LaTeX Lab Report Template

Group 00
Student 70000
Student 70001
Student 70002

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Department of Engineering Cybernetics

Abstract

This document outlines a few important aspects of a lab report. It contains some advice on both content and layout. The LaTeX source for this document is also published, and you can use it as a template of sorts for your own report. You can find an up to date version of the source at `https://github.com/ntnu-itk/labreport`. The main file, "labreport.tex", defines the structure of the document. The "preamble.tex" file is the document preamble, and contains a lot of informative comments. The document is based on work done by Tor Aksel Heirung for TTK4135, and is now under continuous improvement by Andreas L. Flåten and Kristoffer Gryte (happily accepting suggestions and contributions from the community).

When you write your own report, this section (the abstract) should contain a *very* short summary of what the lab is about and what you have done.

Contents

1 Introduction

2 Lab 1

2.1 Motivation

The motivation behind the first lab task was to create a PD regulator to the linearized model of the helicopter **??** for a cause to control its pitch. The equations of motion can be find in **??**. For a simple visualization of the helicopter, see **??**.

$$\ddot{p} = -\frac{K_f l_p V_d}{J_p} = \frac{L_1 V d}{J_p} \tag{1a}$$

$$\ddot{e} = (2m_p g l_h - m_c g l_c)cos(e) + K_f l_h V_s cos(p) = L_2 cos(e) + L_3 V_s cos(p) \tag{1b}$$

$$\ddot{\lambda} = \frac{l_h k_f V_s cos(e) sin(p)}{J_\lambda} = L_4 V_s cos(e) sin(p) \tag{1c}$$

$$\tag{1d}$$

2.2 Lab preperation

There were several tasks which needed to be done as preperation to the first lab day. The first task was to derive the equations of motion from a given diagram of the helicopter **??**. This task required thoroughly analyzing the forces which the helicopter is affected by. When this was done, it was needed to linearize the model around the equlibrium (all system states equal zero) represented by **??** such that creating controllers would be easier. We were now ready to implement a PD controller given by **??** and insert this into the linearized equation of pitch acceleration, to control the pitch to a given reference. The last preperation for lab 1 was to make a test plan for pole placements to the pitch control. We used the equations in **??**.

$$\tilde{V_d} = V_d - V_{d,origo} = V_d \tag{2a}$$

$$\tilde{V_s} = V_s - V_{s,origo} = V_s + \frac{L_2}{L_3} \tag{2b}$$

$$\ddot{p} = \frac{L_1 \tilde{V_d}}{J_p} = K_1 V_d \tag{2c}$$

$$\ddot{e} = \frac{L_3 \tilde{V_s}}{J_e} = K_2 \tilde{V_s} \tag{2d}$$

$$\ddot{\lambda} = \frac{L_4 L_2 \tilde{p}}{L_3 J_\lambda} = K_3 p \tag{2e}$$

$$\tag{2f}$$

$$V_d = K_{pp}(pc - p) - K_{pd}\dot{p} \tag{3a}$$

$$K_{pd} = -\frac{s_2 + s_1}{K_1} \tag{4a}$$

$$K_{pp} = \frac{s_2(s_2 - s_1)}{K_1(\frac{s_2}{s_1} - 1)} \tag{4b}$$

2.3 Hypotheses/Test plan

We wanted to expirement the controller by setting the poles at several different positions, including real and complex numbers. We prepared twelve different tests, with hyoptheses coming from control theory which as for poles in LHS of s plane are exponentially stable, RHS are unstable and poles in origo are stable. The table **??** gives an overview of our tests and their s-values, stability result and hypotheses.

