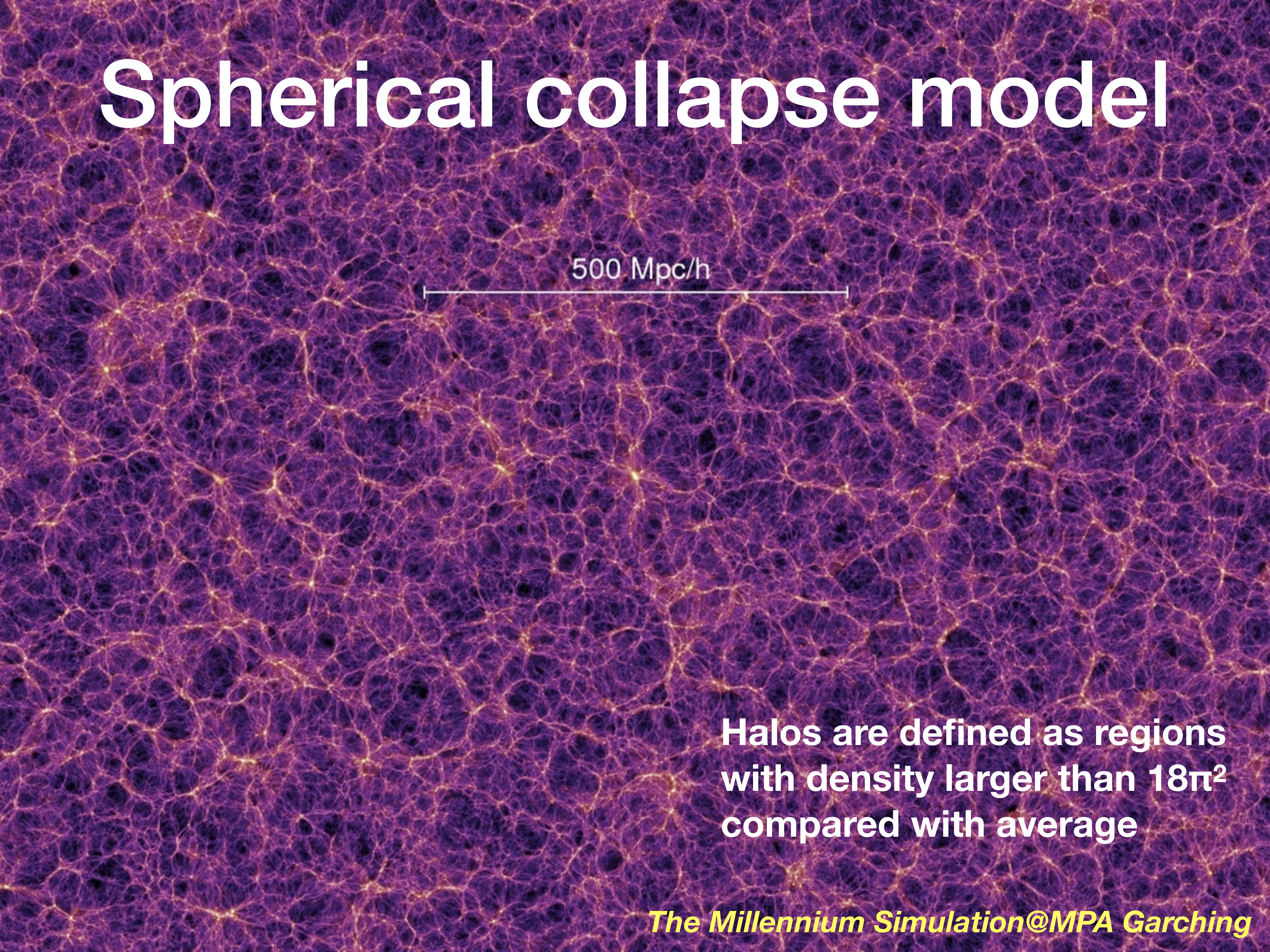
$$\rho_{\rm vir} = \frac{3M}{4\pi R_{\rm vir}^3} = \frac{6M}{\pi R_{\rm ta}^3} = \frac{3\pi}{G t_{\rm col}^2}$$

$$(t_{\rm col} = 2t_{\rm ta})$$

$$\rho_b(t_{\rm col}) = \frac{1}{6\pi G t_{\rm col}^2}$$

$$\frac{\rho_{\rm vir}}{\rho_b(t_{\rm col})} = 18\pi^2$$

Spherical collapse model



Halos are defined as regions with density larger than $18\pi^2$ compared with average

The Millennium Simulation@MPA Garching

Spherical collapse model

- Deriving two **magic numbers** *analytically*

✔ Over-density of virialized halos compared with background

$$\Delta_{\rm vir} = 18\pi^2$$

Useful for simulations to find halos

- Linear extrapolation of over-density for halos that *just* collapsed

$$\delta_c = 1.686$$

Useful for analytic calculations to estimate number of halos

Spherical collapse model

- Deriving two **magic numbers** *analytically*

✔ Over-density of virialized halos compared with background

$$\Delta_{\rm vir} = 18\pi^2 + 82[\Omega_m(z) - 1] - 39[\Omega_m(z) - 1]^2$$

ΛCDM: Bryan & Norman (1998)

- Linear extrapolation of over-density for halos that *just* collapsed

$$\delta_c = 1.686$$

Useful for analytic calculations to estimate number of halos

Analytic model of halo mass function

- It is not possible to describe non-linear evolution of density fully analytically
- However, one can extrapolate behavior in linear regime (that can be solved analytically), as if it continues
    - *What does this $18\pi^2$ collapsed region correspond to, in terms of linear over-density, δ_L?*
- One can estimate the number of halos of given mass (i.e., halo mass function), by using this threshold δ_c and by assuming density distribution is Gaussian (excellent approximation for CMB, hence must be true with linear extrapolation)

Over-density: Linear extrapolation

$$t = \frac{A^3}{\sqrt{GM}}(\theta - \sin\theta)$$

$$R = A^2(1 - \cos\theta)$$

Exact solution:

$$\rho_h = \frac{3M}{4\pi R^3} = \frac{3M}{4\pi A^6}\frac{1}{(1-\cos\theta)^3}$$

$$\rho_b = \frac{1}{6\pi G t^2} = \frac{M}{6\pi A^6}\frac{1}{(\theta - \sin\theta)^2}$$

$$\delta = \frac{\rho_h}{\rho_b} - 1 = \frac{9(\theta - \sin\theta)^2}{2(1-\cos\theta)^3} - 1$$

Over-density: Linear extrapolation

$$t = \frac{A^3}{\sqrt{GM}}(\theta - \sin\theta)$$

$$R = A^2(1 - \cos\theta)$$

$$\xrightarrow[\theta \ll 1]{} \quad t \approx \frac{A^3}{6\sqrt{GM}}\theta^3$$

Linear extrapolation:

$$\delta = \frac{9(\theta - \sin\theta)^2}{2(1-\cos\theta)^3} - 1 \approx \frac{3}{20}\theta^2 \quad \Rightarrow \quad \delta_L \approx \frac{3}{20}\left(\frac{6\sqrt{GM}}{A^3}t\right)^{2/3}$$

At collapse: $(\theta = 2\pi)$

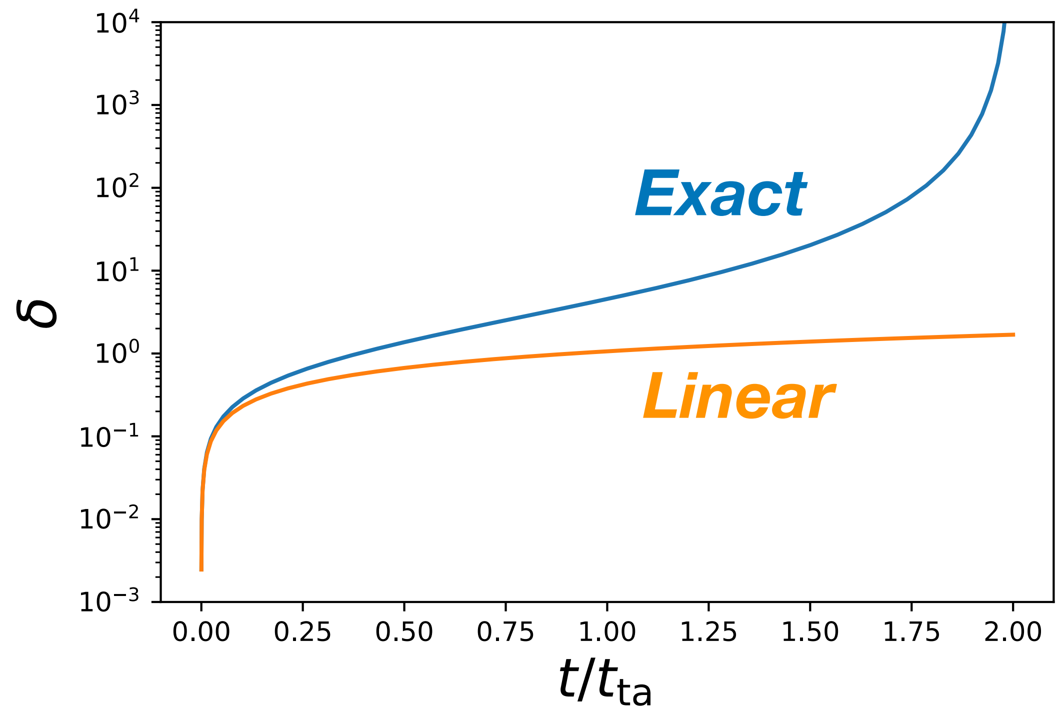
$$t_{\rm col} = \frac{2\pi A^3}{\sqrt{GM}} \quad \Rightarrow \quad \delta_L = \frac{3}{20}(12\pi)^{2/3} \approx 1.686$$

Over-density: Linear extrapolation

Exact

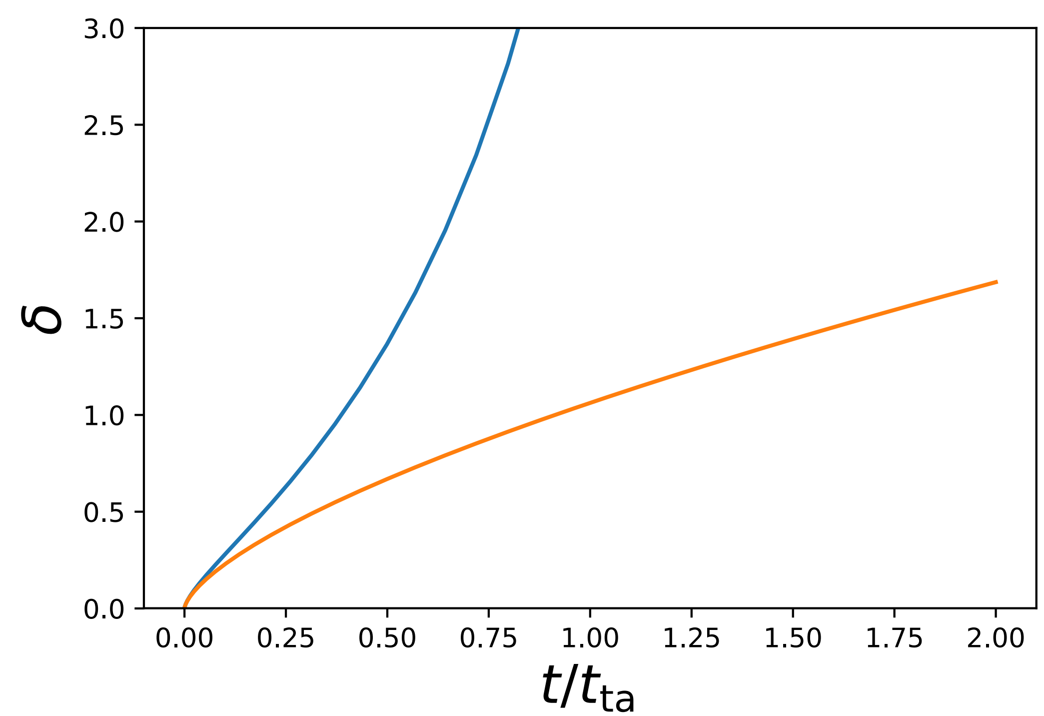
$$t = \frac{A^3}{\sqrt{GM}}(\theta - \sin\theta)$$

$$\delta = \frac{9(\theta - \sin\theta)^2}{2(1-\cos\theta)^3} - 1$$

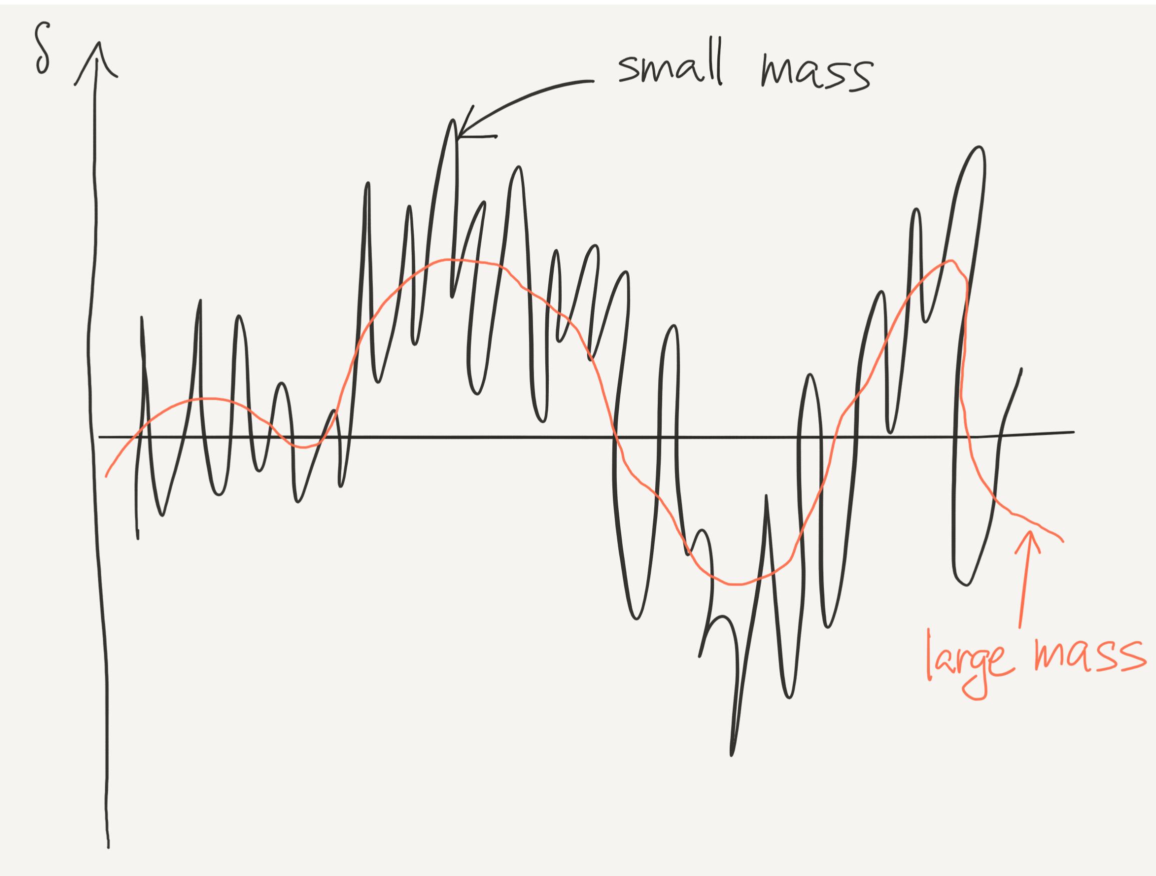


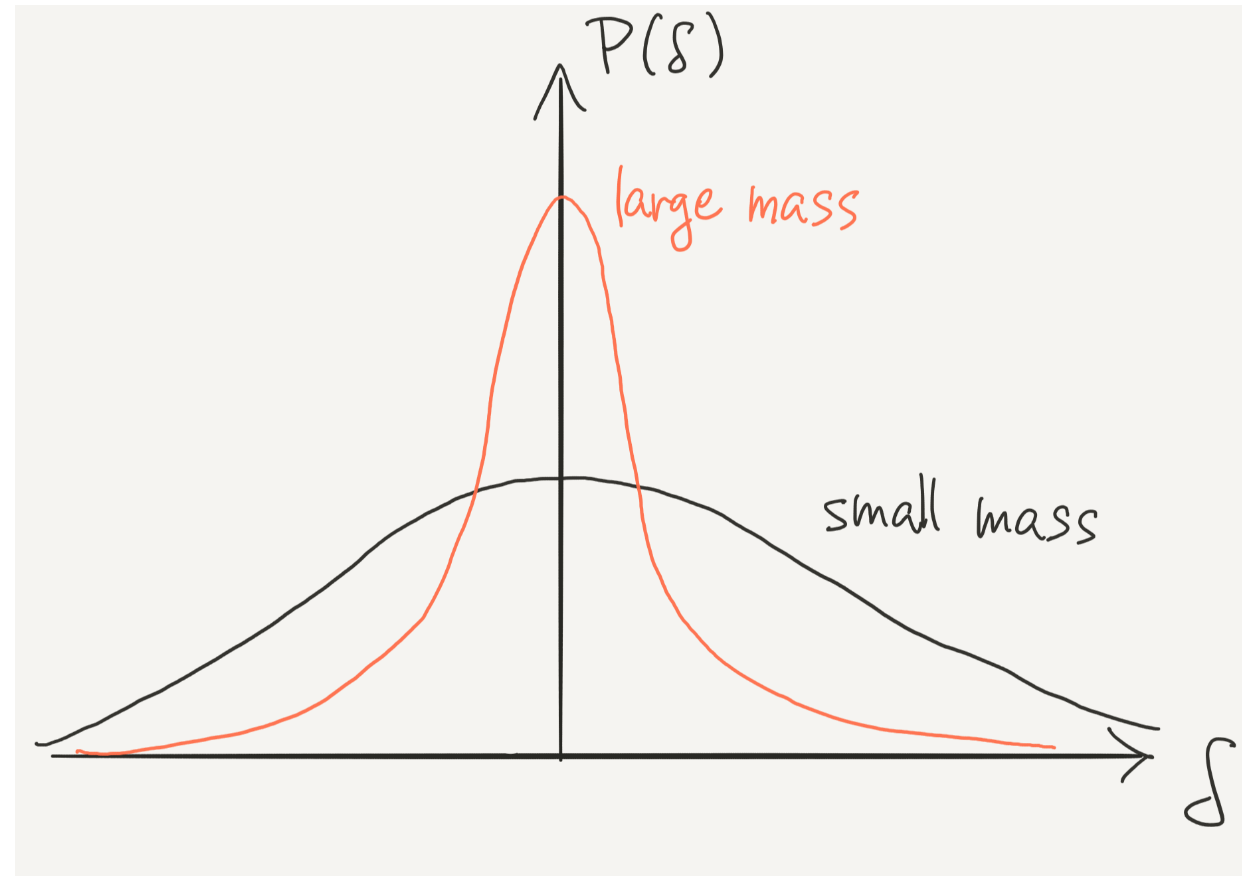
Linear

$$\delta_L \approx \frac{3}{20}\left(\frac{6\sqrt{GM}}{A^3}t\right)^{2/3}$$



Gaussian random field





Density field smeared over *R*, given by

$$M = \frac{4\pi}{3}\bar{\rho}R^3$$

rms over-density:

