1 Wavy heliospheric current sheet

Both components of the drift velocity are modified by the waviness:

- There is a Heaviside step function in the Parker field:

$$\vec{B} = A_c B_0 \left(\frac{r_0}{r}\right)^2 \left(\hat{r} - \frac{\Omega r \sin\theta}{V_{sw}}\hat{\phi}\right) H(\theta' - \theta),$$

  where θ' is the angular extent of the HCS. In Strauss et al 2012 this is given by eq 11:

$$\theta' = \frac{\pi}{2} + \sin^{-1}\left[\sin\alpha \sin\left(\phi - \phi_0(t) + \frac{\Omega r}{V_{sw}}\right)\right],$$

  where $\phi_0(t) = \phi_0 + \Omega t$ and ϕ_0 is an arbitrary phase. In the code, the step function appears when calculating the gradient/curvature part of the drift velocity.

  TODO:

  - How is ϕ_0 determined?
  - α should also be time dependent. How should I implement this? Parameter file should probably point to table of α vs t, which the parameter object can then parse. The date at which the particle was observed at Earth can be specified in the run file.

- The HCS drift velocity is also affected. It takes the form (eq 17)

$$\vec{v}_{hcs} = v_{hcs}\left[\cos A_\xi \xi \sin\Psi \hat{r} + \sin A_\xi \xi \hat{\theta} + \cos A_\xi \xi \cos\Psi \hat{\phi}\right] A_c \operatorname{sgn} q.$$

  The angle ξ (β in the reference) between the tangent to the HCS and the radial line passing through the point on the HCS is given by

$$\tan^2\xi = \left[-r\frac{\partial\theta'}{\partial r}\right]^2 + \left[\frac{1}{\sin\theta}\frac{\partial\theta'}{\partial\phi}\right]^2$$

$$\implies \tan\xi = \frac{\Omega r}{V_{sw}}\frac{1}{\sin\Psi}\frac{\sqrt{\sin^2\alpha - \cos^2\theta'}}{\sin\theta'} \text{ (for Parker field).}$$

  The sign of ξ, $A_\xi = \pm 1$, is equal to the sign of $\partial\theta'/\partial r$:

$$A_\xi = \operatorname{sgn}\cos\left(\phi - \phi_0(t) + \frac{\Omega r}{V_{sw}}\right).$$

  v_{hcs} is given by the usual approximation:

$$v_{hcs} = \left[0.457 - 0.412\frac{L}{r_L} + 0.915\frac{L^2}{r_L^2}\right] v,$$

  where L is the smallest distance from the particle's position to the HCS. If $L > 2r_L$, v_{hcs} is zero. Finally, ξ is evaluated at the HCS point which minimizes L.

The hard part of these modifications is calculating L, which cannot be done analytically and is expensive. If

- $|\theta - \frac{\pi}{2}| \leq \alpha$,

- $\theta < \frac{\pi}{2} - \alpha$ and $L_+^{th} \leq 2r_L$, where $L_+^{th} = r\cos(\alpha+\theta)$ is the distance from the particle to the surface bounding the HCS above,
- Or $\theta > \frac{\pi}{2} + \alpha$ and $L_-^{th} = -r\cos(\theta-\alpha) \leq 2r_L$,

we compute L using the Nelder-Mead simplex algorithm. Here are the steps for minimizing $L(r_s, \phi_s)$, where r_s and ϕ_s are the coordinates of a point in the sheet:

0. Choose an initial set of points $\vec{x}_1$, $\vec{x}_2$, $\vec{x}_3$.
1. Order vertices: $L(\vec{x}_1) \leq L(\vec{x}_2) \leq L(\vec{x}_3)$.
2. Compute $\vec{x}_O = \frac{1}{2}(\vec{x}_1 + \vec{x}_2)$.
3. Compute reflected point $\vec{x}_R = \vec{x}_O + \alpha(\vec{x}_O - \vec{x}_3)$.

   If $L(\vec{x}_1) \leq L(\vec{x}_R) < L(\vec{x}_2)$, replace $\vec{x}_3$ with $\vec{x}_R$ and go to step 1.
4. If $L(\vec{x}_R) < L(\vec{x}_1)$, compute expanded point $\vec{x}_E = \vec{x}_O + \gamma(\vec{x}_R - \vec{x}_O)$.

   If $L(\vec{x}_E) < L(\vec{x}_R)$, replace $\vec{x}_3$ with $\vec{x}_E$ and go to step 1.

   Else, replace $\vec{x}_3$ with $\vec{x}_R$ and go to step 1.

   Else, go to step 5.
5. Compute contracted point $\vec{x}_C = \vec{x}_O + \rho(\vec{x}_3 - \vec{x}_O)$.

   If $L(\vec{x}_C) < L(\vec{x}_3)$, replace $\vec{x}_3$ with $\vec{x}_C$ and go to step 1.

   Else, go to step 6.
6. For all but the $\vec{x}_1$, replace $\vec{x}_i$ with $\vec{x}_1 + \sigma(\vec{x}_i - \vec{x}_1)$. Go to step 1.

Standard values for the constants are $\alpha = 1$, $\gamma = 2$, $\rho = 1/2$, $\sigma = 1/2$.

The only tough part of this is choosing the initial set of points. Since θ' is "periodic" in $r \to r + 4\frac{\Omega}{V_{sw}}$,

$$\left\{\left(r - 2\frac{\Omega}{V_{sw}}, \phi\right), \left(r + 2\frac{\Omega}{V_{sw}}, \phi\right), \left(r, \phi + \frac{\pi}{6}\right)\right\}$$

seem like good guesses. **This will need to be tested, though.**