Test	Poles	Pitch ref	Hypotheses	Result
1	-1,-1	0	Exp. Stability	Exp. Stability
2	-5,-5	0	Exp. Stability	Exp. Stability
3	$-1 \pm j5$	0	Exp. Stability	Exp. Stability
4	$-1 \pm j$	0	Exp. Stability	Exp. Stability
5	$-5 \pm j$	0	Exp. Stability	Exp. Stability
6	$-5 \pm j5$	0	Exp. Stability	Exp. Stability
7	$\pm j$	0	Marginal Stability	Unstable
8	$\pm j5$	0	Marginal Stability	Unstable
9	0	0	Marginal Stability	Unstable
10	$1 \pm j$	0	Unstable	Unstable
11	-1,1	0	Unstable	Unstable
12	1,1	0	Unstable	Unstable

Table 1: Test scheme

2.4 Results

We have decided to plot results of only five of the done tests, such that the plots are not redundant. The plot **??** visualize a time series for test 1,4,7,9 and 11. Y-axis is the pitch measured in radians. X-axis shows each timestep. The reason behind this is to include different pole-values which with also different specific hypotheses and result. For all tests the reference of the pitch was at 0 rad. We added a disturbance at 15s which lasted 0.2 seconds. This is specifically visible for test 1 and test 4.

When looking at plot **??** test 9 and test 11 is cut off early. The reason behind this is that the pitch became so unstable, where at a point it could provide harm to the helicopter. As safety measure we cut its power early. Test 1 and 4 have a similar run, which is expected as they both have same real part. However even though test 4 have an imaginary part, it dosent seem to add more swings than the one without an imaginary part.

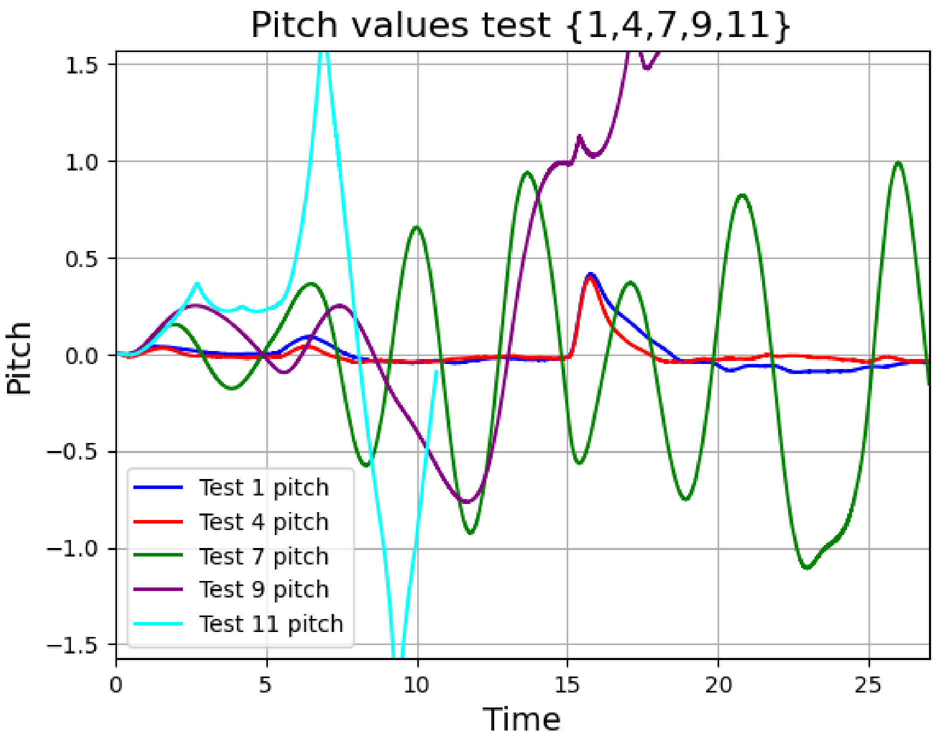


Figure 1: Timeseries of pitch angle in [rad]

Test	Mean	Std	Min value	Max value
1	-0.000549	0.0892	-0.0936	0.416
4	-0.00547	0.0592	-0.0460	0.394
7	-0.0429	0.518	-1.11	0.989
9	0.255	0.632	-0.764	1.72
11	0.0822	0.696	-1.84	1.72