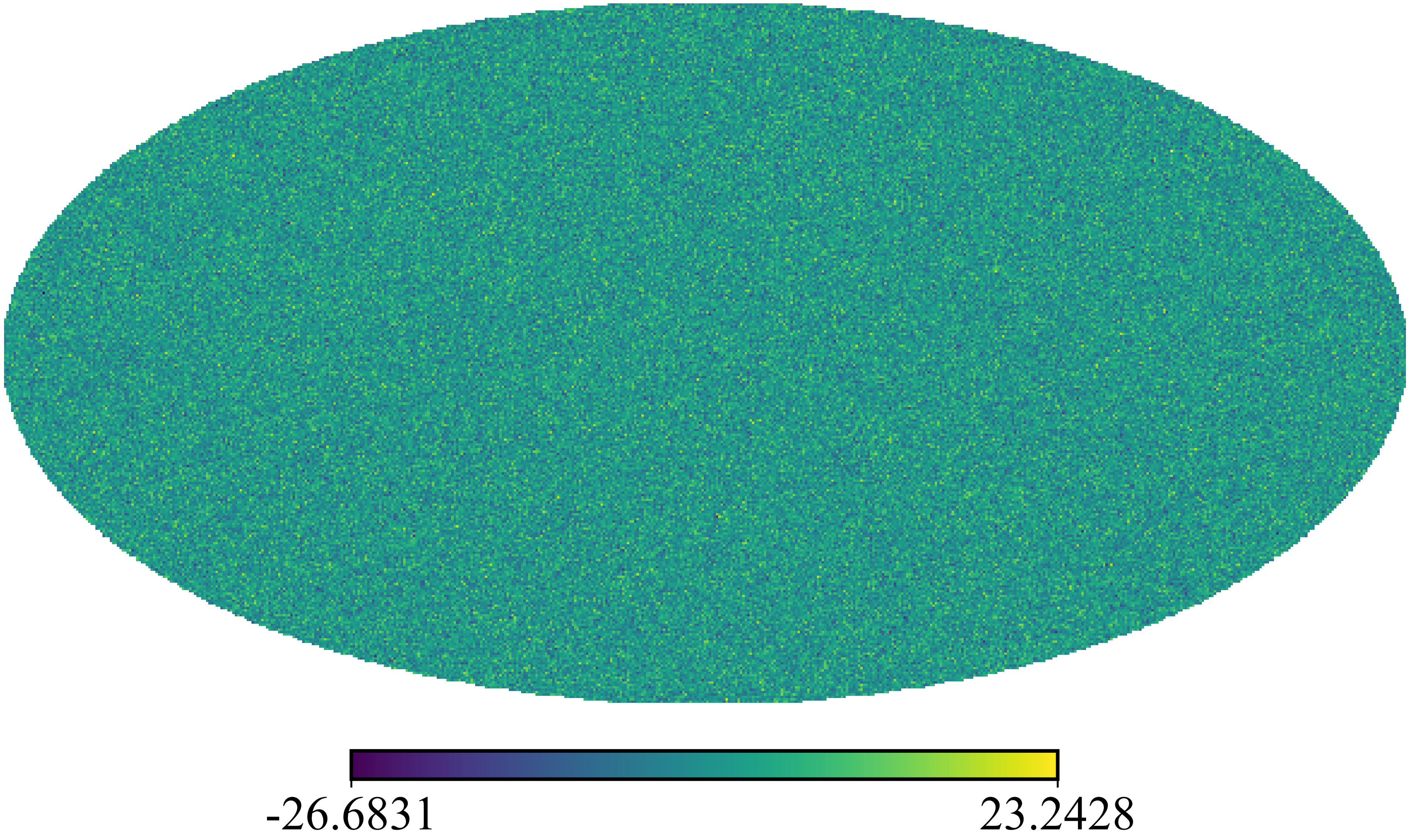
$$\sigma^2(M) = \langle \delta_R^2 \rangle = \int \frac{d^3k}{(2\pi)^3} P_{\rm lin}(k) W_R^2(k)$$

Redshift evolution:

$$\sigma(M, z) = \sigma(M)D(z)$$

Gaussian random field: simple example

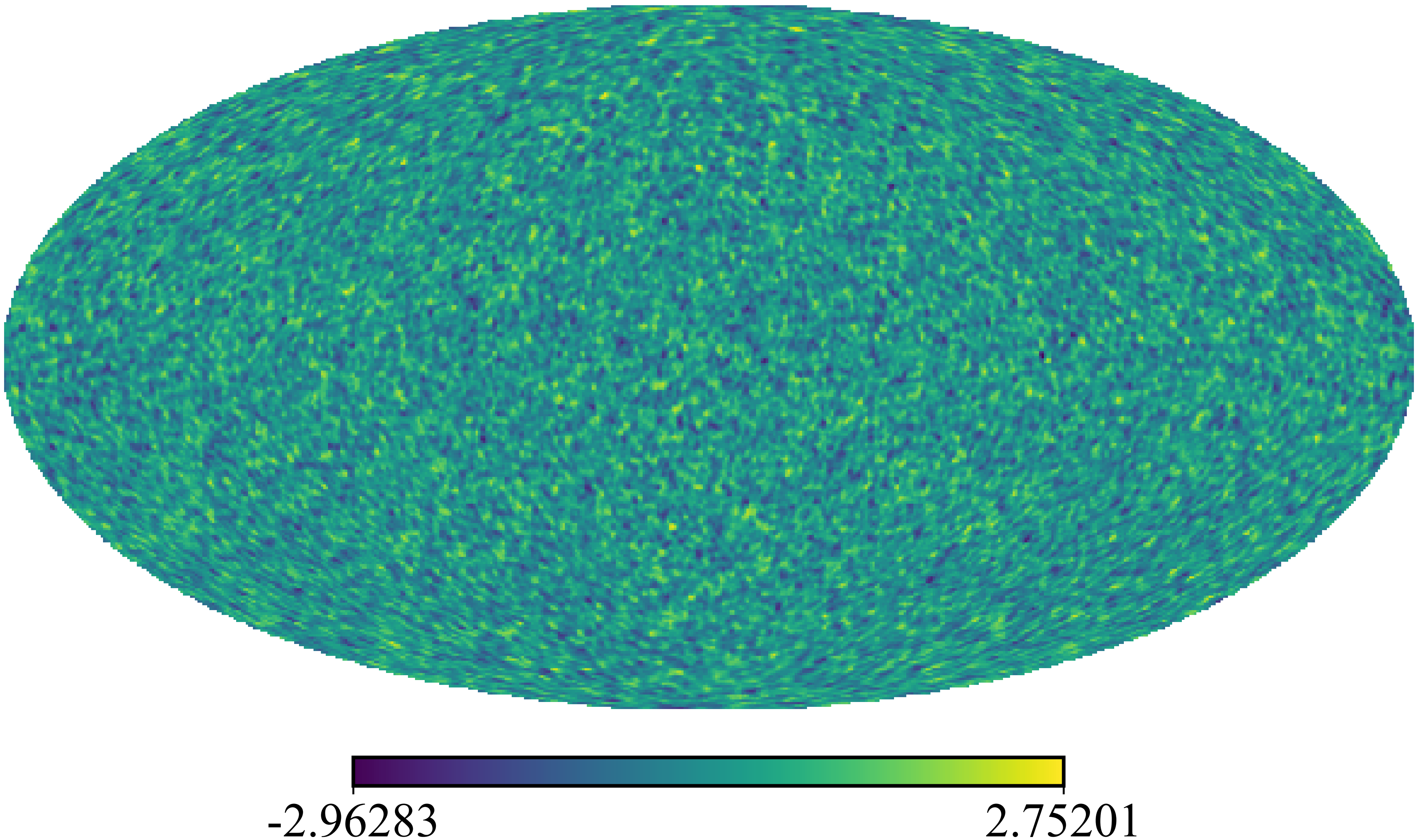
Mean = 0
SD = 5
Pixel size = $(0.23 \text{ deg})^2$



Gaussian random field: simple example

Mean = 0
SD = 5
Pixel size = (0.23 deg)^2

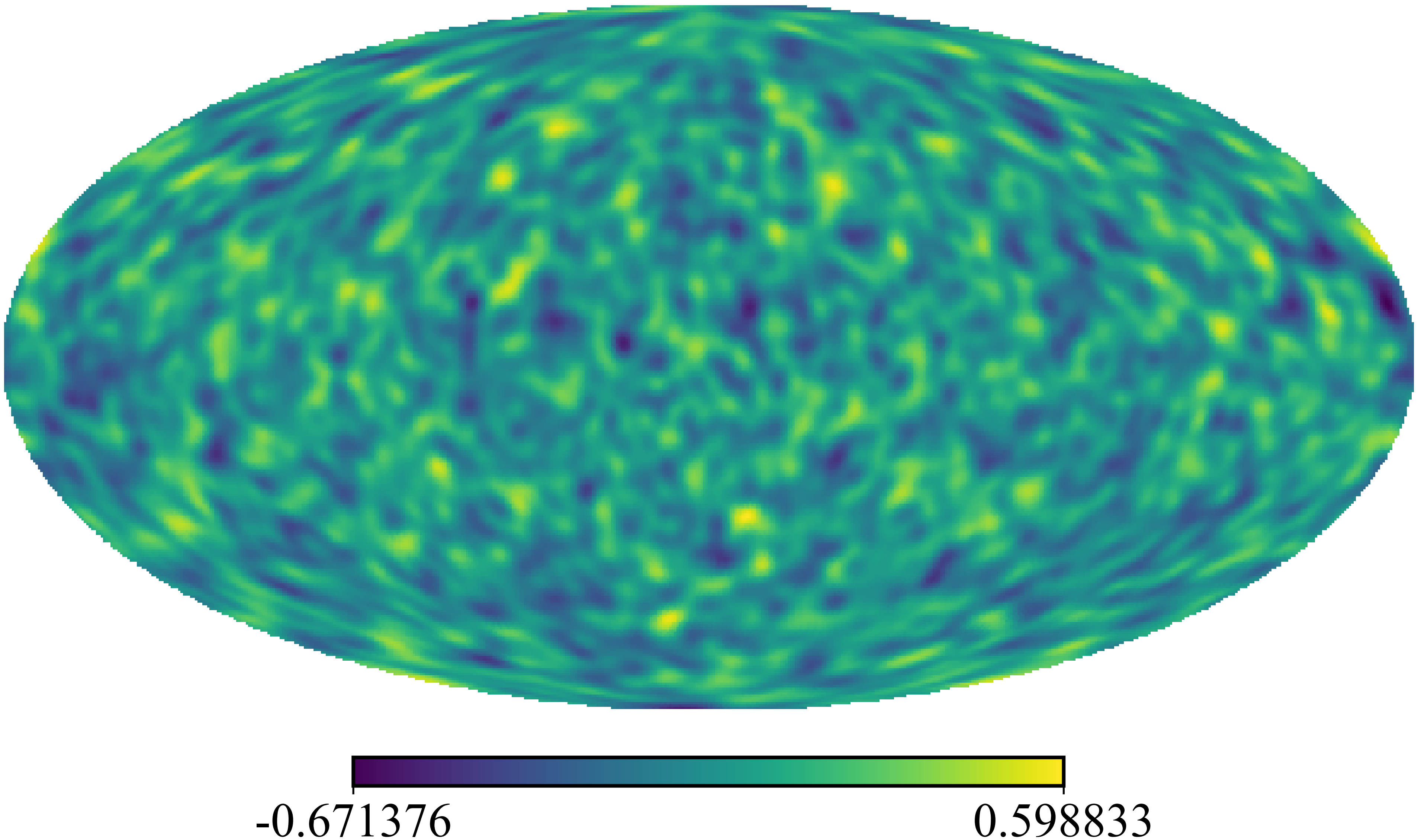
Gaussian smoothing: σ = 0.5 deg



Gaussian random field: simple example

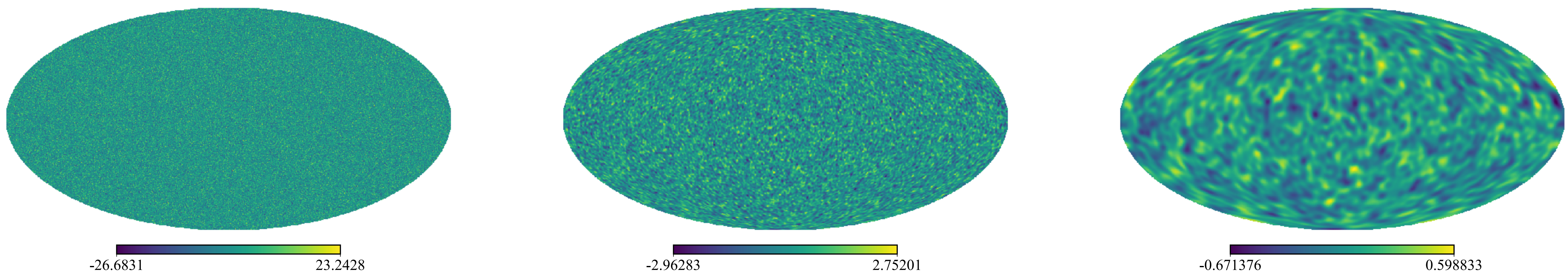
Mean = 0
SD = 5
Pixel size = (0.23 deg)^2

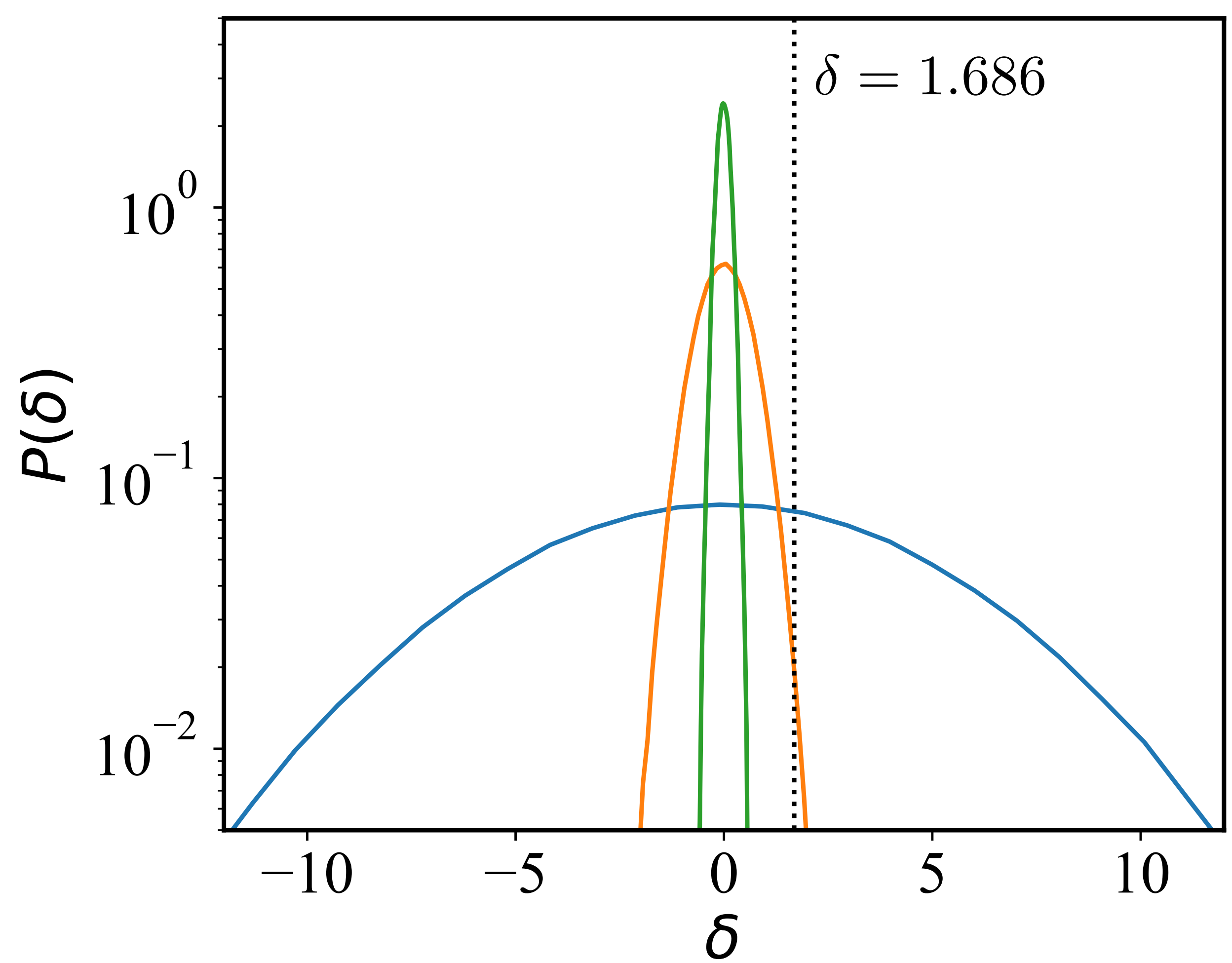
Gaussian smoothing: σ = 2 deg



Gaussian random field: simple example

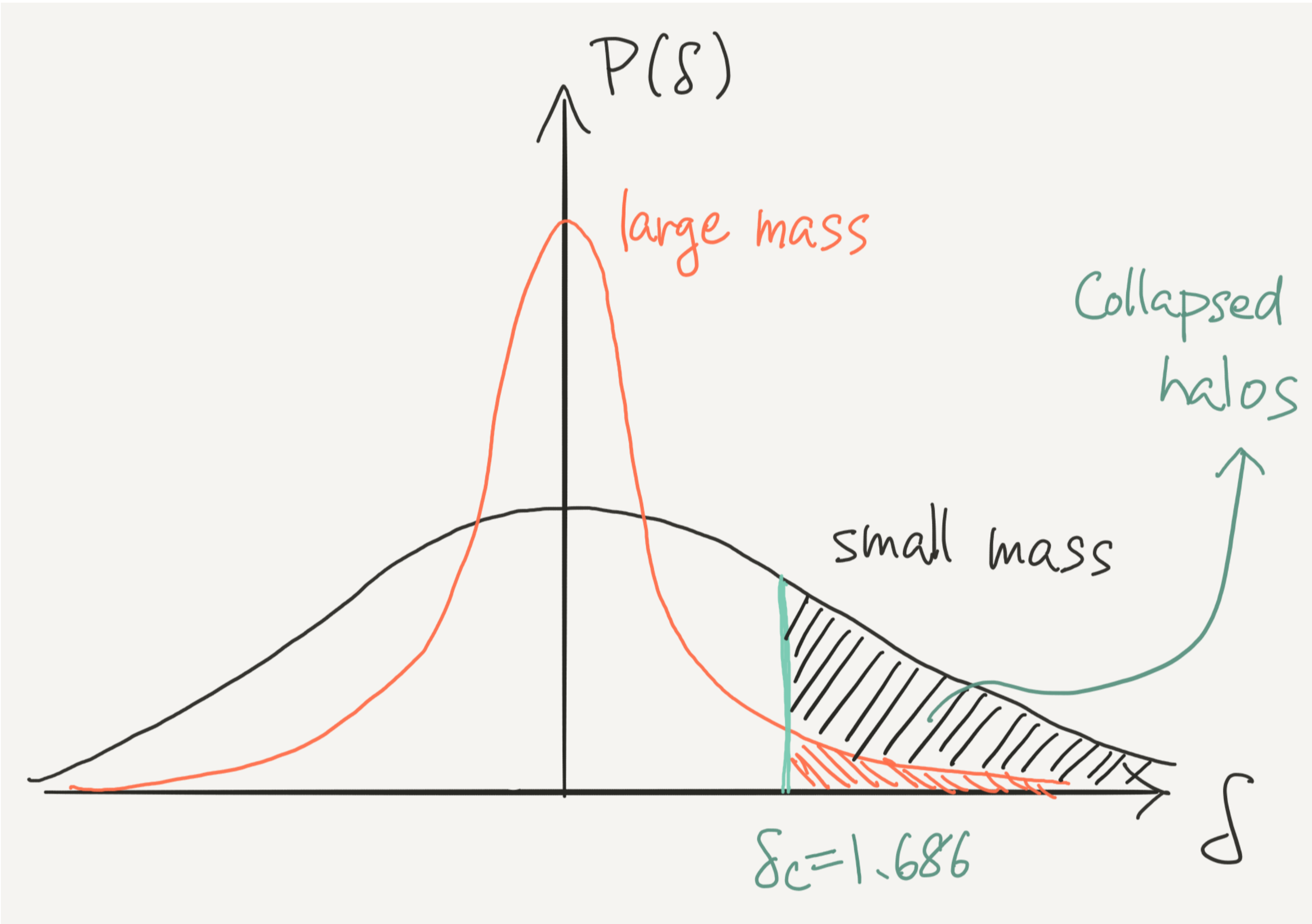
Raw map | **0.5 deg smoothing** | **2 deg smoothing**





Smaller structures form first and then merge and accrete to form larger structures

Press-Schechter mass function



Fraction of collapsed halos

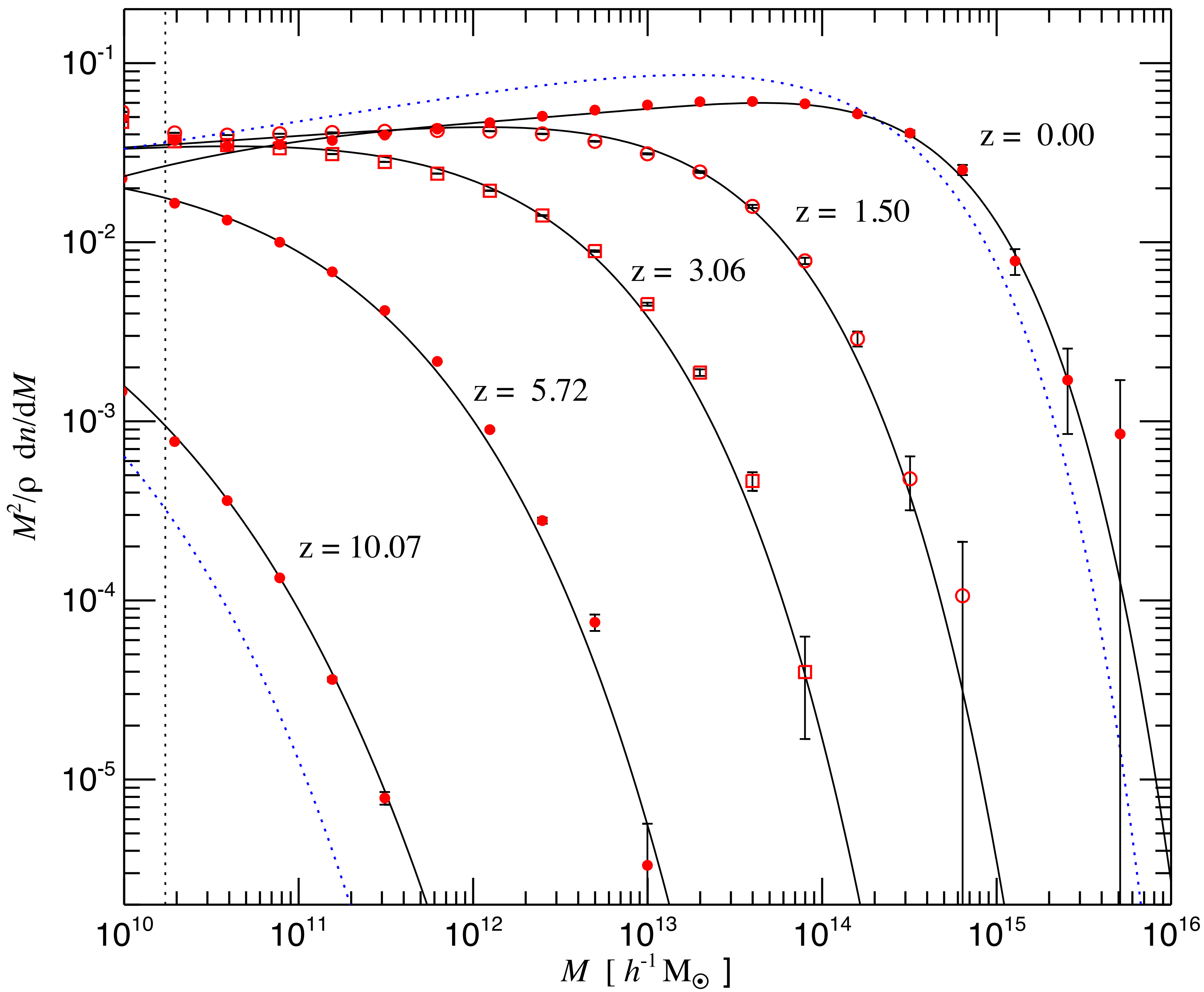
$$\int_{\delta_c}^{\infty} d\delta \ P(\delta|M,z)$$

Press-Schechter mass function $[\nu \equiv \delta_c/\sigma(M,z)]$

$$\frac{dn}{d\ln M} = \sqrt{\frac{2}{\pi}}\frac{\bar{\rho}}{M}\nu \exp\left(-\frac{\nu^2}{2}\right)\frac{d\ln\sigma^{-1}}{d\ln M}$$

Comparison with numerical simulations

Springel et al., *Nature* **435**, 629 (2005)



- Reasonable agreement with the Millennium simulations (**Red** points)
  - **Blue**: Press-Schechter mass function
  - **Black**: Jenkins et al. (2001) mass function
- Other representative models include Sheth & Tormen (2001), Tinker et al. (2008), many of which are based on ***ellipsoidal collapse*** model

Spherical collapse model

- Deriving two **magic numbers** *analytically*

✔ Over-density of virialized halos compared with background

$$\Delta_{\mathrm{vir}} = 18\pi^2$$

Useful for simulations to find halos

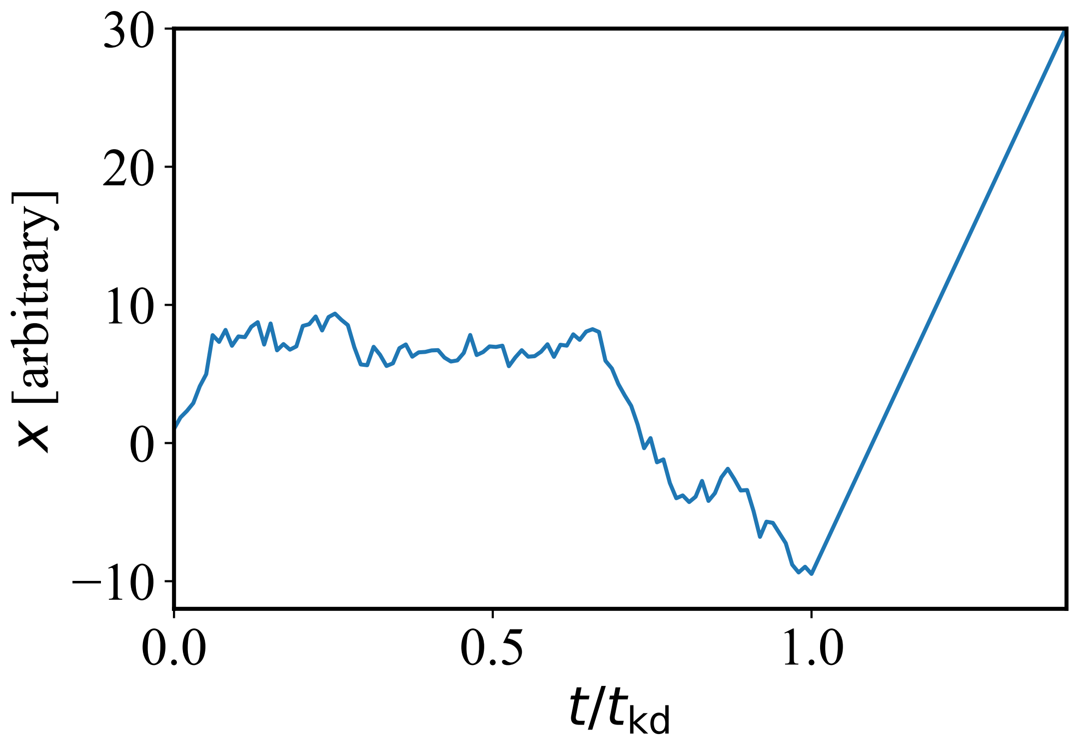
✔ Linear extrapolation of over-density for halos that *just* collapsed

$$\delta_c = 1.686$$

Useful for analytic calculations to estimate number of halos

What is the smallest structure?

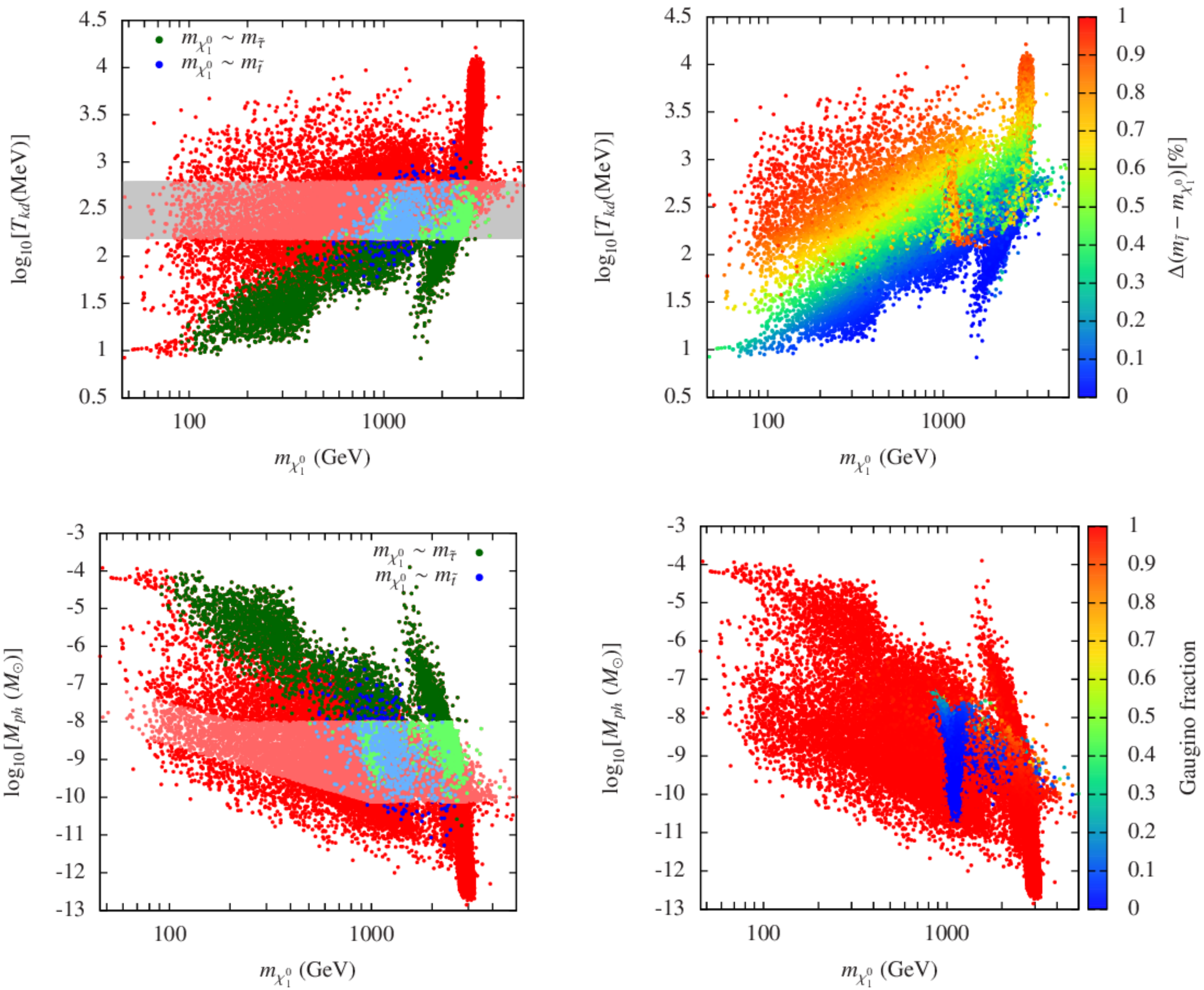
E.g., 1d random walk followed by free-streaming



- In the WIMP scenario:
- After **chemical decoupling**, WIMPs can still interact with baryons and leptons through scattering
- When this gets slower than Hubble expansion (**kinetic decoupling**), WIMPs start free-streaming
- **All the structures below this *kd+free-streaming* scale will be washed away**
- Finding small halos is key to distinguish different dark matter models

What is the smallest structure?

Diamanti, Cabrera-Catalan, Ando, *Phys. Rev. D* **92**, 065029 (2015)

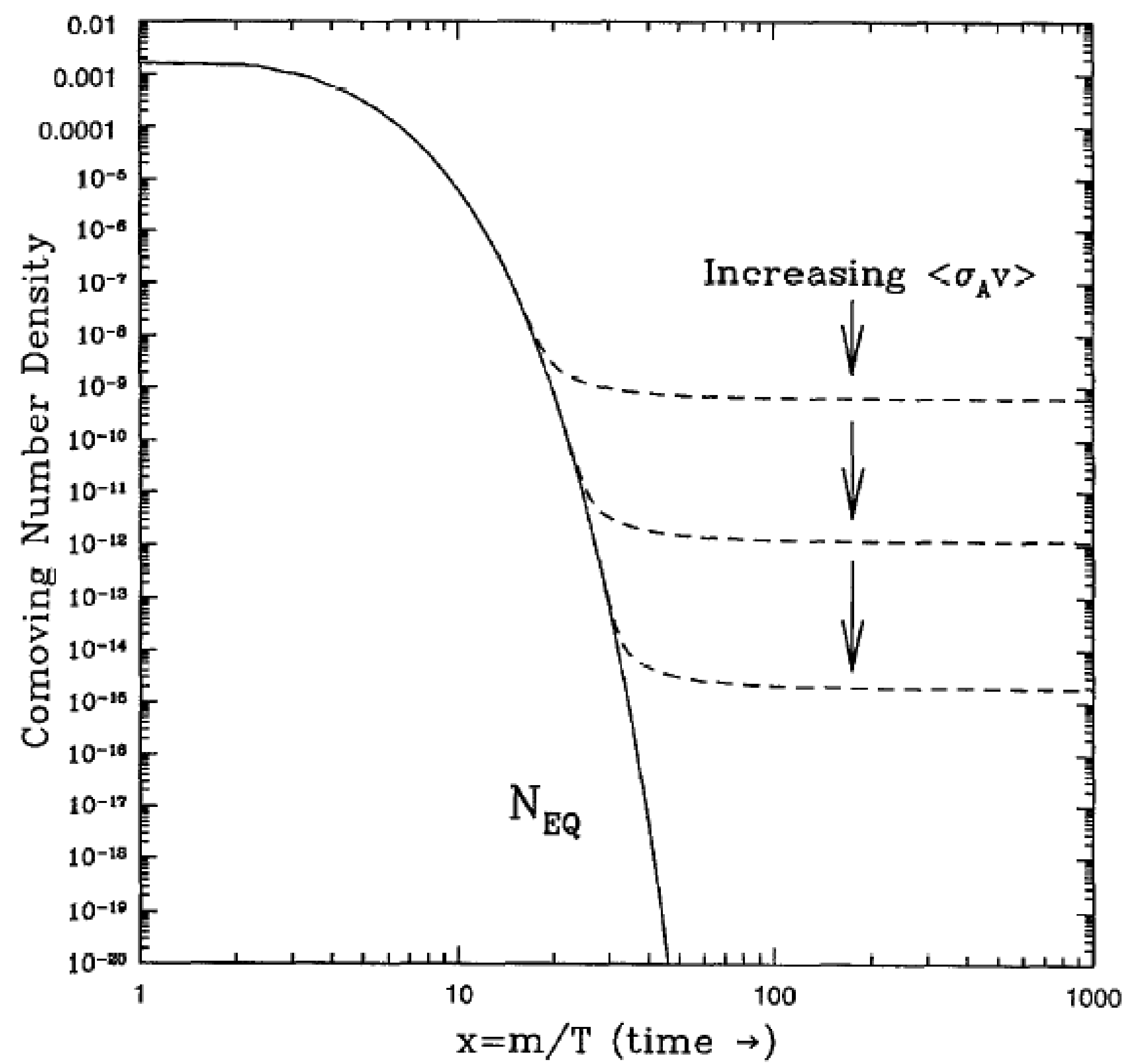


- MCMC parameter scan for 9-parameter MSSM
- Typical kinetic decoupling temperature: a few MeV
- Typical smallest halo mass: $10^{-12} - 10^{-4} M_\odot$

Indirect dark matter searches

Dark matter candidate: WIMP

- Weakly Interacting Massive Particle (**WIMP**)
- Current dark matter density: determined by competition between Hubble expansion and annihilation
  - Later, expansion becomes too fast for WIMPs to annihilate (thermal freeze-out)
- WIMP models can naturally explain the relic abundance
- E.g., neutralino predicted by supersymmetry

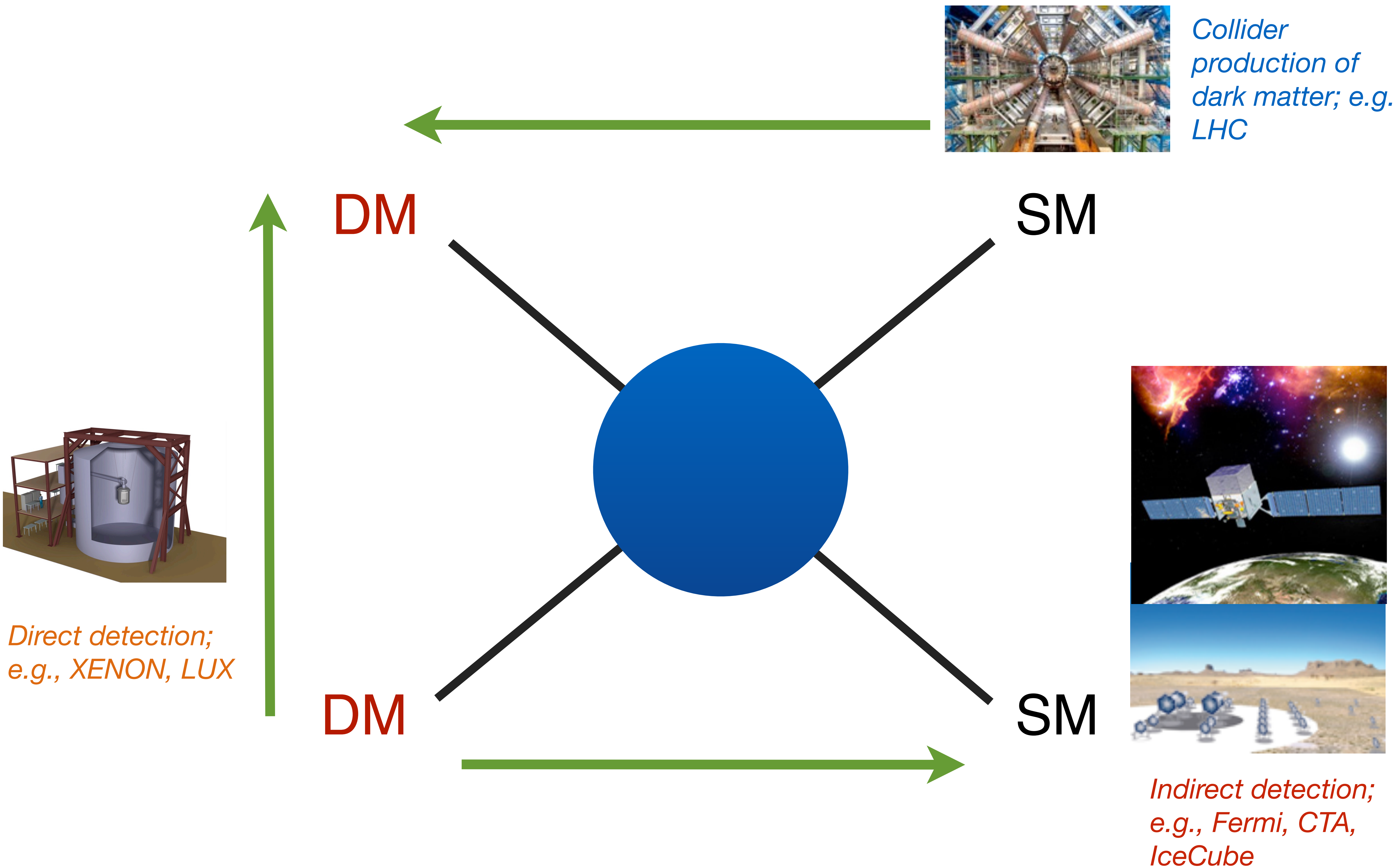


$$\Omega_\chi h^2 \simeq \frac{3\times 10^{-27}\ \mathrm{cm^3\ s^{-1}}}{\langle\sigma_{\rm ann} v\rangle}$$

$$\langle\sigma_{\rm ann} v\rangle \sim \alpha^2 (100\ \mathrm{GeV})^{-2}$$

$$\sim 3\times 10^{-26}\ \mathrm{cm^3\ s^{-1}}$$

Three routes to dark matter

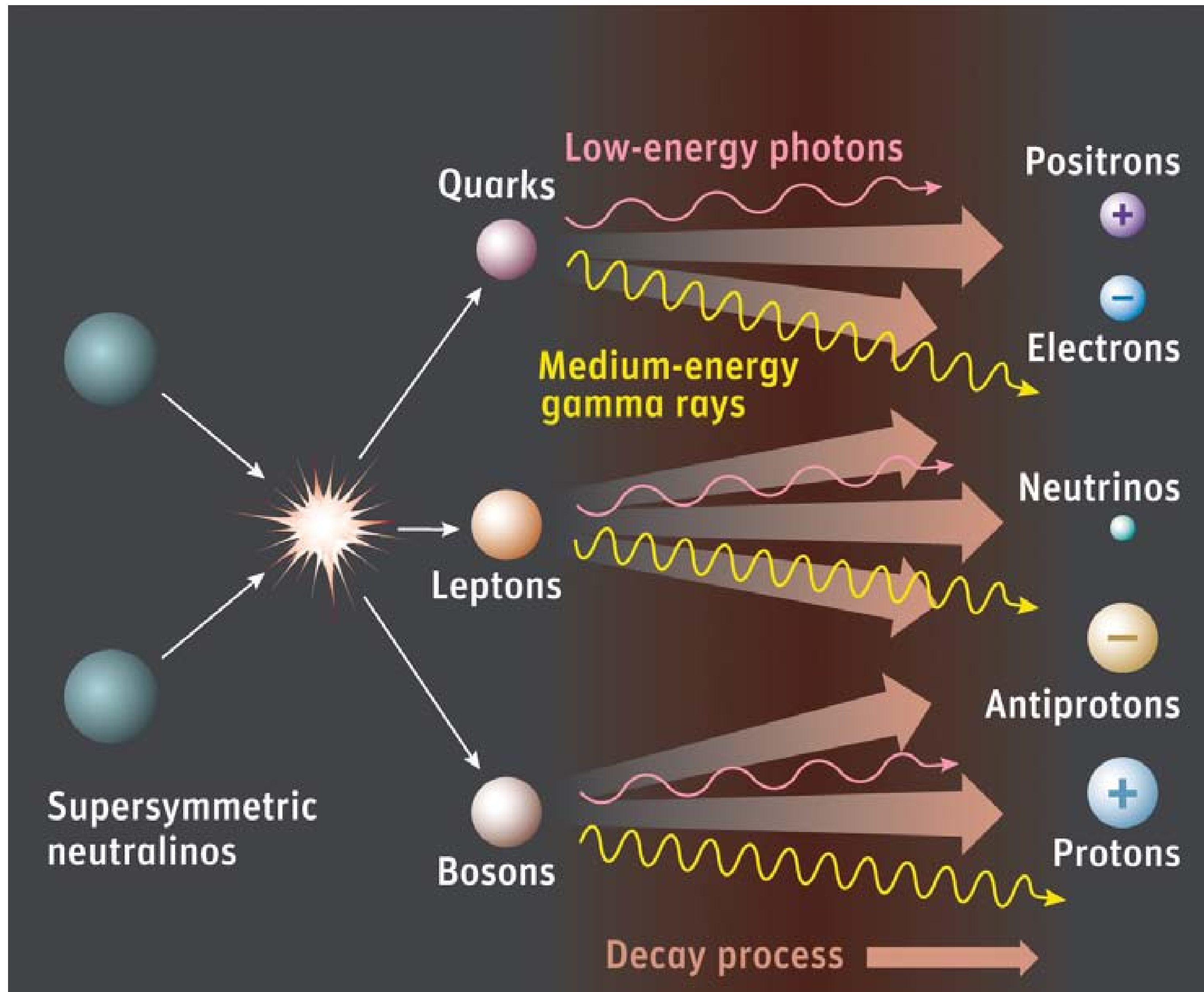


Dark matter annihilation

- WIMPs annihilate into standard model particles (photons, positrons, neutrinos, etc.)