Table 2: Test scheme

2.5 Conclusion

The results match our hypotheses for the tests well. Poles in left hand side of the s plane give exp. stability and poles in right hand side give unstability. However if the real parts of the poles were close to the imaginary axis, the closed loop system became unstable. This is likely a result of the real system being heavily unlinear, and us working with a simple linearized model of it, leaving several margins of error. Sadly when saving data from the lab day we didnt accompany time series of the controller output, which would be a valuable asset to discuss in this report. It would be interesting to visualize and compare values the PD controller generated throughout the tests.

3 Lab 4 - Kalman filter

3.1 Motivation

In this lab we are supposed to dive in to the realm of an other state estimator, Kalman filter. It is tuned based on process noise and measurment noise, and will be used to a stochastic model of the system. Unlike the Luenberger observer which calculates a constant gain, the kalman gain is computed and varies with each timestep. This lab takes us through discretization of the state space system and expirementation of measurement covariance and covariance of disturbance.

3.2 Lab preperation

Our first task in the lab preperation was to derive the discrete system model of the state space model. The discrete system model is at the form **??** where as the matrix equations can be found in **??**.

$$x[k+1] = A_d x[k] + B_d u[k] + \omega_d[k] \tag{5a}$$

$$y[k] = C_d x[k] + v_d[k] \tag{5b}$$

$$\omega_d \sim \mathcal{N}(0, Q_d), \quad v_d \sim \mathcal{N}(0, R_d) \tag{5c}$$

$$A_d = e^{AT} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{K3}{2} & \frac{K3}{6} & 0 & 0 & 1 & 1 \\ K3 & \frac{K3}{2} & 0 & 0 & 0 & 1 \end{bmatrix} \tag{6a}$$

$$B_d = \int_0^T e^{A\alpha}\, d\alpha\, B = \begin{bmatrix} 0 & \frac{K1T^2}{2} \\ 0 & K1T \\ \frac{K2T^2}{2} & 0 \\ K2T & 0 \\ 0 & \frac{K1K3T^4}{24} \\ 0 & \frac{K1K3T^3}{6} \end{bmatrix} \tag{6b}$$

$$C_d = C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{6c}$$

3.3 Hypotheses/Test plan

For the tests we were supposed to change the Qd matrix, which has a role of specifying how much the Kalman filter should trust the model vs measurements. A small Qd means high trust in the measurements, while a high Qd means low trust in measurements and high in model. During the tests we added a step to pi/4 in the pitch reference from 3s to 8s.

Test	Q	Hypotheses	Result
1	5	Medium noise in state est.	Noisy
2	0.1	Low noise in state est.	Noisy
3	1000	High noise in state est.	Noisy

Table 3: Test scheme lab4

3.4 Results

Because of the IMU (which is used to give measurements to the Kalman filter) has a lower sample rate than Simulink, we were needed to adjust the Kalman filter program to that. When there were duplicated measurements as input to the Kalman filter, we made the corrected state estimate and error covariance to be equal to the predicted state estimate and error covariance. See **??**.