- Each of these particles carry a fraction of WIMP mass energy (E ~ GeV–TeV)

- Annihilation rate is proportional to density squared and to annihilation cross section and relative velocity: $\sigma_{ann}v$



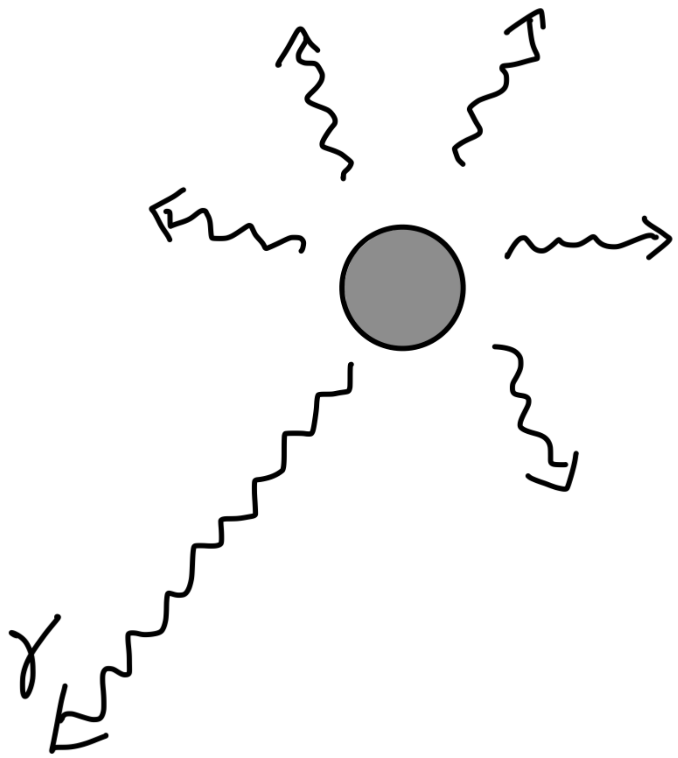
Rate of annihilation: Simple consideration

- Suppose you are a WIMP particle in a region of mass density ρ_χ
- Other WIMP particles are around you with velocity *v*, and if one of them hit you, you are both eliminated
- Incoming flux of the other WIMPs is $n_\chi v = \frac{\rho_\chi v}{m_\chi}$
- You encounter the others with the rate of $n_\chi \sigma v$
- If we look at this region of unit volume, such encounters happen at the rate of

$$\boxed{\frac{n_\chi^2 \sigma v}{2} = \frac{\rho_\chi^2 \sigma v}{2 m_\chi}}$$: rate of annihilation per volume

Gamma-ray flux from dark matter annihilation

Case of single halos



Annihilation rate per volume

$$\frac{\langle \sigma v \rangle \rho_\chi^2}{2m_\chi^2}$$

Differential gamma-ray luminosity

$$\mathcal{L}(E) = \frac{\langle \sigma v \rangle}{2m_\chi^2} \frac{dN_{\gamma,\mathrm{ann}}}{dE} \int dV \rho_\chi^2$$

Differential flux

$$\mathcal{F}(E, z) = \frac{\mathcal{L}((1+z)E)}{4\pi r^2}$$

***r*: comoving distance to the halo**

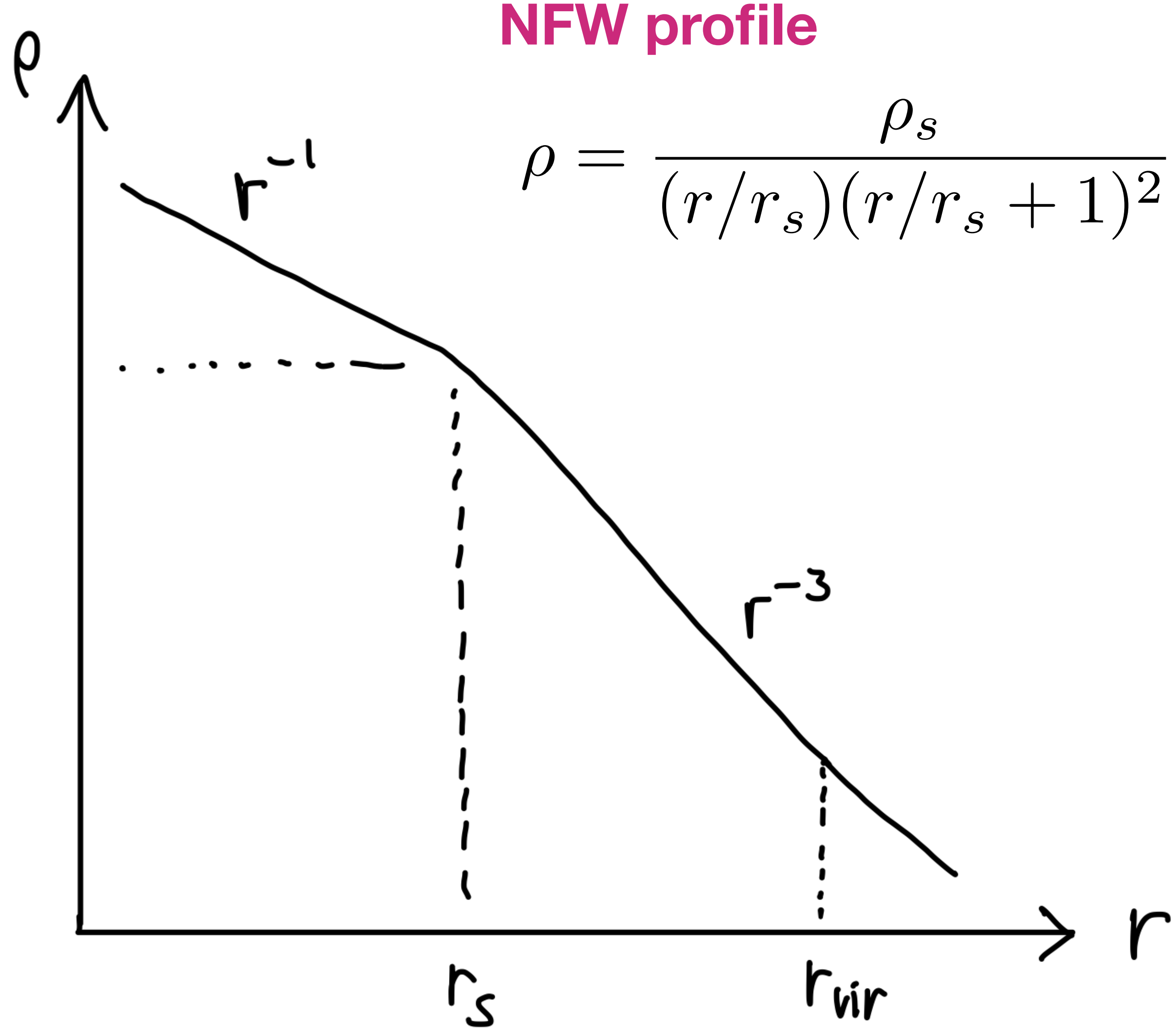
Gamma-ray flux from dark matter annihilation

Case of single halos

Halo mass *M* at redshift *z*

NFW profile

$$\rho = \frac{\rho_s}{(r/r_s)(r/r_s+1)^2}$$



Virial radius

$$r_{\rm vir} = \left(\frac{3M}{4\pi\Delta_{\rm vir}(z)\rho_c(z)}\right)^{1/3}$$

Scale radius

$$r_s = \frac{r_{\rm vir}}{c_{\rm vir}}$$

Characteristic density

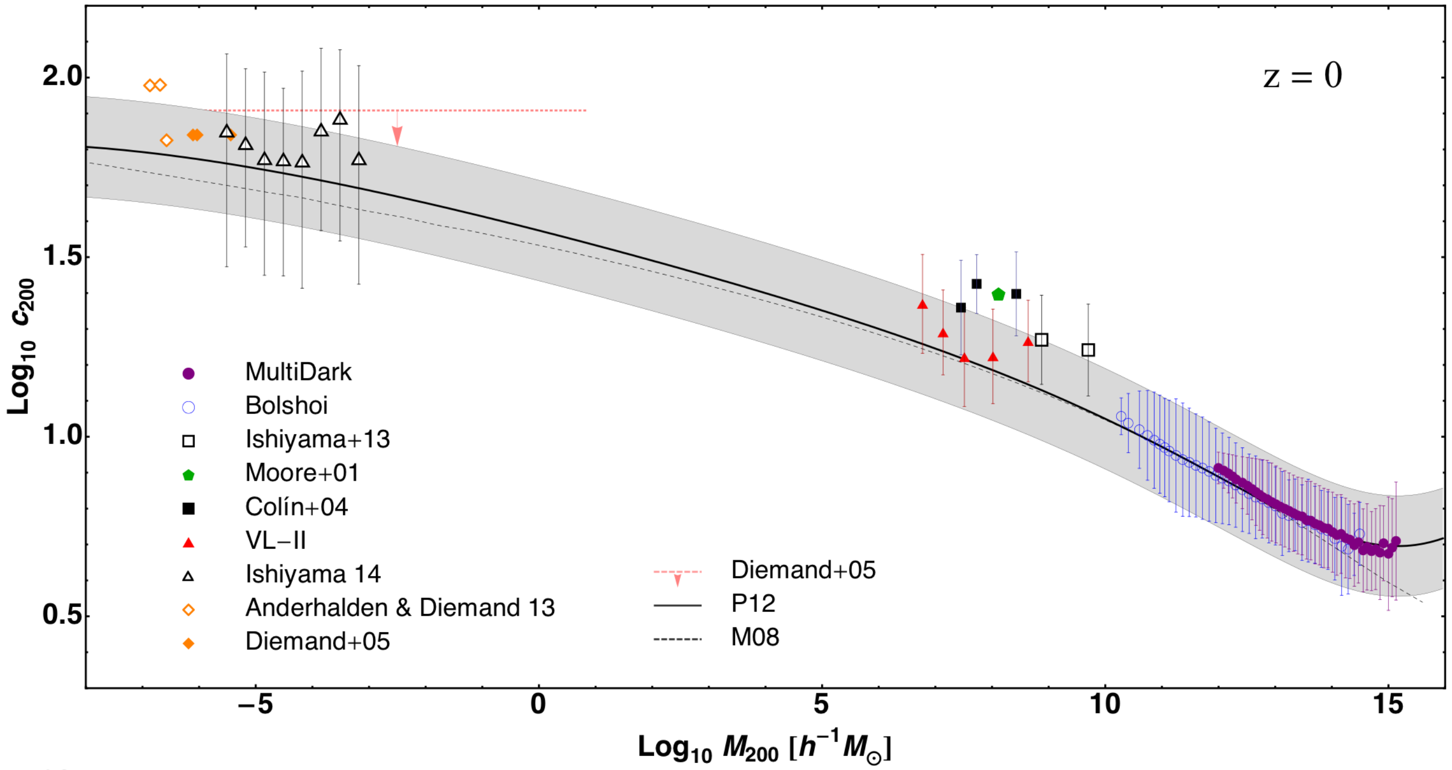
$$\rho_s = \frac{M}{4\pi r_s^3[\ln(1+c_{\rm vir}) - c_{\rm vir}/(1+c_{\rm vir})]}$$

All relevant parameters derived as a function of *M* and *z*

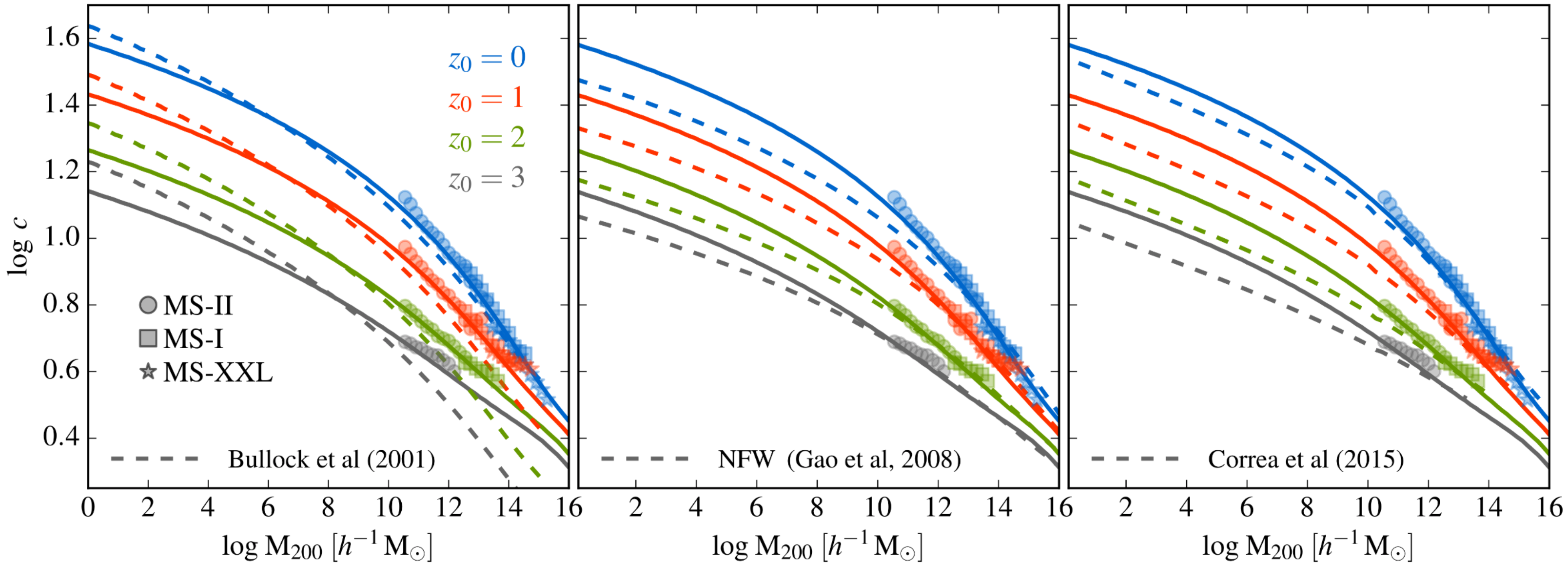
Gamma-ray flux from dark matter annihilation

Case of single halos

Sanchez-Conde, Prada, *Mon. Not. R. Astron. Soc.* **442**, 2271 (2014)



Ludlow et al., *Mon. Not. R. Astron. Soc.* **460**, 1214 (2016)



Halo concentration

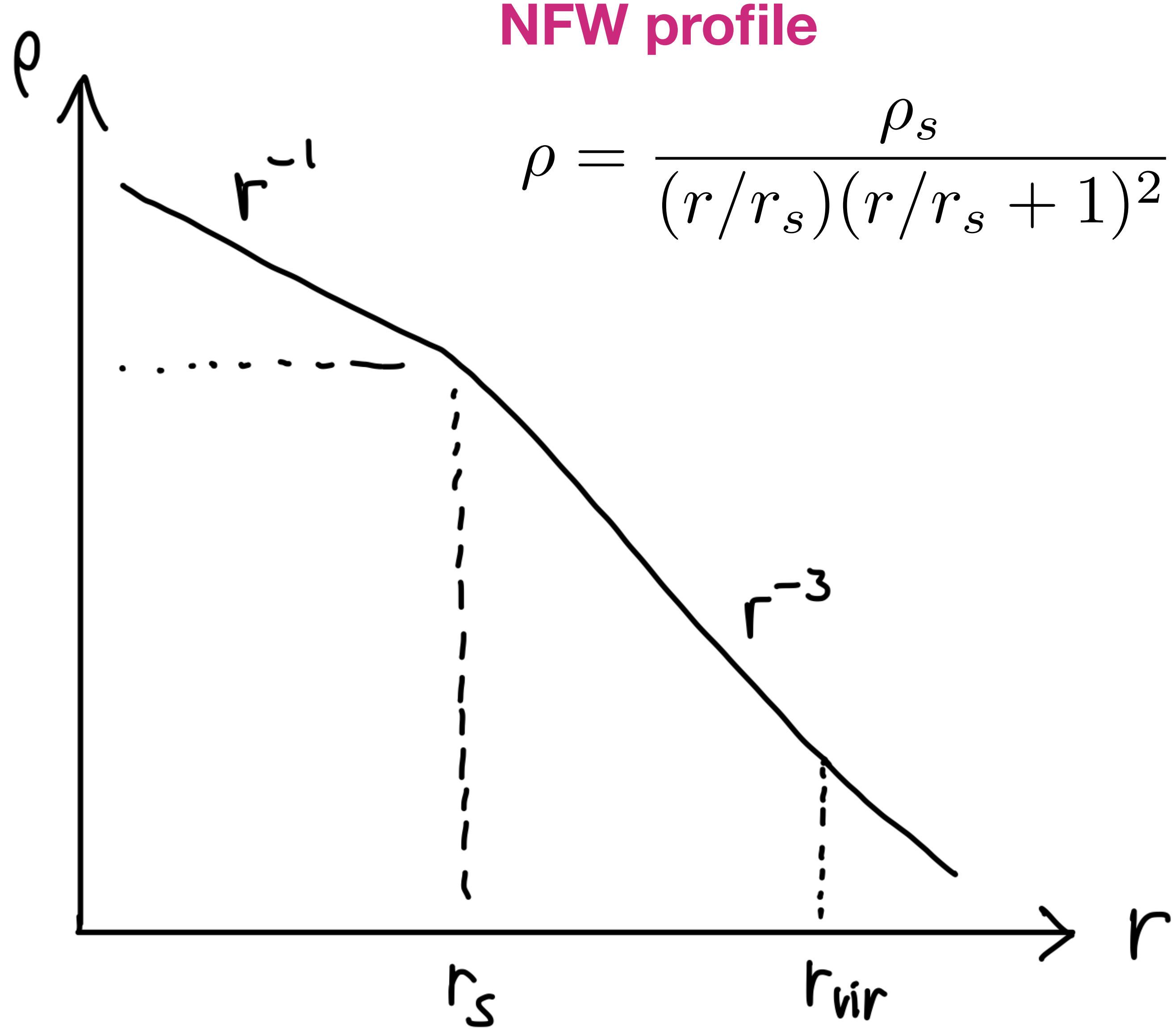
$$c_{\rm vir} = \frac{r_{\rm vir}}{r_s}$$

- Halo concentration-mass relation is well calibrated through simulations
  - From largest to smallest halos
  - From low to high redshifts ($0 < z < 5$)
- About 20-30% scatter from one halo to another

Gamma-ray flux from dark matter annihilation

Case of single halos

Halo mass *M* at redshift *z*



NFW profile

$$\rho = \frac{\rho_s}{(r/r_s)(r/r_s+1)^2}$$

Virial radius

$$r_{\rm vir} = \left(\frac{3M}{4\pi\Delta_{\rm vir}(z)\rho_c(z)}\right)^{1/3}$$

Scale radius

$$r_s = \frac{r_{\rm vir}}{c_{\rm vir}}$$

Characteristic density

$$\rho_s = \frac{M}{4\pi r_s^3[\ln(1+c_{\rm vir}) - c_{\rm vir}/(1+c_{\rm vir})]}$$

$$\boxed{\int dV\rho^2 = \frac{4\pi\rho_s^2 r_s^3}{3}\left[1-\frac{1}{(1+c_{\rm vir})^3}\right]}$$

Gamma-ray flux from dark matter annihilation

Case of single halos

- A quick exercise: ***How many dark matter annihilations are happening per second in the entire Milky Way?***

Milky Way halo: $M = 10^{12} M_\odot$

$$r_{\rm vir} = \left(\frac{3M}{4\pi\Delta_{\rm vir}\rho_c}\right)^{1/3} \sim 200 \text{ kpc}$$

$$r_s = \frac{r_{\rm vir}}{c_{\rm vir}} \sim 20 \text{ kpc} \qquad (c_{\rm vir} = 10)$$

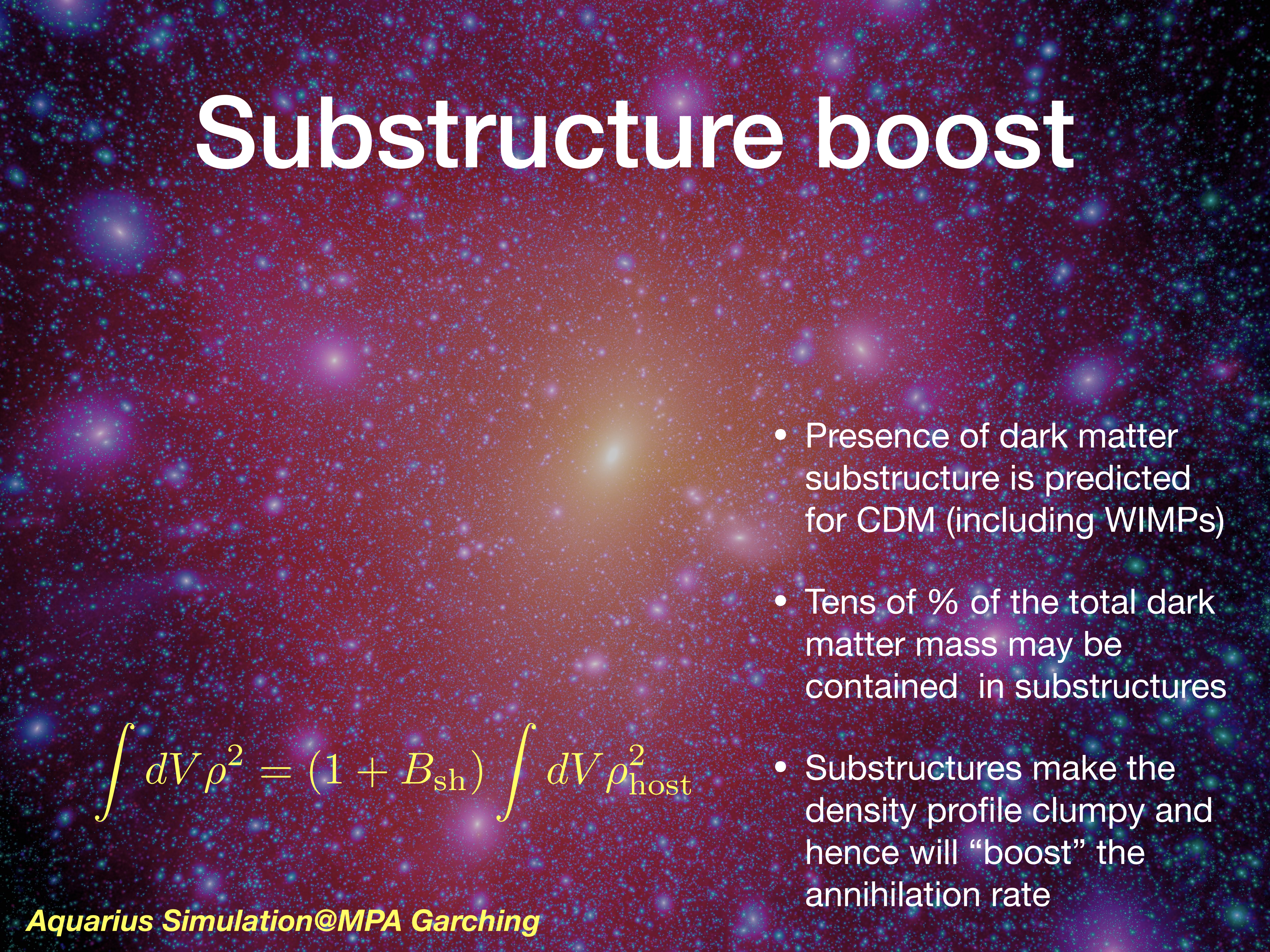
$$\rho_s = \frac{M}{4\pi r_s^3[\ln(1+c_{\rm vir}) - c_{\rm vir}/(1+c_{\rm vir})]}$$

$$\sim 0.3 \text{ GeV cm}^{-3}$$

Annihilation rate:

$$\boxed{\frac{\langle\sigma v\rangle}{2m_\chi^2}\int dV \rho_\chi^2 = 6\times 10^{37} \text{ s}^{-1}\left(\frac{\langle\sigma v\rangle}{3\times 10^{-26} \text{ cm}^3 \text{ s}^{-1}}\right)\left(\frac{m_\chi}{100 \text{ GeV}}\right)^{-2}}$$

Substructure boost



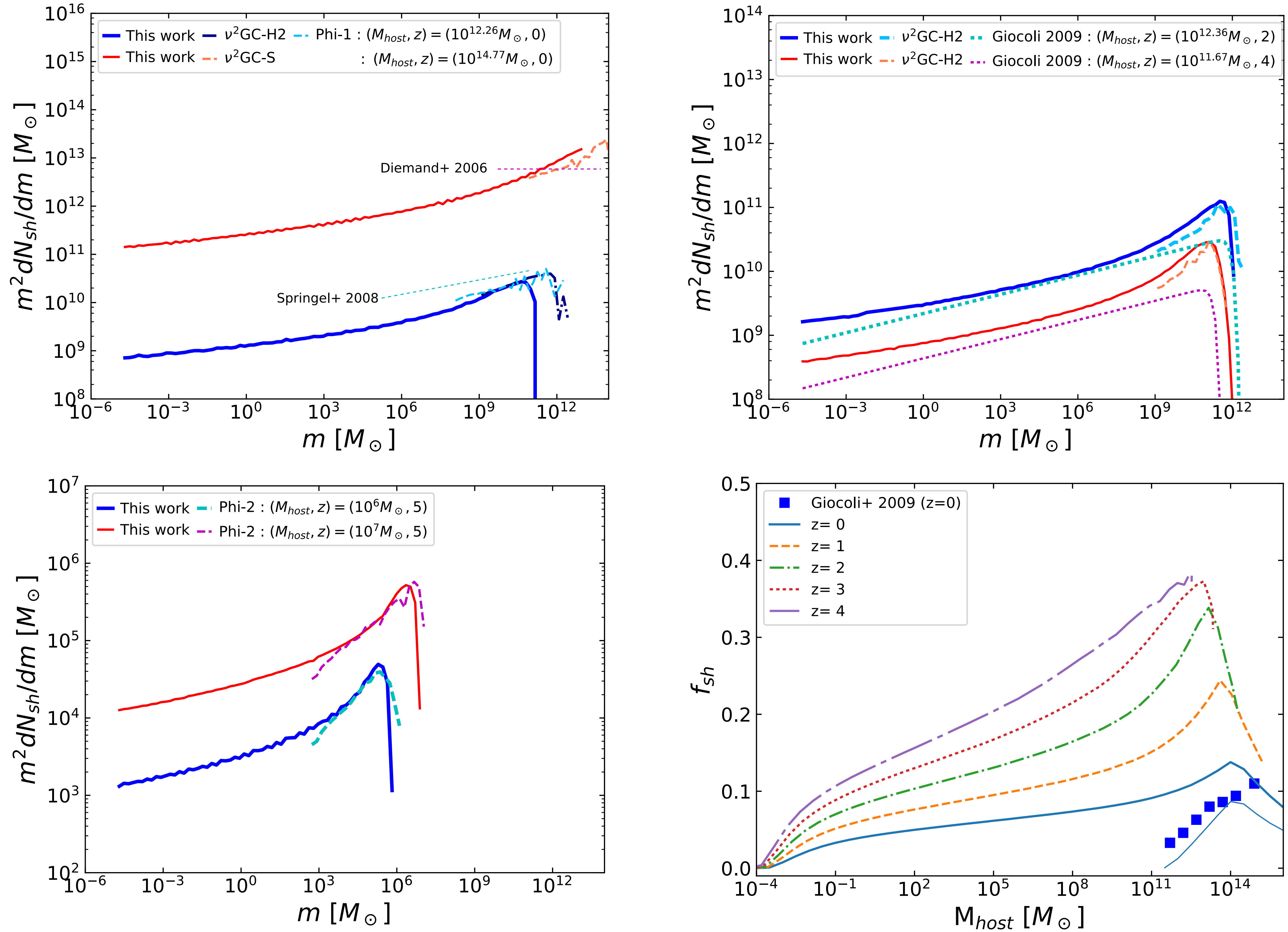
- Presence of dark matter substructure is predicted for CDM (including WIMPs)
- Tens of % of the total dark matter mass may be contained in substructures
- Substructures make the density profile clumpy and hence will “boost” the annihilation rate

$$\int dV\,\rho^2 = (1 + B_{\rm sh}) \int dV\,\rho^2_{\rm host}$$

Aquarius Simulation@MPA Garching

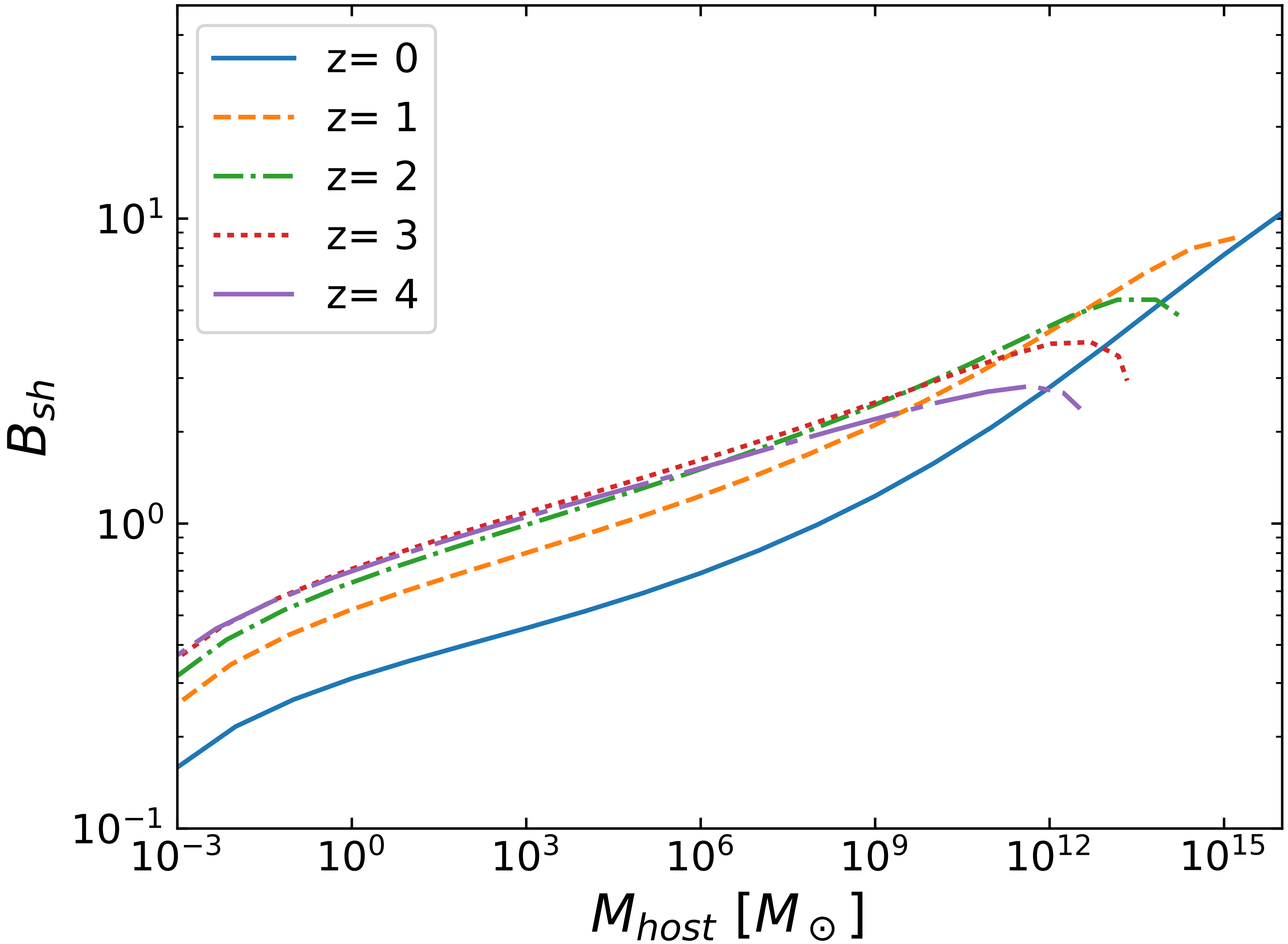
Subhalo mass function

Hiroshima, Ando, Ishiyama, *Phys. Rev. D* **97**, 123002 (2018)



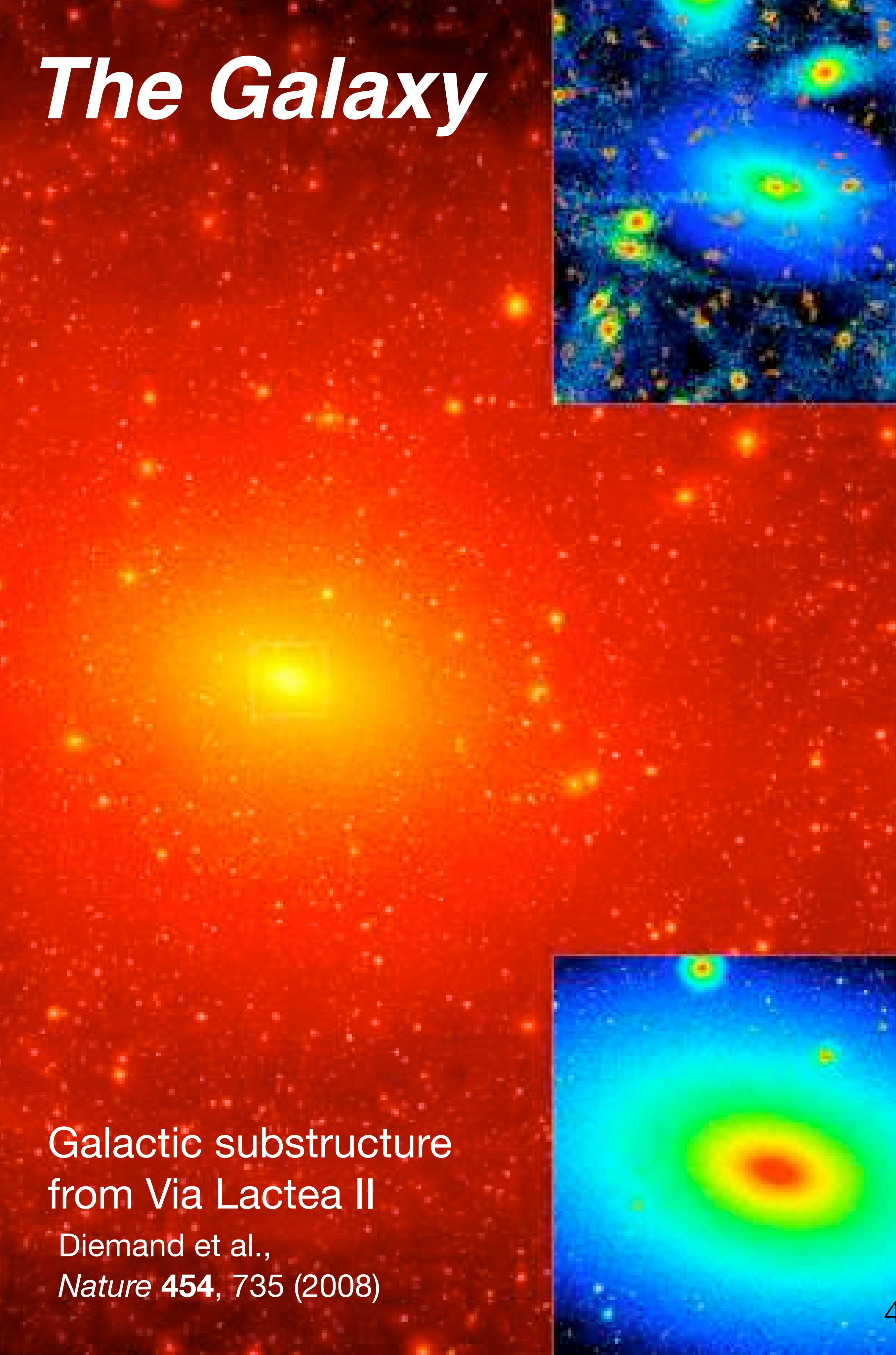
Substructure boost

Hiroshima, Ando, Ishiyama, *Phys. Rev. D* **97**, 123002 (2018)



- Typical substructure boost: a few to ~10 for Milky Way size halos
- Depends on properties of subhalos such as mass function, tidal stripping, etc.
- It might be the only probe of micro-halos that formed the earliest