$$\hat{x} = \bar{x} \tag{7a}$$

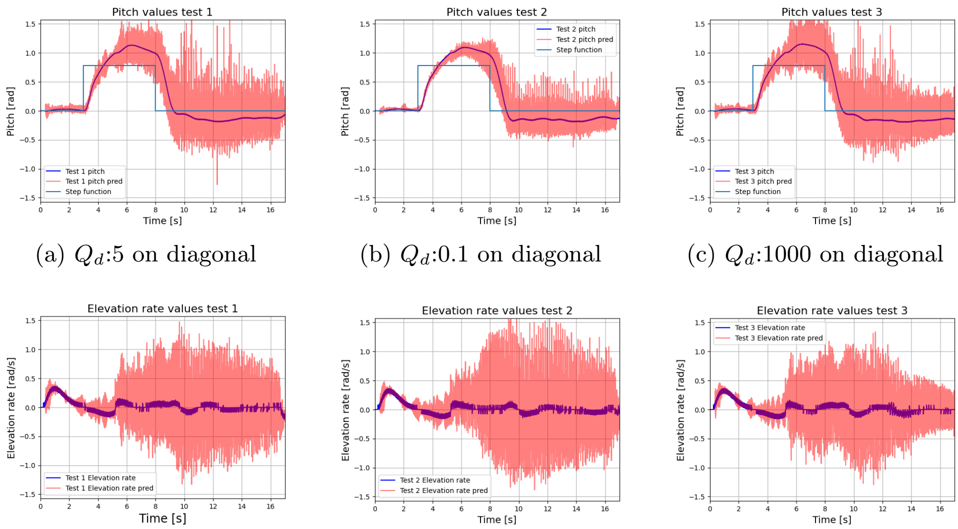
$$\hat{P} = \bar{P} \tag{7b}$$

$$\tag{7c}$$

The first task at the lab was to find an estimate on measurement noise. To do this we created two different time series, one for the helicopter laying still and one for the helicopter in the linearization point. The estimated measurement noise used for the task corresponds to when the helicopter is stationary in its linearization point. The resulted covariance matrices can be seen in **??**. Rg is for helicopter on ground and Rd when helicopter in stationary linearization point. Taking a glance at the two matrices they seem to be nearly identical, with only a few decimal difference in som covariances. From this it seems that it is indifferent which of them is chosen for the Kalman filter.

$$R_f = \begin{bmatrix} 2.9398 & -0.0135 & -0.3548 & 0.0031 & -0.0667 \\ -0.0135 & 0.0001 & 0.0016 & -0.0000 & 0.0003 \\ -0.3548 & 0.0016 & 0.0428 & -0.0004 & 0.0080 \\ 0.0031 & -0.0000 & -0.0004 & 0.0000 & -0.0001 \\ -0.0667 & 0.0003 & 0.0080 & -0.0001 & 0.0015 \end{bmatrix} \tag{8a}$$

$$R_d = \begin{bmatrix} 2.9546 & -0.0147 & -0.3545 & 0.0024 & -0.0668 \\ -0.0147 & 0.0001 & 0.0018 & -0.0000 & 0.0003 \\ -0.3545 & 0.0018 & 0.0425 & -0.0003 & 0.0080 \\ 0.0024 & -0.0000 & -0.0003 & 0.0000 & -0.0001 \\ -0.0668 & 0.0003 & 0.0080 & -0.0001 & 0.0015 \end{bmatrix} \tag{8b}$$



(a) Q_d:5 on diagonal (b) Q_d:0.1 on diagonal (c) Q_d:1000 on diagonal

(d) Q_d:5 on diagonal (e) Q_d:0.1 on diagonal (f) Q_d:1000 on diagonal

Figure 2: Time series of pitch and elevation rate during three tests.

3.5 Conclusion

It was interesting to use Kalman filter to estimate the states of the system. However it seems that the choise of Qd indifferently affects the estimated states. It seems to be a lot of state estimation noise for all three Qd matrix tests.

4 Multivariable control

4.1 Introduction

As the helicopter is a complex system with multiple states, it is reasonable to implement a multivariable control system. The system is controlled by an LQR-controller, as it provides an intuitive way to design a control rule based on minimizing a cost function. Analyzing the controllability matrix of the system showed that the system was indeed controllable. Because the LQR controller provides the feedback gain maktrix K which minimizes a cost function based on the states and input, it will naturally try to drive the states towards 0. In order to actually have the states reach their target reference, a feed forward input is also implemented. Lastly because our system, and also thus our control rules, are based on a linearized approximation of the real system there has been added an integral action to remove the steady-state error.