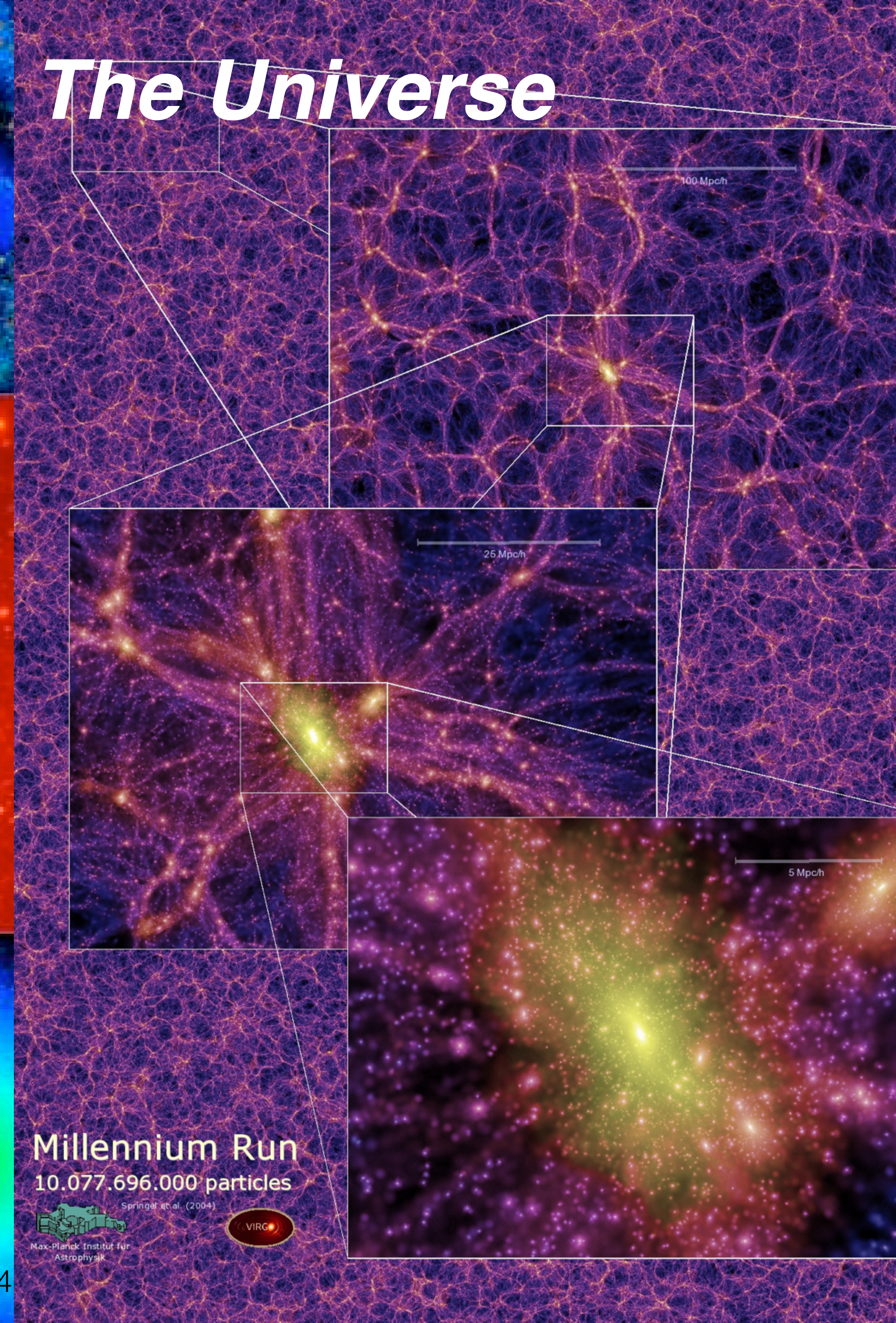
The Galaxy



Galactic substructure
from Via Lactea II

Diemand et al.,
Nature **454**, 735 (2008)

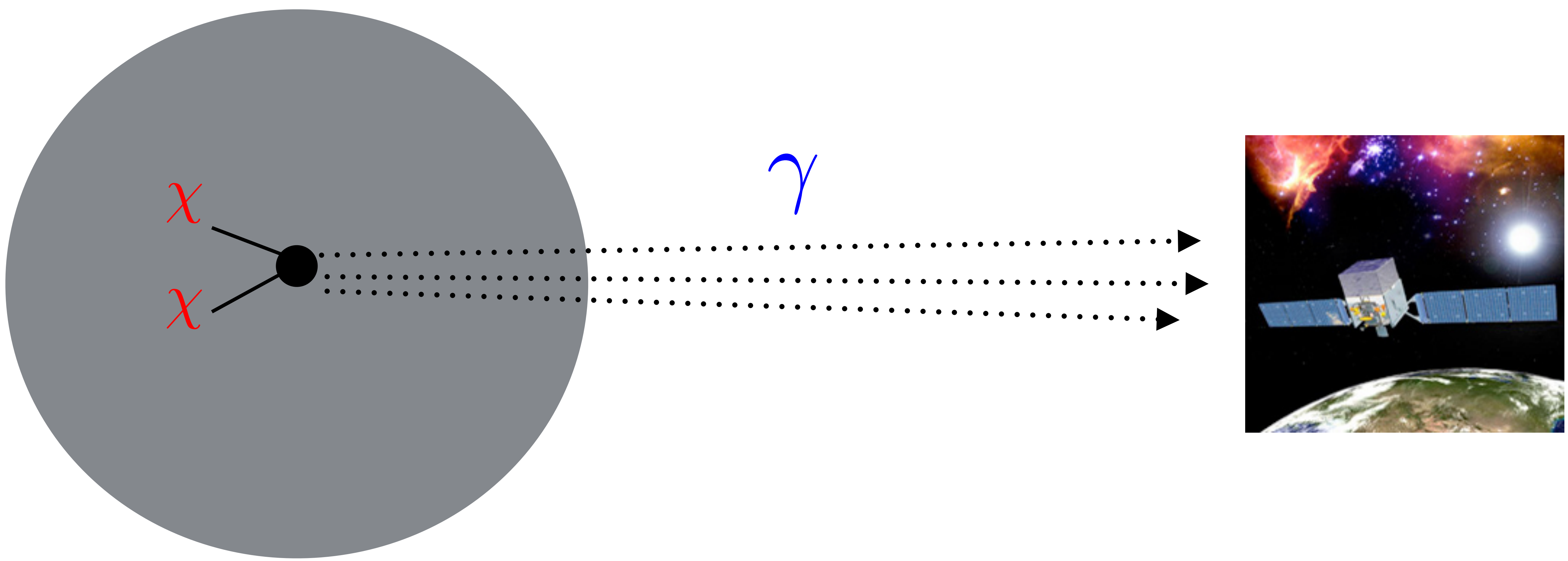
The Universe



Gamma-ray flux from dark matter annihilation

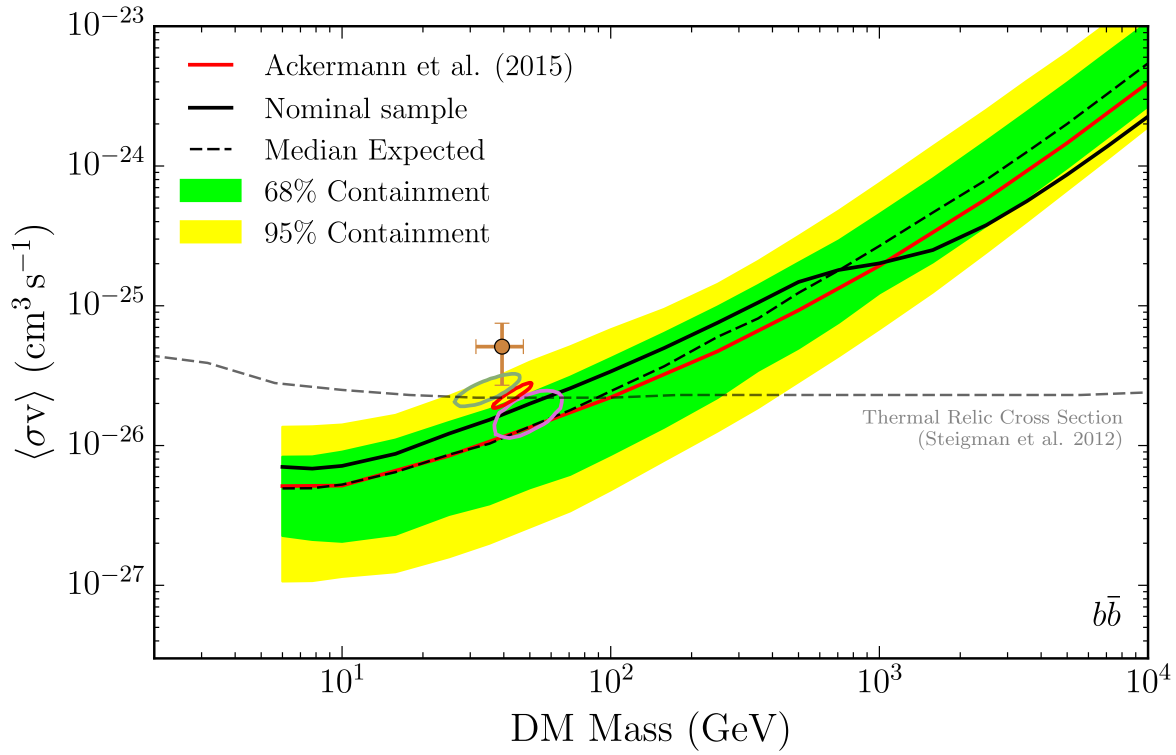
Particle physics **Astrophysics**

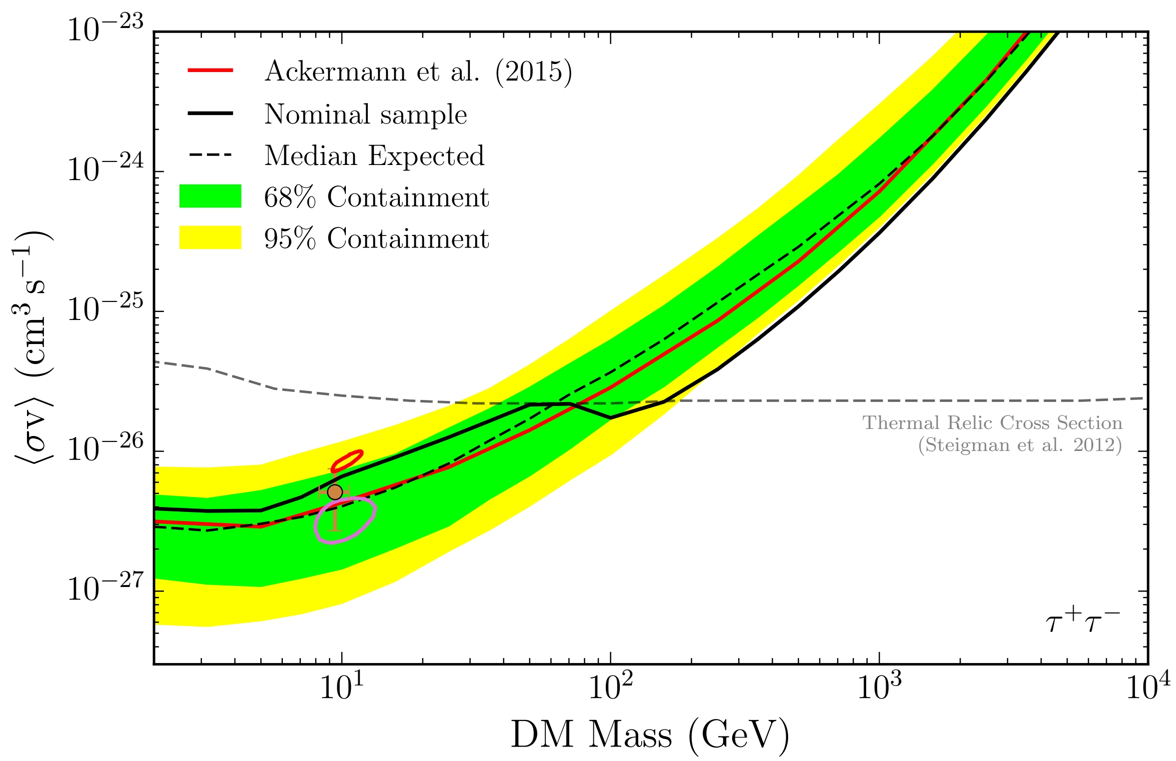
$$I_\gamma(E_\gamma, \psi) = \boxed{\frac{1}{2}\frac{\langle \sigma v \rangle}{m_\chi^2}\frac{dN_{\gamma,\mathrm{ann}}}{dE_\gamma}}\ \boxed{\frac{1}{4\pi}\int d\ell\ \rho_\chi^2(r[\ell, \psi])}$$

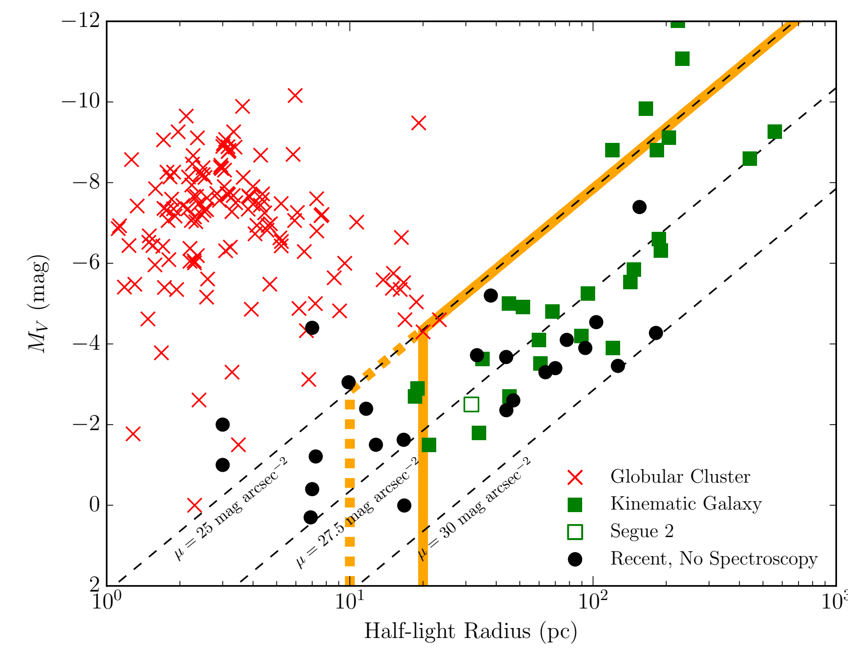


Constraints from dwarf spheroidal galaxies

Fermi-LAT, *Astrophys. J.* **834**, 110 (2017)



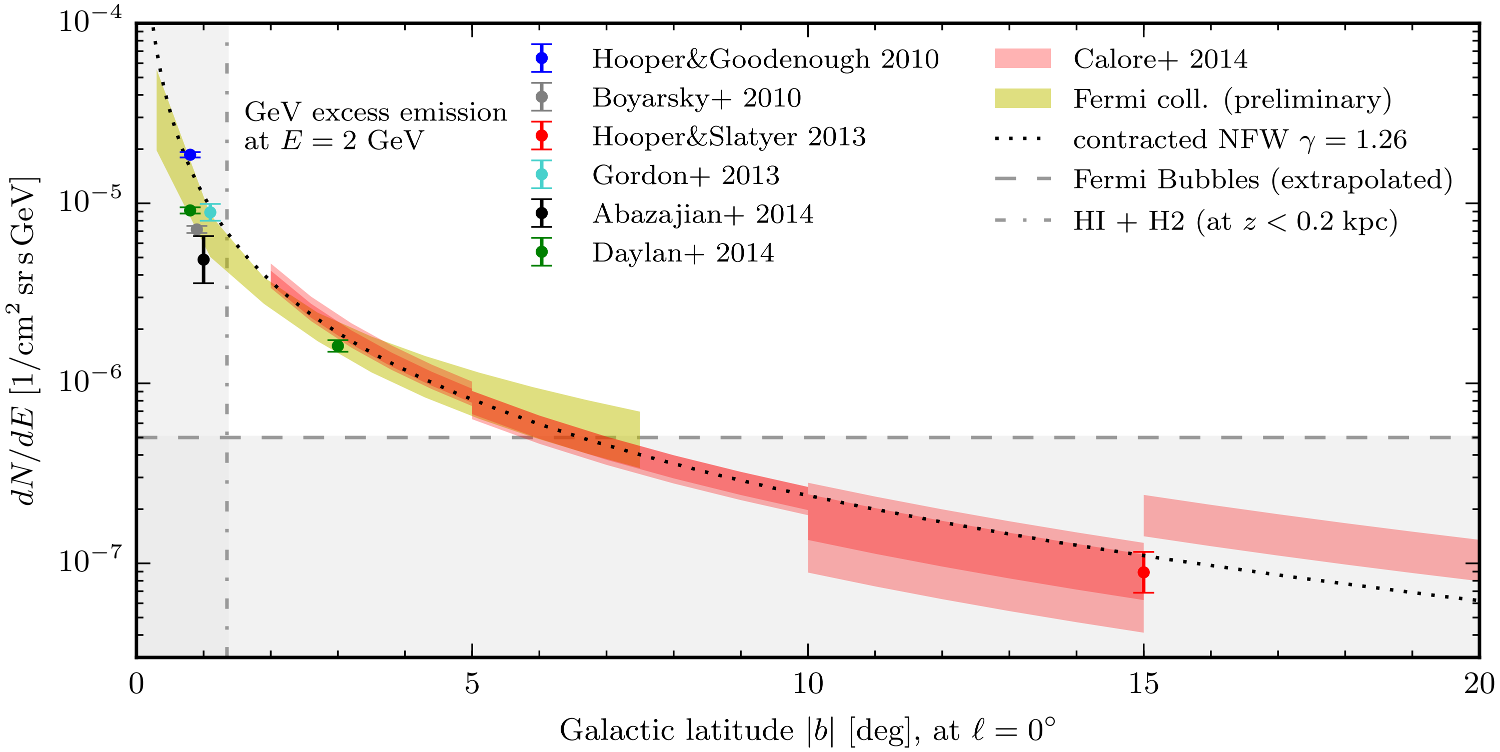


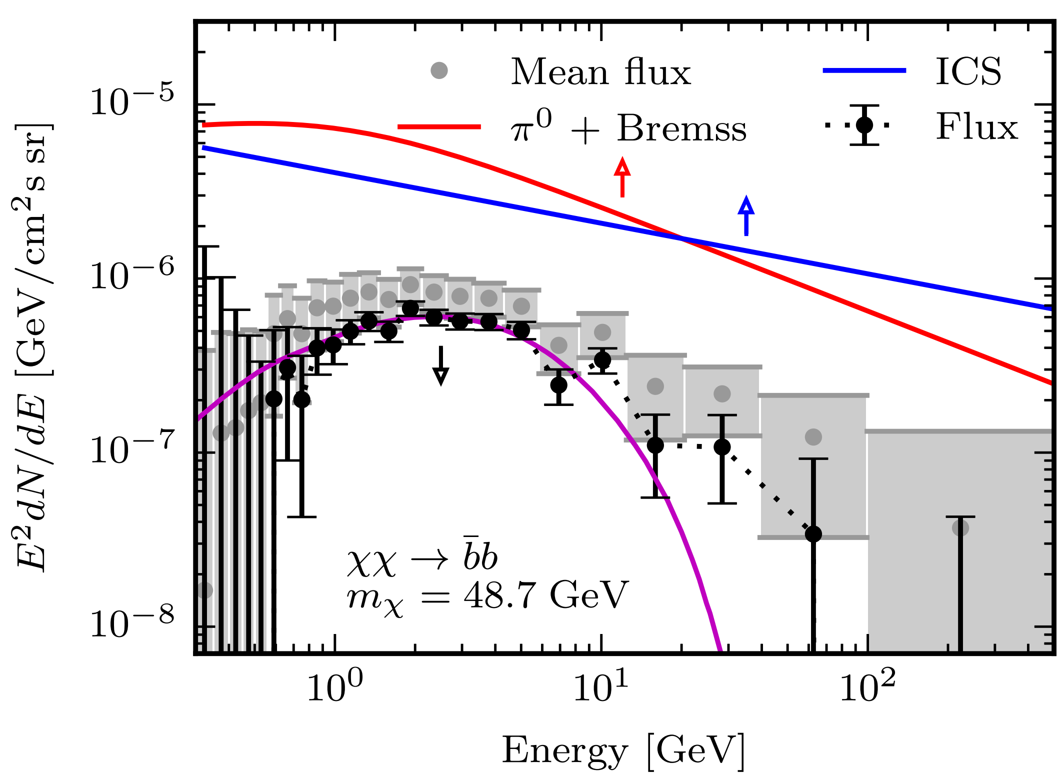


- Highly DM dominated system → suitable environment to test DM annihilation
  - Most robust constraints
- The latest results with PASS 8 data are pretty stringent
- They exclude the canonical cross section for WIMPs lighter than several tens of GeV
  - Nominal sample: 41 dwarfs
  - Ackermann et al. (2015): 15 dwarfs

GeV excess: Signals of dark matter annihilation?

Calore et al., *Phys. Rev. D* **91**, 063003 (2015)

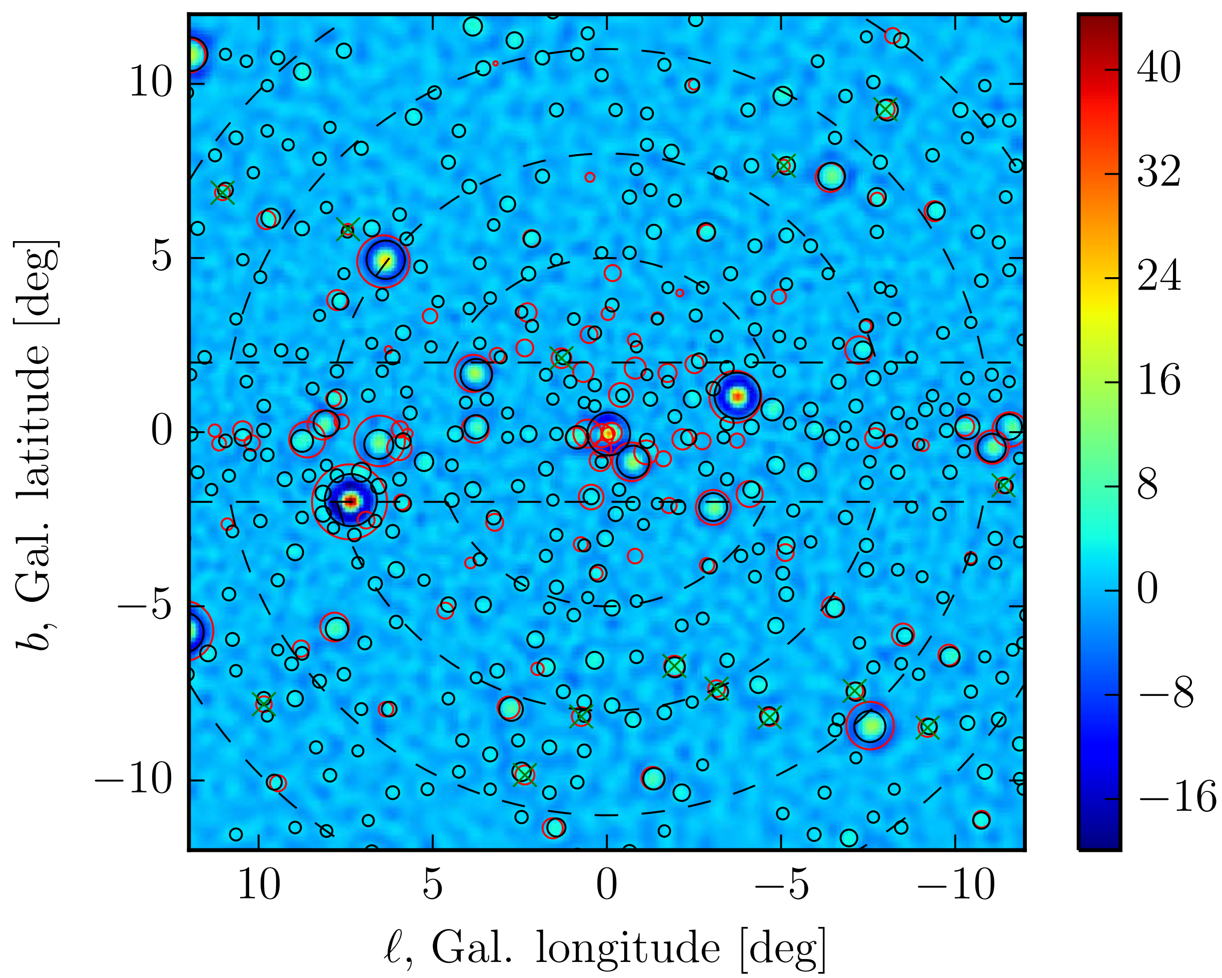




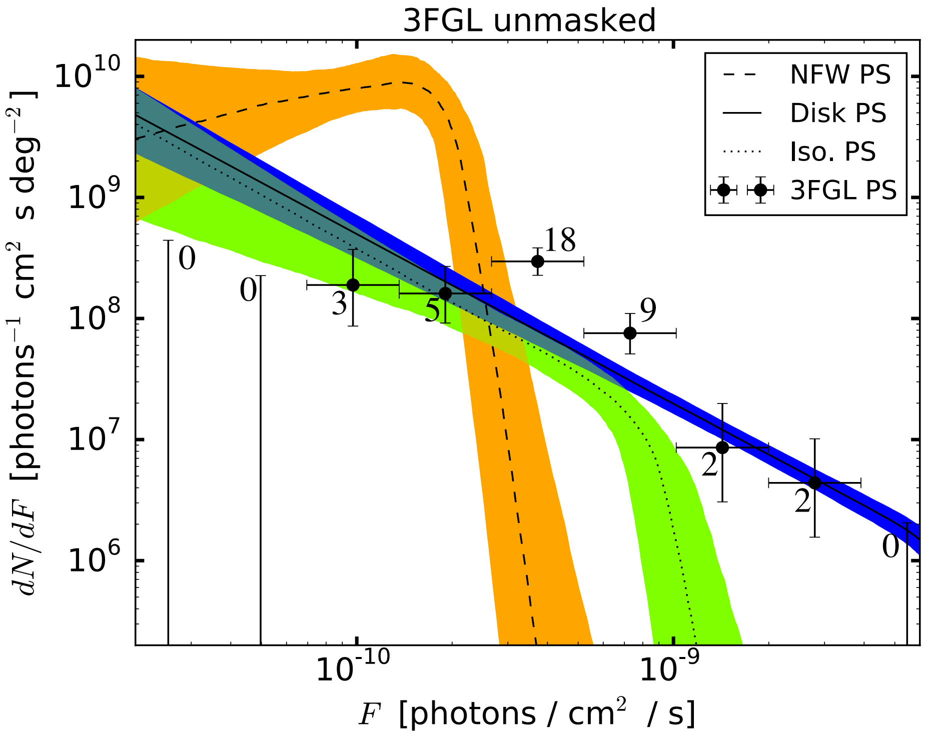
- Gamma-ray excess in GeV regime from the Galactic centre (many sigma) of unknown origin
- Brightness profile is consistent with NFW[2] (with inner slope of −1.26)
- Spectral shape is also consistent with expectation from annihilation
  - mass: ~50 GeV
  - cross section: ~2×10^{-26} cm^3 s^{-1}

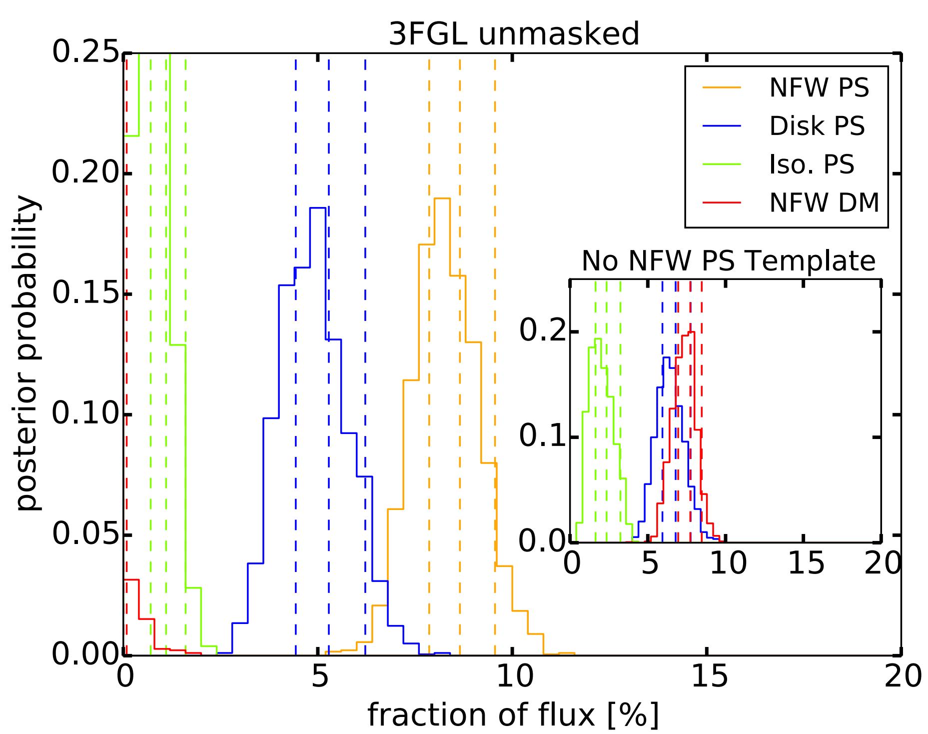
GeV excess: Evidence for astrophysical point sources?

Bartels et al., *Phys. Rev. Lett.* **116**, 051102 (2016)



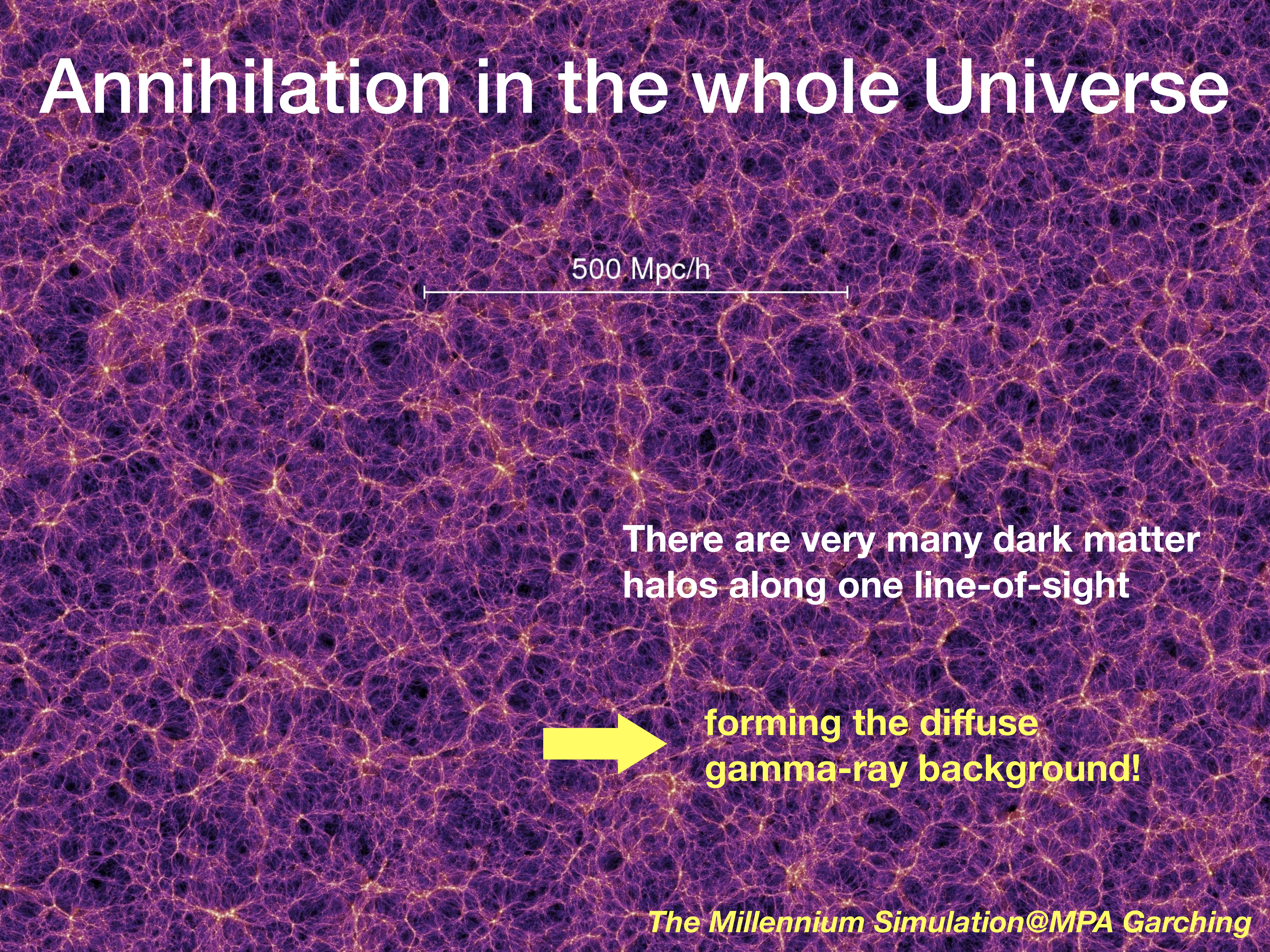
- Two analyses (wavelet/flux distribution) both point towards point-source origin of the GeV excess
- The excess might be caused by unresolved astrophysical sources





Lee et al., *Phys. Rev. Lett.* **116**, 051103 (2016)

Annihilation in the whole Universe

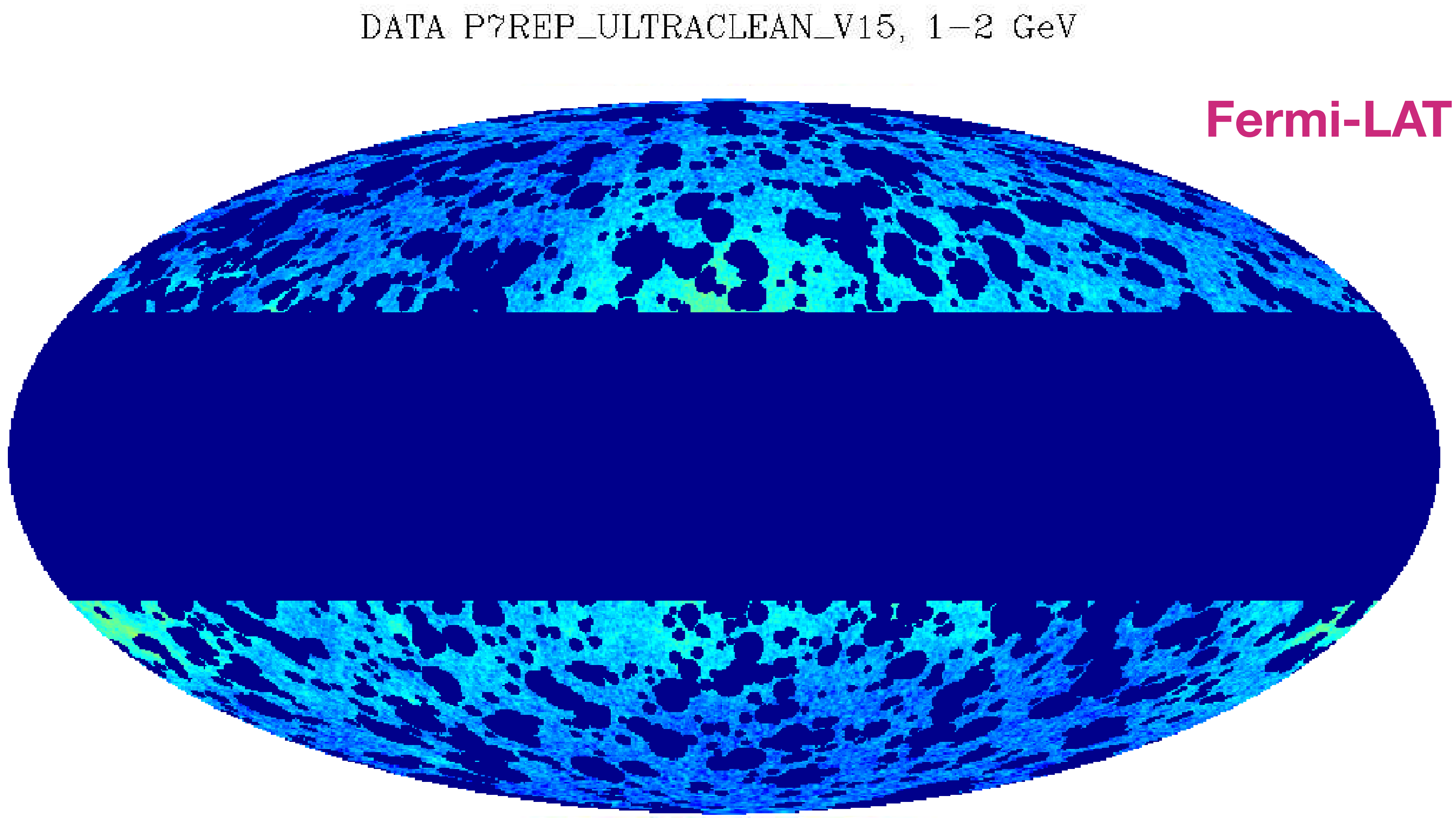


There are very many dark matter halos along one line-of-sight

forming the diffuse gamma-ray background!

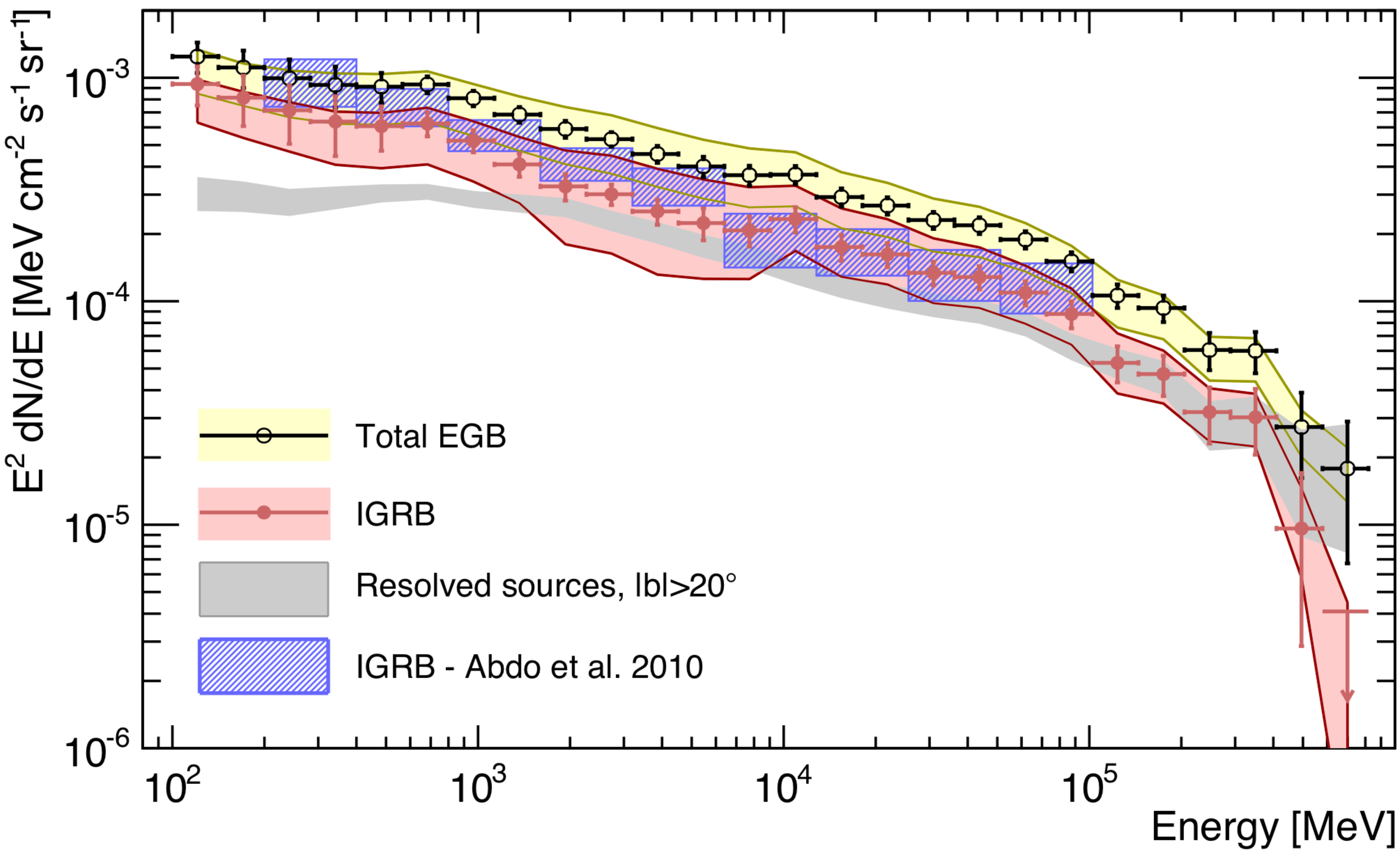
The Millennium Simulation@MPA Garching

Diffuse gamma-ray background



- No dark matter signals have been found around any sources (Galactic center, dwarf galaxies, etc.)
- Hints of dark matter might be hidden in this unresolved map