The system matrices (before the integral action) are shown in **??**.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & K_1 \\ K_2 & 0 \end{bmatrix} \tag{9}$$

Using this we can calculate the controllability matrix **??**, which has a rank of 3 and the system is thus controllable.

$$\mathcal{C} = \begin{bmatrix} 0 & 0 & K_1 & 0 & 0 \\ 0 & K_1 & 0 & 0 & 0 \\ K_2 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{10}$$

Our control rule is then given by **??**, where K is given by LQR and F is derived in **??** from K to drive the system to our target reference r.

$$u = -Kx + Fr \tag{11}$$

$$F = \begin{bmatrix} K_{11} & K_{13} \\ K_{21} & K_{23} \end{bmatrix} \tag{12}$$

The system matrices (after the integral action) are shown in **??**. The G matrix here is being multiplied the the reference.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & K_1 \\ K_2 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad G = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{13}$$

4.2 Hypothesis and test plan

All our test were conducted the same way with a test plan structured as this:

- Start up the system.
- Apply a change in the reference from 0 to $\frac{\pi}{4}$ between time 3s and 5s.
- Simulate a disturbance by applying an external pulse in the input voltage at 13s.

The main things we wanted to test was how different choices of Q and R affected the sytem as well as how the integral action affected the system. We came up with 5 cases to test the relation between Q and R, and tested these 5 cases both with and without integral action:

- $R = 5\mathbf{I},\ Q = \mathbf{I}$
- $R = 100\mathbf{I},\ Q = \mathbf{I}$
- $R = \mathbf{I},\ Q = 5\mathbf{I}$
- $R = \mathbf{I},\ Q = 100\mathbf{I}$
- $R = \mathbf{I},\ Q = \mathbf{I}$

Our Hypothesis were that an R larger than Q would give a slow system response as the cost of the input was weighted higher than the cost of the state, meanwhile the opposite would give a fast response. When both Q and R were equal to an identity matrix we hypothesized that we would get some form of balanced response. As for with or without integral gain our hypothesis was that the integral gain would remove the steady state error of the system.

4.3 Results

Here are the test results for all the cases

4.3.1 $R = 5\mathbf{I},\ Q = \mathbf{I}$

The pitch response fails to even respond to the reference at all without the integral action. The integral helps some but the response is still very sluggish.

4.3.2 $R = 100\mathbf{I},\ Q = \mathbf{I}$

As with the previous test, the response without integral fails to respond at all. THe integral helps some, but is way too slow to even reach the reference point.

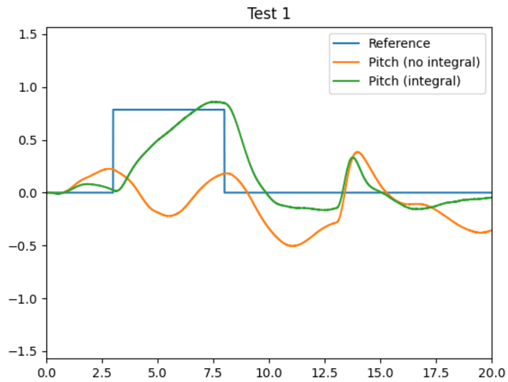


Figure 3: Test result for test 1

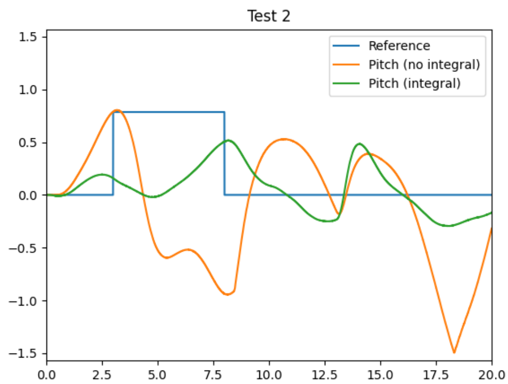


Figure 4: Test result for test 2

4.3.3 $R = \mathbf{I},\ 5Q = \mathbf{I}$

Here the pitch is able to track the reference better, but does converge to a stationary error without the integral. The integral does overshoot a bit, but converges after to the reference. This can also be seen towards the end of the series.