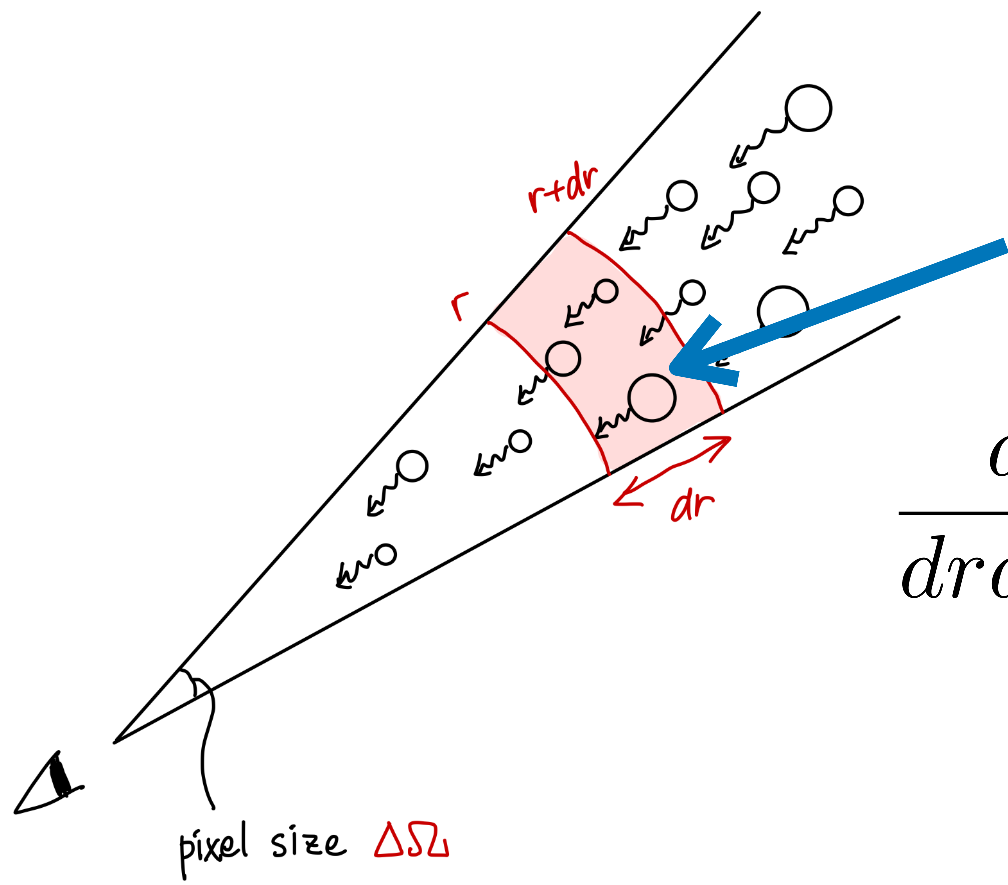
Energy spectrum of the gamma-ray background



Ackermann et al., *Astrophys. J.* **799**, 86 (2015)

Gamma-ray flux from dark matter annihilation

Contribution from all the halos



Number of halos between *M* and *M+dM* in this volume

$$\frac{d^3N}{drdMd\Omega}drdM\Delta\Omega = \Delta\Omega\ r^2dr\frac{dn(M,z)}{dM}dM$$

Intensity (flux per unit solid angle) from all the halos

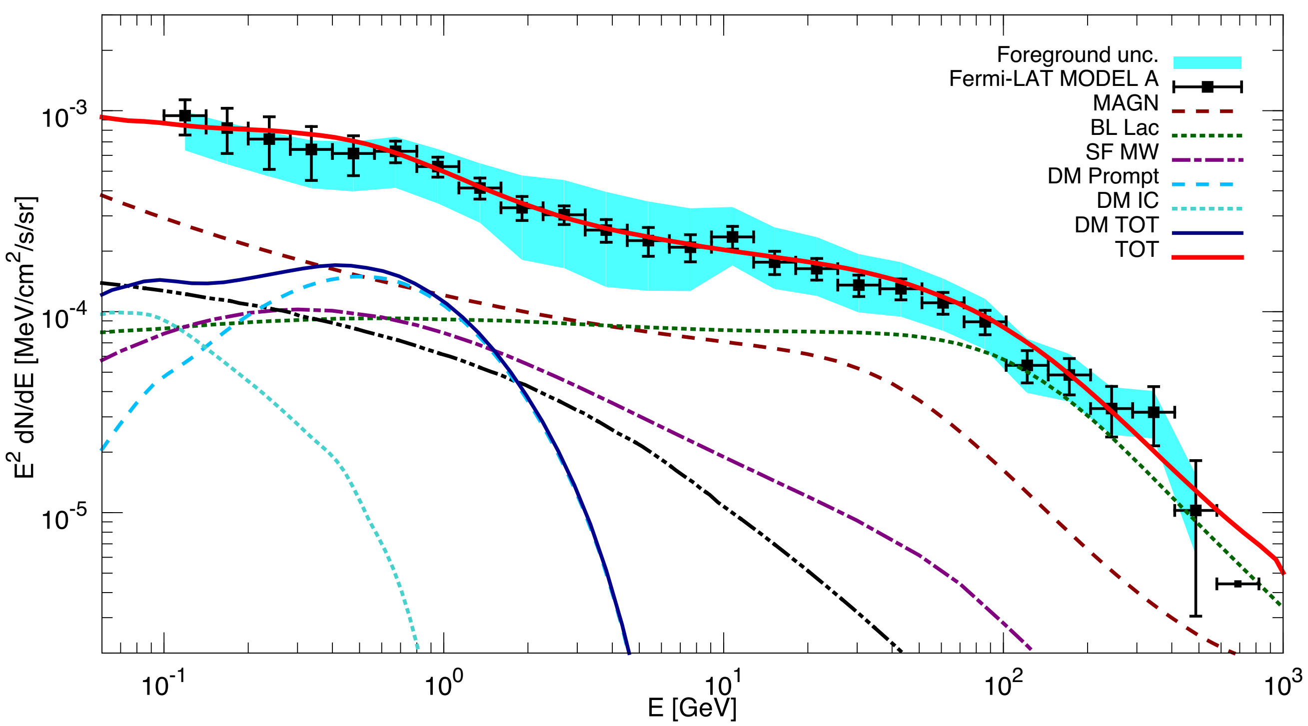
$$I(E) = \int dr \int dM \frac{d^3N}{drdMd\Omega}\frac{1}{4\pi r^2}\mathcal{L}((1+z)E|M,z)$$

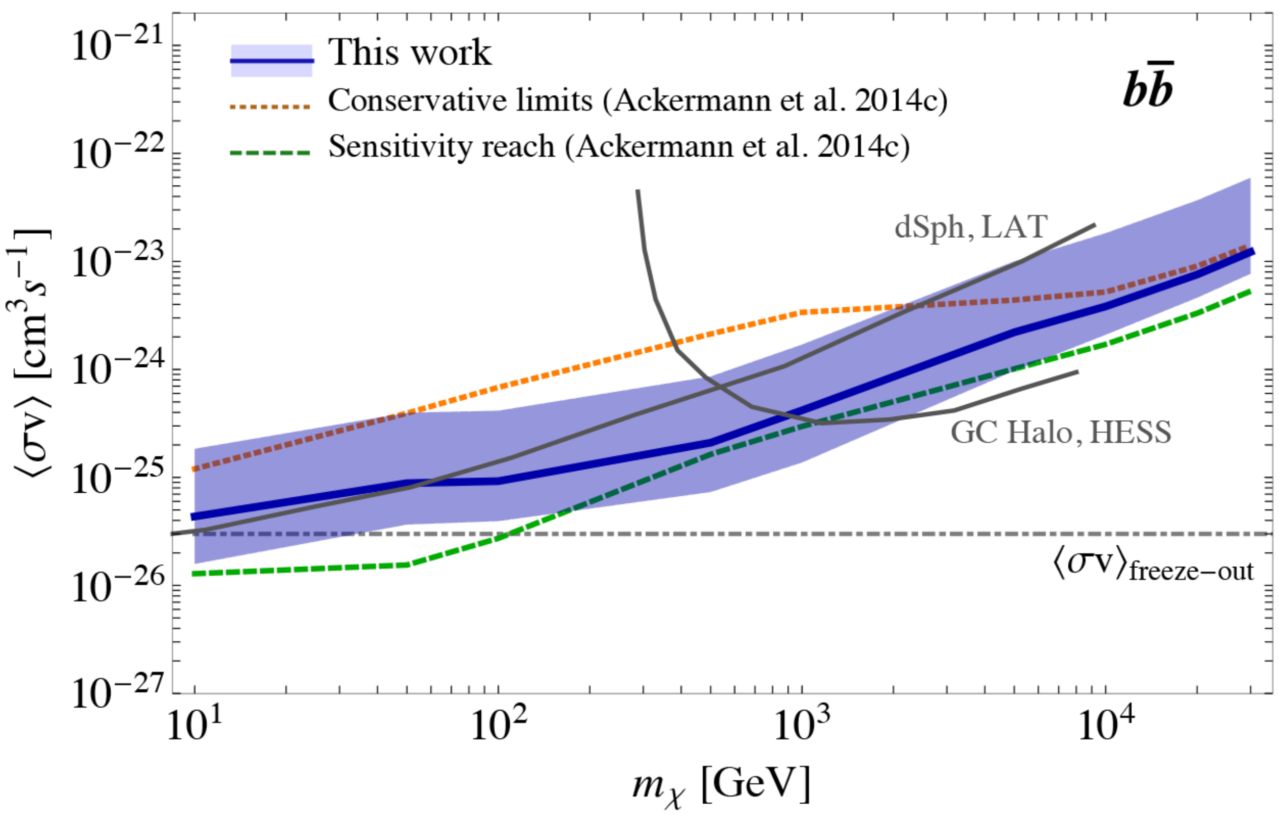
$$= \frac{1}{4\pi}\int dr \int dM \frac{dn(M,z)}{dM}\mathcal{L}((1+z)E|M,z)$$

Gamma-ray background from WIMP annihilation

Di Mauro, Donato, *Phys. Rev. D* **91**, 123001 (2015)

Ajello et al., *Astrophys. J.* **800**, L27 (2015)



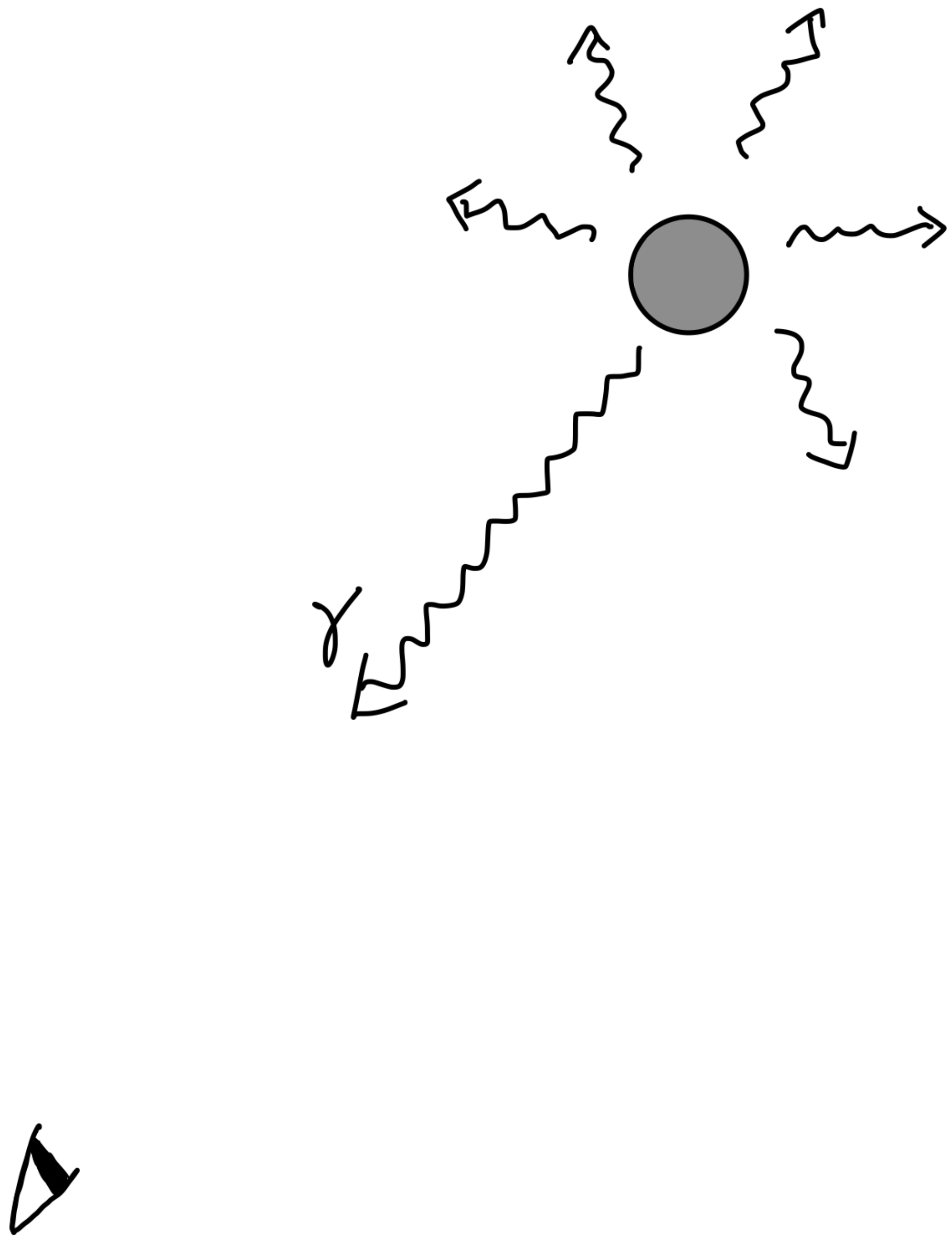


Most, if not all, data can be explained by ordinary astrophysical sources

Stringent constraint on annihilation rate

Gamma-ray flux from dark matter ~~annihilation~~ decay

Case of single halos



~~Annihilation~~ rate per volume

Decay

$$\cancel{\frac{\langle \sigma v \rangle \rho_\chi^2}{2 m_\chi^2}} \qquad \frac{\rho_\chi}{m_\chi \tau_\chi}$$

Differential gamma-ray luminosity

$$\mathcal{L}(E) = \cancel{\frac{\langle \sigma v \rangle}{2 m_\chi^2} \frac{dN_{\gamma,\mathrm{ann}}}{dE} \int dV \rho_\chi^2}$$

$$\frac{M}{m_\chi \tau_\chi} \frac{dN_{\gamma,\mathrm{decay}}}{dE}$$

Differential flux

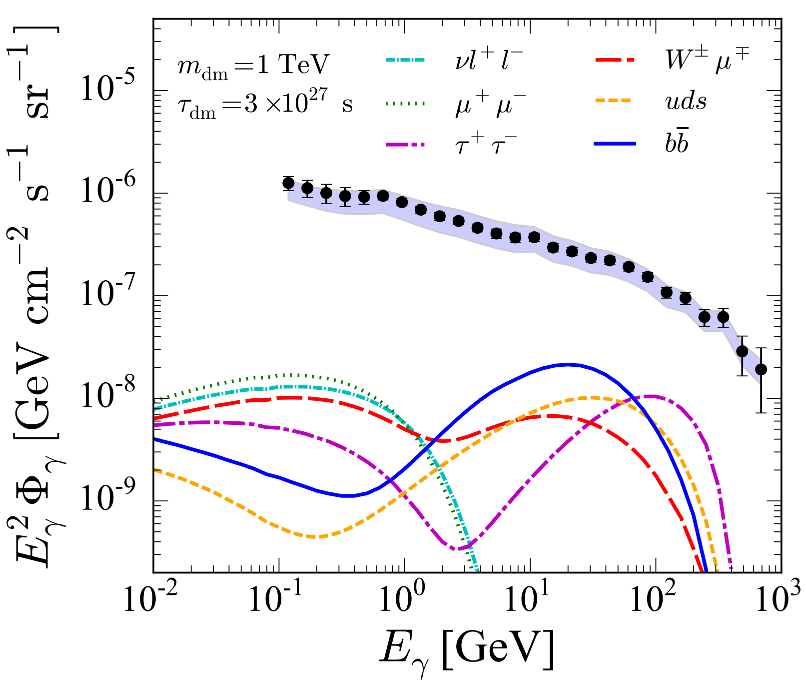
$$\mathcal{F}(E, z) = \frac{\mathcal{L}((1+z)E)}{4\pi r^2}$$

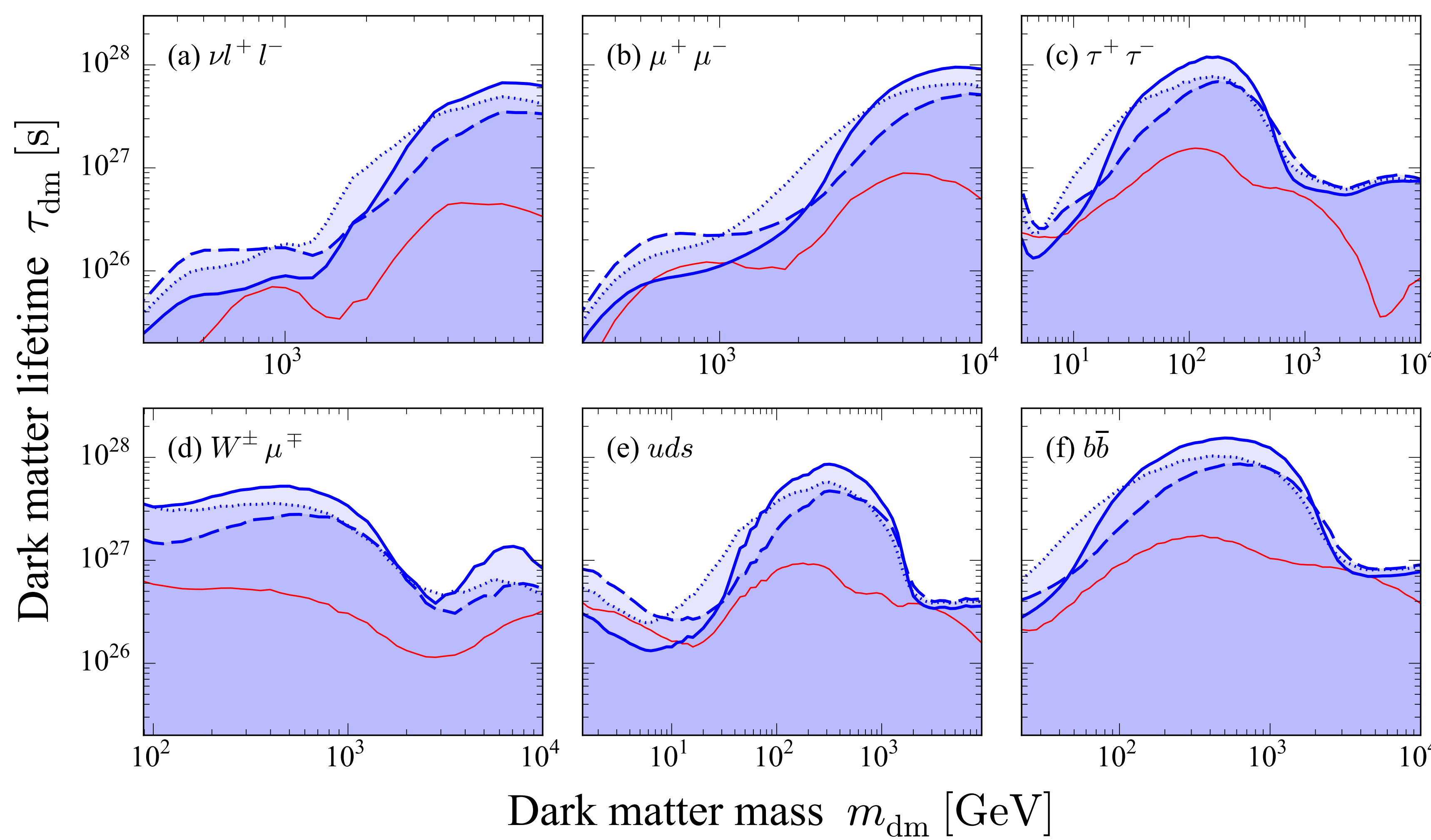
Gamma-ray flux from dark matter decay

Contribution from all the halos

$$I(E) = \frac{\Omega_\chi \rho_c}{4\pi m_\chi \tau_\chi} \int dr \, \frac{dN_{\gamma,\mathrm{decay}}}{dE'}\bigg|_{E'=(1+z)E}$$

Ando, Ishiwata, *JCAP* **1506**, 024 (2015)





- Robust results independent of subhalo abundance

Summary: WIMP annihilation

- **No WIMP has been found** in any search strategies (colliders, direct detection, indirect detection)
- Indirect searches towards dwarf galaxies reject *canonical* annihilation cross section (for thermal freeze-out scenario) for masses below tens of GeV
- Galactic centre excess, although interesting, might be simply explained by astrophysics
- Statistical analysis of the gamma-ray background started to be explored, especially in recent years

Practical notes

- Please submit your first coding assignments to “notebooks/” directory on GitHub repository
- Please start looking for and reading publications for your final presentation!