4.3.4 $R = \mathbf{I},\ 100Q = \mathbf{I}$

The reference is being followed by both the controllers. The integral action eliminates the stationary error.

4.3.5 $R = \mathbf{I},\ Q = \mathbf{I}$

Both signals track the reference, but both controllers are a bit slower to reach it. Also the controller with no integral action has a stationary error.

4.4 Conclusion

All our hypotheses matched with our findings. The tests with high R and low Q had weak, in fact too weak, responses. The tests with high Q values and low R had very fast responses, while the test with equal Q and R values had converging but somewhat slow responses. In addition it was shown that the integral action helped remove the staionary error.

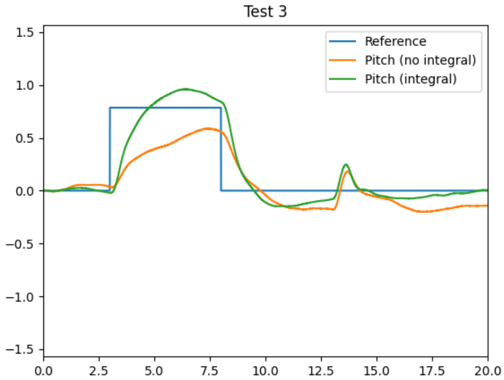


Figure 5: Test result for test 3

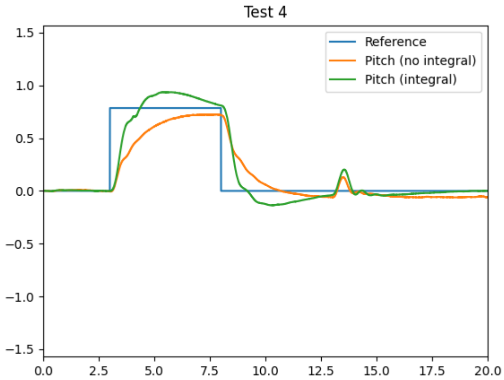


Figure 6: Test result for test 4

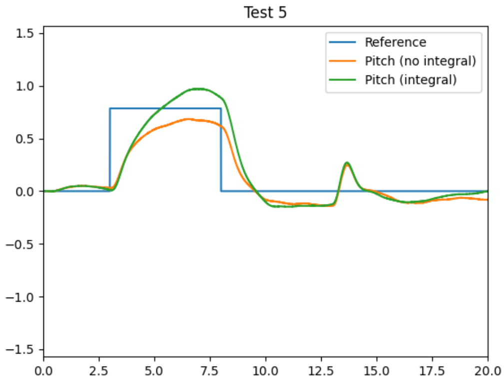


Figure 7: Test result for test 5

5 Conclusion

This does not have to be long, but try to write a few reasonable closing remarks.

A MATLAB Code

This section should contain your MATLAB code. DO NOT attach files posted online (that you didn't write). Note that the method used to input code below does not look as pretty when the lines are too long.

A.1 plot_constraint.m

```
% Plot a figure with some Latex in the labels
l = linspace(70,170)*pi/180;
a = 0.2;
b = 20;
l_b = 2*pi/3;

e = a*exp(-b*(l-l_b).^2);

l_deg = l*180/pi;
e_deg = e*180/pi;

figure(1)
plot(l_deg,e_deg, 'LineWidth', 2)

handles(1) = xlabel('$\lambda$/degrees');
handles(2) = ylabel('$e$/degrees');
set(handles, 'Interpreter', 'Latex');
```

B Simulink Diagrams

This section should contain your Simulink diagrams. Just like the plots, these should be in vector format, like in **??**. Make them tidy enough to understand.